ORIGINAL PAPER - PRODUCTION ENGINEERING

A new method to determine reasonable well pattern density in low-permeability gas reservoirs

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Abstract

The productivity evaluation of gas reservoirs is used to determine the optimal number of well pattern, which is an important part of the preparation of gas reservoir development plan. Using integral transformation and asymptotic analyses to solve the pseudo steady-state flow model in closed rectangular gas reservoir of constant thickness, analytic solution of pressure distribution of eccentric well can be obtained. This paper presents mathematical model of multiple wells producing from a closed gas reservoir by applying the principle of pressure superposition. The pseudo steady-state dimensionless production index is introduced to solve the multiple well mathematical models under the constant pressure, and the number of production wells and cumulative production in different production interval are obtained. The results of the calculation shows that the curve of the cumulative production and economic benefits have a maximum value when the producing time, gas price, and drilling costs are given, so it can determine the optimal number of wells and provide the basis for making gas reservoir development plan.

Keywords Low-permeability gas reservoirs · Pressure superposition principle · Dimensionless production index · Well pattern density · Pressure distribution

Introduction

At present, the supply and demand of oil and gas resources are increasing. Since 2014, the international oil price has fallen. Most of the domestic oil and gas field development have entered a stage of high water cut, and the development effect is not satisfactory. It is necessary to reorganize the well pattern or design infilling wells to enhance oil recovery (Bowers 1981). Due to the existence of non-linear seepage mechanism such as pressure-sensitive effect, the well spacing between the injection well and the production well can not be too large in low-permeability reservoirs, and generally need to do well pattern infilling (Zhang Fenglian and Yuting 2008). Well pattern infilling technology can effectively delay the decline of oil and gas well production. Without taking economic effectiveness into consideration, it is considered that the smaller the well pattern density is, the higher the oil recovery is. However, for each additional infilling well, the investment cost increases and the economic effectiveness decreases. It is not that the smaller the well pattern density is, the better the development benefit is. Choosing a reasonable well pattern density for oil and gas field development is the key to enhance oil recovery and reduce investment costs. How to determine a reasonable well pattern density and obtain maximum economic benefits quickly and accurately becomes an urgent problem to be solved in the development of the oil and gas field.

Since the 1930s, many scholars have begun to study the density of well patterns, and they also have made great progress. There are many methods to calculate the density of well patterns, such as reasonable oil recovery method (Hongbing and Yang 2013; Cunyou and Haibo 2010; Hongge 2014; Jinshan 2013; Song et al. 2014), single well productivity method (Xie and Hongwei 2010; Mengkun et al. 2012; Awotunde 2014; Krisanne et al. 2011), water drive control degree method (Changmin and Xiaoqing 2012; Quanlin et al. 2012), numerical simulation method (John 2010, 2011), and physical simulation method (Lalehrokh

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and Bouma 2014; Yikun and Liang 2010). The above methods mainly have the following shortcomings:

1. The seepage model uses special functions.
2. The derivation and calculation are cumbersome.
3. The well pattern evaluation method is more suitable for the small well group, which is inconsistent with the actual well pattern.
4. The various modeling methods are relatively independent, and the formula expansion is poor.

Therefore, this paper starts from the governing equation of seepage mechanics, and considers the pressure-sensitive effect of low-permeability gas reservoir, and introduces the dimensionless transformation to transform the nonlinear seepage equation into an easy seepage equation. The mathematical model of the unstable seepage flow is solved by means of dimensionless transformation, Laplace transformation, Fourier cosine transformation, and separation variable method. Then, the asymptotic analysis method is used to give the single well pressure approximate solution at any time and any position. Finally, the superposition principle is used to solve the multi-well pressure. The solution model to pressure and average pressure is obtained. The net present value method is used to establish the relationship function between the number of production wells and economic benefits. The extreme value method of multivariate is used to establish the model of well pattern density.

Single well model

The constant flow model

The physical model of low-permeability rectangular gas reservoirs is shown in Fig. 1. To simplify the physical model, the assumptions are made as the followings:

1. The fluid in the reservoir is slightly compressible.
2. The gas reservoir is heterogeneous, but the permeability in the x and y direction at the same position is the same.
3. The flow of fluid in the gas reservoir has a pressure-sensitive effect.
4. The gas density is constant.

The length, width, and height of the rectangular gas reservoir are x_e, y_e, h, and the porosity is φ. The initial viscosity is μ_{gi}. The fluid flow is unstable in the rectangular gas reservoir. The rectangular coordinate system is established, and one well is located at (x_w, y_w). The production well is produced at a flow rate q_{sc}. The gas seepage control equation considering the pressure-sensitive effect is (Jian et al. 2013; Wu 2012):
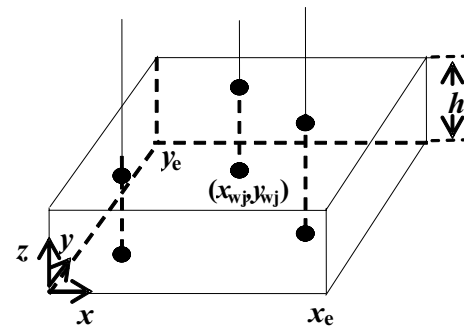
Fig. 1 The physical model of gas reservoir

