A mathematical determination of the pore size distribution and fractal dimension of a porous sample using spontaneous imbibition dynamics theory

Mumuni Amadu¹ · Michal J. Pegg¹

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Abstract

Wallace sandstone has been extensively used by the construction industry for a long time in Nova Scotia. Apart from oxide analysis and a few strength-related parameter data found on some websites, petrophysical data regarding pore-sized distribution and fractal dimension are lacking. In the petroleum engineering literature, the spontaneous imbibition dynamics mechanism has been modeled where imbibition time has been linked to imbibition rise. One of the models links imbibition time to imbibition rise through a group of parameters that integrate the fractal dimension and sediment tortuosity. Based on the assumption of a bundle of parallel capillary tubes model found in the petrophysical literature, we have used the spontaneous imbibition model to derive an equation that links fractal dimension to porosity and permeability. Using literature source data on Wallace sandstone core samples, we have calculated the fractal dimension and pore size distribution using our equation. Results show that this sandstone has a significant level of heterogeneity. Calculation using another literature source data shows that our equation calculates fractal dimensions that are closer to those reported for the capillary pressure method. Although the assumption of bundle of parallel capillary tubes leads to deviations in calculated fractal dimensions using literature source data with experimentally determined porosity, permeability and fractal dimensions, our equation calculates meaningful values of fractal dimensions.

Graphical abstract

Keywords Pore size distribution · Fractal dimension · Spontaneous imbibition · Tortuosity · Heterogeneity · Homogeneity

List of symbols

A_p	Cross-sectional area of pore for spontaneous imbibition, m^2
D_f	Fractal dimension
t	Time, s
z	Imbibition distance, m
ρ	Density of imbibing fluid, $kg\ m^{-3}$
μ	Dynamic viscosity of imbibing fluid, Pa.s
γ_{LV}	Liquid-vapor interfacial tension, Nm^{-1}
λ_{ind}	Pore size distribution index
$\lambda_{min}, \lambda_{max}$	Parameters used in connection with fractal dimension
ϕ	Porosity-fraction
θ	Contact angle-degrees
K	Absolute permeability, m^2

Introduction

The pore size distribution of a porous material such as rock or a ceramic material is an important hydraulic parameter that has a bearing on several physical and physiochemical processes. For instance, the pore size distribution has a direct bearing on the strength of concrete (Kumar and Bhattacharjee 2003) and as such has a direct bearing on its ultimate strength such that the more the proportion of smaller pore sizes, the higher the strength. In the geotechnical literature, the microstructure of sedimentary rocks has been reported to affect its index properties (Sabatakakis et al. 2008). Zorlu et al. (2008) have predicted the uniaxial compressive strength of sandstones using petrographic-based models (Zorlu et al. 2008). Besides, the pore size distribution has an effect on drying shrinkage of activated concrete (Collins 2000).

Apart from the strength-related aspects of the pore size distribution of a porous material, hydraulic properties such as absolute permeability, relative permeability and capillary pressure characteristics are also dependent on pore size distribution (Jerauld and Salter 1990; Bennion 2006). Consequently, empirical relative permeability and capillary pressure functions presented by Brook and Corey (Brooks and Corey 1964) contain the pore size distribution index, which is a statistical measure characterizing the distribution of pore size in a given sample of a porous material.

The impact of pore size distribution on seismic velocity structure of the earth has been reported. In the geophysical literature, the fractal dimension which has a bearing on the pore size distribution index (Li 2010) has been reported to impart bimodal distributions of seismic velocities in rocks (Holliger and Lavender 1994).

Being a macroscopic pore structure parameter, pore size distribution will have a direct impact on the moisture absorption capacity of a facing porous rock such as sandstone slabs

