A density-based material balance equation for the analysis of liquid-rich natural gas systems

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Abstract This study analytically cross examines the consistency among available zero-dimensional material balance equations (MBEs) for liquid-rich gas equations and derive a new simple yet rigorous MBE starting from governing equations applicable to these systems. We propose a new zero-dimensional (tank) material balance equation directly applicable to the analysis of liquid-rich (wet and retrograde) gas reservoirs expressed as a function of an equivalent gas molar density, as well as investigate and critically compare its predictions against other zero-dimensional (tank) models proposed in the past for gas reservoir cases with different amounts of condensate content (lean, intermediate and rich). All models are employed to predict reservoir performance given reservoir original-fluids-in-place and compared against benchmark examples created by numerical simulation. Actual field examples are also analyzed using existing and proposed models to test their ability to provide reliable reserve estimations using straight-line methods. The proposed density-based equation is proven to be straightforward to implement since it is written in terms of density, which allows it be directly expressed as an extension of the dry gas MBE, while not requiring the implementation of two-phase Z-factors.

Keywords Liquid-rich · Natural gas reservoirs · Material balance equation

List of symbols

B_L Reservoir hydrocarbon liquid phase formation volume factor, RB/STB

B_V Reservoir vapor phase formation volume factor, RB/SCF
 $G_{fg,i}$ Initial surface gas in reservoir vapor phase, SCF
 G_p Cumulative gas production, SCF
 G_{pT} Total cumulative gas production, including surface gas and gaseous equivalent of produced condensates, SCF
 G_i Initial gas-in-place, SCF
 G_T Total initial gas-in-place, including surface gas and gaseous equivalent of produced condensates, SCF
 N_i Initial condensate-in-place, STB
 N_p Cumulative oil production, STB
 $N_{fo,i}$ Initial surface oil in reservoir liquid phase, STB
 n_i Total initial hydrocarbon in place, lbmol
 n_p Total cumulative hydrocarbon production, lbmol
 n_g Total moles of surface gas component at reservoir conditions, lbmol
 n_{gp} Cumulative produced moles of surface gas component, lbmol
 n_{op} Cumulative produced moles of surface oil component, lbmol
 n_g^V Moles of surface gas component in reservoir vapor phase, lbmol
 n_g^L Moles of surface gas component in reservoir liquid phase, lbmol
 n_o^V Moles of surface oil component in reservoir vapor phase, lbmol
 n_o^L Moles of surface oil component in reservoir liquid phase, lbmol
 Φ Porosity
 ρ Molar density, lbmol/ft³
 ρ_V Molar density of reservoir vapor phase, lbmol/ft³
 ρ_L Molar density of reservoir liquid hydrocarbon phase, lbmol/RB

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ρ_{gsc}	Molar density of surface gas at standard condition, lbmol/SCF
ρ_{ost}	Molar density of surface oil at stock-tank condition, lbmol/STB
ρ_g^*	Equivalent gas molar density, lbmol/RB
$\bar{\rho}_g^*$	Equivalent gas molar density at average reservoir condition, lbmol/RB
p	Pressure, psia
p_{dew}	Dewpoint pressure, psia
p_i	Initial pressure, psia
t	Time, day
T	Temperature, F
k	Permeability, md
h	Thickness, ft
r_e	Reservoir outer boundary, ft
r_w	Wellbore radius, ft
R	Universal constant
R_s	Solution gas-oil-ratio, SCF/STB
R_{ps}	Cumulative gas-oil-ratio, SCF/STB
R_v	Volatilized oil–gas-ratio, STB/SCF
f_{nL}	Liquid phase molar fraction at reservoir conditions, fraction
f_{nV}	Vapor phase molar fraction at reservoir conditions, fraction
S_V	Vapor phase saturation, %
S_L	Liquid phase saturation, %
q_{gsc}	Gas production rate at standard condition, SCF/D
q_{osc}	Oil production rate at standard condition, STB/D
q_V	Vapor production rate at reservoir condition, ft ³ /D
q_L	Liquid production rate at reservoir condition, RB/D
RV	Relative liquid dropout volume, fraction
v	Velocity, ft/s
v_V	Velocity of reservoir vapor phase, ft/s
v_L	Velocity of reservoir liquid phase, ft/s
V_b	Reservoir bulk volume, ft ³
V_p	Reservoir pore volume, ft ³
V_V	Reservoir vapor volume, ft ³
V_L	Reservoir liquid volume, ft ³
Z_{tp}	Two-phase compressibility factor
$Z_{tp,CCE}$	Two-phase compressibility factor from CCE experimental data
$Z_{tp,CVD}$	Two-phase compressibility factor from CVD experimental data

Introduction

Reliable reserve estimation is a key component of economic evaluation and decision-making in reservoir development. Zero-dimensional material balance

equations (MBEs) are widely deployed during these estimations to perform original fluid in place and reservoir performance predictions. The basic assumption for the zero-dimensional model is that the reservoir system can be depicted as a sealed tank where reservoir changes upon depletion can be represented as a function of volumetrically averaged average reservoir conditions. The first general material balance equation was developed by Schilthuis (1936) through equaling underground fluid withdrawal with total fluid expansion. Schilthuis's equation was limited to the analysis of black oil and dry gas reservoirs.

In this work, zero-dimensional models (tank model) for liquid-rich gases or gas-condensate reservoirs are investigated. Walsh et al. (1994a, b) derived the generalized material balance equation (GMBE) applicable to the full range of conventional reservoir fluid types. When rock and water expansion is neglected, the GMBE for an initially undersaturated gas-condensate reservoir fluid is written as follows:

$$N_p(B_L - B_V R_s) + G_p(B_V - B_L R_v) = G_i[R_{vi}(B_L - B_V R_s) - B_{vi}(1 - R_s R_v) + (B_V - B_L R_v)] \quad (1)$$

The derivation from Walsh et al. (1994a)'s form of GMBE to Eq. 1 is provided in Appendix 1. In all zero-dimensional MBEs, such as the GMBE presented in Eq. 1, reservoir fluids are characterized using black-oil fluid formulations. In these formulations, only two pseudo-components are 'allowed' to be present in the reservoir phases—i.e., surface gas and surface oil. It is further assumed that compositional and volumetric effects can be captured through the concepts of R_s (solution gas-to-oil ratio), R_v (volatilized oil-to-gas ratio), reservoir oil phase formation volume factor (B_L) and reservoir gas phase formation volume factor (B_V)—all sole functions of pressure. Walsh et al. (1994a) showed that the GMBE in Eq. 1 may be collapsed to the well-known p/Z vs. cumulative gas production (G_p) relationship applicable to volumetric dry-gas reservoirs. For gas-condensate reservoir analysis, however, Eq. 1 may not be presented in terms of p/Z vs. G_p straight line plots because it is not pressure or p/Z explicit (Walsh et al. 1994a). Therefore, a single-variable Newton–Raphson procedure would need to be employed to iteratively calculate average pressure for a given resource (G_i).