$$\frac{\partial\left[\rho \frac{K_{gi}e^{-\alpha(p_i-p)}}{\mu_{gi}}\frac{\partial p}{\partial x}\right]}{\partial x} + \frac{\partial\left[\rho \frac{K_{gi}e^{-\alpha(p_i-p)}}{\mu_{gi}}\frac{\partial p}{\partial y}\right]}{\partial y} = \frac{\partial(\rho\varphi)}{\partial t}, \tag{1}$$

where k_{gi} is the original permeability of the reservoir, μm^2. ρ is the reservoir fluid density, g/cm^3. t is the producing time, h. α is the stress sensitivity coefficient, MPa^{-1}. μ_{gi} is the gas viscosity under the original reservoir pressure, mPa s. p_i is the original reservoir pressure, MPa. p is the reservoir fluid pressure, MPa.

Gas properties (such as viscosity, compressibility, compression factor, etc.) are related to gas pressure, which makes the Eq. (1) a nonlinear equation.

Define the pseudo-pressure function:

$$p_p(p) = 2\int_0^p \frac{\mu_{gi}Z_i}{p_i}\frac{\varepsilon e^{-\alpha(p_i-\varepsilon)}}{\mu_g(\varepsilon)Z(\varepsilon)}d\varepsilon,$$

where Z_i is the gas compression factor at the original reservoir pressure. $\mu_g(\varepsilon)$ is a function of gas viscosity, and it can be expressed by the gas pressure function. $Z(\varepsilon)$ is the function of the gas compression factor, and it can be expressed by the gas pressure function.

Substitute the pseudo-pressure function into Eq. (1), we can get:

$$\frac{\partial^2 p_p}{\partial x^2} + \frac{\partial^2 p_p}{\partial y^2} = \frac{1}{\eta(p_p)}\frac{\partial p_p}{\partial t}, \tag{2}$$

where $\eta(p_p) = \frac{K_{gi}}{\varphi\mu_g(p_p)c_g(p_p)}$.

Since $\eta(p_p)$ is still a function of $p_p(p)$, the Eq. (1) is not completely linearized by introducing the pseudo-pressure function. A lot of research have been done on how to linearize $\eta(p_p)$. For example, Kale and Mattar use the perturbation method to give the radial flow approximation solution under constant flow condition (Kale and Mattar 2004). Peres and Reynolds use the Boltzmann transformation method

to study the solution to the vertical well under the infinite atmospheric condition. However, these methods are suitable for self-similarity problems. There are many practical problems of other nonlinear well tests, and it does exist much error to analyze the pressure when using these methods. For the variable flow problem in well test analysis, the Duhamel convolution method is commonly used to solve the equation (Thompson and Reynolds 2009), but $\eta(p_p)$ is treated as a constant and it is mostly used for radial seepage. There are still some shortcomings for conventional two-dimensional seepage. This paper starts with the general 2-D gas reservoir and studies the variable yield and constant yield problems. Through the dimensionless transformation and integral transformation method, the exact solution of vertical well pressure at any position and at any time is obtained.

The dimensionless transformation is a method of converting the seepage equation into a conventional mathematical equation. By dimensionless transformation, the number of comparisons can be greatly reduced, which makes the mathematical physics equation simple, neat, easy to analyze and solve. The following dimensionless transformation is introduced in this paper:

$$p_{pD} = \frac{542.87 K_{gi} h[p_p(p_i) - p_p(p)]}{q_{sc} B_{gi} \mu_{gi}}, \quad x_D = \frac{x}{r_w}, \quad x_{eD} = \frac{x_e}{r_w},$$

$$y_D = \frac{y}{r_w}, \quad y_{eD} = \frac{y_e}{r_w}, \quad t_D = \frac{3.6 K_{gi} t}{\varphi \mu_{gi} c_{gi} r_w^2},$$

where q_{sc} is the production, m^3/d. B_{gi} is the gas volume coefficient under the original reservoir pressure. c_{gi} is the gas compression coefficient under the original reservoir pressure, MPa^{-1}. r_w is the radius of the production wellbore, m. $p_p(p)$ is the pseudo-pressure function, MPa.

Through the dimensionless transformation, the Eq. (2) can be written as follows:

$$\frac{\partial^2 p_{pD}}{\partial x_D^2} + \frac{\partial^2 p_{pD}}{\partial y_D^2} = \frac{\partial p_{pD}}{\partial t_D}. \tag{3}$$

Initial conditions:

$$p_{pD}(x_D, y_D, 0) = 0, \lim_{r_D \to 1}\left[r_D \frac{\partial p_{pD}(x_D, y_D, t_D)}{\partial r_D} \right]$$
$$= -1, \quad r_D^2 = (x_D - x_{wD})^2 + (y_D - y_{wD})^2. \tag{4}$$

Boundary conditions:

$$\frac{\partial p_{pD}(0, y_D, t_D)}{\partial x_D} = 0, \frac{\partial p_{pD}(x_{eD}, y_D, t_D)}{\partial x_D} = 0, \tag{5}$$

$$\frac{\partial p_{pD}(x_D, 0, t_D)}{\partial y_D} = 0, \frac{\partial p_{pD}(x_D, y_{eD}, t_D)}{\partial y_D} = 0, \tag{6}$$

Using Laplace transformation, the Eq. (3) can be written as follows:

$$\frac{\partial^2 \tilde{P}_{pD}}{\partial x_D^2} + \frac{\partial^2 \tilde{P}_{pD}}{\partial x_y^2} = s\tilde{P}_{pD}. \tag{7}$$

Using Fourier cosine transformation of x_D and y_D, the Eq. (7) can be written as follows:

$$\overline{\overline{\tilde{P}}}_{pD}(u_n, v_m, s) = \frac{\cos(u_n x_{eD})\cos(v_m y_{eD})}{s + u_n^2 + v_m^2}, \tag{8}$$

where $u_n = \frac{n\pi}{x_{eD}}$, $v_m = \frac{m\pi}{y_{eD}}$ (m, n is natural number).