and, therefore, on its salt uptake and damage characteristics (Yu and Oguchi 2010). In Nova Scotia, Canada, Wallace sandstone has long been used as a building material, mostly as a facing stone. Consequently, knowledge of the pore size distribution characteristics of this sandstone is essential for the construction industry. What is more, with geological carbon storage being globally accepted as a technically and economically viable option for mitigating global warming (Wilson et al. 2008), knowledge of the effect of pore size distribution on the two-phase flow of anthropogenic carbon dioxide and formation brine is essential for future geological storage planning in Nova Scotia. Wallace sandstone is essentially glauconitic. However, while the glauconitic sandstones of Alberta have been studied (Bennion and Bachu 2008), Wallace sandstone has not yet received the desired attention from people in academia and the likes, apart from test data that was provided by the National Research Council of Nova Scotia (Council 1967). The mineral trapping potential of glauconitic sandstones in Alberta has been studied (Pruess et al. 2004). Therefore, the objective of the present study is to determine the pore size distribution index and the fractal dimension of this sandstone using spontaneous imbibition theory, as a first step towards getting an insight into the pore size distribution and fractal dimension of this geologic system. Different methods have been used for the determination of pore size distribution, ranging from nuclear magnetic resonance approach (Davies and Packer 1990) to capillary pressure methods (Pickell et al. 1996) (Pickell et al. 1996). In the present study, the theoretical basis of the spontaneous imbibition mechanism will be the preferred choice for the determination because of its simplicity.

Theory

Permeability, porosity and fractal dimension are among the macroscopic porous medium parameters (Dullien 2012). All these petrophysical parameters determine strength and hydraulic parameters of the porous medium and will, therefore, control spontaneous imbibition flow. In the literature, porosity has been linked to fractal dimension (Rieu and Sposito 1991). Absolute permeability has also been linked to fractal dimension (Yu and Cheng 2002). Thus, linking porosity and permeability to fractal dimension individually is an impetus for linking fractal dimension to porosity and permeability in a single equation. Therefore, the mathematical description of any hydraulic process of the porous medium that integrates porosity, permeability and fractal dimension in the desired combination can be exploited for developing such an equation.

Spontaneous imbibition is the spontaneous flow of a wetting fluid into a porous medium under capillary pressure

gradient resulting from the combined effects of interfacial energies, pore structure and viscous forces.

In accordance with viscous non-compressible liquid flow along a cylindrical capillary tube, the application of Newton's dynamics gives (Zhud et al. 2000):

$$\rho [zz'' + (z')^2] = \frac{2}{r} \gamma \cos \theta - \mu \frac{8}{r^2} \cos \theta * zz' - \rho g z \quad (1)$$

In this equation, ρ is liquid density [kg m^{-3}], z is the distance advanced by the spontaneous imbibition front [m], z'' is the acceleration [ms^{-2}], $z' = dz/dt$ is the velocity of the imbibition front [ms^{-1}], θ is contact angle, γ interfacial tension between imbibing and resident fluid, r is radius of capillary tube [m], μ dynamic viscosity of imbibing fluid [N s m^{-2}] and g is acceleration due to gravity [ms^{-2}].

The short-term asymptotic solution to this equation is given as:

$$z(t) = \sqrt{\frac{r\gamma \cos \theta t}{2\mu}} \quad (t \rightarrow 0) \quad (2)$$

The bundle of parallel tube model has proven to be useful in analysis of hydraulic problems associated with porous media (Dullien 2012; Kewen 2004) and can be applied to the process. This theory regards a porous sample to consist of a bundle of parallel capillary tubes of different sizes. Consequently, while the cross-sectional area opened to imbibition in a single capillary tube is simple and easy to quantify, the area opened to imbibition in a porous sample is given as (reference 40 of Cai et al. 2010):

$$A_p = \frac{\pi \lambda_{\max}^2 D_f}{4(2 - D_f)} (1 - \phi) \quad (3)$$

In this equation, A_p is area opened to imbibition flow, $\lambda_{\max}$ is maximum pore diameter, D_f is fractal dimension of sediment and ϕ is porosity.

By considering the integral of imbibition volumes from minimum to maximum pore throat in addition to the tortuosity of pore throats and fractal dimension of the sediment, the distance imbibed versus time for early time periods is given as (Cai et al. 2010):

$$z^{2D_T} = \frac{(D_T + D_f - 1)(2 - D_f)}{D_f(2 + D_T - D_f)} \left(\frac{\gamma \cos \theta}{4\mu} \right) \left(\frac{\lambda_{\max}^{D_T}}{\lambda_{\min}^{1-D_T}} \right) \left(\frac{1 - \beta^{2+D_T-D_f}}{1 - \phi} \right) t \quad (4)$$

where D_T is tortuosity dimension of pore throat, and D_f is the fractal dimension of sediment.

