Analytical solution to pressure transient analysis in non-Newtonian/Newtonian fluids in naturally fractured composite reservoirs

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Abstract

During polymer flooding into the reservoir using shear-thinning non-Newtonian fluids, an interface exists between the in situ Newtonian crude oil in the reservoir and the injected polymer solution. This paper examines the application of non-Newtonian well test analysis techniques to develop an analytical solution to non-Newtonian/Newtonian fluids with different flow index composite reservoirs for estimating the interface boundary conditions in such reservoirs. Mathematical models were presented and solved analytically using the Laplace transform. The solution was inverted to real domain with the Stehfest algorithm (Stehfest Commun ACM, 1970). Solutions were obtained for three boundary conditions that are infinite acting, constant pressure and no-flow boundaries. New pressure and pressure derivative type curves are developed for naturally fractured reservoirs. A general solution was obtained, which is adaptable to the case of non-Newtonian/Newtonian fluid interface. Numerical examples are used to estimate the radius of investigation, reservoir and power-law flow parameters using type curve matching and Tiab's direct synthesis technique.

Keywords Power-law (shear thinning) fluids · Well test (pressure transient) analysis · Tiab's direct synthesis (TDS) technique · Naturally fractured reservoirs (NFR) · Non-Newtonian/Newtonian fluid interface and polymer front

List of symbols

B	Formation volume factor, rm^3/stm^3
C_D	Dimensionless wellbore storage
c_t	Total compressibility Pa^{-1}
h	Formation thickness, m
H	Consistency index (power-law parameter) Pa s^n
k	Formation permeability, m^2
m, n	Flow behavior index (power-law parameter), dimensionless
P_D'	Dimensionless wellbore pressure derivative
P_D	Dimensionless wellbore pressure drop
P_i	Initial pressure, Pa
P_{wD}	Dimensionless wellbore flowing pressure
$\underline{P}_D$	Dimensionless pressure drop in Laplace space
q	Volumetric flow rate, m^3/s
r	Radial distance, m
r_D	Dimensionless radial distance at any point, (r/r_w)
R_{eD}	Dimensionless radial distance at boundary conditions (external)

r_w	Wellbore radius, m
S	Skin factor, dimensionless
t	Time, s
$t^* \Delta P'$	Pressure derivative, Pa
t_D	Dimensionless time
$\Gamma(x)$	Gamma (factorial) function
ΔP	Pressure drop
φ	Porosity, fraction of bulk volume
λ	Interporosity flow parameter, dimensionless
μ	Viscosity, Pas
θ	Effective mobility ratio for a two-region composite reservoir
ω	Storage capacity ratio, dimensionless
π	3.142

Subscripts

1	Referring to region 1, inner region
2	Referring to region 2, outer region for a two-region composite reservoir
D	Dimensionless quantity

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Introduction

Enhanced oil recovery (EOR) encompasses processes of improving the recovery of hydrocarbons from oil reservoirs after primary production has declined. Among chemical-based enhanced oil recovery processes, polymer flooding is a highly practiced process in conventional oil reservoirs. Polymer flooding utilizes the injection of polymer solutions into oil reservoirs (Manrique et al. 2017). The presence of a polymer in water increases the viscosity of the injected fluid, which upon injection reduces the water-to-oil mobility ratio and the permeability of the porous media, thereby improving oil recovery. The net result is the increase in oil displacement, reservoir sweep efficiencies, and pressure gradient, especially in heterogeneous oil reservoirs (Manzoor 2020).

Well test analysis is often carried out before, during, and after polymer flooding to obtain an accurate reservoir description and to evaluate fluid mobilities within the project area (Huh and Snow 1985). Many field applications of polymer flooding have been reported as technically and economically successful. However, Non-Newtonian power-law behavior of polymer solutions such as lack of a simple proportionality relation between shear stress and rate of shear, non-uniform polymer distribution within the reservoir, and the decrease of apparent viscosity with increasing shear rates (Tariq Fariss Al-Fariss 1984) has led to various setbacks during polymer flooding. As a result, different works have been done to investigate how to maximize the oil recovery during polymer flooding. Manzoor (2020) studied the effect of injection pressure and polymer concentration on flooding performance.

During polymer flooding, the reservoir is often made up of two or more regions and, as such, is regarded as a composite reservoir. Most times, the two regions, which might be oil and polymer solution/water regions, have different rock and fluid properties. Ikoku and Ramey (1979) derived a linearized partial differential equation for describing the radial flow of a slightly compressible non-Newtonian power-law fluid through a homogeneous porous medium. They obtained approximate analytical solutions to the equation by considering constant rate injection in an infinitely large radial flow system. The solutions provided a new method of well test analysis for non-Newtonian fluids injected into reservoirs. However, they did not consider the discontinuity of fluid in the reservoir at the interface. Lund and Ikoku (1981) addressed this neglect through numerical method by extending the flow of non-Newtonian fluids through a porous media onto composite reservoirs containing non-Newtonian and Newtonian fluids.

Ertekin et al. (1987) studied the pressure transient behavior of non-Newtonian/Newtonian composite fluid

flow in a porous media. Their work employed numerical method and primarily focused on a porous media with a finite-conductivity vertical fracture. Gencer and Ikoku (1984) analyzed injectivity and falloff test data during two-phase flow of non-Newtonian (power-law) and Newtonian fluids to determine the saturation gradients that can develop. They considered simultaneous (multiphase) flow of oil and water, and effects of relative permeability characteristics of the porous media, finite radius of the skin region, skin factor, injection time, viscosity of the Newtonian fluid, and rock compressibility on the pressure response.

Katime-Meindl and Tiab (2001) applied Tiab's direct synthesis (TDS) technique to the case of flow of non-Newtonian fluid through porous media. Pressure and pressure derivative data of a well test data in a homogeneous reservoir are interpreted without the use of type curve matching or regression analysis. The analysis considered the effect of skin factor and wellbore storage, and equations for calculating mobility and radius from wellbore to the nearest boundary were obtained.

Escobar et al. (2010) utilized the pressure derivative curve in interpreting well test data in reservoirs with non-Newtonian power-law fluids. They obtained equations to estimate the permeability, skin factor and non-Newtonian bank radius. The characteristic features found on the pressure and pressure derivative curves enabled the application of Tiab's direct synthesis (TDS) technique to their work. No analytical mathematical model developed to describe the pressure behavior in composite reservoir with non-Newtonian/Newtonian fluids.

In this study, we developed a general analytical solution to describe pressure transient behavior in a reservoir with two immiscible different non-Newtonian fluids having different flow indexes. Figure 1 represents the physical description of the composite reservoir. TDS technique is used for the analysis.

Mathematical model

The linearized partial differential equation for naturally fractured reservoir (NFR) as presented by Omosebi and Igbokoyi (2015) is:

$$\frac{\partial^2 \underline{P}_D}{\partial r_D^2} + \frac{n}{r_D} \frac{\partial \underline{P}_D}{\partial r_D} - r_D^{1-n} s f(s) \underline{P}_D = 0 \quad (1)$$

$$f(s) = \frac{\omega(1-\omega)s + \lambda}{(1-\omega)s + \lambda} \quad (2)$$

All the dimensionless variable and derivation are in "Appendix".

