Derivation of water flooding characteristic curve for offshore low-amplitude structural reservoir with strong bottom water

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Abstract

The water flooding characteristic curve method is one of the essential techniques to predict recoverable reserves. However, the recoverable reserves indicated by the existing water flooding characteristic curves of low-amplitude reservoirs with strong bottom water increase gradually, and the current local recovery degree of some areas has exceeded the predicted recovery rate. The applicability of the existing water flooding characteristic curves in low-amplitude reservoirs with strong bottom water is lacking, which affects the accurate prediction of development performance. By analyzing the derivation process of the conventional water flooding characteristic curve method, this manuscript finds out the reasons for the poor applicability of the existing water flooding characteristic curve in low-amplitude reservoir with strong bottom water and corrects the existing water flooding characteristic curve according to the actual situation of the oilfield and obtains the improvement method of water flooding characteristic curve in low-amplitude reservoir with strong bottom water. After correction, the correlation coefficient between $\frac{k_{rw}}{k_{rw}}$ and S_w is 95.92%. According to the comparison between the actual data and the calculated data, in 2021/3, the actual water cut is 97.29%, the water cut predicted by the formula is 97.27%, the actual cumulative oil production is 31.19×10^4 t, and the predicted cumulative oil production is 31.31×10^4 t. The predicted value is consistent with the actual value. It provides a more reliable method for predicting low-amplitude reservoirs' recoverable ability with strong bottom water and guides the oilfield's subsequent decision-making.

Keywords Strong bottom water reservoir · Low-amplitude structure · Water flooding characteristic curve · Recoverable reserves

Preface

When the water flooding characteristic curve was first put forward, a statistical relationship appeared for its production data and had no theoretical significance. In 1959, the field data were first used to obtain the water flooding characteristic curve's statistical law, from which the water flooding characteristic curve can be used as an intuitive and straightforward statistical expression of dynamic reservoir law. At present, the water flooding characteristic curve method has been widely valued and applied (Muskat et al. 1935, Abass et al. 1988, Guo et al. 1992, Yuanqian 1994). The water flooding characteristic curve method can predict the water

flooding oilfield's related development indexes and effectively predict and judge the water flooding oilfield's movable reserves and original geological reserves.

Because different types of oilfields have their characteristics, the realization of production performance of different types of oilfields would be inconsistent. Therefore, different types of oil fields would have the corresponding water flooding characteristic curve. Different reservoir types, reservoir physical properties, and crude oil properties show different dynamic characteristics. Based on statistical research, some scholars put forward various expressions of the water flooding characteristic curve. It reflects the particularity of oil-field production performance (Cunyou et al. 2011, Liu 2020, Recham 2000). Because of this particularity, some different expressions may be proposed in future. As long as several oilfield examples prove, it is the enrichment and enrichment of the research content. As a hot topic, many scholars are actively engaged in this research and exploration and have made many achievements, which is scientific progress.

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Many scholars in the former Soviet Union have proposed various water flooding characteristic curves (Olabode et al. 2021; Hanqiao et al. 2006; Qitai 1998). At present, the widely used water flooding characteristic curves are Сазонов Б.Ф., Сипачев Н.В., and Назаров С. Н.. In 1978, Tong Xianzhang studied the water flooding characteristic curve expressed by the relationship between the logarithm of cumulative water production and cumulative oil production and established a complete application system. He began applying and researching the water flooding characteristic curve (Jackson et al. 2005; Wenjun et al. 2000; Qitai 1999). Chen Yuanqian combined with the Nonpiston theory of water flooding and made theoretical verification on Максимов М. И. water flooding characteristic curve, Сазонов Б.Ф. water flooding characteristic curve, and Сипачев Н.В. water flooding characteristic curve. The new water flooding characteristic curves proposed in the following research literature have been theoretically deduced or verified. With the deepening of application and research, it is found that different water flooding characteristic curves reflect different oil–water two-phase seepage characteristics.