Using Fourier cosine inverse transformation of u_n and v_m, the Eq. (8) can be written as follows:

$$\tilde{P}_{pD}(x_D, y_D, s) = \sum_{n=1}^{\infty} \frac{2\cos(u_n x_D)\cos(u_n x_{eD})}{s + u_n^2}$$
$$\times \sum_{m=1}^{\infty} \frac{2\cos(v_m y_D)\cos(v_m y_{eD})}{s + u_n^2 + v_m^2}. \tag{9}$$

By introducing the equations:

$$\sum_{k=1}^{\infty} \frac{\cos kx}{k^2 + a^2} = \frac{\pi}{2a}\frac{\cosh a(\pi - x)}{\sinh a\pi} - \frac{1}{2a^2},$$

The Eq. (9) can be written as follows:

$$s\tilde{P}_{pD}(x_D, y_D, s) = \frac{\pi}{x_{eD}}\frac{\cosh \varepsilon_0[y_{eD} - |y_D \pm y_{wD}|]}{\varepsilon_0 \sinh y_{eD}\varepsilon_0}$$
$$+ \frac{2\pi}{x_{eD}} \times \sum_{n=1}^{\infty} \cos\frac{n\pi x_{wD}}{x_{eD}} \cos\frac{n\pi x_D}{x_{eD}} \tag{10}$$
$$\times \frac{\cosh \varepsilon_n(y_{eD} - |y_D \pm y_{wD}|)}{\varepsilon_n \sinh y_{eD}\varepsilon_n},$$

where $\varepsilon_n = \sqrt{s + n^2\pi^2/x_{eD}^2}$.

When t_D is large enough, the dimensionless pressure distribution is obtained by asymptotic analysis:

$$P_{pD}(x_D, y_D, t_D) = \frac{2\pi t_D}{x_{eD} y_{eD}}$$
$$+ 2\pi \frac{y_{eD}}{x_{eD}} \left(\frac{1}{3} - \frac{|y_D \pm y_{wD}|}{2y_{eD}} + \frac{y_D^2 + y_{wD}^2}{2y_{eD}^2} \right)$$
$$+ 2 \sum_{k=1}^{\infty} \frac{1}{k} \frac{\cosh k\pi (y_{eD} - |y_D \pm y_{wD}|)/x_{eD}}{\sinh(k\pi y_{eD}/x_{eD})}$$
$$\times \cos \frac{k\pi x_D}{x_{eD}} \cos \frac{k\pi x_{wD}}{x_{eD}}, \tag{11}$$

The Eq. (11) can be written in dimensional form:

$$P_{pi} - P_p(x, y, t) = \frac{q_{sc} B_{gi} \mu_{gi}}{542.87 k_g h} \left(\frac{2\pi}{x_e y_e} \frac{k_g}{\varphi c_{gi} \mu_{gi}} + F \right), \tag{12}$$

where influence function F is

$$F[x, y, x_w, y_w, x_e, y_e] = 2\pi \frac{y_e}{x_e} \times \left(\frac{1}{3} - \frac{|y \pm y_w|}{2y_e} + \frac{y^2 + y_w^2}{2y_e^2} \right)$$
$$+ 2 \sum_{k=1}^{\infty} \frac{1}{k} \frac{\cosh k\pi (y_e - |y \pm y_w|)/x_e}{\sinh(k\pi y_e/x_e)}$$
$$\times \cos \frac{k\pi x}{x_e} \cos \frac{k\pi x_w}{x_e}. \tag{13}$$

The variable flow model

When the well production changes, the variable flow model of the low-permeability gas reservoir is obtained, with taking the skin factor into consideration.

$$P_{pi} - P_p(x, y, t) = \frac{B_{gi} \mu_{gi}}{542.87 K_g h}$$
$$\times \left[\frac{2\pi}{x_e y_e} \frac{K_g}{\varphi \mu_{gi} c_{gi}} \int_0^t q_{sc}(t) dt + (F + s) q_{sc}(t) \right] \tag{14}$$

According to the conservation of quality, we can get

$$P_{pi} - \bar{P}_p = \frac{B_{gi} \mu_{gi}}{542.87 K_g h} \frac{2\pi}{x_e y_e} \frac{K_g}{\varphi \mu_{gi} c_{gi}} \int_0^t q_{sc}(\tau) \partial\tau. \tag{15}$$

Eliminate the time term by the combination of Eqs. (14) and (15), and introduce the average pressure, we can get

$$\bar{P}_p - P_p(x, y, t) = \frac{\alpha_1 B_{gi} \mu_{gi}}{2\pi K_g h} q_{sc}(t) \times [s + F(x, y, x_w, y_w, x_e, y_e)], \tag{16}$$

where α_1 is the unit conversion factor, and in the SI unit system (Chen 1988), α_1 is 1.1574×10^{-2}.