Fig. 1 Physical representation of the reservoir

$$\omega = \frac{(\varphi c_t)_f}{(\varphi c_t)_f + (\varphi c_t)_m} \quad (3)$$

$$\lambda = \alpha \frac{k_m r_w^2}{k_f l_m^2} \quad (4)$$

The assumptions used in developing this model are: (1) fluid flow in the reservoir is radial, (2) laminar and Darcy's flow is valid, (3) isothermal, single phase and slightly compressible fluid with constant properties, (4) homogenous and isotropic system, (5) pseudoplastic (shear-thinning) fluid obeys Ostwald–de Waele power-law relationship and (6) steady-state effective viscosity. The mathematical model also assumed there is no transition region.

The approximate long-time solution at the wellbore when r_D is unity is:

$$P_{wD}(1, t_D) = \frac{(3-n)^{\frac{2(1-n)}{3-n}}}{(1-n)\Gamma\left(\frac{2}{3-n}\right)} t_D^{\frac{1-n}{3-n}} - \frac{1}{1-n} + S \quad (5)$$

$$t_D \times P'_{wD} = \frac{(3-n)^{\left(\frac{1-n}{3-n}\right)} \left(\frac{1-n}{3-n}\right)}{\Gamma\left(\frac{2}{3-n}\right)} t_D^{\left(\frac{1-n}{3-n}\right)} \quad (6)$$

The solutions to Eq. 1 in Laplace space with various boundary conditions are presented in “Appendix”. Solution with the dimensionless wellbore storage C_D and skin S is:

$$P_{wD}(1, s, C_D, S) = \frac{sP_{wD}(1, s) + S}{\{s + s^2 C_D (sP_{wD}(1, s) + S)\}} \quad (7)$$

Type curve

Apart from the flow index, interporosity transfer and storativity ratio, two key parameters are used to characterize the type curve. They are alpha and theta (see Eqs. 38 and 59 in “Appendix”).

$$\alpha = \frac{m \left(\frac{\varphi c_t \mu_{eff}}{k} \right)_2 \left(\frac{2\pi r_w h}{q} \right)^{n-m}}{n \left(\frac{\varphi c_t \mu_{eff}}{k} \right)_1} \quad (8)$$

and

$$\theta = \frac{k_1 \mu_2}{\mu_1 k_2} \left(\frac{q}{2\pi r_w h} \right)^{m-n} \quad (9)$$

While alpha is a function of diffusivity ratio D , theta is a function of mobility ratio M . Therefore,

$$\alpha = \left(\frac{q}{2\pi r_w h} \right)^{m-n} \frac{m}{n} D \quad (10)$$

$$\theta = \left(\frac{q}{2\pi r_w h} \right)^{m-n} M \quad (11)$$

$$D = \frac{\left(\frac{\varphi c_t \mu_{eff}}{k} \right)_2}{\left(\frac{\varphi c_t \mu_{eff}}{k} \right)_1} \quad (12)$$

$$M = \frac{k_1 \mu_2}{\mu_1 k_2} \quad (13)$$

The effect of mobility and diffusivity ratios is demonstrated in Figs. 2 and 3 as in the work of Ambastha (1988).

Figure 4 is the curve generated for the three outer boundary conditions.

Figures 5 and 6 show the condition when the inner region is Newtonian and the outer region is non-Newtonian. This is to replicate a case when the pressure measurement is taking place at the producer of the Newtonian fluids; for example, crude oil that exhibits Newtonian behavior. The early pressure derivative is 0.5, while the late pressure derivative exhibits non-Newtonian behavior.

Fig. 2 Effect of mobility ratio: $\omega = 0.01$, $\lambda = 10^{-5}$, $n = 1.0$ (inner region) and $m = 1.0$ (outer region), infinite system

Fig. 3 Effect of diffusivity ratio: $\omega = 0.01$, $\lambda = 10^{-5}$, $n = 1.0$ (inner region) and $m = 1.0$ (outer region), infinite system

Figure 7 is a reversed behavior of Fig. 5. However, the non-Newtonian flow regime in the inner region has a strong influence in the outer region with Newtonian flow.

Odeh and Yang (1979) derived the expression for the radius of investigation in non-Newtonian fluid flow in porous media. This expression is simplified to:

$$R = \left[\frac{1}{\Gamma\left(\frac{2n}{2n+1}\right)} \right]^{\frac{2n+1}{2}} \left[(2n+1)^2 \frac{10^4}{n^2} \left(\frac{k}{\phi \mu c_i} \right)_1 t \right]^{\frac{1}{2}} \quad (14)$$

Fig. 4 $\omega=0.01$, $\lambda=10^{-6}$, $n=0.6$ (inner region) and $m=1.0$ (outer region) for $\alpha=1$ and $\theta=100$, $R_{eD}=8000$ and $R_D=400$

Fig. 5 $\omega=0.01$, $\lambda=10^{-6}$, $n=1.0$ (inner region) and $m=0.6$ (outer region) for $\alpha=0.01$ and $\theta=0.1$, infinite system

Ikoku and Ramey (1979) also used a similar approach to arrive at their formula for radius of investigation as:

$$R = \left[\Gamma \left(\frac{2}{3-n} \right) \right]^{\frac{1}{n-1}} \left[\frac{(3-n)^2 t}{G_1} \right]^{\frac{1}{3-n}} \quad (15)$$

Equations 14 and 15 are used in this study to compute the polymer front. Equation 14 gives radius of investigation in centimeter, while Eq. 15 gives it in meter. Ikoku and Ramey (1979) also presented the formula for estimating the effective viscosity of the polymer fluid as:

Fig. 6 $\omega=0.01$, $\lambda=10^{-6}$, $n=1.0$ (inner region) and $m=0.4$, 0.6 and 0.8 (outer region) for $\alpha=0.01$ and $\theta=0.1$, infinite system

Fig. 7 $\omega=0.01$, $\lambda=10^{-6}$, $n=0.6$ (inner region) and $m=1.0$ (outer region) for $\alpha=0.01$ and $\theta=0.1$, infinite system

$$\mu_{eff} = \frac{H}{12} \left[9 + \frac{3}{n} \right]^n [150k\phi]^{\frac{1-n}{2}} \quad (16)$$

where the effective viscosity is not given, Eq. 16 is used to estimate the effective viscosity.

The main purpose of this work is to get a general solution for a composite reservoir with two regions having

different properties with non-Newtonian fluid flow in porous media. The result of the general solution for case of $m=0.9$ in the outer region is shown in Fig. 8. This type of solution is particularly useful for heavy crude that exhibits non-Newtonian behavior and being displaced by polymer fluid for increased recovery.

Fig. 8 $\omega=0.01$, $\lambda=10^{-6}$, $R_D=400$ for $\alpha=0.01$ and $\theta=0.01$, Infinite system

Applications to numerical examples

The two numerical examples used to verify the mathematical models are from Lund and Ikoku's (1981) work. The numerical examples are for homogeneous system with storativity ratio ω equal to 1.0.

Type curve method

Injection test

The match in Fig. 9 was obtained with a flow index n of 0.9 (Table 1).

We estimated the effective viscosity with Eq. 16. The value of the effective viscosity with n of 0.6 is 0.0000402 Pas, while n equals 0.9 giving 0.00424 Pas. The effective viscosity of the polymer cannot be less than that of the Newtonian fluid in the formation; hence, we have confidence in the match with 0.9 as the value of n , and therefore, the value of the polymer effective viscosity used in this work is 0.00424 Pas.

