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ORIGINAL ARTICLE

Comparison of cubic, quadratic, and quintic splines for soil erosion modeling

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Abstract
Approximate curve is constructed using quadratic, quintic, and cubic splines and examination between these splines. The point of this construction is to predict sediment yield index (SYI) corresponding to curve number. This strategy is outlined with a contextual analysis of Manot watershed of Narmada Basin, India. The relation among calculated SYI and observed SYI esteems is associated with a coefficient of determination (R^2) of 0.36 and 0.48 for the corresponding quadratic and quintic splines, while the cubic spline showed R^2 of 0.87 (Meshram et al. Arab J Geosci 10:155–168, 2017b; Appl Water Sci 7:1773–1779, 2017c). Numerical results seemed to indicate that the cubic spline method is more accurate than the quadratic/quintic spline method.

Keywords Sediment · Curve number · Quadratic spline · Quintic spline

Introduction

Rainfall–runoff–sediment yield modeling, being highly complex, dynamic, and nonlinear, exhibits temporal and spatial variability and comprises several physical processes (Meshram et al. 2017a, b, c). Varying complexity from lumped empirical to physically based space and time-distributed, several models are available in the literature to model runoff and subsequent soil erosion/sediment yield, while physically based models have demonstrated exceptionally helpful as an exploration device, yet they are of restricted use in field, particularly in creating nations like India as they require expansive measure of information.

However, search is still continuing for developing new and simple model. In the present research work, an attempt is therefore made to explore new/modified or improved techniques to model major components of the rainfall–runoff–sediment yield process in a more sound theoretical and mathematical environment. However, in the meantime, simplicity of model structure, practical utility in terms of data requirement as well as easiness in use, and parsimony in data, time, and funds are central to the present study. The present research investigates a few important components of process of the hydrological cycle, specifically the process of rainfall–runoff–sediment yield including evapotranspiration, runoff, soil erosion, and sediment yield.

Watersheds or hydrological units are viewed as more proficient and suitable for important overview and examination for the evaluation of these resources and consequent planning and application of different formative projects like soil and water conservation, order territory advancement, disintegration control in catchment waterways, dryland/rainfed cultivating, recovery of gorge lands, and so forth (Gajbhiye and Sharma 2017).

The definition of appropriate watershed management programs for economic advancement requires a stock of the quantitative soil loss, erosion, and need of watershed. There might be different contemplations for the execution of management programs in the few sub-watersheds. It is constantly better to start management measures from the most fundamental sub-watersheds. Prioritization of watershed is the positioning of various basic sub-watersheds along these lines as indicated by the request in which they should be

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taken up for the treatment by soil and water management measures (Ambade 2012).

Sediment yield is one of the primary rules for prioritization of watershed to soil erosion. Be that as it may, this model requires constant observation of sediment test at the catchments' outlet. In India for small watersheds, such information is not really available (Gajbhiye et al. 2014a, b; Gajbhiye et al. 2015; Meshram et al. 2017c; Meshram et al. 2018). In spite of the fact that the sediment yield from large basin can be found from such observations, it is not conceivable to find out the reason for soil loss of small watershed inside a basin. A soil protection program is a costly and bulky procedure, passed out in steps beginning from the most critical area. In this way, there is a requirement to give comparative needs to various areas inside a catchment (Gajbhiye 2015).

The curve number (CN) represents the runoff which is basic for the arrangement of soil erosion control processes. The curve number is at risk to spatial and transient variety (Gajbhiye et al. 2014a, b; Meshram et al. 2017a, b, c). The SCS-CN technique introduced in 1954 processes the surface runoff volume for a given precipitation event from little watersheds (SCS 1956, 1985), which is required for figuring soil erosion.

Spline functions and, all the more by and large, piecewise polynomial functions are the best approximating capacities being used today. A large number of studies have been conducted on the utilization of cubic, quadratic, and quintic splines at territorial and national levels (McAllister and Roulie 1978; Sakai and Usmani 1983; Siddiqi and Akram 2008; Wu and Zhang 2014; Rababah et al. 2017; Herriot and Reinsch 1976; Alayed et al. 2016; Yang and Wang 1994; Holnicki 1996: Tariq and Akram 2016; Meshram et al. 2017a, b, c; Meshram and Powar 2017). Use of spline function is expanding step by step in the areas of different sciences, medicine, farming, and engineering. The cubic spline is the most generally and broadly utilized by spline function; the quadratic spline is also well considered. Thusly, the target of this paper is to apply quadratic and quintic splines to soil erosion modeling and to compare with the cubic spline.