The derivation of the water flooding characteristic curve is based on the semi-logarithmic linear relationship between the ratio of oil to water relative permeability and water saturation (Wenjun and Jun 2007; Yuanqian 1993; Jian et al. 2013; Chuanliang 2005). The relationship between oil and water relative permeability and water saturation only conforms to the linear relationship in the middle part but deviates from the linear relationship in the early development and ultra-high water cut stages. Therefore, the water flooding characteristic curve is generally applicable to the two parameters (water/oil ratio, cumulative oil production, oil recovery) in the rectangular coordinate system after the water cut reaches 40% cumulative water production. When there is a linear relationship or semi-logarithmic linear relationship, in the reservoir with strong bottom water and low amplitude, the water cut rises very fast, there is no water-free oil recovery period, and it would soon enter the high water cut period. When the oilfield enters the ultra-high water cut stage, the relationship between the ratio of oil and water relative permeability and water saturation deviated from the straight line, and the conventional water flooding characteristic curve would also deviate from the straight line. Simultaneously,

applying the conventional water flooding characteristic curve to predict the development index would produce a significant error (Qi-tai 1999; Shaoxian.2012; Hua et al. 2013; Changfu et al. 2011). In this study, a new expression of the relationship between oil–water relative permeability and water saturation is proposed, and a water flooding characteristic curve suitable for low-amplitude reservoirs with strong bottom water is established. On this basis, a new type of water flooding characteristic curve is derived. Through the verification analysis, the new water flooding characteristic curve can reasonably predict the development index of the reservoir with strong bottom water and low amplitude, which is more in line with the actual reservoir development.

Deflection of conventional water flooding characteristic curve method in high water cut stage

In this study, the conventional water flooding characteristic curve (Table 1) predicts the oil recovery in the reservoir's medium and high water cut period with strong bottom water and low amplitude. It is found that there is a big gap with the actual situation, so it is not applicable.

The conventional water flooding characteristic curve method considers that the relative permeability ratio of oil–water two-phase is expressed as $\frac{k_{ro}}{k_{rw}} = ce^{-d \cdot S_w}$. Through actual data discovery, the relationship between $\ln\left(\frac{k_{ro}}{k_{rw}}\right)$ and S_w is only linear for some time (Shuhong et al. 2011; Qiaoyun et al. 2011; Yuanqian and Dang 2004; Welge 1952; Goda et al. 2007; Wang et al. 2013). There is a downward tilt in the later period (see Fig. 1). During the derivation of the conventional water flooding characteristic curve method $\bar{S}_w \approx S_{we} + \frac{1}{mf_w}$. It is assumed that after the oil field enters the high water cut stage ($f_w > 80\%$), $\bar{S}_w \approx S_{we}$. In other words, the water saturation at the high water cut the average saturation of formation can replace stage outlet. But in the actual reservoir, $\bar{S}_w \neq S_{we}$. The formula $\frac{k_{ro}}{k_{rw}} = ce^{-d \cdot S_w}$ needs to be modified according to the actual oilfield.

Table 1 Ordinary water flooding characteristic curve

Types of water flooding characteristic curves	Expression
Water flooding characteristic curve of Максимов М. И.-Xianzhang T	$\log W_p = a + bN_p$
Water flooding characteristic curve of Сазонов Б.Ф	$\log L_p = a + bN_p$
Water flooding characteristic curve of Сипачев Н.В	$L_p/N_p = a + bL_p$
Water flooding characteristic curve of Назаров С. Н	$L_p/N_p = a + bW_p$
Water flooding characteristic curve of Jinqin Z	$W_p/N_p = -a + bW_p/N_p^2$
Water flooding characteristic curve of Qitai Y	$\log N_p = a - b \log (L_p/W_p)$

Fig. 1 Traditional water flooding characteristic curve method would deflect in the high water cut stage

Establishment of water flooding characteristic curve for low-amplitude reservoir with strong bottom water

Derivation of water flooding characteristic curve for low-amplitude reservoir with strong bottom water

According to Buckley–Leverett, water flooding front advancing equation, the functional relationship of $\bar{S}_w$ and S_{we} is as follows (Yuanqian and Cunyou 2014; Can and Kabir 2014):

$$S_{we} = \left(\frac{1}{1+k} \bar{S}_w + \frac{k}{1+k} S_{wi} \right) \left[1 + \left(\frac{\bar{S}_w - S_{wi}}{m} \right)^{-1/k} \right]^{-1} \quad (1)$$

By deriving the two ends of formula (1):

$$\frac{dS_{we}}{d\bar{S}_w} = -\frac{1}{mk} \left[\frac{\frac{1}{1+k} \bar{S}_w + \frac{k}{1+k} S_{wi}}{1 + \left(\frac{\bar{S}_w - S_{wi}}{m} \right)^{-1/k}} - \bar{S}_w \right] \frac{\left(\frac{\bar{S}_w - S_{wi}}{m} \right)^{1/k-1}}{1 + \left(\frac{\bar{S}_w - S_{wi}}{m} \right)^{-1/k}} \quad (2)$$