We used pressure derivative plot to estimate the time at the interface. Lund and Ikoku (1981) used the plot of ΔP against t^v to estimate the time at the interface. The pressure derivatives give better estimate of point of discontinuity. From Fig. 10, we estimated the interface to be at the time

Fig. 9 Type curve match for the injection test

Table 1 Data input of Lund and Ikoku (1981)

Properties	Value	Unit
Permeability	100	md
Newtonian fluid viscosity	0.003	Pas
H	0.02	Pa s ⁿ
n	0.6	
Injection rate	0.00055204	m ³ /s
External radius	800	m
Wellbore radius	0.1	m
External boundary condition	Constant pressure	
Test type	Injection	
Sand thickness	5	m
Polymer front	40	m
Porosity	0.2	Fraction
Total compressibility	1×10^{-9}	Pa ⁻¹
Injection time before shut-in	8.98	Days

$t = 931$ s. Lund and Ikoku's time at interface is 7100 s. This is already in the transition region (Table 2).

The alpha (α) value for both match is 0.003 and theta (θ) is 0.00135.

The effective compressibility in the polymer zone, which is region I, is estimated from the value of alpha and time match.

From the time match:

$$c_t = \frac{k}{\mu_{\text{eff}}} \times \left(\frac{t}{t_D} \right)_{\text{match}} \times \frac{1}{n\theta \left(\frac{2\pi h}{q} \right)^{1-n} r_w^{3-n}}$$

$$c_{t1} = 1.47 \times 10^{-13} \left(\frac{155}{8} \right)_{\text{match}} \times \frac{1}{0.9 \times 0.2 \times \left(\frac{2\pi \times 5}{0.00055204} \right)^{1-0.9} 0.1^{3-0.9}}$$

$$= 1.57 \times 10^{-7} \text{ Pa}^{-1}$$

From the value of alpha:

Fig. 10 Interface determination for the injection test **Table 2** Results from type curve match for the injection test

Parameters	Value
P_D match	3
ΔP match Pa	3,610,000
Skin S	0.45
t_D match	8
Δt at match s	155
Δt at interface s	931
Estimated permeability md	149 or $1.47 \times 10^{-13} \text{ m}^2$
Dimensionless interface radius R_D from match	40
Interface radius from match meter	4.00
Estimated interface radius R from equation 14 m	1.92
Estimated interface radius R from equation 15 m	0.93
If we used Δt at interface 3100 s (3 points ahead of 931 s) Eq. 14 m	3.52
If we used Δt at interface 3100 s (3 points ahead of 931 s) Eq. 15 m	1.66

Fig. 11 Type curve match for the falloff test **Fig. 12** Interface determination for the falloff test

$$c_{t1} = \frac{m \left(\frac{\phi C_t \mu_{eff}}{k} \right)_2 \left(\frac{2\pi r_w h}{q} \right)^{n-m}}{n \alpha \left(\frac{\phi \mu_{eff}}{k} \right)_1} = 1.64 \times 10^{-7} \text{ Pa}^{-1}$$

Falloff test from the same numerical model.

The Agarwal equivalent time is used for all the plots in the falloff test (Figs. 11, 12).

The same values of alpha and theta were used (Table 3).

Table 3 Results from type curve match for the falloff test

Parameters	Value
P_D match	2
ΔP match Pa	2,740,000
Skin S	0.55
t_D match	3
Δt at match s	116
Δt at interface s	3088
Estimated permeability md	131 or $1.29 \times 10^{-13} \text{ m}^2$
Dimensionless interface radius R_D from match	30
Interface radius from match meter	3.00
Estimated interface radius R from equation 14 m	3.51
Estimated interface radius R from equation 15 m	1.65

From the time match:

$$c_{t1} = 1.291 \times 10^{-13} \left(\frac{116}{3} \right)_{\text{match}} \times \frac{1}{0.9 \times 0.2 \times \left(\frac{2\pi \times 5}{0.00055204} \right)^{1-0.9} 0.1^{3-0.9}} = 2.75 \times 10^{-7} \text{ Pa}^{-1}$$

From the value of alpha:

$$c_{r1} = \frac{1.0 \times \left(\frac{0.2 \times 1 \times 10^{-9} \times 0.003}{9.869 \times 10^{-14}} \right)_2 \left(\frac{2\pi \times 0.1 \times 5}{0.00055204} \right)^{0.9-1.0}}{0.9 \times 0.003 \left(\frac{0.2 \times 0.00424}{1.291 \times 10^{-13}} \right)_1} = 1.44 \times 10^{-7} \text{ Pa}^{-1}$$

Tiab's direct synthesis method

Interporosity flow parameter is obtained by substituting the co-ordinates of the minimum point of the trough, $t_{\min}$ and $(t \times \Delta P')_{\min}$ according to Tiab et al (2006). At that point,

$$\lambda = \frac{42.5 \mu r_w^2 h (\varphi c_t)_{m+f} (t \Delta P_w)_{\min}}{qB (t)_{\min}} \quad (17)$$

For storage capacity ratio, the method of calculation is obtained using the pressure derivative co-ordinate of the minimum point and the radial flow regime line, according to Tiab et al (2006). It is given as:

$$\omega = 10^{-0.8684 \left(1 - \left(\frac{(t \Delta P_w)_{\min}}{(t \Delta P_w)_{\text{IARF}}} \right) \right)} \quad (18)$$

At the infinite-acting radial flow regime of non-Newtonian fluid, the log–log plot of the pressure derivative versus time will give a straight line with slope, v . The value of n can then be determined from the slope. By substituting the dimensionless pressure equations into the pressure derivative equation, the mobility ratio (effective permeability–viscosity ratio) is calculated by Igbokoyi and Tiab (2007) as:

$$\frac{k}{\mu_{\text{eff}}} = \left(\frac{0.5 \left(\frac{q}{2\pi h} \right)^{\frac{1+n}{3-n}} t_{\text{IARF}}^{\frac{1-n}{3-n}}}{(t \Delta P'_w)_{\text{IARF}} (n \varphi c_t)^{\frac{1-n}{3-n}}} \right)^{\frac{3-n}{2}} \quad (19)$$

At $t_{\text{IARF}} = 1$ s,

$$\frac{k}{\mu_{\text{eff}}} = \left(\frac{0.5 \left(\frac{q}{2\pi h} \right)^{\frac{1+n}{3-n}}}{(t \Delta P'_w)_{\text{IARF} \text{ 1 s}} (n \varphi c_t)^{\frac{1-n}{3-n}}} \right)^{\frac{3-n}{2}} \quad (20)$$

For the skin factor, the following equation from Omosebi and Igbokoyi (2015) is obtained:

$$S = \frac{k}{\mu_{\text{eff}}} \left(\frac{2\pi h}{q} \right)^n r_w^{1-n} \left[(\Delta P_w)_{\text{IARF}} - \frac{3-n}{1-n} (t \Delta P'_w)_{\text{IARF}} \right] + \frac{1}{1-n} \quad (21)$$

Wellbore storage is computed with the usual equation.

$$C = qB \left(\frac{t}{\Delta P_w} \right) \quad (22)$$

Permeability in region II and the skin factor can be computed with the following equations presented by Tiab (1995).