To maintain the analogy with p/Z vs. G_p MBEs for dry-gas systems, p/Z_{tp} plots versus cumulative production plots have been proposed for the analysis of volumetric liquid-rich gas systems. These models embrace the concept of 'two-phase' Z-factors (Z_{tp}) that replace customary single-phase gas Z-factors in dry-gas MBEs. Hagoort (1988)

initially proposed a zero-dimensional material balance equation for volumetric gas condensate reservoirs analysis in terms of the concept of a “two-phase” Z-factor, written as follows:

$$\frac{\bar{p}}{Z_{tp}} = \frac{p_i}{Z_{tp,i}} \left(1 - \frac{n_p}{n_i} \right) \quad (2)$$

where n_p is total produced hydrocarbon moles and n_i is total initial hydrocarbon moles in place written in terms of total moles using the black-oil fluid formulation—i.e., in terms of surface gas (g) and surface oil (o) components where $n_p = n_{gp} + n_{op}$ and $n_i = n_{gi} + n_{oi}$. Hagoort (1988) rewrote Eq. 2 in terms of cumulative surface gas production (G_p) and surface oil production (N_p) by relating total molar quantities to cumulative production and proposed:

$$\frac{\bar{p}}{Z_{tp}} = \frac{p_i}{Z_{tp,i}} \left[1 - \frac{G_p(1 + R_{MLG_p})}{G_i(1 + R_{MLG_i})} \right] \quad (3)$$

where R_{MLG_i} and R_{MLG_p} are Hagoort's initial and cumulative condensate/gas molar ratios defined as:

$$R_{MLG_i} = \frac{n_{oi}}{n_{gi}} = \frac{R_{vi}G_i\rho_{ost}}{G_i\rho_{gsc}} = \frac{\rho_{ost}}{\rho_{gsc}} \cdot R_{vi} \quad (4)$$

$$R_{MLG_p} = \frac{n_{op}}{n_{gp}} = \frac{N_p\rho_{ost}}{G_p\rho_{gsc}} = \frac{\rho_{ost}}{\rho_{gsc}} \cdot \frac{N_p}{G_p} \quad (5)$$

where the gas molar density at standard conditions (ρ_{gsc} , lbmol/SCF) is a constant equal to 0.002634 lbmol/ft³ for ideal gases and the molar density of the stock tank (ρ_{ost} , lbmol/STB) is estimated directly from stock tank oil API (Walsh and Lake 2003). In black-oil fluid formulations, produced hydrocarbon phases at the surface are ‘pure’ single-component fluids, thus these densities correspond to those of the ‘pure’ fluids containing 100 % of surface gas component and surface oil components, respectively. An identical expression to Eq. 3 is obtained if, instead of using Hagoort's R_{MLG_i} and R_{MLG_p} definitions, one directly substitutes $n_p = \int_0^t (q_{gsc}\rho_{gsc} + q_{osc}\rho_{ost})dt = G_p\rho_{gsc} + N_p\rho_{ost}$ and $n_i = G_i(\rho_{gsc} + \rho_{ost}R_{vi})$ into Eq. 2. The terms $G_p(1 + R_{MLG_p})$ and $G_i(1 + R_{MLG_i})$ in Eq. 3 are also referred as ‘total cumulative gas production’ (G_{pT}) and ‘total initial gas in place’ (G_T), respectively, which are variables that lump the surface gas and the gaseous equivalent of produced condensates together. This leads to the following more compact form of this MBE:

$$\frac{\bar{p}}{Z_{tp}} = \frac{p_i}{Z_{tp,i}} \left(1 - \frac{G_{pT}}{G_T} \right) \quad (6)$$

which has been also presented by Lee and Wattenbarger (1996) as the applicable MBE for volumetric gas-

condensate reservoirs. When coupled with production history, deployment of Eqs. 2, 3, or Eq. 6 requires a priori knowledge of ‘two-phase’ Z-factors (Z_{tp}) and volatilized oil–gas-ratios (R_v) data as a function of pressure. These input variables are routinely estimated from constant volume depletion (CVD) PVT laboratory tests or via a properly tuned equation of state (EOS). From here, it follows that a plot of $\frac{\bar{p}}{Z_{tp,CVD}}$ against total cumulative gas production (G_{pT}) yields total initial gas in place (G_T) at the x -axis intercept resulting from the best-fit straight-line extrapolation. This methodology assumes that $Z_{tp,CVD}$ obtained from the laboratory test is applicable to the study of interest, which tends to be a good approximation when reservoir condensate remains immobile—a basic premise honored by experimental CVD tests. When condensate is highly mobile, this approximation may no longer hold. This is further examined in subsequent sections in this manuscript.

