Productivity evaluation of horizontal well in heterogeneous reservoir with composite water aquifer

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Abstract

Horizontal well is an important way to develop thin reservoir with edge-bottom water aquifer. For the horizontal well near oil–water contact, the reservoir have the features of edge water and bottom water at the same time. The productivity formula of horizontal well in pure edge water or pure bottom water reservoir is not fully applicable to this type of reservoir. For the heterogeneous reservoir with composite water aquifer, the horizontal well is divided into multiple well sections by using the concept of multi-segment well. Based on the seepage mechanics theory and potential superposition principle, considering the mutual interference between multi well sections, the productivity formula of horizontal well in heterogeneous reservoir with composite water aquifer is established. The new productivity formula considers the effect of reservoir heterogeneity, the quasi-linear flow of lateral edge water and the ellipsoidal flow of vertical bottom water. It can effectively determine the influence range and drainage distance of edge water in a heterogeneous reservoir with composite water aquifer. According to the test data of horizontal wells in Weizhou 11-1 oilfield, the drainage distance b of edge water is fitted and used to predict the productivity of new wells, and the result is very good. The research result lays a foundation for productivity prediction of horizontal well in heterogeneous reservoir with composite water aquifer.

Keywords Heterogeneous reservoir with composite water aquifer · Heterogeneity · Drainage distance of edge water · Horizontal well · Productivity evaluation

Introduction

Horizontal well is an important way to develop bottom water reservoir, which can significantly increase oil drainage area, improve well productivity, delay or inhibit coning of bottom water (Al Zarafi 1993; Wibowo et al. 2004; Huo et al. 2010; Luo et al. 2015; Akangbou et al. 2017). In the early research stage of horizontal well development technology, the works mainly focus on the calculation of horizontal well productivity and critical production rate (Chaperon 1986; Joshi 1988; Giger 1989; Fan 1993; Lü 1993; Fan and Lin 1994). With the further research, bottom water coning phenomenon and calculation of breakthrough time are paid more and more attention (Al-Enezi et al. 2010; Permadi and Jayadi 2010;

Nobakht et al. 2013; Adewole 2014; Al-Fatlawi et al. 2019; Sharma et al. 2019; Yue et al. 2019). With the improvement of experimental means, a lot of visualization research on bottom water coning are done, intuitive and mechanism understandings about bottom water movement law are obtained (Wang et al. 2018; You et al. 2019).

Horizontal well productivity is influenced by many factors including reservoir physical property and heterogeneity, horizontal length, pay thickness, offset from water oil contact and skin factor, etc. (Hagoort 2009; Kumar et al. 2017; Su et al. 2018). However, the relationship between horizontal well and water aquifer, flow direction of fluid in reservoir have not been considered comprehensively. In a real reservoir, if a horizontal well is near the reservoir boundary, it is difficult to strictly define it as an edge water or bottom water reservoir (Fig. 1). When the vertical connectivity and permeability of reservoir with composite water aquifer are good, the fluid mainly moves from bottom water to horizontal well [Fig. 2(a)]; when the vertical connectivity is poor or there are interlayers, the fluid mainly moves from edge water to horizontal well [Fig. 2(b)]. In the

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Fig.1 Diagram of reservoir with composite water aquifer

production process of horizontal well in homogeneous bottom water reservoir, the distribution of water saturation is symmetrical along the wellbore of horizontal well. However, in Fig. 2(a), water saturation distribution in the horizontal wellbore is not symmetrical, but shifted to the left, which indicates that the fluid movement to the horizontal wellbore is affected by edge water.

In the production process of horizontal well in a reservoir, both edge water and bottom water have the energy to drive oil. Because bottom water is close to the horizontal well, the bottom water first drives oil to the horizontal well. Along the direction of horizontal wellbore, the thickness of bottom water gradually becomes thinner, and the driving energy gradually depletes, and then the edge water far away from the horizontal wellbore gradually supply energy to the bottom water for oil displacement. In order to calculate the productivity of horizontal well in a heterogeneous reservoir with composite water aquifer, vertical seepage of bottom water and lateral seepage of edge water must be considered. If the energy of bottom water is weak, the influence of edge water is obvious. The effective drainage distance of edge water are directly related with seepage resistance, which will obviously influence the productivity of horizontal well.