$$\left(\frac{k}{\mu_{\text{eff}}} \right)_2 = \frac{0.5 q B}{2\pi h (t * \Delta P')_{\text{IARF2}}} \quad (23)$$

$$s_2 = \frac{1}{2} \left[\left(\frac{\Delta P}{t * \Delta P'} \right)_{\text{IARF2}} - \ln \ln \left(\frac{k_2 t_{r2}}{\varphi \mu_N c_t r_w^2} \right) + 7.43 \right] \quad (24)$$

Calculate the interporosity flow parameter using Eq. 17

Define R as:

$$R = \left(\frac{(t \Delta P_w)_{\min}}{(t \Delta P_w)_{\text{IARF}}} \right) \quad (25)$$

Calculate the storage ratio;

$$\omega = 0.5604R^6 - 1.3322R^5 + 1.7232R^4 - 0.827R^3 + 0.7287R^2 + 0.1469R \quad (26)$$

or

$$\omega = 10^{-0.8684(1-R)} \quad (27)$$

TDS method

Injection test

From Fig. 10, at $t = 931$ s, $(\Delta P_{ws}) = 3610000$ Pa, and $(t * \Delta P'_w) = 724063$ Pa

$$\frac{k}{\mu_{\text{eff}}} = \left(\frac{0.5 \left(\frac{q}{2\pi h} \right)^{\frac{1+n}{3-n}} t_{\text{IARF1}}^{\frac{1-n}{3-n}}}{(tx\Delta P'_w)_{\text{IARF1}} \left(n\phi c_t \right)_1^{\frac{1-n}{3-n}}} \right)^{\frac{3-n}{2}}$$

$$\frac{k}{\mu_{\text{eff}}} = \left(\frac{0.5 \times \left(\frac{0.00055204}{2 \times 3.142 \times 5} \right)^{\frac{1+0.9}{3-0.9}} \times (931)^{\frac{1-0.9}{3-0.9}}}{724063 \times (0.9 \times 0.2 \times 1.57 \times 10^{-7})_1^{\frac{1-0.9}{3-0.9}}} \right)^{\frac{3-0.9}{2}}$$

$$\frac{k}{\mu_{\text{eff}}} = 3.464 \times 10^{-11} \text{ m}^{1.9} / \text{Pa s}^{0.9}$$

From fluid and reservoir parameters,

$$\mu_{\text{eff}} = 4.24 \times 10^{-3} \text{ Pa s}^{0.9} / \text{m}^{1.9}$$

Therefore, permeability:

$$k = \frac{3.464 \times 10^{-11} \times 4.24 \times 10^{-3}}{9.869233 \times 10^{-16}} = 149 \text{ mD}$$

Using the Newtonian region (region II) with average pressure derivative $(t \times \Delta P')_{\text{IARF2}} = 275618 \text{ Pa}$ and $\Delta P_{\text{IARF2}} = 6730000 \text{ Pa}$, $t_{r2} = 63.9 \text{ h}$

$$\left(\frac{k}{\mu_{\text{eff}}} \right)_2 = \frac{0.5 \times 0.00055204 \times 1.0}{2\pi \times 5 \times 275618} = 3.187 \times 10^{-11} \text{ m}^2$$

With Newtonian fluid viscosity of 0.003 Pas, the permeability is 97 md.

Skin factor S using non-Newtonian formula:

$$S = 3.464 \times 10^{-11} \times \left(\frac{2 \times 3.142 \times 5}{0.00055204} \right)^{0.9} \times 0.1^{1-0.9} \times \left[3610000 - 724063 \times \frac{3-0.9}{1-0.9} \right] + \frac{1}{1-0.9} = 3.92$$

Table 4 Summary of injectivity test estimated properties

Parameters	Lund and Ikoku (1981)	This study	
		Type curve	TDS
Flow index, n	0.6	0.9	0.9
Effective viscosity, μ_{eff} (Pa s) Region I		0.00424	0.00424
Effective viscosity, μ_{eff} (Pa s) Region II	0.003	0.003	0.003
Permeability mD region I	100	149	149
Permeability, mD region II	100		97
Radius polymer bank	40 m	4.0	

Table 5 Summary of falloff test estimated properties

Parameters	Lund and Ikoku (1981)	This study	
		Type curve	TDS
Flow index, n	0.6	0.9	0.9
Effective viscosity, μ_{eff} (Pa s) Region I		0.00424	0.00424
Effective viscosity, μ_{eff} (Pa s) Region II	0.003	0.003	0.003
Permeability mD region I	100	131	129
Permeability, mD region II	100		99
Radius polymer bank	40 m	3.0	

Skin factor S using Newtonian formula:

$$S = \frac{1}{2} \left[\left(\frac{6730000}{275618} \right)_{\text{IARF2}} - \ln \ln \left(\frac{97 \times 63.9}{0.2 \times 10^{-6} \times 3 \times 0.33^2} \right) + 7.43 \right] = 3.29$$

Falloff test

From Fig. 10, at $t = 3088 \text{ s}$, $(\Delta P_{ws}) = 4530000 \text{ Pa}$, and $(t \times \Delta P'_w) = 882286 \text{ Pa}$

$$\frac{k}{\mu_{\text{eff}}} = \left(\frac{0.5 \times \left(\frac{0.00055204}{2 \times 3.142 \times 5} \right)^{\frac{1+0.9}{3-0.9}} \times (3088)^{\frac{1-0.9}{3-0.9}}}{882286 \times (0.9 \times 0.2 \times 2.75 \times 10^{-7})_1^{\frac{1-0.9}{3-0.9}}} \right)^{\frac{3-0.9}{2}}$$

$$\frac{k}{\mu_{\text{eff}}} = 3.002 \times 10^{-11} \text{ m}^{1.9} / \text{Pa s}^{0.9}$$

$$k = \frac{3.002 \times 10^{-11} \times 4.24 \times 10^{-3}}{9.869233 \times 10^{-16}} = 129 \text{ mD}$$

Using the Newtonian region (region II) with average pressure derivative $(t \times \Delta P')_{\text{IARF2}} = 268582 \text{ Pa}$ and $\Delta P_{\text{IARF2}} = 7060000 \text{ Pa}$, $t_{r2} = 143.62 \text{ h}$

$$\left(\frac{k}{\mu_{\text{eff}}} \right)_2 = \frac{0.5 \times 0.00055204 \times 1.0}{2\pi \times 5 \times 268582} = 3.271 \times 10^{-11} \text{ m}^2$$

With Newtonian fluid viscosity of 0.003 Pas, the permeability is 99 md (Tables 4, 5).

Skin factor S using non-Newtonian formula:

$$S = 3.002 \times 10^{-11} \times \left(\frac{2 \times 3.142 \times 5}{0.00055204} \right)^{0.9} \times 0.1^{1-0.9} \times \left[4530000 - 882286 \times \frac{3-0.9}{1-0.9} \right] + \frac{1}{1-0.9} = 3.64$$

Skin factor S using Newtonian formula:

$$S = \frac{1}{2} \left[\left(\frac{7060000}{268582} \right)_{\text{IARF2}} - \ln \ln \left(\frac{99 \times 143.62}{0.2 \times 10^{-6} \times 3 \times 0.33^2} \right) + 7.43 \right] = 3.81$$

Discussion of the results

The new analytical model developed was used to analyze simulated data from the work of Lund and Ikoku. The major differences between Lund and Ikoku, and this study results are the value of flow index n in region I and the polymer front. The analytical method for estimating radius of investigation developed by Ikoku and Ramey could not confirm the polymer front estimated by Lund and Ikoku. While Lund and Ikoku used the plot of ΔP versus t^N to estimate the polymer front, we used pressure derivative curve to estimate the polymer front, which should be more accurate. Moreover, we used Odeh and Yang's method to estimate the polymer front, and the results are in good agreement with this study's. Permeability estimated from both type curve and TDS methods for injection and falloff tests is consistent. Therefore, the new analytical method developed is considered valid.

