Applied Water Science (2023) 13:16
https://doi.org/10.1007/s13201-022-01823-z

ORIGINAL ARTICLE

Frequency analysis of extreme daily rainfall over an arid zone of Iran using Fourier series method

Parviz Haghighat jou[1] · Seyed Hassan Mirhashemi[2]

Received: 29 August 2022 / Accepted: 7 November 2022 / Published online: 21 November 2022
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Abstract
In this paper frequency analysis of annual extreme daily rainfall of 14 gauging stations located in an arid zone of Iran were performed using parametric and nonparametric approaches. The parametric methods include normal, two- and three-parameter log-normal, two-parameter gamma, Pearson and log-Pearson type III, extreme value type I (Gumbel), generalized extreme value and generalized logistic distributions. The nonparametric approach is Fourier series method. The data were fitted to all of above mentioned models and the results showed that the goodness of fit of the data to Fourier series is much better than other parametric methods. Thus the Fourier series can be used as an alternative approach for frequency analysis of extreme daily rainfall in an arid zone. In addition, the quantiles can be calculated by the Fourier series.

Keywords Frequency analysis · Extreme daily rainfall · Arid zone · Generalized logistic · Generalized extreme value · Log-Pearson type III · Fourier series method

Introduction

The estimation of annual extreme daily rainfall is essential in the computation of runoff and flood studies, design of water-related structures, dam safety requirements, probable maximum precipitation, drainage system design, soil erosion and sedimentation studies, in agriculture, and monitoring climate change. Events, such as loss of life and property, floods, droughts are caused by changes in the intensity and amount of rainfall (Moazami et al. 2016; Ullah et al. 2018; Rahman and Islam 2019).

In general, frequency analysis of annual extreme daily rainfall is performed using parametric methods in which annual series of data are fitted to a probability distribution function, such as normal, two- and three-parameter log-normal, two-parameter gamma, Pearson and log-Pearson type III, Gumbel or extreme value type I, generalized extreme value and generalized logistic. These methods have been applied in many cases, but have some inconveniences because of not fitting to the observed sample very well, especially in the extreme values. In addition, selection of the best distribution is an important problem. Thus, in this paper Fourier series method as a nonparametric probability distribution function and cumulative distribution function is presented for frequency analysis of extreme daily rainfall. In addition, the Fourier series can be used for calculation of the quantiles.

Kronmal and Tarter (1968) proposed the Fourier series method as a feasible nonparametric approach for the estimation of the probability density and cumulative distribution function. Wu and Woo (1989) applied the Fourier series method to estimate annual flood probability for eight rivers across Canada. Annual flood frequency analyses using Fourier series were compared to results of four parametric and two nonparametric methods. The results show that the Fourier series was the best method for flood frequency analysis. Daily rainfall was accurately predicted by the hybrid SSA-ARIMA-ANN model (Unnikrishnan and Jothiprakash, 2020). The simulated flow was better than the observed rainfall data using interpolation rainfall in the SWAT model (Zhang et al. 2021).

Karmakar and Simonovic (2008) used nonparametric methods based on kernel density estimation and orthonormal

✉ Parviz Haghighat jou
phjou40@gmail.com; parvizhjou@uoz.ac.ir

Seyed Hassan Mirhashemi
hassan.mirhashemi@yahoo.com

1 Department of Water Engineering, Faculty of Soil and Water, University of Zabol, Zabol, Iran

2 Department of Water Engineering, Faculty of Water and Soil, University of Zabol, Zabol, Iran

series to determine the nonparametric distribution functions for peak flow, volume, and duration. They selected the subset of the Fourier series consisting of cosine functions as orthonormal series. It is found that nonparametric method based on orthonormal series is more appropriate than kernel estimation for determining marginal distributions of flood characteristics. The temporal resolution of rainfall time series affects the performance assessment of RWH systems (Zhang et al. 2020).

Haghighat jou et al. (2009) compared seven parametric methods with Fourier series as a nonparametric approach for analyzing frequency of annual precipitation over Iran. It is shown that the nonparametric estimator fitted the real data points better than its parametric counterparts.

Haghighat jou et al. (2013) fitted annual precipitation data from five old rain gauge stations in Iran to nonparametric kernel function using rectangular, triangular, and Gaussian or normal as kernel functions. The findings compared with the results of Haghighat jou et al. (2009) showed that among seven parametric distributions and four nonparametric kernel approaches, Fourier series fitness is the best. Abolverdi and Khalili (2010) analyzed annual maximum daily rainfall at 154 gauging stations in southwest of Iran attempting to develop regional rainfall annual maxima. They proposed generalized logistic and generalized extreme value distribution.

The main objective of the present study is to examine the Fourier series method as an alternative approach for frequency analysis of extreme daily rainfall over arid regions.