Replace (1) with (2):

$$\frac{dS_{we}}{d\bar{S}_w} = -\frac{1}{mk} \left(S_{we} - \bar{S}_w \right) \frac{\left(\frac{\bar{S}_w - S_{wi}}{m} \right)^{1/k-1}}{1 + \left(\frac{\bar{S}_w - S_{wi}}{m} \right)^{-1/k}} \quad (3)$$

From the Welge equation, it can be concluded that:

$$\frac{df_{oe}}{dS_{we}} = \frac{df_{oe}}{d\bar{S}_w} \frac{d\bar{S}_w}{dS_{we}} = \frac{f_{oe}}{S_{we} - \bar{S}_w} \quad (4)$$

By substituting (3) into (4):

$$\frac{df_{oe}}{d\bar{S}_w} = -\frac{1}{mk} \frac{\left(\frac{\bar{S}_w - S_{wi}}{m} \right)^{1/k-1}}{1 + \left(\frac{\bar{S}_w - S_{wi}}{m} \right)^{-1/k}} f_{oe} \quad (5)$$

Solve the above differential equation:

$$f_{oe} = C_1 / \left[1 + \left(\frac{\bar{S}_w - S_{wi}}{m} \right)^{1/k} \right] \quad (6)$$

Then (6) the initial conditions are satisfied $\bar{S}_w = S_{wi}$, $f_{oe} = 1$ (Replace $\bar{S}_w = S_{wi}$ into (1). The initial conditions can be determined $\bar{S}_w = S_{wi}$, then $f_{oe} = 1$), $C_1 = 1$, substitution it into (6):

$$f_{oe} = 1 / \left[1 + \left(\frac{\bar{S}_w - S_{wi}}{m} \right)^{1/k} \right] \quad (7)$$

then

$$(1 - f_{oe}) / f_{oe} = \left(\frac{\bar{S}_w - S_{wi}}{m} \right)^{1/k} \quad (8)$$

From the definition,

$$(1 - f_{oe}) / f_{oe} = Q_w / Q_o \quad (9)$$

$$Q_w = dW_p / dt \quad (10)$$

$$Q_o = dN_p / dt \quad (11)$$

Under the condition that the formation pressure is maintained by water injection and the reservoir is relatively homogeneous, it can be seen from the material balance equation that:

$$\bar{S}_w = \frac{N_p (1 - S_{wi})}{N} + S_{wi} \quad (12)$$

Combining (9) (10) (11) (12) with (8), we can get:

$$dW_p / dN_p = \left[(1 - S_{wi}) \frac{N_p}{mN} \right]^{1/k} \quad (13)$$

(13) When the initial conditions are satisfied $W_p = 0$, $N_p = N_{p0}$. By solving the above differential equations, we can obtain:

$$\ln N_p = A + B \ln(W_p + C) \quad (14)$$

where

$$A = \frac{1}{1+k} \ln \left[\frac{1}{1+k} \left(\frac{mN}{1-S_{wi}} \right)^{1/k} \right]$$

$$B = \frac{1}{1+k}$$

$$C = \frac{1}{1+k} \left(\frac{1-S_{wi}}{mN} \right)^{1/k} N_{p0}^{1+1/k}$$

There is no water-free oil recovery period in low-amplitude structural reservoir with strong bottom water, so $N_{p0}=0$, $C=0$, from (14)

$$\ln \frac{W_p}{N_p} = a + b \ln N_p \quad (15)$$

$$\text{where } b = k; a = \ln \left[\frac{1}{1+k} \left(\frac{mN}{1-S_{wi}} \right)^{1/k} \right]$$

According to the partial flow formula:

$$f_w = 1 - f_{oe} = 1 / [1 + K_{ro} / (\mu_r K_{rw})] \quad (16)$$

The oil–water relative permeability ratio formula is:

$$K_{ro} / K_{rw} = \mu_r / \left[(\bar{S}_w - S_{wi}) / m \right]^{1/k} \quad (17)$$

where

f_{oe}, f_w —They are oil content and water content at the outlet, f ;

k —Constant;

K_{ro}, K_{rw} —They are relative permeability of oil and water, dimensionless;

L_p —Cumulative liquid production, $10^4 m^3$;

m —Constant coefficient;

N, N_p —They are geological reserves and cumulative oil production, $10^4 m^3$;

Q_L, Q_O, Q_W —They are liquid production, oil production, and water production, m^3/d ;

$\bar{S}_w, S_{wi}$ —They are average water saturation and irreducible water saturation, f ;

S_{we} —Water saturation at outlet, f ;

W_p —Cumulative water production, $10^4 m^3$;

μ_r —Oil–water viscosity ratio, dimensionless.