Conclusions

A general solution for a composite reservoir with two regions in a radial system is developed. The solution is applicable to both non-Newtonian and Newtonian composite reservoirs in two-region radial system.

The model was applied to solve simulated numerical examples of injection and falloff tests. Type curve and Tiab's direct synthesis methods were used to analyze the numerical data, and consistent results were obtained.

The radius of investigation for non-Newtonian derived by Ikoku and Ramey, and Odeh and Yang is based on the same assumptions. However, Odeh and Yang's method gives more accurate results.

Appendix: Mathematical derivations

The general partial differential equation (Ikoku & Ramey 1979) is:

$$\frac{\partial^2 P}{\partial r^2} + \frac{n}{r} \frac{\partial P}{\partial r} = Gr^{1-n} \frac{\partial P}{\partial t} \quad (28)$$

$$\frac{\partial^2 P_{D1}}{\partial r_D^2} + \frac{n}{r_D} \frac{\partial P_{D1}}{\partial r_D} = r_D^{1-n} \frac{\partial P_{D1}}{\partial t_D}, \text{ for } P_{D1} (1 \leq r_D \leq R_D) \quad (29)$$

For region II, we have:

$$\frac{\partial^2 P_2}{\partial r^2} + \frac{m}{r} \frac{\partial P_2}{\partial r} = G_2 r^{1-m} \frac{\partial P_2}{\partial t}, t > 0 \quad (30)$$

$$\frac{\partial^2 P_{D2}}{\partial r_D^2} + \frac{m}{r_D} \frac{\partial P_{D2}}{\partial r_D} = r_w^2 G_2 r_D^{1-m} r_w^{1-m} \frac{\partial P_{D2}}{\partial t} \quad (31)$$

where

$$P_D = \frac{P - P_i}{\left(\frac{q}{2\pi h} \right)^n \left(\frac{\mu_{\text{eff}} r_w^{1-n}}{k_r} \right)} \quad (32)$$

$$C_D = \frac{C}{2\pi h n \phi c_i r_w^2} \quad (33)$$

$$t_D = \frac{t}{G_1 r_w^{3-n}} \quad (34)$$

$$G_1 = n \left(\frac{\phi c_i \mu_{\text{eff}}}{k} \right)_1 \left(\frac{2\pi h}{q} \right)^{1-n} \quad (35)$$

$$G_2 = m \left(\frac{\phi c_i \mu_{\text{eff}}}{k} \right)_2 \left(\frac{2\pi h}{q} \right)^{1-m} \quad (36)$$

$$\frac{\partial^2 P_{D2}}{\partial r_D^2} + \frac{n}{r_D} \frac{\partial P_{D2}}{\partial r_D} = r_D^{1-n} r_w^{n-m} \frac{G_2}{G_1} \frac{\partial P_{D2}}{\partial t_D} \quad (37)$$

$$\frac{G_2}{G_1} r_w^{n-m} = \frac{m \left(\frac{\phi c_i \mu_{\text{eff}}}{k} \right)_2 \left(\frac{2\pi r_w h}{q} \right)^{n-m}}{n \left(\frac{\phi c_i \mu_{\text{eff}}}{k} \right)_1} = \alpha \quad (38)$$

$$\frac{\partial^2 P_{D2}}{\partial r_D^2} + \frac{m}{r_D} \frac{\partial P_{D2}}{\partial r_D} = \alpha r_D^{1-m} \frac{\partial P_{D2}}{\partial t_D}, \text{ for } P_{D2} (R_D \leq r_D \leq \infty) \quad (39)$$

Outer boundary condition

Infinite system

$$P_D(r_D = \infty, t_D) = 0 \quad (40)$$

Closed (no-flow) boundary system

$$\frac{\partial P_{D2}}{\partial r_D}(r_D = R_{eD}) = 0 \quad (41)$$

Constant pressure boundary system

$$\underline{P}_{D2}(r_D = R_{eD}) = 0$$

Inner boundary condition

$$\frac{\partial \underline{P}_{D1}}{\partial r_D}(r_D = 1) = -\frac{1}{s}$$

$$q_1 = 2\pi rh \left(-\frac{k_1}{\mu_1} \frac{\partial P_1}{\partial r} \right)^{\frac{1}{n}}$$

$$\frac{\partial P_1}{\partial r} = \left(\frac{q}{2\pi r_w h} \right)^n \frac{\mu_1}{k_1} \frac{\partial P_{D1}}{\partial r_D}$$

and,

$$\frac{q_1}{2\pi rh} = \left(-\frac{k_1}{\mu_1} \frac{\partial P_1}{\partial r} \right)^{\frac{1}{n}}$$

Therefore,

$$\frac{q_1}{2\pi rh} = \left(-\frac{k_1}{\mu_1} \left(\frac{q_1}{2\pi r_w h} \right)^n \frac{\mu_1}{k_1} \frac{\partial P_{D1}}{\partial r_D} \right)^{\frac{1}{n}}$$

$$\left(-\frac{\partial P_{D1}}{\partial r_D} \right)^{\frac{1}{n}} = \frac{r_w}{r} = \frac{1}{r_D}$$

Equation 47 is formerly established by Vongvuthipornchai and Raghavan (1987) in equation B-8 of their paper.

And flow equation for non-Newtonian fluid (flow index m) is:

$$\frac{q_2}{2\pi rh} = \left(-\frac{k_2}{\mu_2} \frac{\partial P_2}{\partial r} \right)^{\frac{1}{m}} \quad (49)$$

But,

$$\frac{\partial P_2}{\partial r} = \left(\frac{q}{2\pi r_w h} \right)^m \frac{\mu_2}{k_2} \frac{\partial P_{D2}}{\partial r_D} \quad (50)$$

Adapting Eq. (12) in Lund and Ikoku (1981) to non-Newtonian/Newtonian fluid flow, at interface R_D (i.e., rates are equal at the interface):

$$\left[-\gamma_1 \frac{\partial P_1}{\partial r} \right]^{\frac{1}{n}} = \left[-\gamma_2 \frac{\partial P_2}{\partial r} \right]^{\frac{1}{m}} \quad (51)$$

where

$$\gamma = \frac{k}{\mu}$$

Therefore,

$$\left[-\frac{k_1}{\mu_1} \left(\frac{q}{2\pi r_w h} \right)^n \frac{\mu_1}{k_1} \frac{\partial P_{D1}}{\partial r_D} \right]^{\frac{1}{n}} = \left[-\frac{k_2}{\mu_2} \left(\frac{q}{2\pi r_w h} \right)^m \frac{\mu_2}{k_2} \frac{\partial P_{D2}}{\partial r_D} \right]^{\frac{1}{m}} \quad (52)$$

$$\left(-\frac{\partial P_{D1}}{\partial r_D} \right)^{\frac{1}{n}} = \left(\frac{q}{2\pi r_w h} \right)^{\frac{n}{m}-1} \left(-\frac{k_2}{\mu_2} \frac{\mu_1}{k_1} \frac{\partial P_{D2}}{\partial r_D} \right)^{\frac{1}{m}} \quad (53)$$