$$\beta = \frac{\lambda_{\min}}{\lambda_{\max}}$$

To seek the equivalent of Eq. (1) for imbibition flow into a porous medium requires invoking the bundle of parallel capillaries in the porous sample. Thus, for a straight capillary tube, the parameter D_T is equal to 1 (Cai et al 2010). Thus, Eq. (9) becomes:

$$z^2 = \frac{(D_T + D_f - 1)(2 - D_f)}{D_f(2 + D_T - D_f)} \left(\frac{\gamma \cos \theta}{4\mu} \right) \left(\frac{\lambda_{\max}^{D_T}}{\lambda_{\min}^{1-D_T}} \right) \left(\frac{1 - \beta^{2+D_T-D_f}}{1 - \phi} \right) t \quad (5)$$

Equation 4, for imbibition through a porous sample is the direct equivalent of Eq. (1) for imbibition through a capillary tube of known radius. It is interesting to see that in all equations, the driving force related to contact angle, surface tension and viscous shear force are present. Referring to Eq. (2), the following can be written in light of this observation:

$$\frac{(D_T + D_f - 1)(2 - D_f)}{D_f(2 + D_T - D_f)} \left(\frac{\gamma \cos \theta}{4\mu} \right) \left(\frac{\lambda_{\max}^{D_T}}{\lambda_{\min}^{1-D_T}} \right)^{1/2} \left(\frac{1 - \beta^{2+D_T-D_f}}{1 - \phi} \right) \equiv \left(\frac{r\gamma \cos \theta}{2\mu} \right) \quad (6)$$

In this regard, all parameters on the left hand side except the contact angle, dynamic viscosity of imbibing fluid and interfacial tension account for an effective radius within the porous sample.

Equation 5 predicts a straight line between the square of imbibition rise and imbibition time. The gradient of the plot will be given as:

$$\frac{dz^2}{dt} = \frac{(D_T + D_f - 1)(2 - D_f)}{D_f(2 + D_T - D_f)} \left(\frac{\gamma \cos \theta}{4\mu} \right) \left(\frac{\lambda_{\max}^{D_T}}{\lambda_{\min}^{1-D_T}} \right) \left(\frac{1 - \beta^{2+D_T-D_f}}{1 - \phi} \right) \quad (7)$$

To exploit this for achieving the principal objective of this paper, an equivalent radius of the porous system used in Leveret J function will be defined as (Gostick et al. 2006):

$$r_e = \sqrt{\frac{K}{\phi}} \quad (8)$$

where r_e is equivalent radius, K is absolute permeability and ϕ is porosity-fraction.

Setting

$$\eta = \frac{(D_T + D_f - 1)(2 - D_f)}{D_f(2 + D_T - D_f)} \left(\frac{1}{4\mu} \right) \left(\frac{\lambda_{\max}^{D_T}}{\lambda_{\min}^{1-D_T}} \right) \left(\frac{1 - \beta^{2+D_T-D_f}}{1 - \phi} \right) \quad (9)$$

$$\frac{dz^2}{dt} = \eta\gamma \cos \theta \quad (10)$$

Equation (2) can be written in terms of the equivalent radius as:

$$z(t) = \sqrt{\frac{K}{\phi}} \sqrt{\frac{\gamma \cos \theta * t}{2\mu}} (t \rightarrow 0) \quad (11)$$

This gives the gradient of the plot as:

$$\frac{dz^2}{dt} = \sqrt{\frac{K}{\phi}} \frac{\gamma \cos \theta}{2\mu} \quad (12)$$

From this, the contact angle for the porous medium is calculated as:

$$\cos \theta = \left(\frac{K}{\phi}\right)^{-0.5} \frac{2\mu}{\gamma} \frac{d(z^2)}{dt} \quad (13)$$

Substitution into Eq. (10) gives:

$$\frac{dz^2}{dt} = \eta\gamma \left(\frac{K}{\phi}\right)^{-0.5} \frac{2\mu}{\gamma} \frac{dz^2}{dt} \quad (14)$$

$$\Leftrightarrow 2\eta\mu \left(\frac{K}{\phi}\right)^{-0.5} = 1$$

A simplified form of this equation can be sought using the bundle of parallel capillary tube model where tortuosity (D_T) is unity. Thus for $D_T = 1$:

$$\eta = \frac{(D_T + D_f - 1)(2 - D_f)}{D_f(3 - D_f)} \left(\frac{1}{4\mu}\right) \left(\frac{\lambda_{\max}^1}{\lambda_{\min}^{1-1}}\right) \left(\frac{1 - \beta^{3-D_f}}{1 - \phi}\right) \quad (15)$$