Vo et al. (1990) showed that “two-phase” Z-factors (Z_{tp}) are molar-weighted averages of the Z-factors of the reservoir gas and oil phases, i.e., $Z_{tp} = f_{nL}Z_L + f_{nV}Z_V$. Two-phase Z-factors, as the Z-factors of the entire hydrocarbon mixture inside a PVT cell, are straightforwardly calculated for any two-phase system on the sole basis of pressure, temperature, volume, and total hydrocarbon moles data obtained from either CVD tests—where gas is continuously produced/removed from the PVT cell—or from constant composition expansion (CCE) tests—where no gas production is realized from the PVT cell. Vo et al. (1990) further illustrated the approximate validity of the following empirical simplification of the Z_{tp} MBE:

$$\frac{\bar{p}}{Z_{tp,CCE}} = \frac{p_i}{Z_{tp,i}} \left(1 - \frac{G_p}{G_i} \right) \quad (7)$$

Equation 7 is written in terms of *surface gas* cumulative production (G_p) rather than *total* produced surface gas moles (n_p) or *total* cumulative gas production (G_{pT}) as Eqs. 2, 3, or Eq. 6 require. To accomplish this, the authors indicate that Z_{tp} values should be collected from CCE tests—and not from CVD tests as with the previous models. The applicability of Eq. 7 for gas-condensate analysis was verified in practice, with satisfactory results, not only by Vo et al. (1990) using numerical cases but also later with field examples (e.g., Zeidouni et al. 2006). As with any approximation, whether Eq. 7 remains generally applicable for all or most cases of interest is debatable. The implications of using of $Z_{tp,CVD}$ instead of the recommended $Z_{tp,CCE}$ in Eq. 7 are not always clear. This is further examined in subsequent sections in this article.

A recent attempt at deploying two-phase Z-factor in liquid-rich MBE analysis has been presented by Sureshjani

et al. (2014). They proposed the use of a *modified* two-phase Z-factor (Z_{tp}^*) along with the following gas-condensate MBE in terms of cumulative gas production (G_p) derived from a statement of surface gas conservation:

$$\frac{\bar{p}}{Z_{tp}^*} = \frac{p_i}{Z_{tp,i}^*} \left(1 - \frac{G_p}{G_i} \right) \quad (8)$$

where Z_{tp}^* , the *modified* two-phase Z-factor, is defined by the authors as:

$$Z_{tp}^* = Z_{tp} \frac{T_{sc} R}{p_{sc}} \left[\frac{\left(\frac{1}{RV} - 1 \right) \rho_V + \rho_L}{\left(\frac{1}{RV} - 1 \right) \frac{1}{B_V} + \frac{R}{B_L}} \right] \quad (9)$$

where Z_{tp} is the CVD two-phase Z-factor, RV is the relative liquid-dropout volume from the CVD test, and ρ_V and ρ_L are the molar density of reservoir gas and oil phases, respectively. As described, Sureshjani et al. (2014) suggest that all parameters required for the calculation of Z_{tp}^* in Eq. 9 should be obtained from CVD data.

This study investigates and compares predictions from these currently available zero-dimensional models for liquid-rich gas (gas-condensate) reservoirs, including the generalized material balance equation (GMBE) and all two-phase Z-factor MBEs described above, along with the presentation of a new model that uses the concept of equivalent gas molar density and does not implement two-phase Z-factors. Assumptions behind each of these models are discussed. They are also deployed to predict reservoir pressure depletion given known values of reservoir original-gas-in-place and for reserve estimations. Model performances are compared against each other and against reported data from finely gridded reservoir compositional simulation and actual field examples.

A density-based liquid-rich MBE

Molar conservation of surface gas component can be expressed in terms of total surface-gas hydrocarbon moles remaining in situ at any point in time (n_g) as follows:

$$n_g = n_{gi} - n_{gp} \quad (10)$$

where $n_g = n_g^V + n_g^L$. By defining an *equivalent gas molar density*, ρ_g^* , as total moles of surface gas present at reservoir conditions per unit reservoir pore volume, we can write its volumetric average $\bar{\rho}_g^*$ as:

$$\bar{\rho}_g^* = \frac{\iiint \rho_g^* dV_p}{V_p} = \frac{n_g}{V_p} = n_g \cdot \frac{\rho_{g,i}^*}{n_{gi}} \quad (11)$$

where $\bar{\rho}_g^*$ represents the equivalent gas molar density evaluated at the average reservoir pressure condition. Substitution of Eq. 10 into Eq. 11 leads to:

$$\frac{\bar{\rho}_g^*}{\rho_{g,i}^*} = 1 - \frac{n_{gp}}{n_{gi}} \quad (12)$$

It is readily noted that an “*equivalent*” gas molar density (ρ_g^*) is *not* the molar density of reservoir gas phase, as it only represents the amount of moles of surface gas component remaining in situ per unit reservoir pore volume as prescribed by Eq. 11. ρ_g^* rather represents the density that the in situ surface gas (n_g , found in the free gas and solution gas) would have if it were single-phase at the given reservoir condition. Using its definition in Eq. 11, this equivalent gas molar density ρ_g^* can also be calculated in terms of standard PVT black-oil properties and saturation data as follows:

$$\bar{\rho}_g^* = \frac{n_g^V + n_g^L}{V_p} = \frac{\rho_{gsc} \frac{V_V}{B_V} + \rho_{gsc} \frac{V_L}{B_L} R_s}{V_p} = \rho_{gsc} \cdot \left(\frac{S_V}{B_V} + \frac{S_L}{B_L} R_s \right) \quad (13)$$

where ρ_{gsc} is the surface gas molar density at standard conditions, and V_V and V_L are the vapor phase and liquid phase volumes at the given reservoir condition, respectively. Given the direct proportionality between moles and surface gas quantities, the proposed liquid-rich MBE expressed in terms of equivalent density is derived from Eq. 12 as shown:

$$\bar{\rho}_g^* = \rho_{g,i}^* \cdot \left(1 - \frac{G_p}{G_i} \right) \quad (14)$$

Equation 14 shows that liquid-rich MBEs can indeed be rigorously written in term of surface gas production G_p and equivalent gas molar density, as per the requirements of surface gas component molar conservation. Equation 14 provides a rigorous black-oil model in term of G_p (surface gas production) in direct analogy with the traditional p/Z (or density) vs. G_p dry-gas models. Indeed, a best-fit straight-line in a $\bar{\rho}_g^*$ vs. G_p plot will intercept x -axis at G_i , as shown in subsequent sections. Also, for reservoir conditions higher than dewpoint, Eq. 14 readily collapses to the density-based dry gas MBE: $\bar{\rho} = \rho_i \left(1 - \frac{G_p}{G_i} \right)$. It is noted that original gas in place G_i can be expressed in terms of $\rho_{g,i}^*$ as:

$$G_i = \frac{\rho_{g,i}^* V_p}{\rho_{gsc}} = \frac{\rho_{g,i}^* A h \phi}{\rho_{gsc}} \quad (15)$$

Since B_L , B_V and R_s are pressure-dependent black-oil PVT properties, Eq. 13 requires a pressure–saturation relationship for the calculation of ρ_g^* . We show that the CVD liquid dropout volume $S_{L,CVD}$ vs. pressure profile is a

good approximations for these purposes. As a result, Eq. 14 can be straightforwardly applied without further manipulation of input data or iterations given that all required input parameters (B_L , B_V and R_s and S_L) are directly obtained from standard CVD tests. In this way, an evenly spaced table of p vs. ρ_g^* can be straightforwardly made prior to analysis. Average pressures may be then explicitly interpolated from this table after $\bar{p}_g^*$ values are obtained using Eq. 14. In addition, being able to connect liquid-rich MBE calculations directly to density estimations readily enables the extension of density-based decline analysis methods (see Ayala and Ye 2013; Zhang and Ayala 2014) to liquid-rich gas condensate wells.