Study area

In order to develop the model, we have taken the data of sediment yield index (SYI) and curve number (CN) from the previous studies of Meshram et al. (2017a, b, c).

Primilaries of splines

Quadratic spline

To infer a scientific model of a quadratic spline, assume the information is $\left\{\left(z_i f_i\right)\right\}_{i=0}^{n}$ where, as for linear splines,

$a = z_0 < z_1 < \cdots\cdots\cdots\cdots z_n = b \quad h \equiv \max\left|z_i - z_{i-1}\right|.$

A quadratic spline $S_{2,n}(z)$ is a C^1 piecewise quadratic polynomial. This implies:$S_{2,n}(z)$ is piecewise quadratic; that is, among continuous knots z_i

$$S_{2,n}(z) = \begin{cases} p_1(z) = a_1 + b_1 z + c_1 z^2, z \in \left[z_{0,} z_1\right] \\ p_2(z) = a_2 + b_2 z + c_2 z^2, z \in \left[z_{1,} z_2\right] \\ \vdots \\ p_n(z) = a_n + b_n z + c_n z^2, z \in \left[z_{n-1,} z_n\right] \end{cases} \tag{1}$$

$S_{2,n}(z)$ is C^1; that is,$S_{2,n}(z)$ is consistent and has ceaseless first subsidiary wherever in the interim [a, b], specifically, at the knots.

For $S_{2,n}(z)$ to be an interpolatory quadratic spline, we should likewise have $S_{2,n}(z)$ which inserts the information, that is,

$$S_{2,n}\left(z_i\right) = f_i, \quad i = 0, 1, \ldots\ldots.., n. \tag{2}$$

Inside every interim $\left(z_{i-1} z_i\right)$, the comparison of quadratic polynomial is ceaseless and has persistent subordinates of all requests. Accordingly, $S_{2,n}(z)$ or one of its subsidiaries can be irregular just at a bunch. Note that the capacity $S_{2,n}(z)$ has two quadratic parts occurrence at the inside knot z_i; to one site of z_i, it is a quadratic $p_i(z)$, while to the correct it is a quadratic $p_{i+1}(z)$.

Hence, an essential and adequate situation for $S_{2,n}(z)$ to have consistent first subordinate is for these two quadratic polynomials episode at the inside bunch to coordinate in first subsidiary esteem. So we have an arrangement of evenness situations: that is, at every inside knot,

$$p_i'\left(z_i\right) = p_{i+1}'\left(z_i\right), \quad i = 1, 2, \ldots\ldots\ldots, n. \tag{3}$$

Furthermore, to insert the information, we have a set of addition conditions: that is, on the ith interim,

$$p_i\left(z_{i-1}\right) = f_{i-1}, \, p_i\left(z_i\right) = f_i, \quad i = 1, 2, \ldots\ldots, n. \tag{4}$$

Along these lines of composing, the insertion conditions additionally power $S_{2,n}(z)$ to be nonstop at the bunches. Since each of the n quadratic pieces has three obscure coefficients, our portrayal of the capacity $S_{2,n}(z)$ includes $3n$ obscure coefficients. Guaranteeing coherence of the principal subsidiary forces $(n-1)$ direct imperatives on its coefficients, and interjection forces an extra $2n$ straight limitations. In this way, there is an aggregate of $3n-1$ straight requirements on the $3n$ obscure coefficients. All together, we have an indistinguishable number of conditions from questions and we require one progressive (straight) limitation.

Quintic spline

Let $z_i = ih\left(i = 0, 1, \dots n, \quad h = L_{n,} n > 0\right)$ be framework purposes of the uniform segment of [0, *L*] into the subintervals $\left[z_{i-1} z_i\right]$. Let $u(z)$ be an adequately differentiable capacity characterized on [0, *L*] and *S* be a quin (*z*) tic polynomial spline to $u(z)$. Consider that every quintic polynomial spline section $P_i(z)$ has the accompanying structure:

$$P_i(z) = a_i\left(z - z_i\right)^5 + b_i\left(z - z_i\right)^4 + c_i\left(z - z_i\right)^3 + d_i\left(z - z_i\right)^2 + e_i\left(z - z_i\right) + f_i \quad (5)$$