Substituting (48) into (53),

$$\frac{1}{R_D} = \left(\frac{q}{2\pi r_w h} \right)^{\frac{n}{m}-1} \left(-\frac{k_2}{\mu_2} \frac{\mu_1}{k_1} \frac{\partial P_{D2}}{\partial r_D} \right)^{\frac{1}{m}} \quad (54)$$

$$\frac{1}{R_D^m} = \left(\frac{q}{2\pi r_w h} \right)^{n-m} \left(-\frac{k_2}{\mu_2} \frac{\mu_1}{k_1} \frac{\partial P_{D2}}{\partial r_D} \right) \quad (55)$$

$$\frac{\partial P_{D2}}{\partial r_D} = -\frac{k_1}{\mu_1} \frac{\mu_2}{k_2} \left(\frac{q}{2\pi r_w h} \right)^{m-n} \frac{1}{R_D^m} \quad (56)$$

Set

$$\theta = \frac{k_1}{\mu_1} \frac{\mu_2}{k_2} \left(\frac{q}{2\pi r_w h} \right)^{m-n} \quad (57)$$

Therefore,

$$\frac{\partial P_{D2}}{\partial r_D} = -\theta \frac{1}{s R_D^m} \quad (58)$$

If we assume that permeability, compressibility and porosity are approximately the same in both regions, then

$$\alpha = \frac{m}{n} \theta \quad (59)$$

From Eq. 6, at the interface;

$$\frac{\partial \underline{P}_{D1}}{\partial r_D} = -\frac{1}{s R_D^n} \quad (60)$$

Combining Eqs. 60 and 62

$$\left(\frac{1}{\theta} \frac{\partial \underline{P}_{D2}}{\partial r_D} \right)^{\frac{1}{m}} = \left(\frac{\partial \underline{P}_{D1}}{\partial r_D} \right)^{\frac{1}{n}} \quad (61)$$

If $m=n$,

$$\frac{\partial \underline{P}_{D2}}{\partial r_D} = \frac{k_1}{\mu_1} \frac{\mu_2}{k_2} \frac{\partial \underline{P}_{D1}}{\partial r_D} \quad (62)$$

A general Laplace solution for the dimensionless pressure drop for region I is:

$$\text{Region 1: Non - Newtonian } (1 \leq r_D \leq R_D) \quad (63)$$

$$\begin{aligned} \underline{P}_{D1} (1 \leq r_D \leq R_D) = & r_D^{\frac{v_1}{\beta_1} + \frac{1}{\beta_1} - 1} \left(C_1 I_{V1} \left(\beta_1 r_D^{\frac{1}{\beta_1}} \sqrt{sfs} \right) \right. \\ & \left. + C_2 K_{V1} \left(\beta_1 r_D^{\frac{1}{\beta_1}} \sqrt{sfs} \right) \right) \end{aligned} \quad (64)$$

$$\frac{\partial \underline{P}_{D1}}{\partial r_D} = r_D^{\frac{v_1}{\beta_1} + \frac{1}{\beta_1} - 1} \sqrt{sfs} \left(C_1 I_{\beta_1} \left(\beta_1 \sqrt{sfs} r_D^{\frac{1}{\beta_1}} \right) - C_2 K_{\beta_1} \left(\beta_1 \sqrt{sfs} r_D^{\frac{1}{\beta_1}} \right) \right) \quad (65)$$

Laplace solution for the dimensionless pressure drop in infinite (non-Newtonian) flow for region II is:

$$\underline{P}_{D2} (R_D \leq r_D \leq \infty) = r_D^{\frac{v_2}{\beta_2} + \frac{1}{\beta_2} - 1} C_3 K_{V2} \left(\beta_2 r_D^{\frac{1}{\beta_2}} \sqrt{\alpha sfs} \right) \quad (66)$$

$$\frac{\partial \underline{P}_{D2}}{\partial r_D} = -r_D^{\frac{v_2}{\beta_2} + \frac{1}{\beta_2} - 1} \sqrt{\alpha sfs} C_3 K_{\beta_2} \left(\beta_2 \sqrt{\alpha sfs} r_D^{\frac{1}{\beta_2}} \right) \quad (67)$$

From (60) and (68),

$$C_3 = \frac{R_D^{-\frac{v_2}{\beta_2} - \frac{1}{\beta_2} + 1 - m} \theta}{s \sqrt{\alpha sfs} K_{\beta_2} \left(\beta_2 R_D^{\frac{1}{\beta_2}} \sqrt{\alpha sfs} \right)} \quad (68)$$

At interface, $r_D = R_D$, $\underline{P}_{D1} = \underline{P}_{D2}$

Therefore,

$$C_3 = \frac{R_D^{\frac{v_1}{\beta_1} - \frac{v_2}{\beta_2}} \left[C_1 I_{V1} \left(\beta_1 R_D^{\frac{1}{\beta_1}} \sqrt{sfs} \right) + C_2 K_{V1} \left(\beta_1 R_D^{\frac{1}{\beta_1}} \sqrt{sfs} \right) \right]}{K_{V2} \left(\beta_2 R_D^{\frac{1}{\beta_2}} \sqrt{\alpha sfs} \right)} \quad (69)$$

Set:

$$s \sqrt{\alpha sfs} K_{\beta_2} \left(\beta_2 R_D^{\frac{1}{\beta_2}} \sqrt{\alpha sfs} \right) = A \quad (70)$$

$$K_{V2} \left(\beta_2 R_D^{\frac{1}{\beta_2}} \sqrt{\alpha sfs} \right) = B \quad (71)$$

$$p = -\frac{v_2}{\beta_2} - \frac{1}{\beta_2} + 1 - m \quad (72)$$

$$u = \frac{v_1}{\beta_1} - \frac{v_2}{\beta_2} \quad (73)$$

where:

$$v_1 = \frac{1-n}{3-n}, v_2 = \frac{1-m}{3-m} \quad (74)$$

$$\beta_1 = \frac{2}{3-n}, \beta_2 = \frac{2}{3-m} \quad (75)$$

Combining Eqs. 69 and 70, we have:

$$\begin{aligned} B R_D^p \theta = & C_1 A R_D^u I_{V1} \left(\beta_1 r_D^{\frac{1}{\beta_1}} \sqrt{sfs} \right) + C_2 A R_D^u K_{V1} \left(\beta_1 r_D^{\frac{1}{\beta_1}} \sqrt{sfs} \right) \\ C_1 = & \frac{B \theta R_D^p - C_2 A R_D^u K_{V1} \left(\beta_1 r_D^{\frac{1}{\beta_1}} \sqrt{sfs} \right)}{A R_D^u I_{V1} \left(\beta_1 r_D^{\frac{1}{\beta_1}} \sqrt{sfs} \right)} \end{aligned} \quad (76)$$