Liquid-rich MBEs: classical vs. numerical models

Zero-dimensional material balance equations (MBEs) describe reservoir dynamics in terms of a volume-averaged condition that neglects (pressure and saturation) gradients (i.e., “tank” model). In numerical simulation, fully dimensional flow models are solved through discretized versions of the applicable partial differential equations. Both approaches are based on mass conservation, thus classical material balance equations (MBEs) must be able to be directly derived from their partial differential counterparts used in numerical simulation. Partial differential equations for multiphase flow in porous media can be written in terms of compositional or black-oil fluid formulations. In the compositional formulation, for example, the conservation principle is applied to each individual component. In this case, the continuity equation for the flow of component (i) becomes:

$$-\nabla \cdot (\rho_V \mathbf{v}_V y_i + \rho_L \mathbf{v}_L x_i) = \frac{\partial}{\partial t} [\phi (\rho_V S_V y_i + \rho_L S_L x_i)] \quad (16)$$

where ρ_V and ρ_L are the molar densities of reservoir vapor and liquid phase, $\mathbf{v}_V$ and $\mathbf{v}_L$ represent fluxes of reservoir vapor and liquid, and y_i and x_i are the molar fraction of component i in vapor and liquid phases, respectively. In these compositional formulations, the entire system is assumed at thermodynamic equilibrium and each of hydrocarbon phases may contain many components. In black-oil formulations, only two pseudo-components are allowed in hydrocarbon phases—namely, surface gas and surface oil. Compositional and volumetric effects in black-oil models are captured through the concepts of R_s (solution gas-to-oil ratio), R_v (volatilized oil-to-gas ratio), reservoir oil formation volume factor (B_L) and reservoir gas formation volume factor (B_V). For such conditions, and for the *surface gas* component, Eq. 16 collapses to:

$$-\nabla \cdot \left(\frac{\mathbf{v}_V}{B_V} + \frac{\mathbf{v}_L}{B_L} R_s \right) = \frac{\partial}{\partial t} \left[\phi \left(\frac{S_V}{B_V} + R_s \frac{S_L}{B_L} \right) \right] \quad (17)$$

For the *surface oil* component, Eq. 16 collapses to:

$$-\nabla \cdot \left(\frac{\mathbf{v}_L}{B_L} + \frac{\mathbf{v}_V}{B_V} R_v \right) = \frac{\partial}{\partial t} \left[\phi \left(\frac{S_L}{B_L} + R_v \frac{S_V}{B_V} \right) \right] \quad (18)$$

All liquid-rich classical material-balance methods are written in terms of black-oil fluid formulations and thus can be derived from either Eqs. 17 or 18, or a combination of both. As a result, zero-dimensional MBEs are bound to reproduce fully dimensional numerical data results when properly applied and for conditions consistent with their assumptions. These connections are demonstrated in the Appendices.

Appendix 1 shows that, upon integration of Eqs. 17 and 18, these PDEs collapse to the GMBE form presented in Eq. 1 as derived by Walsh et al. (1994a). Appendix 2 shows that Hagoort’s Z_{tp} MBE (Eq. 2) is derived from an overall molar conservation statement that also follows from the combined application of the volume-integrated Eqs. 17 and 18. GMBE and Hagoort’s Z_{tp} MBE are both based on enforcing overall mass conservation but: (a) GMBE employs gas and oil component molar conservation statements written independently in term of black-oil properties; (b) Hagoort’s Z_{tp} MBE considers molar conservation of total hydrocarbon (surface gas and oil moles together, undistinguished) expressed in term of total molar recovery.

As discussed previously, Vo’s CCE-based Z_{tp} MBE (Eq. 7) is an empirical simplification with practical applications. It may actually be derived from Hagoort’s Eq. 3 for any hypothetically created condition that would allow $R_{MLG_p} \approx R_{MLG_i}$ —i.e., $\frac{N_p}{G_p} \sim R_{vi}$. Such equivalency $R_{vi} = \frac{N_p}{G_p}$ would be only valid for single-phase depleting systems (such as dry and wet gases or liquid-rich systems with minimal reservoir condensation) which remain *constant-composition* at reservoir conditions throughout depletion. As a result, application of Vo’s Eq. 7 indeed requires the creation of a fictitious *constant-composition* producing stream and Z_{tp} reservoir data consistent with it, which can only be obtained through a CCE test. For such hypothetical stream, $R_{MLG_p, CCE}$ would remain almost unchanged and equal to initial R_{MLG_i} throughout the expansion process. In actual systems, two-phase production leads to significant reservoir composition changes—conditions that are best captured by CVD tests. Therefore, the application of Eq. 7 with CVD-based Z_{tp} data would not be valid and leads to poor predictions.