Substitute the actual relative permeability curve (see Table 2 and Fig. 2) into Formula 17. The relationship between $\frac{K_{ro}}{K_{rw}}$ and S_w is obtained. The correlation coefficient of $\frac{K_{ro}}{K_{rw}}$ and S_w has reached 95.92% (Fig. 3). The higher the water content, the better the fitting degree. It is suitable for low-amplitude reservoirs with strong bottom water.

Through the formula derivation in this section, the relationship between $\frac{K_{ro}}{K_{rw}}$ and S_w is obtained. The correlation coefficient of $\frac{K_{ro}}{K_{rw}}$ and S_w has reached 95.92%. The water flooding

Table 2 Relative permeability under different water saturation

water saturation (Sw)	Relative permeability of water phase (Krw)	Relative permeability of oil phase (Kro)
0.12	0.00	0.93
0.18	0.00	0.76
0.18	0.00	0.73
0.20	0.00	0.70
0.21	0.00	0.67
0.22	0.00	0.64
0.23	0.01	0.61
0.25	0.01	0.55
0.26	0.01	0.52
0.27	0.01	0.50
0.28	0.02	0.47
0.30	0.02	0.41
0.32	0.03	0.36
0.33	0.03	0.35
0.34	0.04	0.31
0.39	0.06	0.22
0.45	0.09	0.13
0.51	0.13	0.07
0.53	0.15	0.05
0.58	0.19	0.02
0.60	0.21	0.01

Fig. 2 Oil–water relative permeability of oilfield

characteristic curve suitable for a low-amplitude reservoir with strong bottom water is obtained.

Using new water flooding characteristic curve to predict cumulative oil production N_p

From Eq. (15), we can get the following results:

Fig. 3 Relation curve of K_{ro}/K_{rw} and $1/(\bar{S}_w - S_{wi})$

$$\ln W_p = a + (b + 1) \ln N_p \quad (18)$$

The derivative of Eq. (18) to N_p is obtained:

$$\frac{1}{W_p} \frac{dW_p}{dN_p} = (b + 1) \frac{1}{N_p} \quad (19)$$

$$\text{By } WOR = \frac{dW_p}{dN_p}:$$

$$W_p = \frac{N_p WOR}{b + 1} \quad (20)$$

Substituting (20) into (18) yields:

$$N_p = e^{\frac{\ln \frac{WOR}{b+1} - a}{b}} \quad (21)$$

Through (21), recoverable reserves under a given water–oil ratio (which can be converted into water content) can be obtained.

This section uses the water flooding characteristic curve in Sect. 3.1 to derive the formula for calculating cumulative oil production N_p .

Prediction of future water cut based on relative permeability

From the yield equation (Zhaojie et al. 2013):

$$Q_o = \frac{0.543KK_{ro}h(P_e - P_{wf})}{\mu_o B_o \left(\ln \frac{r_e}{r_w} + S \right)} \quad (22)$$

$$Q_w = \frac{0.543KK_{rw}h(P_e - P_{wf})}{\mu_w B_w \left(\ln \frac{r_e}{r_w} + S \right)} \quad (23)$$

(22) and (23), the:

$$\frac{Q_w}{Q_o} = \frac{\mu_o B_o K_{rw}}{\mu_w B_w K_{ro}} \quad (24)$$

(17) By substituting (24), the:

$$\frac{Q_w}{Q_o} = \frac{B_o}{B_w} \left[(\bar{S}_w - S_{wi})/m \right]^{1/k} \quad (25)$$

$$N_o = 100Ah\theta(1 - S_{wi})/B_o \quad (26)$$

$$N_{or} = 100Ah\theta(1 - \bar{S}_w)/B_o \quad (27)$$

From (26) and (27), the:

$$\bar{S}_w = (1 - S_{wi})R_o + S_{wi} \quad (28)$$

By substituting (28) into (25), the:

$$\frac{Q_w}{Q_o} = \frac{B_o}{B_w} \left[(1 - S_{wi})R_o/m \right]^{1/k} \quad (29)$$

$$f_w = \frac{Q_w}{Q_o + Q_w} = \frac{1}{\frac{Q_o}{Q_w} + 1} = \frac{1}{\frac{1}{\frac{Q_w}{Q_o}} + 1} \quad (30)$$

By substituting (29) into (30), the:

$$f_w = \frac{1}{\frac{1}{\frac{B_o}{B_w} \left[(1 - S_{wi})R_o/m \right]^{1/k}} + 1} \quad (31)$$

According to formula (31), the future water cut under constant liquid volume can be obtained.

where

K —Effective permeability of reservoir, mD.
 μ_o, μ_w —They are the viscosity of oil and the viscosity of water, mPa·s.
 a, b —Coefficient, f .
 B_o, B_w —They are the volume coefficient of oil and the volume coefficient of water, f .
 R_o —Oil recovery, f .
 A —Reservoir cross section, m^2 .
 h —Valid thickness, m .
 ϕ —Porosity, f .