Hence:

$$\frac{(D_f)(2 - D_f)}{D_f(3 - D_f)} \left(\frac{1}{2}\right) (\lambda_{\max}) \left(\frac{1 - \beta^{3-D_f}}{1 - \phi}\right) \left(\frac{K}{\phi}\right)^{-0.5} = 1 \quad (16)$$

Hence:

$$\frac{(2 - D_f)}{(3 - D_f)} \left(\frac{1 - \beta^{3-D_f}}{1 - \phi}\right) = 2\lambda_{\max}^{-1} \left(\frac{K}{\phi}\right)^{0.5} \quad (17)$$

The right-hand side of this equation has no unit because the reciprocal of length multiplied by length cancels. This agrees with the left-hand side, which is fractal dimension. Porosity is in fraction and the unit of β is dimensionless.

$\beta = 0.01$, (Cai et al. 2010).

Hence:

$$\frac{(2 - D_f)}{(3 - D_f)} \left(\frac{1 - 0.01^{3-D_f}}{1 - \phi}\right) = 2\lambda_{\max}^{-1} \left(\frac{K}{\phi}\right)^{0.5} \quad (18)$$

Characterization of the porous sample enables determination of absolute permeability and porosity. The maximum pore diameter $\lambda_{\max 1}$ is calculated based on a model of an equilateral triangle arrangement (reference 38 Cai et al. 2010), while $\lambda_{\max 2}$, is calculated based on a model of a square arrangement of circular particles (reference 44 of Cai et al. 2010).

The parameter, $\lambda_{\max}$, is calculated as (Cai et al. 2010):

$$\lambda_{\max} = \frac{\lambda_{\max 1} + \lambda_{\max 2}}{2} = \frac{\frac{D_s}{2} \sqrt{\frac{2\phi}{1-\phi}} + \frac{D_s}{2} \left[\sqrt{\frac{\phi}{1-\phi}} + \sqrt{\frac{\pi}{4(1-\phi)}} \right]}{2} \quad (19)$$

where D_s is grain diameter [cm].

This parameter can be substituted into Eq. (18). Using the definition of mean pore radius (Eq. 8), the resulting equation is such that only the fractal dimension is unknown where porosity is given. This is an algebraic equation that can be solved for the fractal dimension of the sediment. The equation will be written as:

$$\frac{(2 - D_f)}{(3 - D_f)} \left(\frac{1 - 0.01^{3-D_f}}{(1 - \theta)}\right) = 2 \left[\left(\frac{2\phi}{1 - \phi}\right)^{0.5} + \left\{ \left(\frac{\phi}{1 - \phi}\right)^{0.5} + \left(\frac{\pi}{4(1 - \phi)}\right)^{0.5} \right\} \right] \quad (20)$$

The relationship between the fractal dimension and the pore size distribution index for a three-dimensional system is given as (Kewen 2004):

$$\lambda = 3 - D_f \quad (21)$$

where λ is the pore size distribution index.

Solving for pore size distribution and substituting into Eq. (22) gives:

$$\frac{(\lambda - 1)}{(\lambda)} \left(\frac{1 - 0.01^\lambda}{(1 - \theta)}\right) = 2 \left[\left(\frac{2\phi}{1 - \phi}\right)^{0.5} + \left\{ \left(\frac{\phi}{1 - \phi}\right)^{0.5} + \left(\frac{\pi}{4(1 - \phi)}\right)^{0.5} \right\} \right] \quad (22)$$

Equation (22) links fractal dimension to porosity as found in another published research work, where fractal dimension has been shown to be a function of porosity and connectivity (Pothuau et al. 2000).

Thus, the petrophysical characterization of samples of Wallace sandstone enables the determinations of porosity and absolute permeability and the substitution of these

together with $\lambda_{\max}$ enables solution for the fractal dimension of this sandstone. Also, porosity data of this sandstone can be substituted into Eq. (24) to solve for the pore size distribution index.

Derivations of fractal dimension

To solve Eq. (18) for the fractal dimension of Wallace sandstone requires testing its suitability. This requires using literature data containing porosity and permeability with corresponding fractal dimensions. In solving this equation, we wrote an objective function as:

$$f(D_f) = \frac{(2 - D_f)}{(3 - D_f)} \left(\frac{1 - 0.01^{3-D_f}}{1 - \phi} \right) - 2\lambda_{\max}^{-1} \left(\frac{K}{\phi} \right)^{0.5} \quad (23)$$

This was written as a formula in excel. The value of this objective function was set to 0 and the fractal dimension was varied by the solver program until the value was close to zero as much as possible. The value of the fractal dimension was then obtained.