The productivity formula of horizontal well in the existing literature is for pure edge water or pure bottom water reservoir, which doesn't consider the influence of edge water on bottom water or the influence of bottom water on edge water. Due to the complexity of edge-bottom water distribution and the variation of reservoir property, conventional productivity formula of horizontal well in pure edge water or pure bottom water reservoir usually generate large errors. In order to improve the accuracy of horizontal well productivity formula, the mutual influence of edge water and bottom water must be considered. Applying the seepage mechanics theory and potential superposition principle, the productivity formula of horizontal well in heterogeneous reservoir with composite water aquifer is established. The new productivity formula considers the effect of reservoir heterogeneity, the quasi-linear flow of lateral edge water and the ellipsoidal flow of vertical bottom water. It can effectively determine the influence range and drainage distance of edge water in a heterogeneous reservoir with composite water aquifer. The calculation results of proposed formula in this paper are in good consistent with the productivity test data of horizontal well.

(a) Fluid mainly moves from bottom water to horizontal well

(b) Fluid mainly moves from edge water to horizontal well

Fig. 2 Fluid movement direction in a heterogeneous reservoir with composite water aquifer

Review of horizontal well productivity formula

Joshi's formula

Joshi (1988) divided the three-dimensional seepage field of horizontal well into the superposition of two-dimensional seepage fields in horizontal and vertical directions, and proposed the formula to calculate production rate of horizontal well.

$$Q = \frac{0.543k_h h \cdot \Delta P}{\mu B \left[\ln \frac{a + \sqrt{a^2 - (L/2)^2}}{L/2} + \frac{\beta h}{L} \ln \left(\frac{\beta h}{2r_w} \right) \right]} \quad (1)$$

where

$$a = \frac{L}{2} \sqrt{0.5 + \sqrt{0.25 + (2R_{eh}/L)^4}}, \quad \beta = \sqrt{k_h/k_v}$$

$$r_{eh} = \sqrt{A/\pi}.$$

Q is production rate of horizontal well, m³/d; k_h , k_v are horizontal and vertical permeability of reservoir, mD; h is reservoir thickness, m; ΔP is producing pressure difference,

MPa; a is long half axis of ellipsoidal discharge area in horizontal well, m; L is length of horizontal well, m; r_{eh} is discharge radius of horizontal well, m; A is discharge area of horizontal well, m²; r_w is well radius, m; β is permeability anisotropy coefficient; μ is oil viscosity, mPa s; B is oil volume factor, m³/m³.

Fan's formula

Fan (1993) applied the conformal transform and potential function theory to derive the productivity formula of horizontal well in bottom-water reservoir with infinite extension in the horizontal direction.

$$Q = \frac{0.543kL}{\mu B} \frac{\Delta P}{\ln \frac{4\beta h}{\pi r_w} + \ln \tan \frac{\pi z_w}{2h}} \quad (2)$$

where, $k = \sqrt{k_h \cdot k_v}$, z_w is vertical position of horizontal wellbore, m.

Fan and Lin (1994) also derived the productivity formula of horizontal well in edge-water reservoir.

$$Q = \frac{0.7238kL}{\mu B} \frac{\Delta P}{\ln \frac{\beta h}{2\pi r_w} + \frac{1.333\pi b}{\beta h} - \ln \sin \frac{\pi z_w}{h}} \quad (3)$$

where, b is the distance from edge water to horizontal wellbore, m.