According to the relative permeability productivity formula, this section deduces the formula for calculating water cut when the cumulative oil production N_p is known (Sect. 3.2).

Case analysis

The geological reserve of a strong bottom water sandstone reservoir is $2000 \times 10^4 m^3$; the adequate thickness is 6 m, the viscosity of crude oil is 50 mPa·s, the viscosity of water is 0.51 mPa·s, the density of oil is $0.923 g/cm^3$, and the volume coefficient of oil is 1.13. Take A2H well in this oilfield as an example (see Table 3). According to Eq. (15), there is an excellent linear relationship between well $\ln \frac{W_p}{N_p}$ and $\ln N_p$ (Fig. 4). The corresponding a and b values can be obtained, and the water content calculated by the formula (31) is compared with the actual water content (Fig. 5). The calculated cumulative oil production is in good agreement with the actual cumulative oil production (Fig. 6). In this way, the recovery degree of a single well under different water cuts can be obtained.

The recoverable reserves under the given water–oil ratio (which can be converted into water content) are obtained.

Figure 4 shows $a = -21.175, b = 1.8649$. It can be concluded from Eq. 15:

$$\ln \frac{W_p}{N_p} = -21.175 + 1.8649 \ln N_p \quad (32)$$

By $W_p + N_p = L_p$, Eq. 32 shows that:

$$\ln \frac{L_p - N_p}{N_p} = -21.175 + 1.8649 \ln N_p \quad (33)$$

Through Eq. 33, under the condition of constant liquid production (see Table 3. The actual production data from 2021/3, 48405t), in future, the accumulated liquid production L_p at any time in future can be obtained; thus, the accumulated oil production N_p at any time in future can be obtained. Through the accumulated oil production N_p , the

monthly oil production can be obtained, and the corresponding water content can be obtained. That is to say, through Eq. 33, knowing L_p and finding N_p , we can get the output of different water content at any time in future under constant liquid quantity.

According to the comparison between the actual data and the calculated data, in 2021/3, the actual water cut is 97.29%, the water cut predicted by the formula is 97.27%, the actual cumulative oil production is $31.19 \times 10^4 t$, and the predicted cumulative oil production is $31.31 \times 10^4 t$. The predicted value is consistent with the actual value.

Conclusions

In this study, the derivation process of the conventional water flooding characteristic curve is studied. At the same time, according to the Buckley–Leverett water flooding front advancing equation, combined with the characteristics of strong bottom water and low-amplitude structural reservoir without a water-free oil recovery period, the water flooding characteristic curve is modified:

(1) The conventional water flooding characteristic curve method considers that the relative permeability ratio of oil–water two-phase is expressed as $\frac{k_{ro}}{k_{rw}} = ce^{-d \cdot S_{wo}}$. Through the actual data, it is found that the relationship between $\ln \left(\frac{k_{ro}}{k_{rw}} \right)$ and S_w is only a straight line in a period, and there is a downward tilt in the later period.

(2) During the derivation of the conventional water flooding characteristic curve method, $\overline{S_w} \approx S_{we} + \frac{1}{mf_w}$. It is assumed that after the oil field enters the high water cut stage ($f_w > 80\%$), $\overline{S_w} \approx S_{we}$. In other words, the water saturation at the high water cut the average saturation of formation can replace stage outlet. But in the actual reservoir, $\overline{S_w} \neq S_{we}$.

(3) After correction, the correlation coefficient between $\frac{k_{ro}}{k_{rw}}$ and S_w is 95.92%.

(4) According to the comparison between the actual data and the calculated data, in 2021/3, the actual water cut is 97.29%, the water cut predicted by the formula is 97.27%, the actual cumulative oil production is $31.19 \times 10^4 t$, and the predicted cumulative oil production is $31.31 \times 10^4 t$. The predicted value is consistent with the actual value.

