



# Reliability analysis for seismic capacity of casing strings

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## Abstract

Based on the analysis of seismic intensity and randomness, an earthquake intensity probability model is built, and the relationship between seismic intensity and seismic acceleration is presented. The distribution law of casing parameters, including outer diameter, wall thickness, Young's modulus and Poisson's ratio, is described by analyzing the randomness of casing geometry and mechanical properties. With the setting up a mechanical model of the casing strings in earthquakes, and using the spectral modal analysis theory and the equivalent static load method, the calculation method of the equivalent load of casing strings under seismic action is studied. The limit state equations of various failure forms of casing strings in earthquakes are established. Based on the theory of structural reliability, the reliability model of casing strings under seismic action is built. The seismic reliability of casing strings is calculated, and the calculation results are verified by field data. The calculation results show that influence of earthquake on safety and reliability of casing strings is very serious for long service life oil and gas wells in areas where the seismic fortification intensity is high.

**Keywords** Casing strings · Seismic · Randomness · Structural reliability

## Introduction

Casings play an important role in oil and gas production. With the passage of time, the possibility of casings failure increases, which has become an important factor restricting the sustainable development of many oil and gas fields in China and other places of the world.

Earthquake is one important cause for casings failure (Xu 1992). However, there have been little researches on the reaction of casing strings under seismic action by so far. In long service life oil and gas wells, the randomness of seismic action, and the randomness of pipe geometry and mechanical properties will be not negligible, and their influences on casings failure will also be not negligible. Therefore, seismic reliability evaluation is of important engineering value.

## Seismic intensity and randomness

Earthquake intensity and probability of occurrence are two basic parameters for an earthquake. The former is specified by the term of seismic intensity, while the latter is described by the seismic randomness. The seismic intensity and randomness have a strong correlation.

## Earthquake intensity probabilistic model

The stochastic probability distribution function of seismic intensity is subject to the law of extremum III distribution, as follows (Gao and Bao 1985):

$$F_t(I) = [f_T(I)]^{\frac{t}{T}} = \exp \left[ -\frac{t}{T} \left( \frac{12-I}{12-I_s} \right)^K \right] \quad (1)$$

Mean and variance is as follows (Dai and Wang 1987):

$$\mu_I = 12 - (12 - I_s) \Gamma(1 + 1/K) \quad (2)$$

$$\sigma_I = (12 - I_s) \sqrt{\Gamma(1 + 2/K) - \Gamma^2(1 + 1/K)} \quad (3)$$

where  $t$  stands for time, a (years);  $T$  for base period,  $T = 50a$ ;  $I$  for seismic intensity,  $I = 1, 2, 3, \dots, 12$ ; and  $I_s$  stands for seismic portrait intensity (Chen et al. 2016), which means the

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seismic intensity with exceeding possibility of 63.2% within 50a,  $I_s = I_0 - 1.55$ ;  $I_0$  stands for seismic fortification intensity. Seismic fortification intensity means the seismic intensity which is regarded as the basis of seismic fortification in a region according to relevant national regulations (Architecture Design Handbook Editorial Board 1994);  $K$  stands for shape factor.

Distribution parameters of each fortification intensity level in accordance with the literature (Gao and Bao 1985) are shown in Table 1.

For a better understanding about the relationship between fortification intensity and actual earthquake possibility, this paper gives the actual earthquake possibility in areas of different fortification intensity levels in 20 years, as is shown in Table 2.

It can be concluded that seismic fortification intensity is safe enough as a design basis (Tao et al. 2018). For example, for areas of fortification intensity of 6, the occurrence possibility of magnitude-6 earthquake is only 4.1268%, while the occurrence possibility of magnitude-7 earthquake is only 0.7%. In following discussion of this paper, seismic fortification intensity will be adopted as the design and calculation basis.

### The acceleration description of earthquake intensity

Seismic acceleration is used to describe the ground motion acceleration in earthquake. There is correspondence between seismic acceleration and seismic intensity which can be represented by fortification intensity. And among the acceleration in three directions, one of the accelerations in the horizontal direction is the maximum (Construction Department of P.R.C. 2001; Wu et al. 2002), so maximal horizontal acceleration is adopted as the design base seismic acceleration. According to Chinese National Construction Earthquake Fortification Criteria, the design value of maximal horizontal acceleration shall follow Table 3.