Zhang and Weller (2014) studied the fractal dimension of sandstone samples from the Eocene sandstone formation in China. Table 6 of Appendix 1 at the end of this paper shows data on porosities, permeabilities and fractal dimensions of corresponding samples using capillary pressure, nuclear magnetic resonance and specific surface area methods. Using Eq. (18), the fractal dimensions for different samples in the appendix have been calculated and compared to those reported in the appendix. Following this, porosity and permeability data for Wallace sandstone have been extracted from literature source (Tables 7 and 8 of Appendix 2) for

the calculation of the fractal dimensions. Accordingly, mean pore radii were calculated using porosity and permeability data in accordance with Eq. (8).

Results and discussion

Table 1 shows results of calculated fractal dimensions of samples with petrophysical data from Table 6 of Appendix 1. Column 7 of the table shows actual fractal dimensions calculated using Eq. (18) of the present work. Generally, Eq. (18) calculates fractal dimension below values indicated by the maximum values of nuclear magnetic resonance (NMR), and above values indicated by other methods. A better idea about the accuracy of the spontaneous imbibition

Table 2 Deviations of fractal dimensions from calculated fractal in the present study

CP	NMR max	NMR min	SS
0.42	0.35	− 0.34	− 0.28
− 0.20	0.28	− 0.24	− 0.20
− 0.23	0.27	− 0.09	− 0.23
− 0.34	0.23	− 0.29	− 0.34
− 0.26	0.22	− 0.02	− 0.26
− 0.22	0.36	− 0.36	− 0.22
− 0.17	0.32	− 0.33	− 0.17
− 0.14	0.42	− 0.32	− 0.14
− 0.09	0.35	− 0.37	− 0.09
− 0.11	0.43	− 0.39	− 0.11
− 0.18	0.39	− 0.31	− 0.18
− 0.14	0.39	− 0.29	− 0.14

Table 1 Comparison of calculated fractal dimensions with literature values from Table 6 in Appendix 1 (converted to 3 decimal places)

Permeability	Porosity	Fractal dimension from capillary pressure (CP)	Fractal dimension from nuclear magnetic resonance (NMR) max	Fractal dimension from nuclear magnetic resonance (NMR) min	Fractal dimension from specific internal surface	Calculated fractal dimension in the present work
0.129	0.125	2.984	2.908	2.224	2.277	2.560
18.279	0.166	2.337	2.817	2.296	2.337	2.535
25.056	0.17	2.305	2.803	2.438	2.305	2.533
60.502	0.162	2.196	2.77	2.248	2.196	2.536
45.68	0.163	2.274	2.753	2.515	2.274	2.537
0.081	0.137	2.337	2.914	2.190	2.337	2.554
0.13	0.139	2.384	2.875	2.226	2.384	2.553
0.227	0.147	2.413	2.968	2.231	2.413	2.550
0.364	0.132	2.465	2.910	2.188	2.468	2.558
0.432	0.143	2.451	2.991	2.169	2.451	2.550
0.624	0.156	2.364	2.934	2.237	2.364	2.542
1.297	0.15	2.405	2.934	2.256	2.405	2.546

max maximum value, *min* minimum value

approach in the present study can be obtained by calculating the deviations from reported fractal dimensions shown in Table 6 of Appendix 1.

Table 2 shows calculated deviations, where deviation is measured as the difference between reported fractal dimension and calculated fractal dimension in this work. Overall, fractal dimensions calculated using Eq. (18) show less deviations compared to the capillary pressure method. This means predictions of fractal dimension using Eq. (18) are closer to values of the capillary pressure method.

Table 3 shows the calculated fractional deviations using Eq. (18). This table again shows that Eq. (18) calculates fractal dimensions closer to those reported for the capillary pressure method and this is clearly seen in Table 1.