Multi-well model

Transient production

When there are n wells producing at the same time, according to the pressure drop superposition principle [superimpose the Eq. (16)], we can get

$$\bar{P}_p - P_p(x, y, t) = \frac{\alpha_1 B_{gi} \mu_{gi}}{2\pi K_g h} \sum_{j=1}^{n} q_{scj}(t) \times [F_j(x, y, x_{wj}, y_{wj}, x_e, y_e) + s_j]. \tag{17}$$

Rewrite the Eq. (17) in a matrix form as

$$\vec{d} = \frac{\alpha_1 B_{gi} \mu_{gi}}{2\pi K_g h} (F + D_s) \vec{q}_{sc}. \tag{18}$$

There are two basic vectors in the Eq. (18): the pressure drop vector $\vec{d}$ and the production vector $\vec{q}_{sc}$.

$$\vec{d} = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_n \end{bmatrix} = \begin{bmatrix} \bar{P}_p - P_{pwf,1} \\ \bar{P}_p - P_{pwf,2} \\ \vdots \\ \bar{P}_p - P_{pwf,n} \end{bmatrix}, \quad \vec{q}_{sc} = \begin{bmatrix} q_{sc1} \\ q_{sc2} \\ \vdots \\ q_{scn} \end{bmatrix}. \tag{19}$$

The influence function matrix F and the diagonal matrix D_s are shown in the Eq. (20).

$$F = \begin{bmatrix} F_{11} & F_{12} \cdots F_{1n} \\ F_{21} & F_{22} \cdots F_{2n} \\ \vdots \\ F_{n1} & F_{n2} \cdots F_{nn} \end{bmatrix}, \quad D_s = \begin{bmatrix} s_1 & 0 \cdots 0 \\ 0 & s_2 \cdots 0 \\ \vdots \\ 0 & 0 \cdots s_n \end{bmatrix}. \tag{20}$$

Rewrite the Eq. (18), we can get

$$\vec{q}_{sc} = \frac{2\pi K_g h}{\alpha_1 B_{gi} \mu_{gi}} (F + D_s)^{-1} \vec{d}. \tag{21}$$

Define the quasi-steady stateless dimension production index:

$$J_{Dj} = \frac{\alpha_1 q_{scj}(t) B_{gi} \mu_{gi}}{2\pi K_g h [\bar{P}_p(t) - p_{pwf}]}. \tag{22}$$

Combining the Eqs. (21) and (22), we can get

$$\begin{cases} 1 = J_{D1}\left[F_1(y_{w1}) + s_1\right] + J_{D2}F_2(y_{w1}) + \cdots + J_{DN}F_N(y_{w1}) \\ 1 = J_{D1}F_1(y_{w2}) + J_{D2}\left[F_2(y_{w2}) + s_2\right] + \cdots + J_{DN}F_N(y_{w2}) \\ \cdots \\ 1 = J_{D1}F_1(y_{wn}) + J_{D2}F_2(y_{wn}) + \cdots + J_{DN}\left[F_n(y_{wn}) + s_n\right] \end{cases}.$$

(23)

By combining the Eqs. (13) and (23), and by following the operation of 'three step method', the transient productivity of the gas reservoirs can be obtained.

The 'three step method' is present as the following:

Step 1 The required data, such as reservoir size and well location coordinates are collected, and then according to the Eq. (13), the influence function can be obtained.

Step 2 The Eq. (23) is solved by Gauss method, and the dimensionless production index of each well is obtained.

Step 3 By substitute the dimensionless production index of each well into the Eq. (22), the production of each well $f(n, q_k)$ is obtained.

Whether it is a regular well pattern or an irregular well pattern, the production of the gas wells after well pattern infilling technology can be obtained.

Average production

In the actual gas reservoir development, it is often required to take the average production of a single well for a period of time, which is used for gas well dynamic analysis and production evaluation. Therefore, it is necessary to give a calculation model of average production by being based on formula (23).

The average pressure drop rate is

$$\frac{d\bar{P}_p}{dt} = -\frac{\alpha_2 \sum_{j=1}^{n} q_{scj}(t)B_{gi}}{Ah\varphi c_{gi}},$$

(24)

where α_2 is the unit conversion factor, and in the SI unit system (Chen 1988), α_2 is 0.04167.

Substitute the Eq. (22) in (24), we can get

$$\frac{d\bar{P}_p}{dt} = -c_1\bar{P}_p + c_2,$$

(25)

where

$$c_1 = \frac{\alpha_2 2\pi K_g \sum_{j=1}^{n} J_{Dj}}{\alpha_1 A\varphi c_{gi}\mu_{gi}}, \quad c_2 = \frac{\alpha_2 2\pi K_g \sum_{j=1}^{n} J_{Dj}P_{pwf}}{\alpha_1 A\varphi c_{gi}\mu_{gi}}.$$

(26)

Solve the Eq. (26), and the average pressure is obtained as the following:

$$\bar{P}_p = \frac{c_2}{c_1} + \left(P_{pi} - \frac{c_2}{c_1}\right)e^{-c_1 t}.$$

(27)

By combining the Eqs. (22), (26) and (27), the average production $\bar{f}(n, q_k)$ can be obtained.