Lü's formula

Lü (1993) assumed that the reservoir is a rotating ellipsoid whose focal point is in the two ends of the horizontal well and the length of short half axis is reservoir thickness h . He derived the productivity formula of horizontal well in isotropic reservoir

$$Q = \frac{1.0857kL \cdot \Delta P}{\mu B \ln \left[\frac{2(\sqrt{L^2+h^2}-L)L^2}{(\sqrt{L^2+h^2}+L)r_w^2 \ln \frac{2L}{r_w}} \right]} \quad (4)$$

In anisotropic reservoir, the productivity formula is

$$Q = \frac{1.0857k_v^{0.75}k_h^{0.25} \cdot L}{\mu B} \cdot \frac{\Delta P}{\ln \frac{1}{2} + 2 \ln \left[\frac{4L}{(\sqrt{k_h/k_v}+1)r_w} \right]} + A \quad (5)$$

$$A = \ln \frac{\sqrt{k_v L^2 + k_h h^2} - \sqrt{k_v} L}{\sqrt{k_v L^2 - k_h h^2} + \sqrt{k_v} L} - \ln \ln \frac{4L}{(\sqrt{k_h/k_v}+1)r_w} \quad (6)$$

The above formulas take into account the influence of anisotropy, but don't consider the influence of heterogeneity.

They also don't consider the common effects of edge water and bottom water. These formulas are not fully applicable to the horizontal well in a heterogeneous reservoir with composite water aquifer.

Derivation of horizontal well productivity calculation model in heterogeneous reservoir with composite water aquifer

The horizontal well model in heterogeneous reservoir with composite water aquifer is in Fig. 3. The top of reservoir is closed. The bottom water is below the horizontal well and the edge water is on the lateral side. Edge and bottom water act on horizontal well at the same time. Consider the common influences of edge water and bottom water, the three-dimensional flow field of horizontal well can be approximately divided into internal and external two-dimensional flow fields. The external seepage field supplies oil from the drainage area with effective edge-water distance b and boundary pressure of P_e to the "straight tunnel" with length of L . The internal seepage field supplies oil from bottom water containing "straight tunnel" to the horizontal well with bottom hole pressure of P_{wf} (Fig. 3).

In Fig. 3, the total length of horizontal section is L ; the total production rate is Q . The steady seepage of incompressible fluid obeys Darcy's law. The production section of horizontal well is divided into m sections from toe to heel. The length, permeability and production rate of section i are $L(i)$, $k(i)$, $Q(i)$ ($i = 1, 2, \dots, m$) respectively. The coordinates of two ends for segment i are $A_i(x_{i1}, y_{i1}, z_{i1})$ and $B_i(x_{i2}, y_{i2}, z_{i2})$.

Fig. 3 The heterogeneous reservoir model with composite water aquifer

The potential of horizontal well section i in infinite three-dimensional space is (Lü 1993; Qi et al. 2015):

$$\Phi(i) = \frac{k(i)}{\mu B} P(i) + C = \frac{Q(i)}{1.0857L(i)} \ln \frac{D(i) - L(i)}{D(i) + L(i)} + C \quad (7)$$

where, $k(i)$ is permeability of horizontal well section i , mD; $P(i)$ is pressure of horizontal well section i , MPa; $L(i)$ is length of horizontal section i , m; $Q(i)$ is production rate of horizontal well section i , m³/d; $D(i)$ is the sum of distances from any point in the reservoir to two ends of horizontal well section $L(i)$.

$$S_1(x, y, z, i) = \sum_{n=-\infty}^{\infty} \left[\ln \frac{D_1(x, y, z, n, i) + L_i}{D_1(x, y, z, n, i) - L_i} + \ln \frac{D_2(x, y, z, n, i) + L_i}{D_2(x, y, z, n, i) - L_i} - \ln \frac{D_3(x, y, z, n, i) + L_i}{D_3(x, y, z, n, i) - L_i} - \ln \frac{D_4(x, y, z, n, i) + L_i}{D_4(x, y, z, n, i) - L_i} \right] \quad (12)$$

According to the superposition principle of potential, the potential generated at any point in the reservoir for the whole horizontal well is:

$$\Phi = \sum_{i=1}^m \Phi(i) + C \quad (8)$$

The potential function is obtained in the external seepage field and internal seepage field respectively.