Equation 7 (Vo’s CCE-based Z_{tp} MBE), Eq. 8 (Sureshijani’s CVD-based Z_{tp}^* MBE), and Eq. 14 (proposed

Table 1 Gas reservoir fluid PVT properties

	Lean gas A	Mid gas B	Rich gas C	Rich gas D
Initial pressure, p_i (psia)	3500	3000	3000	3500
Dew point pressure, p_{dew} (psia)	3275.0	2912.7	2787.7	3145.2
Temperature, T (F)	200	200	250	250
Maximum CVD liquid dropout, % S_o	4.6	12.5	22.2	33.2
Initial volatilized-oil content (R_{vis} , STB/MMSCF)	47.3	82.4	129.8	166.8

density-based MBE) exemplify available MBEs written in term of surface gas production G_p only—i.e., without requiring cumulative oil production data, N_p —thus providing direct liquid-rich gas analogs to the dry-gas model. Vo et al. (1990)'s model only approximately enforces overall conservation of all components, and the absence of oil cumulative production (N_p) in the equation is a consequence of the constant cumulative condensate/gas molar ratios characteristic of CCE processes. Both Sureshijani's CVD-based Z_{tp}^* MBE (Eq. 8) and the proposed equivalent gas density MBE (Eq. 14) enforce species conservation (surface gas), rather than overall molar conservation, and thus predictions from these models must be consistent. The proposed formulation, however, does not require the definition of two-phase Z-factors and more directly connects to physical quantities (density) and principles. Notably, the equivalent gas molar density ρ_g^* expression in Eq. 13 directly tracks the surface gas-component storage term $\left(\frac{S_V}{B_V} + \frac{S_L}{B_L} R_s\right)$ found on the right-hand-side of the surface-gas PDE (Eq. 17) and thus better bridges MBE calculations with multiphase performance applications. Well performance predictions, for example, can indeed be directly linked to ρ_g^* time changes by combining the integrated version of Eq. 17 (see Eq. 19, Appendix 1) with the equivalent gas molar density ρ_g^* expression in Eq. 13.

Case studies: numerical examples

In this section, four different natural gases of varying degrees of initial volatilized condensate contents are considered, as presented in Table 1. These fluids include Lean gas A, Mid gas B, Rich gas C, and Rich gas D with initial volatilized oil-to-gas ratios (R_{vi}) of 47.3, 82.4, 129.8 and 166.8 STB/MMSCF and maximum liquid dropout saturations of 4.6, 12.5, 22.5 and 33.2 %, respectively, as depicted in Fig. 1. All relevant CVD and CCE versus pressure data for these fluids are presented in Appendix 3. These PVT tests and resulting data were simulated for the fluids of interest using the same Peng–Robinson equation of state (PR-EOS) employed by the compositional simu-

Fig. 1 CVD liquid drop-out curve of liquid-rich gases A, B, C and D

lator, to maintain consistency in performance comparisons. Table 2 displays the various combinations of depletion scenarios considered for each of these liquid-rich gas fluids, including type of production specification (constant bottomhole pressure, BHP, or constant surface gas specification) along with type of relative permeability curve inputted to the compositional numerical simulator. Three sets of relative permeability curves are considered (Set A, B, and C), as displayed in Fig. 2. k_r -curves from Set A ($S_{oc} = 0.40$) corresponds to a multiphase system where the condensate would stay mostly immobile for all fluids under consideration, given the maximum liquid saturations of Fig. 1. Condensate would mobilize when Set B of the k_r -curves ($S_{oc} = 0.10$) is used for rich C and D gases. The highest degree of condensate mobility would be attained when Set C of the k_r -curves is used. No connate water is considered.

For all case studies presented in Table 2, reservoir pressure responses were numerically generated via finely ridded compositional simulations for a 1-D radial-cylindrical system using the reservoir properties listed in

Table 2 Numerical case studies

Case study	Fluid	Production specification	k_r -curves
1	Lean gas A	Constant BHP (500 psia)	Set A
2	Mid gas B	Constant q_{gsc} (500 MSCFD)	Set B
3	Rich gas C	Constant BHP (500 psia)	Set A
4	Rich gas D	Constant BHP (500 psia)	Set B
5	Rich gas C	Constant q_{gsc} (500 MSCFD)	Set C
6	Rich gas D	Constant q_{gsc} (500 MSCFD)	Set C

Fig. 2 Relative permeability for synthetic cases **Fig. 3** Pressure prediction comparison—Case Study 1: Lean gas A Set A**Table 3** Reservoir properties and initial fluids in place

Absolute permeability, k (md)	1
Porosity, ϕ (fraction)	0.30
Wellbore radius, r_w (ft)	0.25
Reservoir thickness, h (ft)	50
Wellbore outer boundary, r_e (ft)	500
Gas-in-place, G_i (BSCF)	
Lean gas A	2.48
Mid gas B	2.42
Rich gas C	2.15
Rich gas D	2.28
Condensate-in-place, N_i (MMSTB)	
Lean gas A	0.12
Mid gas B	0.20
Rich gas C	0.28
Rich gas D	0.38

Table 3 and the reported initial fluids in place. These pressure responses were volumetrically averaged for direct comparison against average pressure predictions from

available zero-dimensional liquid-rich gas MBEs discussed above: Walsh's GMBE (Eq. 1), Hagoort's Z_{tp} MBE (Eq. 3), Vo's CCE-based Z_{tp} MBE (Eq. 7), Sureshijani's CVD-based Z_{tp}^* MBE (Eq. 8), and the proposed density-based MBE (Eq. 14). Figures 3, 4, 5 and 6 compare predictions from each zero-dimensional model (p_{cal}) versus actual average pressure reported by finely gridded numerical simulation (p_{avg}) for Case Study 1 (Fig. 3), Case Study 2 (Fig. 4), Case Study 3 (Fig. 5) and Case Study 4 (Fig. 6). In these figures, p_{cal}/p_{avg} ratios are reported as a function of pressure depletion. In these figures, the best-performing zero-dimensional model should be able to remain as close as possible to the unit (1) ratio. It is observed that, for all case studies, all MBEs are able to remain within less than 4 % error and are thus able to provide predictions quite close to the benchmark, though the difference becomes more noticeable as the fluid gets richer. Notably, Vo et al. (1990)'s model (Eq. 7) tends to yield the largest, yet still relatively small, errors for the first three case studies; while the proposed density-based MBE provides results quite consistent with GMBE and Sureshijani's Z_{tp}^* MBE with the advantages noted earlier. Hagoort's Z_{tp} MBE, although not

Fig. 4 Pressure prediction comparison—Case Study 2: Mid gas B, Set B

Fig. 6 Pressure prediction comparison—Case Study 4: Rich gas D, Set B

Fig. 5 Pressure prediction comparison—Case Study 3: Rich gas C, Set A

Fig. 7 Pressure prediction comparison for Case Study 5: Rich gas C, Set C

consistently the closest to the unit (1) ratio in all these figures, tends to perform slightly better as the reservoir fluid gets enriched.