Well pattern density determination

Due to the low natural productivity of low-permeability gas reservoirs and the large seepage resistance of reservoir fluids, the reasonable well pattern density is the key to the success of developing low-permeability gas fields. The economic benefits, cumulative production, and the number of wells have optimal values by considering economic factors, such as gas prices and drilling and completion costs. Through these optimal values, the reasonable well pattern density and limit well pattern density can be determined.

Here, take the average production $\bar{f}(n, q_k)$ as an example to show how to determine the well pattern density.

When there are n wells producing in the gas reservoir at the same time, the total sales income V_1 can be obtained by the Eq. (28).

$$V_1 = Ct\sum_{k=1}^{n}\bar{f}(n, q_k)[1 + (1+j) + (1+j)^2 \\ + \cdots (1+j)^{t-1}] = Ct\sum_{k=1}^{n}\bar{f}(n, q_k)\left[\frac{(1+j)^t - 1}{j}\right],$$

(28)

where C is the price of natural gas, yuan/ton. j is the discount rate, decimal.

The total development investment V_2 can be obtained by the Eq. (29).

$$V_2 = nM[1 + (1+j)j + (1+j)^2 j + \cdots + (1+j)^{t-2}j] = nM(1+j)^{t-1},$$

(29)

where M is the total investment for each well, yuan/well.

The total cost of development and maintenance management V_3 can be obtained by the Eq. (30).

$$V_3 = nG[1 + (1+j) + (1+j)^2 + \ldots + (1+j)^{t-1}] = nG[\frac{(1+j)^t - 1}{j}],$$

(30)

where G is the single well maintenance management fee, yuan / ton.

The net income V can be obtained by the Eq. (31).

$$V = V_1 - V_2 - V_3$$

(31)

when $V = 0$, the limit well number n_{jx} under the current conditions is obtained, and the ultimate well pattern density is obtained.

When $dV/dn = 0$, that is to solve the Eq. (32)

$$Ct\sum_{k=1}^{n}\bar{f}'(n, q_k)\left[\frac{(1+j)^t - 1}{j}\right] = M(1+j)^{t-1} + G\left[\frac{(1+j)^t - 1}{j}\right].$$
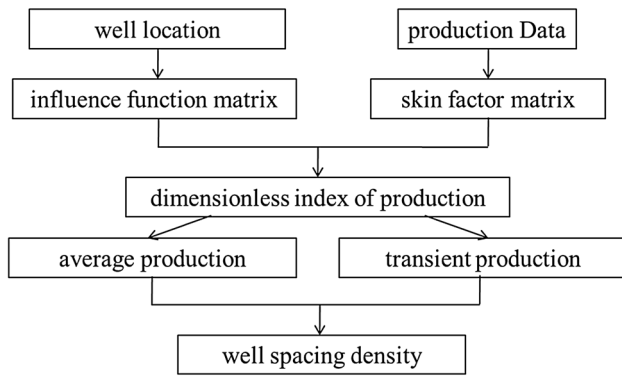
(32)

Fig. 2 The calculation diagram of the well spacing density

The reasonable number of wells n_{hl} under current conditions can be obtained, and the reasonable well pattern density can be obtained.

The Eq. (32) is a method for determining the well pattern density, which is based on the dynamic analysis of gas wells in low-permeability gas reservoirs. Using this method, the number of production wells can be adjusted according to the production dynamics of low-permeability gas reservoirs, which can realize efficient and stable development of low-permeability gas reservoirs. The specific calculation steps are shown in Fig. 2.

According to Serkachev's formula (Yikun and Liang 2010), there is a certain relationship between well pattern density and gas recovery. Being based on the dynamic production rate of single well, the gas recovery can be obtained by the Eq. (33)

$$E_R = \frac{t \sum_{k=1}^{n} \overline{f}(n, q_k)}{N} \times 100\%, \tag{33}$$

where E_R is the gas recovery, %. N is the geological reserves, m^3.

Through the formula (33), the gas recovery rate under the economical or reasonable well pattern density can be obtained.

Table 2 The Coordinates of well position and skin factor

Time	Coordinates	Skin factor
$t = 0$ d	$(x_1, y_1) = (896.13$ m$, 1407.96$ m$)$	$s_1 = 9.1$
	$(x_2, y_2) = (2816.37$ m$, 1408.14$ m$)$	$s_2 = 0.3$
$t = 100$ d	$(x_3, y_3) = (2048.23$ m$, 640.08$ m$)$	$s_3 = -2.7$

Case analysis

The temperature of a rectangular sealed gas reservoir remains unchanged ($T = 361$K). The reservoir and fluid characterisics are shown in Table 1.

In the initial stage, there were two production wells. When the production time was 100 days, an infill well was added. The well parameters are shown in Table 2.