External flow field

As shown in Fig. 3, the external seepage field is from the effective drainage distance of edge water $y + b$ to the "straight tunnel" with length of L . For the external seepage field, the top and bottom of reservoir are seen as closed boundaries, and the reservoir is an edge-water reservoir. Considering the interferences between well segments i and j and referring to the form of Eq. (7), the potential difference function is

$$D_3(x, y, z, n, i) = \sqrt{[x - x_{i1}(i)]^2 + [y - (y_{i1} + 2b)]^2 + [z - (2nh + z_{i1})]^2} + \sqrt{[x - x_{i2}(i)]^2 + [y - (y_{i2} + 2b)]^2 + [z - (2nh + z_{i2})]^2} \quad (15)$$

$$D_4(x, y, z, n, i) = \sqrt{[x - x_{i1}(i)]^2 + [y - (y_{i1} + 2b)]^2 + [z - (2nh - z_{i1})]^2} + \sqrt{[x - x_{i2}(i)]^2 + [y - (y_{i2} + 2b)]^2 + [z - (2nh - z_{i2})]^2} \quad (16)$$

$$\Delta\Phi_1(i, j) = \frac{k(j)}{\mu B} \cdot \Delta P_1(i, j) = \frac{Q(i)}{1.0857L(i)} [W_1(i, j) - E_1(i, j)] \quad (9)$$

For the permeability $k(j)$, the symbol j is used to reflect the influence of well segment j on well segment i . Where,

$$W_1(i, j) = S_1[x(j), y_{i1}, 0, i] \quad (10)$$

$$E_1(i, j) = S_1[x(j), y_{i1} + 2b, 0, i] \quad (11)$$

Equation (10) represents the function at the "straight tunnel" with length L , and Eq. (11) represents the function at the effective drainage distance of edge water ($y + 2b$). where, the S_1 function is:

In Eq. (12), D_1 and D_2 are the functions of production well segment i after mirror reflection through the top and bottom boundary of reservoir.

$$D_1(x, y, z, n, i) = \sqrt{[x - x_{i1}(i)]^2 + (y - y_{i1})^2 + [z - (2nh + z_{i1})]^2} + \sqrt{[x - x_{i2}(i)]^2 + (y - y_{i2})^2 + [z - (2nh + z_{i2})]^2} \quad (13)$$

$$D_2(x, y, z, n, i) = \sqrt{[x - x_{i1}(i)]^2 + (y - y_{i1})^2 + [z - (2nh - z_{i1})]^2} + \sqrt{[x - x_{i2}(i)]^2 + (y - y_{i2})^2 + [z - (2nh - z_{i2})]^2} \quad (14)$$

The image of real production well segment i through the edge-water interface is an injection well segment with injection rate of $-Q(i)$, whose x -coordinate and z -coordinate on both ends are the same as that of real production well segment i , while y coordinate is $y + 2b$. For the image injection well segment i , after mirror reflection through the top and bottom boundary of reservoir, the functions of D_3 and D_4 are

where, $x(j) = x_{i1}(j) + \frac{L}{2m}$, $x_{i1}(i) = (i-1)L/m$, $x_{i2}(i) = iL/m$.

Internal seepage field

As shown in Fig. 3, the internal seepage field is from bottom water containing a "straight tunnel" to a horizontal well with bottom hole pressure of P_{wf} . For the internal seepage field,

$$D'_4(x, y, z, n, i) = \sqrt{[x - x_{i1}(i)]^2 + (y - y_{i1})^2 + [z - (4nh - 2h + z_{i1})]^2} + \sqrt{[x - x_{i2}(i)]^2 + (y - y_{i2})^2 + [z - (4nh - 2h + z_{i2})]^2} \quad (24)$$

the top of the reservoir is a closed boundary, and the bottom is constant pressure boundary.

Considering the interferences between well sections i and j , the potential difference function is

$$\Delta\Phi_2(i, j) = \frac{k(j)}{\mu B} \cdot \Delta P_2(i, j) = \frac{Q(i)}{1.0857L(i)} [W_2(i, j) - E_2(i, j)] \quad (17)$$

where,

$$W_2(i, j) = \begin{cases} S_2[x(j), y_{i1}, \frac{z_{i1} + z_{i2}}{2} + r_w, i] & i = j \\ S_2[x(j), y_{i1}, \frac{z_{i1} + z_{i2}}{2}, i] & i \neq j \end{cases} \quad (18)$$