Two additional figures (Figs. 7, 8) display the combination of rich gas condensate (C and D) with highly mobile condensate (Set C) for the examination of more extreme scenarios with significant condensate mobility (i.e., Case studies 5 and 6 in Table 2). An interesting trend emerges by directly comparing them against Figs. 5, 6 given that the only difference among them is the set of relative permeability curves being used. Hagoort's Z_{tp} MBE (Eq. 3)

predictions worsen when condensate becomes highly mobile. This is an indication that the (Z_{tp}, R_{MLGP}) coupling assumed by Eq. 3 at the given pressure depletion step from CVD data becomes more tenuous as condensate mobility increases. Figures 7 and 8 also show that the proposed density model using surface gas production data only, also closely followed by GMBE and Sureshijani's Z_{tp}^* MBE, proves to be more reliable for the analysis of these higher oil mobility cases. The effect of condensate mobility in the proposed model is introduced by pressure–saturation relationship used when evaluating ρ_g^* in Eq. 13: as condensate

Fig. 8 Pressure prediction comparison—Case Study 6: Rich gas D, Set C

Fig. 9 Average p - S_o relationship for Rich gas D versus CVD and CCE data

mobility increases, the average pressure–saturation profile differs from the one reported by the CVD test. Figure 9 illustrates such condensate mobility effect on the pressure–saturation relationship for Rich Gas D for the cases when condensate is nearly immobile (k_r -curves Set B) and highly mobile (Set C) compared to results reported by numerical simulation. When condensate is close to immobile (Set B), actual average S_o - p tend to more closely follow CVD-reported trends (Fig. 7). As condensate mobilizes (Set C), actual average S_o - p data deviate from CVD trends. Figures 7 and 8 also show that the proposed MBE is less

Fig. 10 Production history—Field case A

affected by this pressure–saturation disparity than the inputs used by Hagoort's Eq. 3.

Case studies: field examples

Two field examples are analyzed in this section to illustrate the applicability of proposed MBE for reserve estimations. Field Case A considers a lean gas condensate reservoir in Algeria presented and studied by Bengherbia and Tiab (2002) using compositional simulation. Starting at an initial pressure of 2683 psia, slightly higher than dewpoint pressure (2619 psia), two-phase flow occurs after a short production time. Reported production history is shown in Fig. 10 and reported original gas- and condensate-in-place are 67.1 BSCF and 1.9 MMSTB, respectively. For the purpose of zero-dimensional MBE analysis, CVD and CCE test data was generated using the Peng Robinson EOS for the reported fluid composition. Results of the analysis using the proposed density-based MBE (Eq. 14) is shown in Fig. 11 as $G_i = 65.1$ BSCF. Original condensate-in-place estimation corresponds to $G_i \times R_{vi} = 1.7$ MMSTB, both estimations showing good agreement with reported values. For comparison purposes, and for the same data, Hagoort's Z_{tp} MBE (Eq. 3) and Vo's CCE-based Z_{tp} MBE (Eq. 7) analysis (not displayed here) yield similar G_i estimations of 65.3 BSCF and 65.5 BSCF, respectively. Field Case B is a gas condensate reservoir located in East Texas reported by Allen and Roe (1950). Reservoir pressure dropped below dew point from the beginning of production. This gas reservoir underwent volumetric depletion throughout its entire production life and was nearly depleted by the end of the reported production history shown in Fig. 12. CVD data for this intermediate gas

Fig. 11 Density-based material-balance analysis—Field case A

Fig. 13 Density-based material-balance analysis—Field case B

Fig. 12 Production history—Field case B

condensate retrograde shows a 15 % maximum liquid saturation dropout. Figure 13 displays the analysis result using the density-based MBE (Eq. 14), from where original gas and condensate resource are calculated to be 23.0 BSCF and 1.4 MMSTB—consistent with ultimate recovery values of 21 BSCF gas and 0.85 MMSTB condensate at an average pressure of 500 psia reported by the authors. For the same data, Hagoort's Z_{tp} MBE (Eq. 3) and Vo's CCE-based Z_{tp} MBE (Eq. 7) analysis (not displayed here) yield similar G_i estimations of 23.0 and 22.3 BSCF, respectively.

Concluding remarks

We have reviewed and summarized all available zero-dimensional material balance models applicable to liquid-rich gas reservoirs and compared their assumptions. Widely used models—GMBE (Eq. 1) and CVD-based two-phase Z-factor (Eq. 3)—are shown to be consistent with overall molar hydrocarbon conservation and associated black-oil PDEs. It is shown why the application of a simplified two-phase Z-factor model (Eq. 7) requires the formulation of a fictitious constant-composition producing stream and CCE-based Z_{tp} data. A density-based MBE (Eq. 14) is proposed on the basis of species (surface gas component) conservation. The proposed MBE is written in term of equivalent gas molar density and cumulative surface gas production, and thus can be visualized as a direct extension of the gas density MBE applicable to dry gas reservoirs. The density-based MBE is shown to be easy to implement, to be readily related to CVD test parameters, and to provide reliable average pressure predictions and reserve estimation for a number of numerical and field case studies. The methodology is consistent with other available modified two-phase Z-factor approaches (Eq. 8), but does not require the definition of two-phase Z-factors and more closely connects to physical quantities (density) and physical principles, directly tracking the evolution of the surface gas-component storage term on the right-hand-side of the surface-gas PDE.