When $t = 100$ days, the influence function matrix and the skin factor matrix are shown as below.

$$F = \begin{bmatrix} 9.61877 & -1.13693 \\ -1.13693 & 9.41553 \end{bmatrix}, \quad D_s = \begin{bmatrix} 9.1 & 0 \\ 0 & 0.3 \end{bmatrix}. \tag{34}$$

Substitute (34) in (21), the dimensionless production index can be obtained.

$$J_{D1} = 0.052551, \quad J_{D2} = 0.093237.$$

When $t = 100$ days, the average reservoir pressure is 7.07 MPa, and using Eq. (19), the production can be obtained, shown as below:

$$q_1 = 4.18 \times 10^6 \text{m}^3/\text{day}, \quad q_2 = 7.82 \times 10^6 \text{m}^3/\text{day}.$$

When $t = 500$ days, the influence function matrix and the skin factor matrix are shown as below.

$$F = \begin{bmatrix} 9.61877 & -1.13693 & -0.44266 \\ -1.13693 & 9.41553 & -0.05773 \\ -0.44261 & -0.05769 & 8.64801 \end{bmatrix},$$

$$D_s = \begin{bmatrix} 9.1 & 0 & 0 \\ 0 & 0.3 & 0 \\ 0 & 0 & -2.7 \end{bmatrix}. \tag{35}$$

Table 1 Gas reservoir parameters for the example 1

| Parameter	Values	Parameter	Values
Reservoir length	$x_e = 4061.6$ m	Volume factor	$B_{gi} = 0.0053$
Reservoir width	$y_e = 1020.2$ m	Compression coefficient	$c_{gi} = 0.0413$ MPa^{-1}
Reservoir height	$h = 11.43$ m	Wellbore radius	$r_w = 0.0831$ m
Temperature	$T = 361$K	Original pressure	$p_i = 18.6201$ MPa
Permeability	$k_{gi} = 0.11 \mu m^2$	Bottom hole flowing pressure	$p_{wf} = 6.2051$ MPa
Porosity	$\varphi = 0.14$	Viscosity	$\mu_{gi} = 0.0241$cp
Gas saturation	$S_g = 51\%$	Effective area	$A = 2.03$ km^2
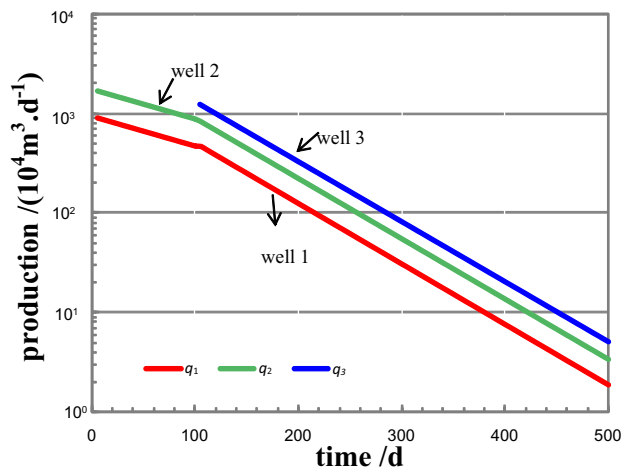
Fig. 3 The curves of flow rate versus time

Substitute (35) in (21), the dimensionless production index can be obtained.

$$J_{D1} = 0.052551, \quad J_{D2} = 0.093237, \quad J_{D3} = 0.14099.$$

When $t = 500$ days, the average reservoir pressure is 6.36 MPa, and using Eq. (19), the production can be obtained, shown as below:

$$q_1 = 1.7038 \times 10^4 m^3/day, \quad q_2 = 3.0229 \times 10^4 m^3/day,$$
$$q_3 = 4.5713 \times 10^4 m^3/day.$$

In the same way, the other time step production can be obtained, as shown in Fig. 3.

Figure 3 shows that when the production time is 100 days, the production deceleration rate of well 1 and well 2

changes, and the deceleration rate increases, which is caused by the mutual influence of pressure as the infill (well 3) is producing. In the quasi-steady state, if there is no other infill well, the production deceleration rate is a fixed value.

The pressure distribution at 100 and 500 days is shown in Fig. 4. Figure 4 shows that the new infill well will aggravate formation energy depletion, and it is necessary to replenish energy to the formation under the premise of ensuring production.

Sensitivity analysis

Production

Through calculation, the relationship between production (q), cumulative production (Q), and the number of wells (n) is obtained, as shown in Fig. 5.

Figure 5 shows that as the number of wells increases, the production first increases and then decreases. When the number of wells is the same, the initial production increases rapidly, and after reaching the peak value, the production is slowly reduced. This makes the cumulative production increase in the early stage. When the number of wells exceeds the number of reasonable production wells, the cumulative output gradually becomes flat. Therefore, there is a reasonable well pattern density (reasonable number of wells) in the development of the gas reservoir. When exceeding the reasonable number of wells, the cumulative production tends to be stable and the economic benefits reduce.