$$E_2(i, j) = S_2[x(j), y_{i1}, 0, i] \quad (19)$$

Equations (18) and (19) are the functions at horizontal wellbore and the oil–water interface of bottom water ($z=0$). The S_2 function is:

$$S_2(x, y, z, i) = \sum_{n=-\infty}^{\infty} \left[\ln \frac{D'_1(x, y, z, n, i) + L_i}{D'_1(x, y, z, n, i) - L_i} + \ln \frac{D'_2(x, y, z, n, i) + L_i}{D'_2(x, y, z, n, i) - L_i} - \ln \frac{D'_3(x, y, z, n, i) + L_i}{D'_3(x, y, z, n, i) - L_i} - \ln \frac{D'_4(x, y, z, n, i) + L_i}{D'_4(x, y, z, n, i) - L_i} \right] \quad (20)$$

D'_1 and D'_2 are the functions of production well segment i after mirror reflection through the top and bottom boundary of reservoir.

$$D'_1(x, y, z, n, i) = \sqrt{[x - x_{i1}(i)]^2 + (y - y_{i1})^2 + [z - (4nh + z_{i1})]^2} + \sqrt{[x - x_{i2}(i)]^2 + (y - y_{i2})^2 + [z - (4nh + z_{i2})]^2} \quad (21)$$

$$D'_2(x, y, z, n, i) = \sqrt{[x - x_{i1}(i)]^2 + (y - y_{i1})^2 + [z - (4nh + 2h - z_{i1})]^2} + \sqrt{[x - x_{i2}(i)]^2 + (y - y_{i2})^2 + [z - (4nh + 2h - z_{i2})]^2} \quad (22)$$

D'_3 and D'_4 are the functions of injection well segment i after mirror reflection through the top and bottom boundary of reservoir.

$$D'_3(x, y, z, n, i) = \sqrt{[x - x_{i1}(i)]^2 + (y - y_{i1})^2 + [z - (4nh - z_{i1})]^2} + \sqrt{[x - x_{i2}(i)]^2 + (y - y_{i2})^2 + [z - (4nh - z_{i2})]^2} \quad (23)$$

Calculation of production rate

For the horizontal well with m segments in a heterogeneous reservoir with composite water aquifer, considering the interferences of well section j on well section i , the total potential difference function is

$$\Delta\Phi(i, j) = \Delta\Phi_1(i, j) + \Delta\Phi_2(i, j) \quad (25)$$

The inverse matrix of productivity index is obtained from Eqs. (9) and (17)

$$A(i, j) = \frac{\mu B}{1.0857k(j)} \left\{ \frac{m}{L} [W_1(i, j) - E_1(i, j)] + \frac{m}{L} [W_2(i, j) - E_2(i, j)] \right\} \quad (26)$$

Production pressure difference matrix of well section i is

$$C(i) = P_e - P_w(i) \quad (27)$$

where, P_e is formation pressure, MPa; P_w is bottom hole pressure, MPa.

Production rate matrix of well section i is

$$Q(i) = A(i, j)^{-1} \cdot C(i) \quad (28)$$

Actually, Eq. (28) is a matrix expression form of productivity equation for well section i . The total production rate of horizontal well is

$$Q = \sum_{i=1}^m Q(i) \quad (29)$$

If the reservoir is anisotropic, let $\beta = \sqrt{k_h(j)/k_v(j)}$, and substitute $\sqrt{k_h(j) \cdot k_v(j)}$, βh and βz for $k(j)$, h and z in above

formulas. Then, the potential difference and production rate of horizontal well in anisotropic reservoirs can be obtained.

Result analysis

Analysis of theoretical curve features

Because of the influence of reservoir heterogeneity, the distribution of flow rate along horizontal well section is non-uniform. Accurate prediction of flow rate distribution is helpful to adopt reasonable production technology for horizontal well. Basic parameters of a reservoir are as follows: permeability $k=400\text{mD}$, pay thickness $h=15\text{ m}$, oil viscosity $\mu_o=40\text{ mPa.s}$, volume factor $B_o=1.05\text{ m}^3/\text{m}^3$, horizontal

well length $L=600\text{ m}$, well radius $r_w=0.1\text{ m}$, edge water influence distance $b=60\text{ m}$. Because the influence of permeability on flow rate distribution of horizontal well is more obvious than other parameters, firstly the sensitivity analyses on permeability are done.