$i = 0, 1, \dots \dots, n - 1$, alongside the necessity that

$$P_i(z) \in c^4[0, L] \quad (6)$$

$$S(z) = P_i(z), \forall z\left[z_i z_{i+1}\right], \quad i = 0, 1, \dots \dots \dots, n - 1. \quad (7)$$

To build up the uniformity relations among the estimations of spline and its subordinates at ties, let

$$\begin{aligned} P_i\left(z_i\right) &= S_i, \quad P_i\left(z_{i+1}\right) = S_{i+1} \\ P_i^2\left(z_i\right) &= M_i, \quad P_i^2\left(z_{i+1}\right) = M_{i+1} \\ P_i^4\left(z_i\right) &= F_i, \quad P_i^4\left(z_{i+1}\right) = F_{i+1}. \end{aligned} \quad (8)$$

It is to be seen that the spline *S* can be composed as far as $S_i s$ and any three derivatives at the limits of each subinterval. To characterize spline as far as $S_i s$ and $F_i s$, the coefficients presented in Eq. (5) are figured as:

$$a_i = \frac{1}{120h}\left(F_{i+1} - F_i\right)$$

$$b_i = \frac{1}{24} F_i$$

$$C_i = 16h\left(M_{i+1} - M_i\right) - h36\left(F_{i+1} + 2F_i\right)$$

$$d_i = 12M_i$$

$$e_i = 1h\left(S_{i+1} - S_i\right) - h6\left(M_{i+1} + 2M_i\right) + h360\left(7F_{i+1} + 8F_i\right)$$

$$f_i = S_i. \quad (9)$$

Applying the first and third subordinate coherencies at the knots,

$$P_i^\rho\left(z_i\right) = P_{i-1}^\rho\left(z_i\right), \quad (10)$$

where $\rho = 1$ and 3, and the accompanying helpful relations are gotten as

$$M_{i+1} + 4M_i + M_{i-1} = 6h2\left(S_{i+1} - 2S_i + S_{i-1}\right) + h260\left(7F_{i+1} + 16F_i + 7F_{i-1}\right) \quad (11)$$

$$M_{i+1} - 2M_i + M_{i-1} = h26(F_{i+1} + 4F_i + F_{i-1}. \quad (12)$$

Utilizing conditions in Eqs. (11) and (12), the accompanying consistency connection regarding the fourth subordinate of spline F_i and S_i $i = 0, 1, \dots \dots \dots, n$ is determined as:

$$S_{i+2} - 4S_{i+1} + 6S_i - 4S_{i-1} + S_{i-2} = \frac{h^4}{120}\left(F_{i+2} + 26F_{i+1} + 66F_i + 26F_{i-2} + F_{i-2}\right), \quad (13)$$

for $i = 2, 3, \dots \dots, n - 2$. As Eq. (13) gives $(n-3)$ straight arithmetical conditions in the $(n - 1)$ unknowns $\left(S_{i,} i = 1, 2, \dots \dots, n - 1\right)$, along these lines two more conditions (end conditions) are required. The two end situations can be gotten utilizing Taylor arrangement and the technique for undetermined coefficients. Two end situations are:

$$-2S_0 + 5S_1 - 4S_2 + S_3 = -h2M_0 + \frac{h^4}{120}\left(18F_0 + 65F_1 + 26F_2 + F_3\right) \quad (14)$$

and

$$S_{n-3} - 4S_{n-2} + 5S_{n-1} - 2S_n = -h^2 M_n + \frac{h^4}{120}(F_{n-3} + 26F_{n-2} + 65F_{n-1} + 18F_n. \quad (15)$$

Construction of splines

The SYI demonstrate, in view of the spline, was adjusted utilizing the information of Manot watersheds (Table 1). The sub-watersheds 4, 5, 7, 8, 9, 12, 13 were utilized for spline development, and the rest of the watersheds were utilized for the approval of the spline approximation.