Gas recovery

Substitute the calculated cumulative production at different production time in Eq. (33), and the relationship between gas recovery and well pattern density can be obtained, as
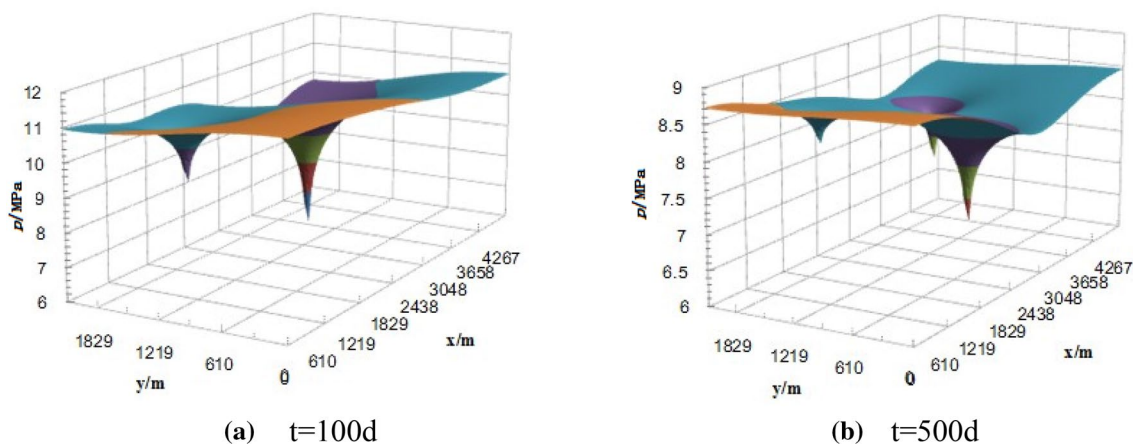

(a)   t=100d

(b)   t=500d

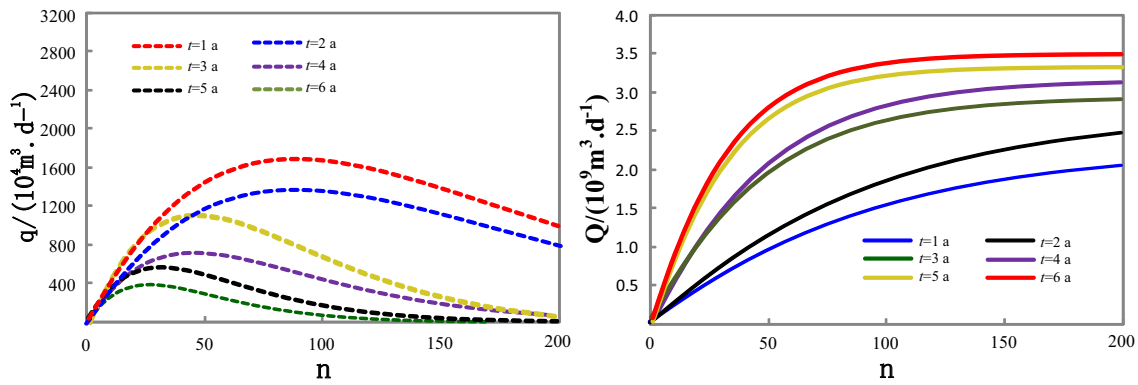**Fig. 4** The pressure distribution at 100 and 500 days

Fig. 5 The curves of flow rate and cumulative production versus well number

shown in Fig. 6. Figure 6 shows that as the density of the well pattern decreases, the gas recovery rate increases. When the well pattern density is 45–60 km²/well, the increase rate of the gas recovery is not obvious. Because the controllable reserves of a single well are certain. When the well pattern density exceeds the technical limit well density of a single well, it is usually not used to reduce the well spacing to improve gas recovery. When it is compared with the Sherka's empirical formula, the trend of this model in the paper is generally the same, except that the nodes and amplitudes of the curve rise are different.

Gas price

The gas price C is 1.0, 1.5, 2.0, 2.5, 3.0, 3.5 yuan/m³, and the reasonable number of wells in the low-permeability gas reservoir is calculated, as shown in Fig. 7. Figure 7 shows
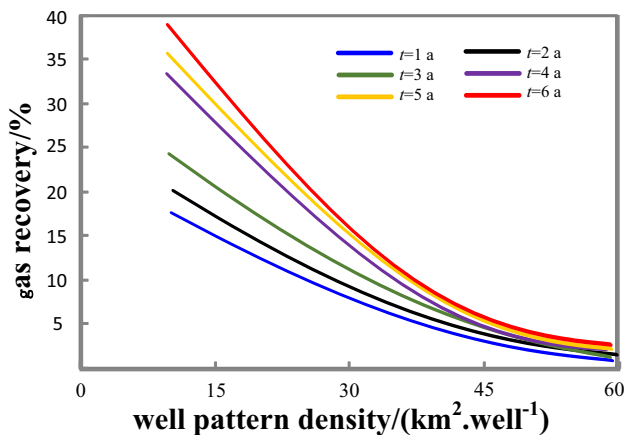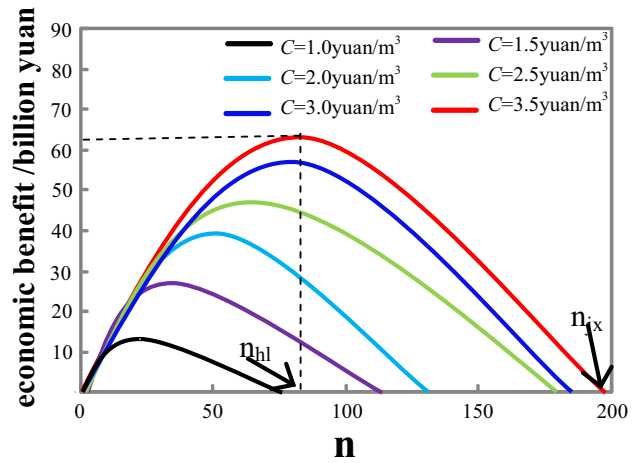

Fig. 7 The curves of cumulative production and economic benefits versus time

that when the gas price is 3.5 yuan/m³, the reasonable number of wells in the gas reservoir is 84, and the number of economic limit wells is 193. The calculated reasonable well pattern density is 20.28 well/km², and the economic limit well density is 46.61 well/km².