Under homogeneous condition, the productivity index of horizontal well is $75.17\text{m}^3/(\text{d.MPa})$. The distribution of productivity index along the horizontal well section is shown in Figs. 4, 5, 6 and 7. The results show that under homogeneous conditions (Fig. 4), the distribution of flow rate along horizontal well section is U-shaped, with less flow rate in the middle and more at both ends, which is caused by larger oil drainage area at both ends of horizontal well. Setting the permeability increasing gradually from heel to toe of horizontal well section but the average permeability unchanged,

Fig. 4 The relationship between productivity index and permeability of horizontal well section in homogeneous reservoir

Fig. 5 The relationship between productivity index and permeability of horizontal well section in heterogeneous reservoir (Permeability increases from heel to toe)

Fig. 6 The relationship between productivity index and permeability of horizontal well section in heterogeneous reservoir (Permeability from heel to toe increases first and then decreases)

Fig. 7 The relationship between productivity index and permeability of horizontal well section in heterogeneous reservoir (Permeability from heel to toe decreases first and then increases)

the distribution of flow rate along horizontal well section is no longer U-shaped, but gradually increases from heel to toe (Fig. 5). At the end of horizontal well section, the productivity index rises sharply. The productivity index of the whole horizontal well section is $75.55 \text{ m}^3/(\text{d MPa})$, which is approximately equal to that under homogeneous condition.

If the permeability from heel to toe increases first and then decreases (Fig. 6), the distribution of flow rate along horizontal well section tends to be high in the middle. Compared to the homogeneous condition, the high flow rate at the end of horizontal well is restrained, resulting in the decrease of productivity index of the whole horizontal well section, which is $69.0 \text{ m}^3/(\text{d MPa})$. If the permeability from heel to toe decreases first and then increases, the distribution of flow rate along horizontal well section presents a more

obvious U-shaped trend (Fig. 7). Compared to the homogeneous condition, the distribution of bigger flow rate at the end of horizontal well section is enhanced, resulting in the increase of productivity index of the whole horizontal well section, which is $80.41 \text{ m}^3/(\text{d MPa})$.

It can be seen that under the condition of equal average permeability along horizontal well section, the productivity index of the whole horizontal well will also change obviously due to the change of permeability distribution.

Under homogeneous condition, the relationship between the productivity index of horizontal well and the ratio of edge water drainage distance to offset from water oil contact is shown in Fig. 8. With the increase of edge water drainage distance, the productivity index of horizontal well first decreases sharply, then tends to be stable gradually. When

Fig. 8 The production index of horizontal well under different drainage distance of edge water **Table 1** Comparison of productivity calculation results of horizontal wells in Weizhou 11-1 Oilfield

Well name	Parameters			Actual productivity index $J_h, m^3/(d.MPa)$	Calculated productivity index $J_h, m^3/(d.MPa)$		
	Permeability k, mD	Pay thickness h, m	Length of horizontal well L, m		Joshi's formula	Fan's formula	Lü's formula
A13H	165.5	12.2	487	39.88	15.60	176.45	51.51
A14H	362.0	12.3	719	58.05	41.81	569.64	167.00
A22H	273.0	8.2	626	59.59	22.78	400.17	122.77
A23H	438.7	8.2	284	36.63	21.12	292.06	88.23
A8S2	438.4	11.5	385	53.73	30.64	373.41	109.19

the ratio of edge water drainage distance to offset from water oil contact is 0~10, the change of horizontal well productivity index is very sensitive. For the horizontal well in a reservoir with composite water aquifer, if the drainage distance of edge water is several times of offset from water oil contact, the process that edge water supplies energy to bottom water for oil displacement cannot be ignored.

Case analysis

Weizhou 11-1 oilfield is located in the Weixinan Sag, Beibu Gulf Basin, South China Sea. It is an anticline structural reservoir with low amplitude and shallow buried depth of 942~980 m. The horizontal wells in Weizhou 11-1 oilfield have obvious features of edge-bottom water drive. Oil viscosity is 38.50 mPa s, and oil volume factor is 1.05 m³/m³. Reservoir permeability, pay thickness and effective length of horizontal well section are shown in Table 1. Considering the anisotropy of reservoir, take $k_v/k_h = 0.1$.