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Appendix 1: Relationship between generalized PDEs and GMBE

Integrating the *surface gas* component PDE in Eq. 17 over the entire reservoir volume V yields:

$$\begin{aligned} & - \iiint_V \nabla \cdot \left(\frac{\mathbf{v}_V}{B_V} + \frac{\mathbf{v}_L}{B_L} R_s \right) dV \\ & = - \iiint_V \nabla \cdot \frac{\partial}{\partial t} \left[\phi \left(\frac{S_V}{B_V} + R_s \frac{S_L}{B_L} \right) \right] dV \end{aligned} \quad (19)$$

Following Gauss divergence theorem, the volume integral of the divergence over the region inside the surface equals the outward flux of the vector through the closed surface. Therefore, the left-hand-side of Eq. 19 yields total surface gas production out of the control volume:

$$\begin{aligned} & - \iiint_V \nabla \cdot \left(\frac{\mathbf{v}_V}{B_V} + \frac{\mathbf{v}_L}{B_L} R_s \right) dV = - \iint_S \left(\frac{\mathbf{v}_V}{B_V} + \frac{\mathbf{v}_L}{B_L} R_s \right) \\ & \cdot \hat{\mathbf{n}} dS = -q_{gsc} \end{aligned} \quad (20)$$

Application of Leibniz's rule to the right-hand-side of Eq. 19 yields:

$$\begin{aligned} & \iiint_V \frac{\partial}{\partial t} \left[\phi \left(\frac{S_V}{B_V} + R_s \frac{S_L}{B_L} \right) \right] dV \\ & = \frac{d}{dt} \iiint_V \left[\phi \left(\frac{S_V}{B_V} + R_s \frac{S_L}{B_L} \right) \right] dV = V_b \frac{d}{dt} \left[\phi \left(\frac{S_V}{B_V} + R_s \frac{S_L}{B_L} \right) \right] \end{aligned} \quad (21)$$

Thus, Equation 19 becomes:

$$-q_{gsc} = V_b \frac{d}{dt} \left[\phi \left(\frac{S_V}{B_V} + R_s \frac{S_L}{B_L} \right) \right] \quad (22)$$

where volumetric average is applied for $\phi \left(\frac{S_V}{B_V} + R_s \frac{S_L}{B_L} \right)$ term. Since tank models assume average value representing

entire reservoir condition, all of these parameters (B_L , B_V , R_s , R_v , S_L and S_V) stated in this study imply average condition and “—” is dropped subsequently. Further integration of Eq. 22 allows eliminating the time-dimension:

$$- \int_0^t q_{gsc} dt = V_b \int_0^t \left[\phi \left(\frac{S_V}{B_V} + R_s \frac{S_L}{B_L} \right) \right] dt \quad (23)$$

which leads to:

$$\begin{aligned} -G_p &= V_b \left[\phi \left(\frac{S_V}{B_V} + R_s \frac{S_L}{B_L} \right) \right]_t - V_b \left[\phi \left(\frac{S_V}{B_V} + R_s \frac{S_L}{B_L} \right) \right]_{t=0} \\ &= V_b \left[\phi \left(\frac{S_V}{B_V} + R_s \frac{S_L}{B_L} \right) \right] - G_i \end{aligned} \quad (24)$$

Thus,

$$(-G_p + G_i) = V_b \phi \left(\frac{S_V}{B_V} + R_s \frac{S_L}{B_L} \right) \quad (25)$$

Following the same procedure for the surface-oil flow equation, Eq. 18 can also be integrated over the entire reservoir volume to yield:

$$\int_0^t q_{osc} dt = V_b \int_0^t \left[\phi \left(\frac{S_L}{B_L} + R_v \frac{S_V}{B_V} \right) \right] dt \quad (26)$$

which leads to the following final expression in terms of cumulative stock-tank oil production:

$$(-N_p + N_i) = V_b \phi \left(\frac{S_L}{B_L} + R_v \frac{S_V}{B_V} \right) \quad (27)$$

In this development, in situ water and water production are neglected but could be included without change in the methodology. Equations 25 and 27 can be written for initial condition as:

$$G_i = V_b \phi_i \left(\frac{S_{Vi}}{B_{Vi}} + R_{si} \frac{S_{Li}}{B_{Li}} \right) \quad (28)$$

$$N_i = V_b \phi_i \left(\frac{S_{Li}}{B_{Li}} + R_{vi} \frac{S_{Vi}}{B_{Vi}} \right) \quad (29)$$

Removing S_L in Eqs. 25 and 27 ($S_V = 1 - S_L$) leads to:

$$V_b \phi = \frac{(G_i - G_p)(B_V - B_L R_v) + (N_i - N_p)(B_L - B_V R_s)}{(1 - R_s R_v)} \quad (30)$$

Removing S_{Li} in Eqs. 28 and 29 ($S_{Vi} = 1 - S_{Li}$) yields:

$$V_b \phi_i = \frac{G_i(B_{Vi} - B_{Li} R_{vi}) + N_i(B_{Li} - B_{Vi} R_{si})}{(1 - R_{si} R_{vi})} \quad (31)$$

If rock compressibility is assumed to be negligible ($\phi_i \cong \phi$), Eqs. 30 and 31 lead to:

Fig. 14 **a** Fluid properties of Lean gas **A**. **b** Fluid properties of Mid gas **B**. **c** Fluid properties of Rich gas **C**. **d** Fluid properties of Rich gas **D**. **e** CVD two-phase Z-factors for fluids A, B, C, and D. **f** CCE two-phase Z-factors for fluids A, B, C, and D

$$\frac{(G_i - G_p)(B_V - B_L R_v) + (N_i - N_p)(B_L - B_V R_s)}{(1 - R_s R_v)} = \frac{G_i(B_{Vi} - B_{Li} R_{vi}) + N_i(B_{Li} - B_{Vi} R_{si})}{(1 - R_{si} R_{vi})} \quad (32)$$

For an initial reservoir pressure equal or above dewpoint pressure:

$$V_b \phi_i = B_{Vi} G_i \quad (33)$$

$$N_i = R_{Vi} G_i \quad (34)$$

Substitution of Eqs. 33 and 34 in 30 yields:

$$B_{Vi} G_i = \frac{(G_i - G_p)(B_V - B_L R_v) + (R_{Vi} G_i - N_p)(B_L - B_V R_s)}{(1 - R_s R_v)} \quad (35)$$

Or,

$$\begin{aligned} N_p(B_L - B_V R_s) + G_p(B_V - B_L R_v) \\ = G_i[R_{Vi}(B_L - B_V R_s) - B_{Vi}(1 - R_s R_v) + (B_V - B_L R_v)] \end{aligned} \quad (36)$$