Physical parameter

By changing the effective thickness and the original permeability of the gas reservoir, the reasonable well pattern density under different physical conditions is studied, and a reasonable well pattern density is obtained, as shown in Fig. 8. Figure 8 shows that when the effective thickness of the reservoir is small, the reasonable well pattern density has a linear relationship with effective thickness. After the inflection point, the reasonable well pattern density increases
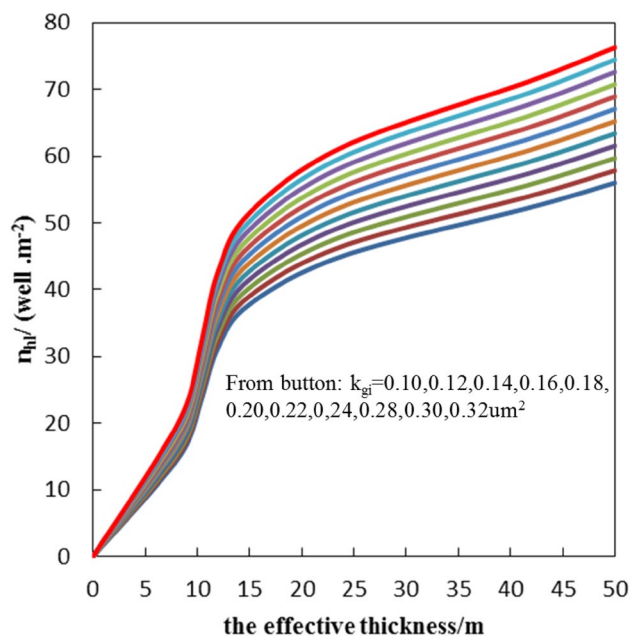

Fig. 6 The curves of gas recovery and well pattern density versus time

Fig. 8 The curve of reasonable well spacing density in different physical properties

fast first and then increases slowly. For the same reservoir thickness, as the original permeability increases, the reasonable well pattern density also increases.

Methods comparison

The method is compared with the empirical formula method (Cunyou and Haibo 2010), the net present value method (Mengkun et al. 2012), the numerical simulation method (Yikun and Liang 2010), and the actual data. The comparison results are shown in Table 3. Table 3 shows that the calculation results of this method in this paper and the numerical simulation method are very close, but this method is much closer to the actual data, which indicates that this method in this paper is reliable. The calculation results of the net present value method are generally lower than the method of this paper, mainly because the net present value method does not consider the influence of cross-well interference and skin factor on single well productivity, and ignores the ratio of injection–production wells to recovery. When the same recovery is achieved, more infill wells are

needed for the net present value method. In the middle and late stages of low-permeability gas reservoir development, due to the comprehensive production data, the calculation error of empirical formula method is reduced. From the perspective of economy and time saving, the method in this paper is the best solution to obtain reasonable well pattern density.

Conclusion

1. At present, there are two kinds of ways to determine the well pattern density, namely gas reservoir engineering and numerical simulation. The gas reservoir engineering method is mainly based on the empirical formula of Serkachev's formula, and it is not suitable for low-permeability gas reservoirs with low porosity and low permeability characteristics, and it does not consider the pressure-sensitive effect. The numerical simulation method needs to combine geological model, and it takes a long time to obtain the well pattern density. For the numerical simulation method, the corresponding model should be built based on the research area, which cannot be universal. In this paper, a new method to determine the economical and reasonable well pattern density based on dynamic production of low-permeability gas reservoirs is established. This method can adjust the number of production wells according to the production dynamics, and provide a theoretical basis for the reorganization of the old gas field well pattern and the design of the new gas field well pattern.

2. When the infilling wells are producing, the formation energy consumption increases, and the production deceleration rate of the old wells are different before and after well pattern infilling. To maintain the efficient and stable development of the low-permeability gas reservoir, energy must be added to the formation, such as water injection and gas injection.

3. With the number of production wells changing, the economic benefit and cumulative production have the maximum value. Under the same reservoir physical property, the reasonable well pattern density increases with the increase of natural gas price. Under the same natural gas price, the reasonable well pattern density increases

Table 3 comparison table of calculation results using different ways (km²/well)

Time/a	Empirical formula method	Net present value method	Numerical simulation method	This method in this paper	Actual data
1	32.61	18.64	21.47	20.28	20.01
3	25.43	16.29	19.86	18.72	18.91
5	18.03	15.05	17.92	17.26	17.19
7	16.99	14.82	16.63	16.05	16.11

with the increase of reservoir permeability and effective thickness, and decreases with the increase of recovery factor. The method is a simple and reliable method for determining the economical and reasonable well pattern density.

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