Table 2 Fitting of effective edge water drainage distance in Weizhou 11-1 Oilfield

Well name	Actual productivity index $J_h, m^3/(d.MPa)$	Parameter fitting Edge water drainage distance b, m
A13H	39.88	27.5
A14H	58.05	80
A22H	59.59	28
Average		45.17

The calculation results of different formulas show that Fan's and Lü's formulas don't consider the seepage resistance of edge water, therefore the calculation results are bigger than actual productivity index. The pseudo-radial flow in the plane is considered in Joshi's formula, but its drainage radius is larger than the effective drainage distance of edge water, resulting in larger seepage resistance and lower productivity index of horizontal wells.

Table 3 Prediction results of horizontal well productivity index in Weizhou 11-1 oilfield

Well name	Actual productivity index J_h , $\text{m}^3/(\text{d MPa})$	Predicted productivity index J_h , $\text{m}^3/(\text{d MPa})$		
		Prediction parameter	Prediction results	Error, %
A23H	36.63	$b = 45.17$	34.23	−6.56
A8S2	53.73		57.67	7.34

The productivity formula proposed in this paper takes into account the influence range and drainage distance b of edge water in a heterogeneous reservoir with composite water aquifer. If the water is mainly at the bottom of oil well or the energy of bottom water aquifer is strong, water mainly flows in the vertical direction and the effective edge-water drainage distance b is small. If the water is mainly at the edge of oil well or the energy of edge water aquifer is strong, water mainly flows in the horizontal direction and the effective edge-water drainage distance b is big. In actual reservoir, due to the unclear recognitions about oil–water contact relationship and energy of composite edge-bottom water aquifer, the parameter b is difficult to determine. In the process of practical application, the drainage distance b of edge water can be fitted according to existing test data, and then it can be used as a known parameter to predict the productivity index of new wells. As shown in Table 2, according to test data of productivity index, the edge water drainage distances of well A13H, A14H and A22H are 27.5 m, 80 m and 28 m respectively, with an average of 45.17 m. The prediction errors of well A23H and A8S2 are −6.56% and 7.34% (Table 3). Compared with Table 1, the prediction accuracy is greatly improved.

Conclusions

1. Based on the seepage mechanics theory and potential superposition principle, considering the mutual interference between multi well sections, the productivity formula of horizontal well in heterogeneous reservoir with composite water aquifer is established. The new productivity formula considers the influence of reservoir heterogeneity, the quasi-linear flow of lateral edge water and the ellipsoidal flow of vertical bottom water. It can effectively determine the influence range and drainage distance of edge water in a heterogeneous reservoir with composite water aquifer.
2. Under homogeneous condition, the distribution of flow rate at both ends of horizontal well section is high but it is low in the middle. If the permeability from heel to toe increases first and then decreases, the high flow

rate at the end of horizontal well is restrained, resulting in the decrease of productivity index of the whole horizontal well section. If the permeability from heel to toe decreases first and then increases, the distribution of bigger flow rate at the end of horizontal well section is enhanced, resulting in the increase of productivity index of the whole horizontal well section.

3. In the reservoir with composite water aquifer, with the increase of edge water drainage distance, the productivity index of horizontal well first decreases sharply, then tends to be stable gradually. When the ratio of edge water drainage distance to offset from water oil contact is 0~10, the change of horizontal well productivity index is very sensitive. For the horizontal well in a reservoir with composite water aquifer, if the drainage distance of edge water is several times of offset from water oil contact, the process that edge water supplies energy to bottom water for oil displacement cannot be ignored.
4. According to the test data of horizontal wells in Weizhou 11-1 oilfield, the average edge water drainage distance of well A13H, A14H and A22H is 45.17 m. It is used to predict the productivity index of well A23H and A8S2, whose accuracy is greatly improved with errors of −6.56% and 7.34%. The method has good application prospect in the productivity prediction of horizontal well in similar reservoir.

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