Study area

The study area is 181578 km^2, covering province of Sistan and Balouchestan and is located in southeast of Iran. This area is surrounded by northern latitude of 25–31.5 deg and eastern longitude of 58.8–63.3 deg. The climate of the province is hyper-arid and arid. The climate of this province is considered to be BWh by Koppen and Geiger classification.

The province comprises two parts, Sistan in the north and Balouchestan in the south. Sistan is a flat plain formed by Hirmand River alluvium. The 120-day northern wind, which blows from June to September, is a distinguishing feature of this region. The mean annual precipitation in the north of the province is 55 mm.

The southern part is mostly mountainous and borders the Oman Sea from the south. There are two distinct sources of precipitation over the southern part of the province which are westerly winds blowing from Mediterranean Sea. These winds produce rainfall on various parts of the province both in northern and southern parts in winters. Southeasterly Monsoon winds blowing from Indian Ocean sometimes produce considerable amounts of rainfall over this region in summers. The average annual precipitation in the province is 110 mm.

Materials and methods

The extreme daily rainfall data of 16 gauging stations in an arid zone located in southeast of Iran were selected for analysis. Data for two stations were too short to analyze and therefore were omitted. Finally, 14 stations were selected for analysis. These stations include: Zabol, Zahedan, Zahak, Nosrat Abad, Khash, Iranshahr, Bampour, Saravan, Sarbaz, Karvandar, Bahookalat, Ghasreghand, Konarake Chabahar, Chabahar. The record length of these stations range between 17 and 43 years. The data were collected from Meteorological year books of Iran published by Iranian Meteorological Organization. The sample sizes, geographical location, minimum, median and maximum of the annual extreme daily rainfall for each of the stations are given in Table 1.

Basic statistical characteristics such as the mean, the standard deviation, the coefficient of variation, the coefficient of skew and the coefficient of kurtosis were calculated for the annual extreme daily rainfall data of the 14 gauging stations. These statistics are listed in Table 2.

The descriptions of the parametric distributions and parameter estimation methods are not presented in this paper, because they are available in other publications such as (Kite 1988; Haan 1977; Rao and Hamed 2000). Furthermore, the Fourier series method as a nonparametric approach is described in Kronmal and Tarter (1968), Wu and Woo (1989), and Haghighat jou et al. (2009).

In this paper for comparison of the parametric and nonparametric methods, the mean relative deviation (M.R.D) and the mean square relative deviation (M.S.R.D) were used to measure the goodness of fit of above mentioned methods, these statistical terms are defined as follows:

$$\text{M.R.D} = \frac{1}{n}\sum_{i=1}^{n}\frac{\left|x_i - \hat{x}_i\right|}{x_i} \times 100 \quad (1)$$

$$M.S.R.D = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i - \hat{x}_i}{x_i} \times 100\right)^2 \quad (2)$$

where x and $\hat{x}$ are observed and estimated annual extreme daily rainfall, respectively and n is sample size.

Results and discussion

The annual extreme daily rainfall data of 14 mentioned gauging stations were fitted to the nine parametric functions including normal, two and three-parameter

Table 1 Station's name, sample size, geographical location, minimum, median and maximum daily rainfall

Station's Name	Sample size	Lat	Long	Elev	Min	Med	Max
Zabol	39	31° 02'	61 ° 29'	489	4.6	13.3	45
Zahedan	40	29 ° 28'	60 ° 53'	1370	6	15	52
Zahak	17	30° 54'	61 ° 41'	495	4	14	29
Nosrat Abad	31	29 ° 51'	59 ° 58'	1000	5	13	45
Khash	43	28 ° 13'	61 12'	1430	7	31.3	124
Iranshahr	41	27 ° 12'	60 ° 42'	566	4	26	59
Bampour	27	27 ° 12'	60 ° 27'	360	3	21	64
Saravan	24	27 ° 20'	62 ° 20'	1195	4	27.2	59
Sarbaz	34	26 ° 38'	61 ° 16'	880	8.5	32	112
Ghasre ghand	23	26 ° 13'	60 ° 44'	500	8	34	100
Karvandar	40	27 ° 51'	60 ° 46'	1065	8	30.3	76
Bahookalat	34	25 ° 43'	61 ° 25'	120	3	30	121
Konarake Chabahar	26	25 ° 26'	60 ° 22'	12	11	32.2	116
Chabahar	39	25 ° 17'	60 ° 37'	8	10	27	73

Table 2 Basic statistics of the annual extreme daily rainfall data of the stations