In the derivation of Eq. (18), the assumption of bundle of parallel capillary tubes with varying radii was the basis. However, in reality, the effect of overburden compaction in the sedimentary column and the attendant pressure solution effect (Yang 2000) suggest real sediments will have significant levels of tortuosity. This means the parameter, D_T , corresponding to the tortuosity fractal dimension, which measures the convolutedness of the capillary path ways in Eq. (4) is an essential parameter. What is more, the effect of sediment deformation under stress heterogeneity and grain size evolution (Iverson et al. 1996) is to cause collapse of original large pore spaces leading to varied pore size distribution, which causes higher fractal dimensions or heterogeneities. In the published work of Zhang and Weller (2014), (Table 6 in Appendix 1), the age of the formation is Eocene and the original depths of the samples are between 3332 and 3738 meters. Under such overburden conditions, the tortuosity factor of the sediment cannot be neglected. Therefore, any model that neglects tortuosity will obviously calculate fractal dimensions different from actual or expected values. That is why our assumption of bundle of parallel capillary tubes without tortuosity has led to calculated fractal dimensions

with deviation, though values of calculated fractal dimensions fall within those expected for sandstones.

Comparing calculated values of fractal dimensions using Eq. (18) to those reported for the capillary pressure method in Table 1 gave impetus for calculation of the fractal dimension of Wallace sandstone samples using this equation. This is because we have data on absolute permeability, which is correlated with pore size distribution generated from capillary pressure data (Marshall 1958). Zakir Ullah (2013) studied the effect of waste cooking oil on rock wettability in his graduation project work at Dalhousie University (Ullah 2013). He used four core samples of Wallace sandstone. Tables 7 and 8 in Appendix 2 give the details of characterization results of this sandstone samples under ambient conditions.

Accordingly, Table 4 shows fractal dimensions of Wallace sandstone calculated using Eq. (18). The mean value is 2.550. Table 5 shows results of pore size distribution index calculation using Eq. (21) and values of fractal dimensions from Table 4. Accordingly, values of pore size distribution index with a mean of 0.446 fall within those reported elsewhere for sandstones (Bloomfield et al. 2001).

All values of fractal dimensions calculated using Eq. (18) found in Table 1 and Table 3 fall within those for sandstones that have been reported by other workers (Zhang and Weller 2014; Radlinski et al. 2004; Dathe et al. 2001). This testifies to the suitability of this equation.

Apart from its relationship to varying physiochemical processes reviewed in Sect. 1, the fractal dimension and pore size distribution give indication about the heterogeneities or homogeneities of sediments (Pfeifer and Avnir 1983; Wheatcraft and Tyler 1988). Accordingly, the bigger the fractal dimension of a sediment, the more heterogeneous it is and vice versa. Bigger values of fractal dimensions will have wider values of wetting phase saturation and a

Table 3 Fractional deviations from calculated values

CP	NMR max	NMR min	SSA
0.14	0.12	− 0.15	− 0.12
− 0.09	0.10	− 0.10	− 0.09
− 0.10	0.10	− 0.04	− 0.10
− 0.15	0.08	− 0.13	− 0.15
− 0.12	0.08	− 0.01	− 0.12
− 0.09	0.12	− 0.16	− 0.09
− 0.07	0.11	− 0.14	− 0.07
− 0.05	0.14	− 0.14	− 0.05
− 0.04	0.12	− 0.17	− 0.04
− 0.04	0.15	− 0.18	− 0.04
− 0.05	0.16	− 0.10	− 0.05
− 0.06	0.13	− 0.13	− 0.06

Table 4 Calculated fractal dimensions for Wallace sandstone core samples using Eq. (18)

Sample 1	Sample 2	Sample 3	Sample 4	Mean value
2.564	2.553	2.549	2.550	2.550

Table 5 Fractal dimension and pore size distribution index

Sample no.	Fractal dimension	Pore size distribution index
Sample 1	2.564	0.436
Sample 2	2.553	0.447
Sample 3	2.549	0.451
Sample 4	2.550	0.450

broader range of capillary pressure data, whereas smaller values will be characterized by bigger values of pore size distribution as given by Eq. (21). Such rocks are characterized by narrow ranges of capillary pressure and wetting phase saturation. From Tables 4 and 5, the mean values of fractal dimension and pore size distribution index are 2.554 and 0.446, respectively. In the literature, typical values of pore size distribution vary from 0.5 for materials with larger range of pore sizes with a value of 0.45 being characteristic of consolidated formations (Bloomfield et al. 2001). Therefore, values calculated for Wallace sandstone in this paper are characteristic of eminent heterogeneity which is further supported by oxide analysis. The oxide analysis shows 82% (Quary 2001) of silica and varying compositions of other minerals and these point to heterogeneity in mineralogy, which reflects the pore size distribution index calculated in our work. Based on evidence of deviations of fractal dimension reported in Table 1, we admit that calculation of the fractal dimension of Wallace sandstone reported in Table 4 also has this deviation, but calculated fractal dimensions are appreciable because they reflect those of sandstones.