We built the quadratic and quintic splines as discussed in "Quadratic spline" section for the Manot watershed as:

Table 1 Manot dataset utilized in the present study for quadratic/quintic spline

Sub-watershed	SYI	CN
MN 1	991.61	73.87
MN 2	987.42	77.73
MN 3	986.82	72.03
MN 4	1032.24	84.95
MN 5	1173.33	79.51
MN 6	1088.42	78.23
MN 7	1031.08	64.75
MN 8	1099.92	59.36
MN 9	940.97	76.71
MN 10	1030.19	66.42
MN 11	1347.6	76.71
MN 12	1238.36	67.84
MN 13	1007.85	73.76
MN 14	925.38	74.15

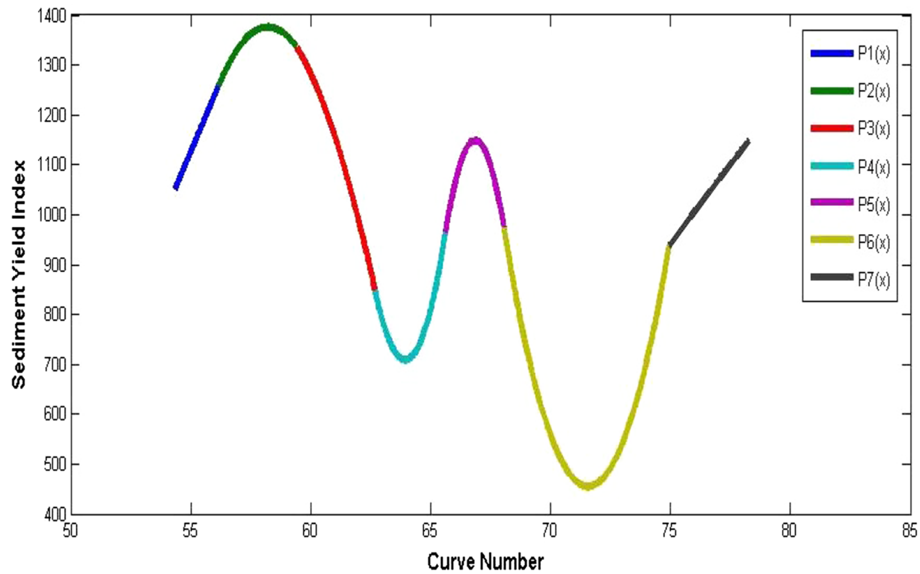

Fig. 1 Quadratic splines approximation of given dataset

Consider the CN value esteems as nodal focuses and SYI value esteems as information focuses. Here, the nodal focuses are organized as:

$n_1 < n_2 < n_3 < n_4 < n_5 < n_6 < n_7$.

Given information at the knots $\delta(n_1), \delta(n_2), \delta(n_3), \delta(n_4), \delta(n_5), \delta(n_6), \delta(n_7)$, we built a piecewise quadratic polynomial.

Quadratic spline

We decided the polynomial pieces as discussed in "Quadratic spline" section. The piecewise quadratic polynomials appeared in Fig. 1 are:

For the interval [59.36–64.75]

$$d_1(y) = 1858.0540 - 12.7718x + 0x^2. \tag{16}$$

For the interval [64.75–67.84]

$$d_2(y) = 110203.5103 - 3359.3496x + 25.8423x^2. \tag{17}$$

For the interval [67.84–73.76]

$$d_3(y) = -153227.6271 + 4406.8986x - 31.3971x^2. \tag{18}$$

For the interval [73.76–76.71]

$$d_4(y) = 390381.6870 - 10333.0503x + 68.5212x^2. \tag{19}$$

For the interval [76.71–79.51]

$$d_5(y) = -215587.4299 + 5465.9098x - 34.4573x^2. \tag{20}$$

For the interval [79.51–84.95]