Name	Mean	standard deviation	coefficient of variations	coefficient of skew	coefficient of kurtosis
Zabol	15.8	9.3	59	1.465	5.500
Zahedan	18.3	9.6	52	1.753	3.585
Zahak	14.0	6.2	44	0.552	4.952
Nosrat Abad	17.1	9.5	56	1.305	4.724
Khash	34.4	21.6	63	2.008	9.628
Iranshahr	27.8	12.7	46	0.637	3.640
Bampour	22.9	12.6	55	1.528	6.846
Saravan	28.2	12.7	45	0.494	4.009
Sarbaz	34.6	21.3	62	1.624	7.365
Ghasre ghand	38.7	23.0	59	1.325	5.082
Karvandar	34.1	16.3	48	0.803	3.368
Bahookalat	35.4	25.1	72	1.727	6.803
Konarake Chabahar	40.4	26.6	66	1.440	4.922
Chabahar	30.0	14.9	50	1.103	4.400

log-normal, two-parameter gamma, Pearson and log-Pearson type III, extreme value type I (Gumbel), generalized extreme value and generalized logistic distributions and also to the Fourier Series as a nonparametric approach to compare their suitability. The parameters of the parametric distribution functions were estimated by the method of moments. Then the mean relative deviation (M.R.D) and the mean square relative deviation (M.S.R.D) test statistics were calculated to measure the goodness of fit of the above mentioned parametric and nonparametric functions and comparing their ability to fit to the observed data.

As Table 3 shows the values of the mean relative deviation (M.R.D) and the mean square relative deviation (M.S.R.D) test statistics for Fourier series is significantly less than comparing with these values for the parametric distribution functions**.** Thus, fitness of the data to Fourier series is much better than other parametric distribution functions.

Also, Table 3 shows, for frequency analysis of annual extreme daily rainfall data of 14 stations using parametric distribution functions**,** the data of each station fits to a given distribution function, and one needs to apply eight different distributions for analyzing frequency of data of the stations, for example for Zabol, Konarake- Chabahar and Chabahar the suitable distribution is log-Pearson type III, for Zahedan, Bampour and Sarbaz the suitable distribution is three-parameter log-normal, for Zahak and Saravan, Normal distribution, for Karvandar and Bahookalat, generalized extreme value distribution and for Ghasre-ghand, Iranshahr, Khash and Nosratabad, extreme value type I or Gumbel, generalized logistic, two-parameter gamma and Pearson type III, are suitable distributions, respectively. But, on the other hand,

Table 3 Values of the M.R.D and M.S.R.D test statistics for the parametric distribution functions and Fourier series method

Station	Parametric function	M.R.D.[1]	M.S.R.D.[1]	M.R.D.[2]	M.S.R.D.[2]
Zabol	LP III	4.7686	40.5312	2.2394	12.2158
Zahedan	LN3	5.6720	50.1547	2.3377	16.8159
Zahak	Normal	4.8526	41.7297	3.4858	33.5286
Nosrat Abad	P III	6.2304	56.3059	2.7291	20.4812
Khash	Gamma 2	8.6258	117.8482	2.0650	6.5569
Iranshahr	GLO	5.3116	49.7105	1.7853	14.1892
Bampour	LN3	5.7797	53.2905	2.5084	16.2676
Saravan	Normal	5.8723	129.1046	2.7916	31.1012
Sarbaz	LN3	6.8392	81.1188	2.5594	15.9699
Ghasre- ghand	EV I	5.9573	50.5302	3.8187	38.3146
Karvandar	GEV	4.7606	29.9585	1.4302	4.5987
Bahookalat	GEV	12.4288	440.7500	3.9002	78.7554
Konarake Chabahar	LP III	7.5850	83.5648	2.4298	20.7805
Chabahar	LP III	4.3563	32.0033	1.6599	9.2696

[1]These values are for the parametric distribution functions as listed in column 2 of the Table

[2]These values are for the Fourier series method

the results show that all of the datasets fit to Fourier series very well and this method performs as a unique and suitable approach for frequency analysis of all of the datasets.

Quantiles for 2, 5, 10, 20, 25, 50, 100 and 200 years as return period are listed in Tables 4 and 5 for both the best parametric method and the Fourier series method. Comparison of Tables 4 and 5 show that the quantiles estimated by the Fourier series method are less sensitive to both very low (these are not tabulated) and very high return periods compared to the parametric methods. In addition, the ratio of the quantiles with return period of 200 year to 100 year and 100 year to 50 year due to parametric methods are higher than that of obtained by Fourier series method.

The results of the previous studies also show that this method performs very well for frequency analysis of observed data of monthly and annual precipitation (Haghighat jou et al. 2009, 2013) and annual floods (Wu and Woo 1989).