(5) The existing water flooding characteristic curve is corrected, and the improved water flooding characteristic curve in the high water cut period of low-amplitude structural reservoir with strong bottom water is obtained. The practical application is in line with the oilfield's actual situation, which provides a more reliable method for predicting the recoverability of low-amplitude structural reservoirs with strong bottom water and provides guidance for the subsequent decision-making of the oilfield.

Table 3 Actual production data and forecast data of A2H

Well Name	Date	Monthly oil production,t	Monthly liquid production,t	Actual water cut,%	Actual cumulative oil production, 10 ⁴ t	Calculate cumulative oil production, 10 ⁴ t	Calculate water cut,%	ln(Np)	ln(Wp/Np)
A02H	2011/4	16,588	18,093	8.32	1.66	1.48	10.13	9.72	− 2.40
A02H	2011/5	18,037	26,963	33.10	3.46	3.53	32.32	10.45	− 1.20
A02H	2011/6	12,420	20,989	40.83	4.70	4.18	46.56	10.76	− 0.91
A02H	2011/7	9343	17,447	46.45	5.64	4.69	55.41	10.94	− 0.73
A02H	2011/8	7179	15,428	53.47	6.36	5.42	61.12	11.06	− 0.59
A02H	2011/9	6176	14,863	58.45	6.97	6.01	65.35	11.15	− 0.46
A02H	2011/10	5368	13,658	60.70	7.51	6.30	68.56	11.23	− 0.36
A02H	2011/11	4837	13,798	64.94	7.99	6.91	71.14	11.29	− 0.27
A02H	2011/12	4458	13,314	66.52	8.44	7.16	73.27	11.34	− 0.19
A02H	2012/1	3823	11,981	68.09	8.82	7.43	74.94	11.39	− 0.12
A02H	2012/2	2997	10,776	72.19	9.12	8.21	76.15	11.42	− 0.06
A02H	2012/3	4187	15,332	72.69	9.54	8.31	77.71	11.47	0.02
A02H	2012/4	2362	15,026	84.28	9.78	11.88	78.53	11.49	0.12
A02H	2012/5	2824	15,631	81.93	10.06	10.91	79.46	11.52	0.20
A02H	2012/6	2598	16,608	84.36	10.32	11.92	80.26	11.54	0.28
A02H	2012/7	2012	16,905	88.10	10.52	14.00	80.86	11.56	0.37
A02H	2012/8	1078	8061	86.63	10.63	13.08	81.16	11.57	0.40
A02H	2012/9	1705	9714	82.45	10.80	11.11	81.64	11.59	0.43
A02H	2012/10	3220	17,620	81.73	11.12	10.83	82.49	11.62	0.49
A02H	2012/11	5219	28,467	81.67	11.64	10.81	83.75	11.67	0.56
A02H	2012/12	4117	27,302	84.92	12.06	12.18	84.65	11.70	0.63
A02H	2013/1	3851	25,323	84.79	12.44	12.12	85.44	11.73	0.69
A02H	2013/2	4528	29,124	84.45	12.89	11.96	86.29	11.77	0.75
A02H	2013/3	3695	27,052	86.34	13.26	12.92	86.93	11.80	0.81
A02H	2013/4	3258	22,259	85.36	13.59	12.40	87.46	11.82	0.84
A02H	2013/5	3168	21,578	85.32	13.91	12.38	87.95	11.84	0.88
A02H	2013/6	2607	17,912	85.45	14.17	12.44	88.33	11.86	0.90
A02H	2013/7	3767	30,743	87.75	14.54	13.77	88.85	11.89	0.95
A02H	2013/8	3085	31,954	90.35	14.85	15.78	89.25	11.91	1.00
A02H	2013/9	3228	34,880	90.75	15.17	16.16	89.65	11.93	1.06
A02H	2013/10	2904	34,393	91.56	15.46	17.01	89.99	11.95	1.11
A02H	2013/11	2659	35,117	92.43	15.73	18.07	90.29	11.97	1.16
A02H	2013/12	2519	34,035	92.60	15.98	18.30	90.56	11.98	1.20
A02H	2014/1	2373	32,743	92.75	16.22	18.51	90.80	12.00	1.24
A02H	2014/2	2387	34,185	93.02	16.46	18.90	91.04	12.01	1.28
A02H	2014/3	2263	33,175	93.18	16.68	19.14	91.25	12.02	1.32
A02H	2014/4	2235	33,307	93.29	16.91	19.31	91.46	12.04	1.36
A02H	2014/5	2109	32,103	93.43	17.12	19.54	91.65	12.05	1.39
A02H	2014/6	2120	32,756	93.53	17.33	19.70	91.83	12.06	1.42
A02H	2014/7	2211	33,603	93.42	17.55	19.52	92.02	12.08	1.45
A02H	2014/8	2250	32,835	93.15	17.78	19.09	92.20	12.09	1.48
A02H	2014/9	2375	33,514	92.91	18.01	18.74	92.38	12.10	1.50
A02H	2014/10	1865	26,213	92.89	18.20	18.70	92.52	12.11	1.52
A02H	2014/11	2373	33,321	92.88	18.44	18.69	92.70	12.12	1.55
A02H	2014/12	2187	31,135	92.98	18.66	18.84	92.85	12.14	1.57
A02H	2015/1	2184	30,201	92.77	18.88	18.54	93.00	12.15	1.59
A02H	2015/2	2510	34,496	92.72	19.13	18.47	93.17	12.16	1.61
A02H	2015/3	2395	33,418	92.83	19.37	18.63	93.32	12.17	1.63