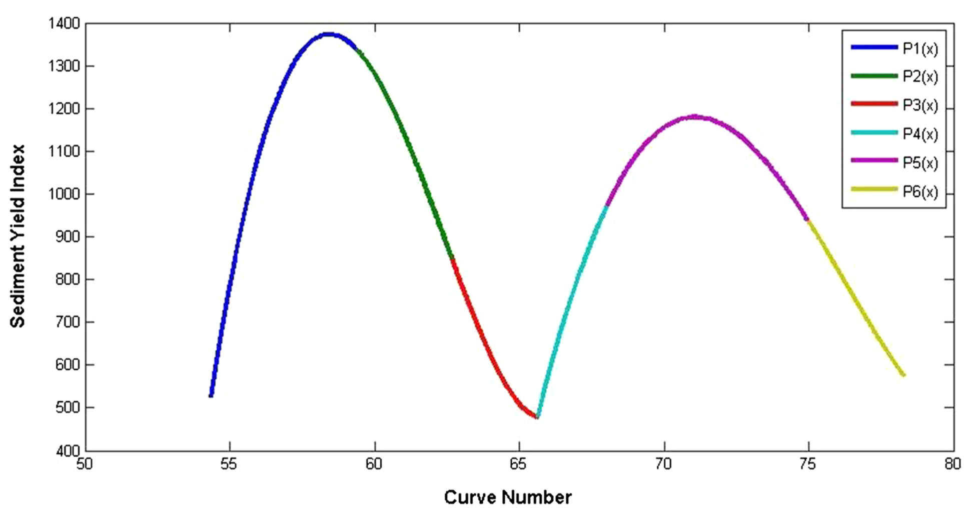

Fig. 2 Quintic splines approximation of given dataset

Table 2 Values of the coefficient and constructed quadratic spline

CN	Coefficient			Splines
	C_1	C_2	C_3	
59.36–64.75	0.0000	−12.7718	1858.0540	$p_1(x) = 1858.0540 - 12.7718x + 0x^2$
64.75–67.84	25.8423	−3359.3496	110,203.5103	$p_2(x) = 110203.5103 - 3359.3496x + 25.8423x^2$
67.84–73.76	−31.3971	4406.8986	−153,227.6271	$p_3(x) = -153227.6271 + 4406.8986x - 31.3971x^2$
73.76–76.71	68.5212	−10,333.0503	390,381.6870	$p_4(x) = 390381.6870 - 10333.0503x + 68.5212x^2$
76.71–79.51	−34.4573	5465.9098	−215,587.4299	$p_5(x) - 215587.4299 + 5465.9098x - 34.4573x^2$
79.51–84.95	−2.2869	350.1714	−12,211.2477	$p_6(x) = -12211.2477 + 350.1714x - 2.2869x^2$

$$d_6(y) = -12211.2477 + 350.1714x - 2.2869x^2. \tag{21}$$

Quintic splines

We decided the polynomial pieces as discussed in "Quintic spline" section. The piecewise quintic polynomials, shown in Fig. 2, are:

For the interval [59.36–64.75]

$$d_1(y) = -177.4029 - 2292.8540x + 168.2128x^2 - 4.5962x^3 + 0.0554x^4 - 0.0002x^5. \tag{22}$$

For the interval [64.75–67.84]

$$d_2(y) = 120.8112 + 1238.0507x - 12.7640x^2 - 1.2753x^3 + 0.0302x^4 - 0.0002x^5. \tag{23}$$

For the interval [67.84–73.76]

$$d_3(y) = 91.5073 + 1240.5799x - 12.8494x^2 - 1.2739x^3 + 0.0302x^4 - 0.0002x^5. \tag{24}$$

For the interval [73.76–76.71]

$$d_4(y) = 1.2750 + 20.4722x + 35.6846x^2 - 1.6269x^3 + 0.0239x^4 - 0.0001x^5. \tag{25}$$

For the interval [76.71–79.51]

$$d_5(y) = -0.1627 - 0.9618x + 36.8027x^2 - 1.6488x^3 + 0.0241x^4 - 0.0001x^5. \tag{26}$$

For the interval [79.51–84.95]

$$d_6(y) = -0.9337 - 13.9050x + 37.4543x^2 - 1.6611x^3 + 0.0242x^4 - 0.0001x^5. \tag{27}$$

Sediment yield index (SYI) approximation validation

For justification, the CN value and the above parameters (C_1–C_7) (Tables 2, 3) were utilized in Eqs. 16–27 for the calculation of SYI. This figured SYI named as computed sediment yield index (SYI_C), and it was compared with the conventionally derived sediment yield index named as observed sediment yield index (SYIo). The observed SYI values are plotted against the computed SYI (Figs. 3, 4) and compared through a line of perfect fit. The coefficient of determination (R^2) is appeared in Table 4. It is observed that the quadratic and quintic splines have not predicted well the SYI from CN.

Inter-comparison of models

In order to compare the applied spline (quadratic/quintic) with each other and another spline, i.e., cubic spline (Meshram et al. 2017a, b, c), Table 4 shows the R^2 criteria of the selected splines and cubic spline. According to Table 4, it is evident that, as it is expected, the cubic spline was the best model based on coefficient of determination (i.e., R^2). This confirms that the cubic spline can be a powerful tool for sediment estimation at daily scales. The quintic spline (with a value of $R^2 = 0.48$) and the quadratic spline (with a value of $R^2 = 0.36$) ranked as the second and the third best models, respectively.