Conclusion

The fractal dimension and pore size distribution of a sediment is an important petrophysical parameter that has a strong and direct bearing on its hydraulic and physiochemical properties. In Canada, the glauconitic sandstones of Alberta have been studied and data for petrophysical parameters exist. In Nova Scotia, Canada, Wallace sandstones are glauconitic sandstones that have been used by the construction industries for long. Apart from test data on mechanical strength and reports of oxide compositions of this sandstone, no data on pore size distribution and fractal dimension exist to the best of our knowledge. We have used analytical approach based on spontaneous imbibition dynamics to derive an equation that integrates pore size distribution, porosity and permeability of sediment. In the application of this equation to Wallace sandstone, our principal assumption was that a cylindrical sample of Wallace sandstones core sample can be regarded as a bundle of parallel capillary tubes without tortuosity. Based on this equation, we have calculated the fractal dimension and pore size distribution index for this sandstone, hoping that it will provide an insight into its hydraulic and

petrophysical characteristics. The following sums up the conclusion of this study:

1. Wallace sandstone has a fractal dimension with a mean value of 2.554 and a pore size distribution index with a mean value of 0.446;
2. Values of pore size distribution and fractal dimension deduced in this study show that this sandstone is characterized by a significant level of heterogeneity and it will, therefore, exhibit a wider range of capillary pressure versus wetting phase saturation;
3. Mean values of pore size distribution index and fractal dimension calculated in this study fall within those reported in the literature for sandstones;
4. Our assumption of bundle of parallel capillary tubes for sediment samples has led to deviations of fractal dimensions from a literature source data but deviations are not alarming.

Future work

Our observation that the bundle of parallel capillary tubes of varying radius assumption leads to calculation of fractal dimension with deviations has motivated future work. In this regard, our objective is to use a more rigorous and robust mathematical model that will integrate the pore tortuosity dimension into the spontaneous imbibition model. This will enable us to deduce more representative values of fractal dimension and pore size distribution index of Wallace sandstone.

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Appendix 1

See Table 6.

Table 6 Data from literature source (Zhang and Weller 2014)

Permeability	Porosity	Fractal dimension from capillary pressure CP	Fractal dimension from nuclear magnetic resonance (NMR) max	Fractal dimension from nuclear magnetic resonance (NMR) min	Fractal dimension from specific internal surface
0.129	0.125	2.984	2.908	2.224	2.277
18.279	0.166	2.337	2.817	2.296	2.337
25.056	0.17	2.305	2.803	2.438	2.305
60.502	0.162	2.196	2.77	2.248	2.196
45.68	0.163	2.274	2.753	2.515	2.274
0.081	0.137	2.337	2.914	2.190	2.337
0.13	0.139	2.384	2.875	2.226	2.384
0.227	0.147	2.413	2.968	2.231	2.413
0.364	0.132	2.465	2.910	2.188	2.468
0.432	0.143	2.451	2.991	2.169	2.451
0.624	0.156	2.364	2.934	2.237	2.364
1.297	0.15	2.405	2.934	2.256	2.405

Appendix 2

See Tables 7 and 8.

Table 7 Petrophysical characteristics of core samples (Ullah 2013): Core dimensions

Sample no.	Length, cm	Diameter, cm	Bulk volume, cm ³
Sample 1	7.16	3.78	80.30
Sample 2	7.20	3.78	80.75
Sample 3	7.14	3.78	80.08
Sample 4	7.14	3.78	80.28

Table 8 Test results (Ullah 2013)

Sample	Dry weight gm	Wet weight gm	Porosity %	Permeability mD	Permeability, 10 ⁻¹² m ²	Mean radius, 10 ⁻⁶ m
1	187	199	15.55	1.88	1.86	3.49
2	188	199	13.85	1.89	1.87	3.67
3	185	197	14.55	1.85	1.83	3.54
4	187	199	14.40	1.84	1.82	3.55

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Affiliations

Mumuni Amadu¹ · Michal J. Pegg¹

✉ Mumuni Amadu
mm846771@dal.ca

Michal J. Pegg
Michael.Pegg@dal.ca

¹ Department of Process Engineering and Applied Science, Dalhousie University, Halifax, Canada