Table 3 (continued)

Well Name	Date	Monthly oil production,t	Monthly liquid production,t	Actual water cut,%	Actual cumulative oil production, 10 ⁴ t	Calculate cumulative oil production, 10 ⁴ t	Calculate water cut,%	ln(Np)	ln(Wp/Np)
A02H	2015/4	2189	30,624	92.85	19.58	18.66	93.46	12.19	1.64
A02H	2015/5	2003	28,444	92.96	19.78	18.81	93.58	12.20	1.66
A02H	2015/6	1790	30,706	94.17	19.96	20.85	93.69	12.20	1.68
A02H	2015/7	1757	31,085	94.35	20.14	21.20	93.79	12.21	1.70
A02H	2015/8	1722	32,238	94.66	20.31	21.86	93.88	12.22	1.72
A02H	2015/9	1412	26,498	94.67	20.45	21.88	93.96	12.23	1.73
A02H	2015/10	1983	36,677	94.59	20.65	21.71	94.07	12.24	1.75
A02H	2015/11	1960	36,492	94.63	20.85	21.79	94.17	12.25	1.77
A02H	2015/12	1810	34,002	94.68	21.03	21.90	94.26	12.26	1.79
A02H	2016/1	1536	29,702	94.83	21.18	22.24	94.34	12.26	1.80
A02H	2016/2	1837	34,716	94.71	21.37	21.97	94.43	12.27	1.82
A02H	2016/3	1796	33,380	94.62	21.55	21.77	94.51	12.28	1.83
A02H	2016/4	1898	35,189	94.61	21.73	21.74	94.60	12.29	1.85
A02H	2016/5	1843	34,480	94.65	21.92	21.85	94.69	12.30	1.86
A02H	2016/6	2551	47,232	94.60	22.17	21.73	94.80	12.31	1.88
A02H	2016/7	2740	50,897	94.62	22.45	21.76	94.92	12.32	1.90
A02H	2016/8	2710	50,432	94.63	22.72	21.79	95.03	12.33	1.92
A02H	2016/9	4458	50,744	91.21	23.17	16.64	95.21	12.35	1.93
A02H	2016/10	2983	29,821	90.00	23.46	15.47	95.32	12.37	1.94
A02H	2016/11	2501	28,352	91.18	23.71	16.60	95.41	12.38	1.94
A02H	2016/12	2823	36,060	92.17	24.00	17.74	95.51	12.39	1.95
A02H	2017/1	2121	36,245	94.15	24.21	20.81	95.59	12.40	1.96
A02H	2017/2	1987	42,620	95.34	24.41	23.51	95.65	12.41	1.98
A02H	2017/3	2236	40,430	94.47	24.63	21.45	95.73	12.41	1.99
A02H	2017/4	1445	24,067	94.00	24.77	20.52	95.77	12.42	2.00
A02H	2017/5	1877	32,515	94.23	24.96	20.96	95.83	12.43	2.00
A02H	2017/6	11	123	91.06	24.96	16.48	95.83	12.43	2.00
A02H	2017/7	1564	26,567	94.11	25.12	20.74	95.88	12.43	2.01
A02H	2017/8	1976	36,294	94.56	25.32	21.63	95.94	12.44	2.02
A02H	2017/9	2055	38,231	94.62	25.52	21.78	96.00	12.45	2.03
A02H	2017/10	1065	38,986	97.27	25.63	31.19	96.03	12.45	2.05
A02H	2017/11	851	30,888	97.24	25.71	31.06	96.06	12.46	2.06
A02H	2017/12	983	41,418	97.63	25.81	33.58	96.09	12.46	2.08
A02H	2018/1	1456	37,873	96.16	25.96	26.05	96.13	12.47	2.09
A02H	2018/2	1748	36,298	95.18	26.13	23.11	96.18	12.47	2.10
A02H	2018/3	1724	38,431	95.51	26.31	24.00	96.22	12.48	2.11
A02H	2018/4	1833	42,816	95.72	26.49	24.61	96.27	12.49	2.12
A02H	2018/5	1908	37,232	94.88	26.68	22.35	96.32	12.49	2.13
A02H	2018/6	1917	39,779	95.18	26.87	23.10	96.37	12.50	2.14
A02H	2018/7	386	7144	94.60	26.91	21.72	96.38	12.50	2.14
A02H	2018/8	1913	42,194	95.47	27.10	23.87	96.43	12.51	2.15
A02H	2018/9	2170	44,344	95.11	27.32	22.91	96.48	12.52	2.16
A02H	2018/10	1800	35,468	94.93	27.50	22.47	96.53	12.52	2.17
A02H	2018/11	1808	34,676	94.79	27.68	22.14	96.57	12.53	2.17
A02H	2018/12	1666	34,886	95.22	27.85	23.21	96.61	12.54	2.18
A02H	2019/1	1583	34,753	95.44	28.00	23.81	96.65	12.54	2.19
A02H	2019/2	1519	33,460	95.46	28.16	23.85	96.68	12.55	2.20
A02H	2019/3	1568	33,865	95.37	28.31	23.60	96.71	12.55	2.20