At wellbore condition, $r_D = 1$:

$$\frac{\partial \underline{P}_{D1}}{\partial r_D} = -\frac{1}{s} = \sqrt{sfs} \left[C_1 I_{\beta} \left(\beta \sqrt{sfs} \right) - C_2 K_{\beta} \left(\beta \sqrt{sfs} \right) \right] \quad (77)$$

$$C_1 = \frac{\frac{-1}{s \sqrt{sfs}} + C_2 K_{\beta_1} \left(\beta_1 \sqrt{sfs} \right)}{I_{\beta_1} \left(\beta_1 \sqrt{sfs} \right)} \quad (78)$$

Combine Eqs. 77 and 79, we have:

$$C_2 = \frac{B \theta R_D^p I_{\beta_1} \left(\beta_1 \sqrt{sfs} \right) + \frac{A R_D^u}{s \sqrt{sfs}} I_{V1} \left(\beta_1 R_D^{\frac{1}{\beta_1}} \sqrt{sfs} \right)}{A R_D^u I_{V1} \left(\beta_1 R_D^{\frac{1}{\beta_1}} \sqrt{sfs} \right) K_{\beta_1} \left(\beta_1 \sqrt{sfs} \right) + I_{\beta_1} \left(\beta_1 \sqrt{sfs} \right) K_{V1} \left(\beta_1 R_D^{\frac{1}{\beta_1}} \sqrt{sfs} \right)} \quad (79)$$

Laplace solution for the dimensionless pressure drops for constant pressure boundary system for region II is

Region II: Non - Newtonian ($R_D \leq r_D \leq R_{eD}$)

$$\underline{P}_{D2} = \left[C_3 I_{V2} \left(\beta_2 r_D^{\frac{1}{\beta_2}} \sqrt{\alpha sfs} \right) + C_4 K_{V2} \left(\beta_2 r_D^{\frac{1}{\beta_2}} \sqrt{\alpha sfs} \right) \right] r_D^{\frac{v_2}{\beta_2}} \quad (80)$$

$$\begin{aligned} \frac{\partial \underline{P}_{D2}}{\partial r_D} = & r_D^{\frac{v_2}{\beta_2} + \frac{1}{\beta_2} - 1} \sqrt{\alpha sfs} \left[C_3 I_{\beta_2} \left(\beta_2 \sqrt{\alpha sfs} r_D^{\frac{1}{\beta_2}} \right) \right. \\ & \left. - C_4 K_{\beta_2} \left(\beta_2 r_D^{\frac{1}{\beta_2}} \sqrt{\alpha sfs} \right) \right] \end{aligned} \quad (81)$$

Since $P_{D2} = 0 @ R_{eD}$,

$$C_4 = \frac{-C_3 I_{V2} \left(\beta_2 R_{eD}^{\frac{1}{\beta_2}} \sqrt{\alpha s f s} \right)}{K_{V2} \left(\beta_2 R_{eD}^{\frac{1}{\beta_2}} \sqrt{\alpha s f s} \right)} \quad (82)$$

$$\frac{\partial P_{D2}}{\partial r_D} = r_D^{\frac{v_2}{\beta_2} + \frac{1}{\beta_2} - 1} \sqrt{\alpha s f s} C_3 \left[I_{\beta_2} \left(\beta_2 \sqrt{\alpha s f s} r_D^{\frac{1}{\beta_2}} \right) + \frac{C_3 I_{V2} \left(\beta_2 R_{eD}^{\frac{1}{\beta_2}} \sqrt{\alpha s f s} \right)}{K_{V2} \left(\beta_2 R_{eD}^{\frac{1}{\beta_2}} \sqrt{\alpha s f s} \right)} K_{\beta_2} \left(\beta_2 r_D^{\frac{1}{\beta_2}} \sqrt{\alpha s f s} \right) \right] \quad (83)$$

At interface R_D , $\frac{\partial P_{D2}}{\partial r_D} = -\frac{\theta}{s R_{D1}^m}$ and $P_{D1} = P_{D2}$.

Therefore,

$$C_3 = \frac{-\theta}{R_D^{\frac{v_2}{\beta_2} + \frac{1}{\beta_2} - 1 + m} s \sqrt{\alpha s f s} I_{\beta_2} \left[\left(\beta_2 \sqrt{\alpha s f s} R_D^{\frac{1}{\beta_2}} \right) + \frac{I_{V2} \left(\beta_2 R_{eD}^{\frac{1}{\beta_2}} \sqrt{\alpha s f s} \right)}{K_{V2} \left(\beta_2 R_{eD}^{\frac{1}{\beta_2}} \sqrt{\alpha s f s} \right)} K_{\beta_2} \left(\beta_2 R_D^{\frac{1}{\beta_2}} \sqrt{\alpha s f s} \right) \right]} \quad (84)$$

Also,

$$C_3 = \frac{C_1 I_{V1} \left(\beta_1 R_D^{\frac{1}{\beta_1}} \sqrt{s f s} \right) + C_2 K_{V1} \left(\beta_1 R_D^{\frac{1}{\beta_1}} \sqrt{s f s} \right)}{R_D^{\frac{v_2}{\beta_2} - \frac{v_1}{\beta_1}} \left[I_{V2} \left(\beta_2 R_D^{\frac{1}{\beta_2}} \sqrt{\alpha s f s} \right) - \frac{I_{V2} \left(\beta_2 R_{eD}^{\frac{1}{\beta_2}} \sqrt{\alpha s f s} \right)}{K_{V2} \left(\beta_2 R_{eD}^{\frac{1}{\beta_2}} \sqrt{\alpha s f s} \right)} K_{V2} \left(\beta_2 R_D^{\frac{1}{\beta_2}} \sqrt{\alpha s f s} \right) \right]} \quad (85)$$

Set

$$R_D^{\frac{v_2}{\beta_2} + \frac{1}{\beta_2} - 1 + m} s \sqrt{\alpha s f s} \left[I_{\beta_2} \left(\beta_2 \sqrt{\alpha s f s} R_D^{\frac{1}{\beta_2}} \right) + \frac{I_{V2} \left(\beta_2 R_{eD}^{\frac{1}{\beta_2}} \sqrt{\alpha s f s} \right)}{K_{V2} \left(\beta_2 R_{eD}^{\frac{1}{\beta_2}} \sqrt{\alpha s f s} \right)} K_{\beta_2} \left(\beta_2 R_D^{\frac{1}{\beta_2}} \sqrt{\alpha s f s} \right) \right] = A \quad (86)$$

$$R_D^{\frac{v_2}{\beta_2} - \frac{v_1}{\beta_1}} \left[I_{V2} \left(\beta_2 R_D^{\frac{1}{\beta_2}} \sqrt{\alpha s f s} \right) - \frac{I_{V2} \left(\beta_2 R_{eD}^{\frac{1}{\beta_2}} \sqrt{\alpha s f s} \right)}{K_{V2} \left(\beta_2 R_{eD}^{\frac{1}{\beta_2}} \sqrt{\alpha s f s} \right)} K_{V2} \left(\beta_2 R_D^{\frac{1}{\beta_2}} \sqrt{\alpha s f s} \right) \right] = B \quad (87)$$