Conclusion

We anticipate the rough estimation of SYI of a given set of CN ahead of time without doing any examination. What's more, we have settled the issue of labor and time limitation up to some degree. In this way, the SYI (observed) information of Manot watershed shows a solid relationship with SYI inferred utilizing cubic spline ($R^2 = 0.87$), and some connection with SYI determined utilizing the quadratic spline ($R^2 = 0.36$) contrasted with quintic spline ($R^2 = 0.48$). Numerical outcomes appeared to demonstrate that the cubic spline technique is more exact than the quadratic/quintic spline strategy.

Table 3 Values of the coefficient and constructed quintic spline

CN	Coefficient						Splines
	C_1	C_2	C_3	C_4	C_6	C_7	
59.36–64.75	−0.0002	0.0554	−4.5962	168.2128	−2292.8540	−177.4029	$p_1(x) = -177.4029 - 2292.8540x + 168.2128x^2 - 4.5962x^3 + 0.0554x^4 - 0.0002x^5$
64.75–67.84	−0.0002	0.0302	−1.2753	−12.7640	1238.0507	120.8112	$p_2(x) = 120.8112 + 1238.0507x - 12.7640x^2 - 1.2753x^3 + 0.0302x^4 - 0.0002x^5$
67.84–73.76	−0.0002	0.0302	−1.2739	−12.8494	1240.5799	91.5073	$p_3(x) = 91.5073 + 1240.5799x - 12.8494x^2 - 1.2739x^3 + 0.0302x^4 - 0.0002x^5$
73.76–76.71	−0.0001	0.0239	−1.6269	35.6846	20.4722	1.2750	$p_4(x) = 1.2750 + 20.4722x + 35.6846x^2 - 1.6269x^3 + 0.0239x^4 - 0.0001x^5$
76.71–79.51	−0.0001	0.0241	−1.6488	36.8027	−0.9618	−0.1627	$p_5(x) = -0.1627 - 0.9618x + 36.8027x^2 - 1.6488x^3 + 0.0241x^4 - 0.0001x^5$
79.51–84.95	−0.0001	0.0242	−1.6611	37.4543	−13.9050	−0.9337	$p_6(x) = -0.9337 - 13.9050x + 37.4543x^2 - 1.6611x^3 + 0.0242x^4 - 0.0001x^5$

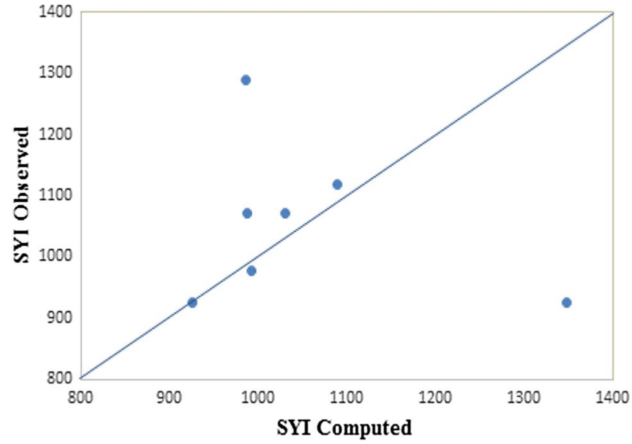

Fig. 3 Scatter plot between predicted and actual SYI

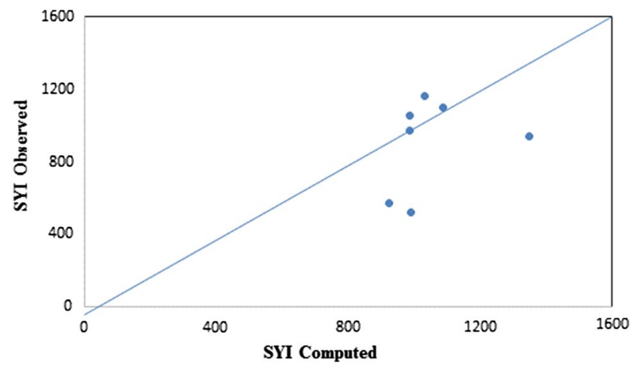

Fig. 4 Scatter plot between predicted and actual SYI

Table 4 Performance evaluation of splines

Spline	R^2
Quadratic spline	0.36
Quintic spline	0.48
Cubic spline	0.87

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