Table 3 (continued)

Well Name	Date	Monthly oil production,t	Monthly liquid production,t	Actual water cut,%	Actual cumulative oil production, 10 ⁴ t	Calculate cumulative oil production, 10 ⁴ t	Calculate water cut,%	ln(Np)	ln(Wp/Np)
A02H	2019/4	1696	33,735	94.97	28.48	22.58	96.75	12.56	2.21
A02H	2019/5	1647	33,894	95.14	28.65	23.00	96.79	12.57	2.22
A02H	2019/6	1760	33,897	94.81	28.82	22.19	96.82	12.57	2.22
A02H	2019/7	1440	32,688	95.59	28.97	24.23	96.85	12.58	2.23
A02H	2019/8	1358	32,516	95.82	29.10	24.93	96.88	12.58	2.24
A02H	2019/9	1211	35,604	96.60	29.22	27.80	96.91	12.59	2.24
A02H	2019/10	1568	34,664	95.48	29.38	23.89	96.94	12.59	2.25
A02H	2019/11	1031	30,549	96.63	29.48	27.91	96.96	12.59	2.26
A02H	2019/12	817	27,549	97.03	29.57	29.88	96.97	12.60	2.27
A02H	2020/1	287	8714	96.71	29.59	28.27	96.98	12.60	2.27
A02H	2020/2	374	8721	95.71	29.63	24.58	96.99	12.60	2.27
A02H	2020/3	1333	45,609	97.08	29.76	30.11	97.01	12.60	2.28
A02H	2020/4	1307	43,835	97.02	29.90	29.79	97.04	12.61	2.29
A02H	2020/5	1091	37,235	97.07	30.00	30.07	97.06	12.61	2.30
A02H	2020/6	765	24,575	96.89	30.08	29.13	97.07	12.61	2.30
A02H	2020/7	1319	44,480	97.03	30.21	29.88	97.10	12.62	2.31
A02H	2020/8	1464	42,795	96.58	30.36	27.71	97.12	12.62	2.32
A02H	2020/9	1696	48,651	96.51	30.53	27.44	97.15	12.63	2.33
A02H	2020/10	1740	46,277	96.24	30.70	26.36	97.18	12.63	2.34
A02H	2020/11	1456	47,325	96.92	30.85	29.31	97.21	12.64	2.35
A02H	2020/12	912	31,246	97.08	30.94	30.13	97.23	12.64	2.36
A02H	2021/1	627	24,113	97.40	31.00	32.01	97.24	12.64	2.36
A02H	2021/2	531	21,286	97.51	31.06	32.71	97.24	12.65	2.37
A02H	2021/3	1313	48,405	97.29	31.19	31.31	97.27	12.65	2.38

Fig. 4 Relation curve of $\ln \frac{W_p}{N_p}$ and $\ln N_p$ (the data of middle and high water cut stage were selected for linear regression) **Fig. 5** Comparison between calculated water content and actual water content

Fig. 6 Comparison between calculated cumulative oil production and actual cumulative oil production

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