The conversion relation between earthquake intensity and maximal horizontal acceleration is also presented in the literature (Wu et al. 2002) according to theory of Liu et al. (1981), shown as follows:

$$A_{IH} = 0.81 \times 2^I \tag{4}$$

**Table 1** Seismic intensity distribution parameters

Seismic fortification intensity $I_0$	Shape factor $K$	Equalizing value $\mu_I$	Standard deviation $\sigma_I$	Coefficient of variation $C_I$
6	9.7933	4.8237	0.8806	0.1826
7	8.3339	5.8195	0.8826	0.1517
8	6.8714	6.8133	0.8866	0.1301
9	5.4029	7.8037	0.8954	0.1147

**Table 2** Possibility of earthquake in areas with different seismic fortification intensity levels

Seismic fortification intensity $I_0$	Magnitude			
	6 (%)	7 (%)	8 (%)	9 (%)
6	4.1268	0.7043	0.0794	0.0047
7	17.5175	4.1268	0.6541	0.0597
8	49.5133	17.7385	4.1267	0.5820
9	83.1875	48.6147	18.0791	4.128

where  $A_{IH}$  is the maximal horizontal acceleration at seismic intensity  $I$ ,  $\text{cm s}^{-2}$ . The calculation results are in accordance with Table 3.

Furthermore, the maximal vertical acceleration value is between 1/2 and 2/3 of the maximal horizontal acceleration value. 1/2 is used as the coefficient in the calculation of maximal vertical acceleration calculation.

$$A_{IV} = 0.5A_{IH} = 0.41 \times 2^I \tag{5}$$

$A_{IV}$  is the maximal vertical acceleration at seismic intensity of  $I$ ,  $\text{cm s}^{-2}$ .

### Randomness of casing dimension and mechanical parameters

The factors affecting the strength of the casing are mainly the physical dimensions and mechanical properties of the casing, including the outer diameter, wall thickness, elastic modulus and Poisson’s ratio of the casings. According to the statistical test of 2000 joints of API casings from five manufacturers (Adams et al. 1993), combined with the research of Galambos and Ravindra (1978), it is found that the above parameters are in accordance with the normal distribution, as is illustrated in Eq. 6, and the corresponding distribution parameters are shown in Table 4:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma_x} \exp\left[-\frac{(x - \mu_x)^2}{2\sigma_x^2}\right] \tag{6}$$

where  $x$  stands for random variables, which generally refers to actual physical dimension and mechanical property;  $\mu_x$  stands for mean value of random variable;  $\sigma_x$  stands for standard deviation of random variables.

**Table 3** Maximal horizontal acceleration at difference intensity

Seismic fortification intensity	6	7	8	9	10
Horizontal acceleration $\text{cm/s}^2$	50	100	200	400	800

**Table 4** Mean values and variation coefficients of casing parameters

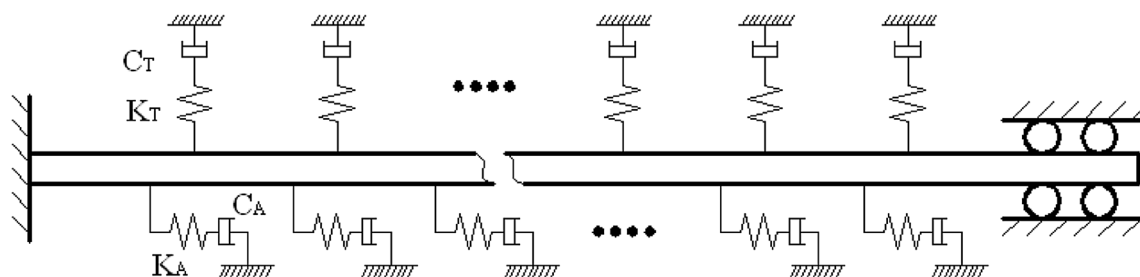
Parameters	Casing' outside diameter ( $D$ )	Wall thickness ( $t$ )	Yield strength ( $Y_p$ )	Elastic modulus ( $E$ )	Poisson's ratio ( $\nu$ )
Mean value/nominal value	1.005	1.00	1.09	1.00	1.00
Variation coefficient	0.0013	0.018	0.022	0.035	0.025

## Casing reaction calculation in earthquakes

### The mechanical model of casing strings in earthquakes

Since the axial dimension of the casing strings is much larger than the radial dimension, the casing string is regarded as a continuous structure. The casing head structure is negligible (Xue et al. 2000). The casing string is an axisymmetric structure, so it can be considered that the seismic impact does not apply torques on the casing strings. Due to the low gas density and low viscous drag, the effect of gas in the casings is negligible for gas wells. And because of the low flow rate in the casings of the oil wells, it can also be considered that the fluid influence in the casings of oil wells is negligible. There is the cementing force and friction between the cement rings and the casings, and these forces dampen the vibration of the casings. Meanwhile, taking into account the linear elastic properties of the cement rings and the adjacent formation rock within a certain range, the constraints of the cement rings and the formation rock on the casings can be simplified to an elastic-damping constraint. Since the seismic acceleration generally consists of acceleration in three directions—two horizontal directions and one vertical direction, there are three orthogonal seismic loads on casing strings in this mechanical model.