Combining Eqs. (85) and (86),

$$-B\theta = C_1 A I_{V1} \left(\beta_1 R_D^{\frac{1}{\beta_1}} \sqrt{s f s} \right) + C_2 A K_{V1} \left(\beta_1 R_D^{\frac{1}{\beta_1}} \sqrt{s f s} \right) \quad (88)$$

$$C_1 = \frac{- \left[B\theta + C_2 A K_{V1} \left(\beta_1 R_D^{\frac{1}{\beta_1}} \sqrt{s f s} \right) \right]}{A I_{V1} \left(\beta_1 R_D^{\frac{1}{\beta_1}} \sqrt{s f s} \right)} \quad (89)$$

From Eq. (78),

$$C_1 = \frac{\frac{-1}{s \sqrt{s f s}} + C_2 K_{\beta_1} \left(\beta_1 \sqrt{s f s} \right)}{I_{\beta_1} \left(\beta_1 \sqrt{s f s} \right)} \quad (90)$$

Combining Eqs. (90) and (91),

Similarly, we have:

$$C_2 = \frac{\frac{A}{s\sqrt{sfs}} I_{V1} \left(\beta_1 R_D^{\frac{1}{\beta_1}} \sqrt{sfs} \right) - B\theta I_{\beta 1} \left(\beta_1 \sqrt{sfs} \right)}{A \left[I_{V1} \left(\beta_1 R_D^{\frac{1}{\beta_1}} \sqrt{sfs} \right) K_{\beta 1} \left(\beta_1 \sqrt{sfs} \right) + I_{\beta 1} \left(\beta_1 \sqrt{sfs} \right) K_{V1} \left(\beta_1 R_D^{\frac{1}{\beta_1}} \sqrt{sfs} \right) \right]} \quad (91)$$

Laplace solution for the dimensionless pressure drops for closed (no-flow) boundary system for region II is

$$C_1 = \frac{- \left[B\theta + C_2 A K_{V1} \left(\beta_1 R_D^{\frac{1}{\beta_1}} \sqrt{sfs} \right) \right]}{A I_{V1} \left(\beta_1 R_D^{\frac{1}{\beta_1}} \sqrt{sfs} \right)} \quad (97)$$

Region II: Non - Newtonian ($R_D \leq r_D \leq R_{eD}$)

$$\frac{\partial P_{D2}}{\partial r_D} = r_D^{\frac{\nu_2}{\beta_2} + \frac{1}{\beta_2} - 1} \sqrt{\alpha sfs} C_3 \left[I_{\beta_2} \left(\beta_2 \sqrt{\alpha sfs} r_D^{\frac{1}{\beta_2}} \right) - \frac{I_{\beta 2} \left(\beta_2 R_{eD}^{\frac{1}{\beta_2}} \sqrt{\alpha sfs} \right)}{K_{\beta 2} \left(\beta_2 R_{eD}^{\frac{1}{\beta_2}} \sqrt{\alpha sfs} \right)} K_{\beta 2} \left(\beta_2 r_D^{\frac{1}{\beta_2}} \sqrt{\alpha sfs} \right) \right] \quad (92)$$

At interface R_D , $\frac{\partial P_{D2}}{\partial r_D} = -\frac{\theta}{s R_D^m}$ and $P_{D1} = P_{D2}$

$$C_3 = \frac{-\theta}{R_D^{\frac{\nu_2}{\beta_2} + \frac{1}{\beta_2} + m - 1} s \sqrt{\alpha sfs} \left[I_{\beta_2} \left(\beta_2 \sqrt{\alpha sfs} R_D^{\frac{1}{\beta_2}} \right) - \frac{I_{\beta 2} \left(\beta_2 R_{eD}^{\frac{1}{\beta_2}} \sqrt{\alpha sfs} \right)}{K_{\beta 2} \left(\beta_2 R_{eD}^{\frac{1}{\beta_2}} \sqrt{\alpha sfs} \right)} K_{\beta 2} \left(\beta_2 R_D^{\frac{1}{\beta_2}} \sqrt{\alpha sfs} \right) \right]} \quad (93)$$

$$C_3 = \frac{C_1 I_{V1} \left(\beta_1 R_D^{\frac{1}{\beta_1}} \sqrt{sfs} \right) + C_2 K_{V1} \left(\beta_1 R_D^{\frac{1}{\beta_1}} \sqrt{sfs} \right)}{R_D^{\frac{\nu_2}{\beta_2} - \frac{\nu_1}{\beta_1}} \left[I_{V2} \left(\beta_2 R_D^{\frac{1}{\beta_2}} \sqrt{\alpha sfs} \right) + \frac{I_{\beta 2} \left(\beta_2 R_{eD}^{\frac{1}{\beta_2}} \sqrt{\alpha sfs} \right)}{K_{\beta 2} \left(\beta_2 R_{eD}^{\frac{1}{\beta_2}} \sqrt{\alpha sfs} \right)} K_{V2} \left(\beta_2 R_D^{\frac{1}{\beta_2}} \sqrt{\alpha sfs} \right) \right]} \quad (94)$$

Set

$$R_D^{\frac{\nu_2}{\beta_2} + \frac{1}{\beta_2} + m - 1} s \sqrt{\alpha sfs} \left[I_{\beta_2} \left(\beta_2 \sqrt{\alpha sfs} R_D^{\frac{1}{\beta_2}} \right) - \frac{I_{\beta 2} \left(\beta_2 R_{eD}^{\frac{1}{\beta_2}} \sqrt{\alpha sfs} \right)}{K_{\beta 2} \left(\beta_2 R_{eD}^{\frac{1}{\beta_2}} \sqrt{\alpha sfs} \right)} K_{\beta 2} \left(\beta_2 R_D^{\frac{1}{\beta_2}} \sqrt{\alpha sfs} \right) \right] = A \quad (95)$$

$$R_D^{\frac{\nu_2}{\beta_2} - \frac{\nu_1}{\beta_1}} \left[I_{V2} \left(\beta_2 R_D^{\frac{1}{\beta_2}} \sqrt{\alpha sfs} \right) + \frac{I_{\beta 2} \left(\beta_2 R_{eD}^{\frac{1}{\beta_2}} \sqrt{\alpha sfs} \right)}{K_{\beta 2} \left(\beta_2 R_{eD}^{\frac{1}{\beta_2}} \sqrt{\alpha sfs} \right)} K_{V2} \left(\beta_2 R_D^{\frac{1}{\beta_2}} \sqrt{\alpha sfs} \right) \right] = B \quad (96)$$

$$C_1 = \frac{\frac{-1}{s\sqrt{sfs}} + C_2 K_{\beta 1} \left(\beta_1 \sqrt{sfs} \right)}{I_{\beta 1} \left(\beta_1 \sqrt{sfs} \right)} \quad (98)$$

$$C_2 = \frac{\frac{A}{s\sqrt{sfs}} I_{V1} \left(\beta_1 R_D^{\frac{1}{\beta_1}} \sqrt{sfs} \right) - B \theta I_{\beta 1} \left(\beta_1 \sqrt{sfs} \right)}{A \left[I_{V1} \left(\beta_1 R_D^{\frac{1}{\beta_1}} \sqrt{sfs} \right) K_{\beta 1} \left(\beta_1 \sqrt{sfs} \right) + I_{\beta 1} \left(\beta_1 \sqrt{sfs} \right) K_{V1} \left(\beta_1 R_D^{\frac{1}{\beta_1}} \sqrt{sfs} \right) \right]} \quad (99)$$

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Declarations

Conflict of interest Not applicable.

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