To relate Eq. 32 to the form proposed by Walsh et al. (1994a), we write:

$$\begin{aligned} N_i &= G_{fgi} R_{vi} + N_{foi} \\ G_i &= G_{fgi} + N_{foi} R_{si} \end{aligned} \quad (37)$$

where G_{fgi} is initial surface gas present in reservoir vapor phase, and N_{foi} is initial surface oil present in reservoir liquid phase. Substitution of Eqs. 37 in 32 results in:

$$\begin{aligned} N_p \frac{B_L(1 - B_V R_{ps}) + B_V(R_{ps} - R_s)}{(1 - R_s R_v)} \\ = G_{fgi} \frac{(B_V - B_{Vi}) + B_L(R_{vi} - R_v) + R_s(B_{Vi} R_v - B_V R_{vi})}{(1 - R_s R_v)} \\ + N_{foi} \frac{(B_L - B_{Li}) + B_V(R_{si} - R_s) + R_v(B_{Li} R_s - B_L R_{si})}{(1 - R_s R_v)} \end{aligned} \quad (38)$$

Equation 38 is identical Walsh et al. (1994a)'s GMBE written in terms of fluid withdrawal and expansion. For an initially undersaturated gas condensate reservoir ($p_i > p_{dew}$), $N_{foi} = 0$ and $G_i = G_{fgi}$; thus Eq. 38 collapse to Eq. 1 as:

$$\begin{aligned} N_p \frac{B_L(1 - B_V R_{ps}) + B_V(R_{ps} - R_s)}{(1 - R_s R_v)} \\ = G_i \frac{(B_V - B_{Vi}) + B_L(R_{vi} - R_v) + R_s(B_{Vi} R_v - B_V R_{vi})}{(1 - R_s R_v)} \end{aligned} \quad (39)$$

And it is the same as Eq. 36 after substituting $R_{ps} = G_p/N_p$.

Appendix 2: Relationship between generalized PDEs and two-phase Z-factor MBE

As shown in Appendix 1, the integration of the surface gas component PDE (Eq. 17) over the entire reservoir volume leads to the total gas flow rate equation in Eq. 22. To derive the two-phase-Z MBE first proposed by Hagoort (1988), and thus showcase its consistency with numerical PDEs and GMBE, we express Eq. 22 in terms of molar conservation by multiplying both sides by the gas molar density at standard condition (ρ_{gsc}):

$$-q_{gsc} \rho_{gsc} = V_b \rho_{gsc} \frac{d}{dt} \left[\phi \left(\frac{S_V}{B_V} + R_s \frac{S_L}{B_L} \right) \right] \quad (40)$$

The integration of Eq. 40 over time yields:

$$\begin{aligned} -n_{gp} &= \int_0^t (q_{gsc} \rho_{gsc}) dt = V_b \left[\phi \rho_{gsc} \left(\frac{S_V}{B_V} + R_s \frac{S_L}{B_L} \right) \right]_t \\ &\quad - V_b \left[\phi \rho_{gsc} \left(\frac{S_V}{B_V} + R_s \frac{S_L}{B_L} \right) \right]_{t=0} \end{aligned} \quad (41)$$

The first term in the right-hand side of this equation represents the total amount of moles of surface gas left in the reservoir at time 't', and the last term represents initial moles of surface gas. Thus, Eq. 41 is just a simple surface-gas molar conservation statement:

$$n_{gp} = n_{gi} - n_g \quad (42)$$

where $n_g = n_g^V + n_g^L$. Following the same development, Eq. 26 can be multiplied by surface oil molar density ρ_{osc} to write:

$$\begin{aligned} -n_{op} &= - \int_0^t (q_{osc} \rho_{ost}) dt = V_b \left[\phi \rho_{ost} \left(\frac{S_L}{B_L} + R_v \frac{S_V}{B_V} \right) \right]_t \\ &\quad - V_b \left[\phi \rho_{ost} \left(\frac{S_L}{B_L} + R_v \frac{S_V}{B_V} \right) \right]_{t=0} \end{aligned} \quad (43)$$

Similarly, this yields the simple molar statement for surface-oil:

$$n_{op} = n_{oi} - n_o \quad (44)$$

where $n_o = n_o^V + n_o^L$. The combination of Eqs. 42 and 44 leads to:

$$n_p = n - n_i \quad (45)$$

where n_p and n_i are produced total moles of hydrocarbon and initial hydrocarbon moles in place, respectively. The definition of the single-phase Z-factor concept, applied to both the vapor and liquid reservoir phases, leads to:

$$pV_V = n_V Z_V RT \quad (46)$$

$$pV_L = n_L Z_L RT \quad (47)$$

which, when applied to the overall two-phase system in terms of total volume ($V = V_V + V_L$) and total moles ($n = n_V + n_L$), allows writing:

$$pV = nZ_{tp}RT \quad (48)$$

where Z_{tp} is the concept of the two-phase Z-factor. Addition of Eqs. 46 and 47 and comparison against 48 reveals:

$$p(V_V + V_L) = (n_V Z_V + n_L Z_L)RT \quad (49)$$

from where the observation $n_V Z_V + n_L Z_L = nZ_{tp}$ follows from the inspection of Eqs. 48 and 49. This leads to the identification of the two-phase compressibility factor as a molar-weighted average of single-phase Z-factors:

$$Z_{tp} = f_{nV} Z_V + f_{nL} Z_L \quad (50)$$

where $f_{nV} = n_V/n$ and $f_{nL} = n_L/n$. For the initial condition, we write:

$$p_i V = n_i Z_{tp,i} RT \quad (51)$$

Finally, substitution of Eqs. 48 and 51 into Eq. 45, when solved in terms n and n_i , yields:

$$\frac{p}{Z_{tp}} = \frac{p_i}{Z_{tp,i}} \left(1 - \frac{n_p}{n_i} \right) \quad (52)$$

Equation 52 is Hagoort's two-phase Z-factor MBE proposed for gas-condensate reservoirs.

Appendix 3: Fluid CVD and CCE data

Four different natural gases of varying degrees of initial volatilized condensate contents were considered in this study: Lean gas A, Mid gas B, Rich gas C, and Rich gas D (see Table 1). CVD B_L , B_V , R_s and R_v data for these fluids are presented in Fig. 14a, b, c, and d, respectively. Z_{tp} data from CVD and CCE tests for the same fluids is presented in Fig. 14e and f, respectively.

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