The casing strings mechanical model in earthquakes is shown in Fig. 1, where  $K_A$  and  $K_T$  stand for axial and transverse elastic stiffness of the constraint, and  $C_A$  and  $C_T$  stand for axial and transverse damping coefficients, respectively. It is obvious that the model is multi-particle and multi-degree of freedom model.

**Fig. 1** Casing strings mechanical model in earthquakes

### Calculation of casing strings equivalent stress in earthquakes

For the response of casing strings in earthquake, the most concerned issue shall be its maximum dynamic response, especially the maximum stress of casing strings caused by earthquakes.

For the vibrational mechanical model of casing strings with multiple degrees of freedom, the equivalent static stress calculation of casing strings under seismic action can be carried out according to the spectral modal analysis theory and seismic equivalent static load (Shen et al. 2000).

The  $j$ th order force exerted by earthquakes on casing strings structure can be expressed as:

$$\{F\}_j = [M]\{\Phi\}_j \gamma_j A_a(\omega_j, C_j, \xi_j) \quad j = 1, 2, \dots, N \quad (7)$$

where  $\{F\}_j$  stands for the  $j$ th order force of the maximal vibration mode;  $[M]$  stands for mass matrix of casing strings structure;  $\{\Phi\}_j$  stands for the  $j$ th order vibration of the casing strings structure;  $\gamma_j$  stands for the  $j$ th order mode participation coefficient;  $\omega_j$  stands for the  $j$ th order of natural frequency of the casing strings;  $C_j$  stands for the  $j$ th order elastic stiffness coefficient of casing constraint, corresponding with  $K_A$  and  $K_T$  in Fig. 1;  $\xi_j$  stands for the  $j$ th damping of casing constraint;  $A_a(\omega_j, C_j, \xi_j)$  stands for the  $j$ th seismic acceleration response spectrum. The calculation of acceleration response spectrum and its parameters is detailed as follows (Fu 1990):

$$A_a(\omega_j, C_j, \xi_j) = A(\omega) [H(\omega_j, C_j, \xi_j)] \quad (8)$$

where  $[H(\omega_j, C_j, \xi_j)]$  stands for the  $j$ th order transfer function array of casing strings structure;  $A(\omega)$  stands for the seismic acceleration  $j$ th order response spectrum. The calculations of natural frequency, mode of vibration, mode of vibration participation coefficient and transfer function array have been introduced by many studies (Gao et al. 1995; Shuai et al. 1999).

Seismic response stress of various modes in the casing strings  $\{S\}_j$  and corresponding equivalent stress  $S$  can be calculated by statics methods:

$$[K]\{S\}_j = \{F\}_j \quad (9)$$

$$S = \sqrt{\sum_{i=1}^N \sum_{j=1}^N \rho_{o,ij} S_i S_j} \quad (10)$$

where  $[K]$  stands for stiffness matrix of casing strings structure;  $S$ ,  $S_i$  and  $S_j$  stand for components of  $\{S\}$ ,  $\{S\}_i$  and  $\{S\}_j$  respectively;  $\rho_{o,ij}$  stands for mode of vibration cross-correlation function. Proximate solution is as follows (Shen et al. 2000):

$$\rho_{o,ij} = \frac{8(\omega_i \xi_i + \omega_j \xi_j)(\omega_i \omega_j)^{3/2} (\xi_i \xi_j)^{1/2}}{2(\omega_i \xi_i + \omega_j \xi_j)^2 (\omega_i^2 + \omega_j^2) + (\omega_i^2 - \omega_j^2)} \quad (11)$$

The casing maximal axial load, transverse load and equivalent stress in earthquake effect can be solved by this method.

## Seismic reliability evaluation of casing strings

### The reliability model of casing strings under seismic action

Under seismic action, the failure modes of the casing include strength failure, axial elastic instability and lateral deformation (He et al. 2018). The limit state equations are as follows.

1. The casing strings strength failure.

$$Z_S = Y_p - S = 0 \quad (12)$$

2. The casing strings axial elastic instability.

$$Z_A = F_{LJ} - F_{AY} = 0 \quad (13)$$

3. The casing strings lateral deformation.

$$Z_f = f_{lim} - f = 0 \quad (14)$$

where  $Y_p$  stands for casing yield strength, MPa;  $F_{LJ}$  stands for the critical load of the casing strings axial elastic instability, KN;  $F_{AY}$  stands for the actual axial

load of the casing strings, KN;  $f_{lim}$  stands for casing strings allowable lateral deformation mm;  $f$  stands for the actual lateral deformation of the casing strings, mm.