It seems that none of the parametric methods (for example generalized extreme value and generalized logistic distributions as cited by Abolverdi and Khalili 2010) can

Table 4 Quantiles calculated by the best parametric method mentioned in Table 3

Return period, years								
	2	5	10	20	25	50	100	200
Station	Probability of exceedance (percent)							
	50	20	10	5	4	2	1	0.5
Zabol	13.3	21.5	28.0	34.9	37.3	45.1	53.7	63.2
Zahedan	16.2	24.5	30.4	36.4	38.4	44.6	51.1	57.8
Zahak	14.0	19.3	22.0	24.3	25.0	26.9	28.6	30.1
Nosrat Abad	14.8	23.5	29.7	35.7	37.6	43.5	49.5	55.4
Khash	30.0	49.9	63.1	75.6	79.5	91.5	103.3	114.9
Iranshahr	27.0	37.0	43.3	49.4	51.4	57.6	64.1	70.8
Bampour	20.1	31.1	38.9	46.7	49.2	57.3	65.6	74.2
Saravan	28.2	38.9	44.4	49.0	50.4	54.2	57.7	60.9
Sarbaz	30.1	48.7	61.8	74.9	79.2	92.6	106.4	120.7
Ghasre ghand	34.9	55.2	68.7	81.6	85.7	98.3	110.8	123.3
Karvandar	32.0	46.6	55.7	64.1	66.7	74.4	81.7	88.7
Bahookalat	31.2	52.3	67.5	83.1	88.3	104.8	122.4	140.9
Konarake Chabahar	32.1	54.9	74.7	97.8	106.1	134.8	168.5	208.3
Chabahar	26.9	40.5	50.0	59.5	62.5	72.1	81.9	92.0

Table 5 Quantiles calculated by Fourier series method

Return period, years								
	2	5	10	20	25	50	100	124.8
Station	Probability of exceedance (percent)							
	50	20	10	5	4	2	1	0.5
Zabol	13.4	22.7	27.0	40.5	40.9	44.7	45.1	45.3
Zahedan	14.9	24.3	29.2	41.4	41.9	51.6	52.1	52.4
Zahak	13.9	18.0	20.6	28.5	28.8	29.2	29.5	29.7
Nosrat Abad	13.5	24.5	31.0	36.9	37.3	44.9	45.3	45.5
Khash	30.4	49.6	55.5	78.2	80.4	123.0	124.2	124.80
Iranshahr	26.1	33.8	49.0	55.9	56.8	58.4	59.1	59.5
Bampour	21.6	29.1	36.5	46.8	50.3	63.9	64.5	64.8
Saravan	26.8	38.3	43.9	50.5	51.7	59.1	59.7	60.0
Sarbaz	31.8	51.5	61.7	63.8	64.3	111.5	112.4	112.9
Ghasre ghand	33.8	51.1	72.1	87.6	98.1	100.2	101.2	101.7
Karvandar	30.6	49.5	61.2	67.2	67.8	75.4	76.2	76.6
Bahookalat	31.0	43.1	72.2	91.1	91.7	120.3	121.4	122.1
Konarake Chabahar	31.9	60.3	81.3	91.0	106.2	115.9	116.9	117.4
Chabahar	27.0	40.8	50.3	63.4	64.3	72.5	73.2	73.5

perform so well as the Fourier series method for frequency analysis of annual extreme daily rainfall data.

Because of fitting the floods and annual precipitation data to Fourier series very well, this method can be generalized for frequency analysis both in hydrology, meteorology and water resources engineering (Wu and Woo 1989; Haghighat jou et al. 2009). Application of the Fourier series for frequency analysis of various datasets has many advantages over parametric methods, especially, in small samples, parameters are not precisely estimated and this leads to large errors in quantile estimation. Thus, applying Fourier series for frequency analysis is justified for reliable estimation of quantiles.

Conclusion

One of the most important findings of the current study is that by applying the Fourier series method, there is no need for regionalization in order to frequency analysis of the data. As the results of this study show, observed data for all of the 14 stations fit to the Fourier series very well. Furthermore, both small and large samples fit to Fourier series very well. The sample sizes of the data used in this study are between 17 and 43. It must be noted that even the low and high outliers or extreme values in each dataset, also fit to the Fourier series very well.

By applying the nonparametric Fourier series method the problems encounter in selecting a parameter estimation method, which sometimes are confusing tasks, are excluded. When applying parametric methods, one can't choose a unique probability distribution function for frequency analysis of all of the datasets or observations, but all of the observed datasets of the 14 stations located in an arid zone of Iran, fit to the Fourier series very well, which is an important and excellent feature of this method.

Thus, the Fourier series method is easy to apply and it is a suitable, robust and alternating nonparametric approach for frequency analysis of annual extreme daily rainfall in the arid regions.

Acknowledgements This paper has been provided by supporting and financing of the vice chancellor of research of the University of Zabol by Grant no. IR-UOZ-GR- 0303. The authors are grateful to all of the individuals and staffs of the research unit of University of Zabol.

Declarations

Conflict of interest The corresponding author states that there is no conflict of interest.

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