Based on seismic fortification intensity for the region, the probability of failure in multiple forms of casing strings in earthquakes within 1 year can be expressed as:

$$P_{fn} = \text{MAX}_{j=1}^{12} \left\{ P_{fn}^j (Z_n < 0) \cdot P_t(I_j) \right\} \quad (15)$$

where  $P_{fn}$  stands for casing strings failure probability in the form of  $n$ ;  $P_{fn}^j (Z_n < 0)$  stands for casing strings failure probability in the form of  $n$  in earthquake of magnitude  $j$ ;  $Z_n$  generally refers to  $Z_S$ ,  $Z_A$  and  $Z_f$  in formulation (12) to (14), standing for casing strings failure form;  $P_t(I_j)$  stands for the occurrence probability of earthquakes exceeding magnitude  $I_j$  during  $t$  years;  $\text{MAX}\{\cdot\}$  stands for max function.

Since the casing strings are a typical series system, the casing strings will fail as long as one failure form occurs. The overall reliability of the casing strings during its service life is as follows (Liu 1996).

$$R = \prod_{n=1}^3 (1 - P_{fn}) \quad (16)$$

### Calculation example of casing string seismic reliability

The length of a production casing is 3000 m, the material is N-80, the outer diameter is 177.8 mm, and the wall thickness is 13.7 mm. The elastic modulus of cement ring and formation rock is 20 Gpa, and there is strong binding force between the two layers. The designed service life of oil well is 20 years. The seismic fortification intensity is 7. According to Housner and Jennings's advice (1964), the seismic acceleration frequency for hard ground conditions is 15.6 Hz, and the damping is 0.64. In this paper, the seismic acceleration frequency will be changed in the range of 0.1–20 Hz.

In the service life of the casing string, the failure probability in earthquakes of different magnitudes is calculated, and the results are shown in Table 5.

It can be seen from Table 5 that the overall seismic reliability of the casing strings is 99.19%. When the magnitude is below 6, the casing string has high reliability and the multi-form failure probability is low. When earthquake intensity is magnitude 9 or above, the casing string is almost destroyed. However, the low fortification intensity in this area indicates that the probability of a high-intensity earthquake is very low. Generally, the casing string still has high reliability and can meet the seismic requirements.

**Table 5** Casing string failure probability in earthquakes of different magnitudes during service life (%)

Casing failure form	Magnitude				Failure probability in the service life
	6	7	8	9	
a. Strength Failure	$9.12 \times 10^{-2}$	10.33	39.07	87.59	0.43
b. Axial Elastic Instability	$3.27 \times 10^{-4}$	3.70	42.35	94.94	0.03
c. Lateral deformation	0.17	8.47	50.01	99.96	0.35
Overall seismic reliability	99.74	79.04	17.56	<0.01	99.19

## Field data verification

There are very few statistical data of casing failure cases caused by earthquakes in the literature and reports. This paper analyzes one case data obtained after Wenchuan earthquake.

On May 12th, 2008, a magnitude 8.0 earthquake occurred in Wenchuan, Sichuan Province of China, causing nearly 80,000 deaths and huge property losses. In Sinopec Puguang Gas Field which is located in the seismic zone, some gas wells were found to have casing failure of varying degrees. Sinopec used caliper logging tool to detect casing failure in Puguang gas field. A total of 22 wells were checked, and casing failure was found in 20 wells (Ge et al. 2010). In this case, the ratio of casing failure wells to statistical wells shows that the seismic failure rate of casing in Wenchuan M8.0 earthquake is 90.9%, which is relatively close to the calculated failure possibility of 82.44% according to Table 5.

## Conclusions

In addition to the traditional calculation method for casing strength, this paper fully considers the randomness of earthquakes, as well as randomness of casing physical dimensions and mechanical properties. In order to analyze the load on the casing strings in earthquakes, a seismic reliability analysis for the casing strings of oil and gas wells based on reliability theory is carried out. And the conclusions are as below.

1. The occurrence possibility of failure inducing factors and corresponding probability of leading to failure should be comprehensively calculated in casing strings reliability analysis.
2. Other than traditional nominal strength and safety factor calculation, reliability theory is introduced into the seismic resistance analysis of casings. The seismic randomness and casing resistance are taken into consideration in the strength design. A more reasonable seismic reliability analysis method is established.
3. A sample calculation is carried out based on the proposed seismic reliability analysis method. Compared with the data of casing failure in Puguang gas field after

Wenchuan earthquake, the reliability of the calculation model is verified.

4. Though the casing strings are generally reliable in areas with low seismic fortification intensity, the seismic influence should still be seriously considered in the casing strings designing for long service life oil and gas wells, especially in areas with high seismic fortification intensity.

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