PAPER IN PHILOSOPHY OF THE NATURAL SCIENCES

On the status of quantum tunnelling time

Grace E. Field[1] ⬤

Abstract

How long does a quantum particle take to traverse a classically forbidden energy barrier? In other words, what is the correct expression for quantum tunnelling time? This seemingly simple question has inspired widespread debate in the physics literature. I argue that we should not expect the orthodox interpretation of quantum mechanics to provide a unique correct expression for quantum tunnelling time, because to do so it would have to provide a unique correct answer to a question whose assumptions are in tension with its core interpretational commitments. I explain how this conclusion connects to time's special status in quantum mechanics, the meaningfulness of classically inspired concepts in different interpretations of quantum mechanics, the prospect of constructing experimental tests to distinguish between different interpretations, and the status of weak measurement in resolving questions about the histories of subensembles.

1 Introduction

In both classical and quantum physics, it is easy to calculate the time taken by a free particle to travel from point A to point B. Motivated by this simple picture, a whole community of quantum theorists have asked: how long does a quantum particle take to traverse a classically forbidden energy barrier? Or, in their terminology, *what is the correct expression for quantum tunnelling time*? Unlike its free-particle counterpart, this question does not seem to admit a straightforward answer, and has inspired widespread debate in the literature.

✉ Grace E. Field
gef30@cam.ac.uk

[1] Department of History and Philosophy of Science, University of Cambridge, Free School Lane, Cambridge, CB2 3RH, UK

Various expressions for quantum tunnelling time have been proposed. Some track internal properties of the tunnelling system, while others rely on coupling between the tunnelling particle and an external physical system. The proposals are weighed against each other on mostly pragmatic grounds, and tunnelling time, as a result, is often described as a topic riddled with "confusion" and "controversy" (see e.g. Sokolovski and Connor 1993, 4677 and Winful 2006, 1).

Many authors see this ambiguity in the definition of tunnelling time as part of the fallout from how quantum mechanics treats time in general: as a parameter, not an operator (see e.g. Challinor et al. 1997; Lunardi et al. 2011 and Peres 1980). Others have gone so far as to describe the whole concept as meaningless within the trajectory-free 'orthodox' interpretation of quantum mechanics (see e.g. Dumont & Marchioro 1993; Leavens & Aers 1996 and Leavens 1993): tunnelling time does have a clear and unique definition within the deterministic de Broglie-Bohm 'pilot-wave' picture. A few authors have even speculated about whether quantum tunnelling time could thereby provide evidence for de Broglie-Bohm theory (see e.g. Cushing 1995; Acuña 2019 and Leavens 1990).

Against this varied backdrop, recent high-profile experiments at the University of Toronto claim to have used weak measurement to extract a traversal time of 0.61ms for Bose-condensed ^{87}Rb atoms tunnelling through a 1.3-micrometre-thick optical barrier (Ramos et al., 2020).

The state of the literature on quantum tunnelling time therefore motivates four questions of both physical and philosophical import. First, does the confusion about tunnelling time really stem from the more general "problem of time" in quantum mechanics—namely, the fact that time lacks an operator? Second, is tunnelling time really a meaningless concept in the orthodox interpretation of quantum mechanics? If so, why, and in what sense? Third, is it possible to use quantum tunnelling time to gain concrete evidence for the de Broglie-Bohm interpretation? And finally, are weak measurement techniques—and experimental realisations thereof—able to resolve the decades-long debate and provide clear, unique and experimentally verifiable values for tunnelling time?

This paper aims to answer each question in turn. Section 2 sets up the problem, describing the physical scenario at the basis of the tunnelling time debate, and identifying in precise terms the question to which its proponents are seeking an answer. Section 3 reviews the history of quantum tunnelling time and the state of the existing literature, from the concept's inception in 1931 to 1962 (Section 3.1) and from 1962 to the present (Section 3.2).

In Section 4.1, I assess the status of quantum tunnelling time in the trajectory-free 'orthodox' interpretation of quantum mechanics, and I argue that we should not expect that interpretation (or, in fact, any trajectory-free interpretation) to provide a clear or unique definition of quantum tunnelling time. In Section 4.2, I introduce the clear and unique definition of quantum tunnelling time that is provided by the de Broglie-Bohm interpretation, and I explain why we should not be surprised to find such clarity within an interpretation that provides particle trajectories.

Answers to the four questions follow in Section 5. As to whether the apparent confusion and ambiguity surrounding tunnelling time can be traced back to the more general problem of time in quantum mechanics, I argue 'No': the real source of

the ambiguity is superposition, specifically the role that superposition plays in any trajectory-free interpretation (Section 5.1). As to whether tunnelling time is meaningless in the orthodox interpretation, I argue that it is an attempt to answer a question which the orthodox interpretation is not well-posed to answer (Section 5.2). On the third question—whether we can or should use quantum tunnelling time as evidence for the de Broglie-Bohm interpretation—I argue that we cannot and should not (Section 5.3). Finally, as to whether weak measurement procedures are capable of resolving the controversy, I suggest that they offer another perspective, not a resolution (Section 5.4).

These questions and answers are not all new. Each has been raised in some form in the literature, but they have not yet been tied together, and where they do appear they are inserted as brief comments within much longer technically-focused expositions. Furthermore, they are all controversial: some existing works agree, either implicitly or explicitly, with the position I present here, and some stand in opposition.

To the extent that I am drawing together ideas that have already been expressed, I aim to offer a simple and focused explanation of why those ideas—and not their proposed alternatives—are correct. To the extent that I am presenting new ideas, I aim to show how quantum tunnelling time can shed new light on the relationship between de Broglie-Bohm theory and other, trajectory-free, interpretations of quantum mechanics.

2 Setting up the problem

Here I summarise the physical system on which discussions of tunnelling time are based. The system has two components: a free particle with momentum $k_0 > 0$ and initial position $x_0 \ll x_1$, and a classically forbidden barrier extending from $x = x_1$ to $x = x_2$.

For $x \ll x_1$, the wavefunction $\psi(x, t)$ of the particle is a wavepacket sharply peaked around some value of momentum k_0, but containing various momentum components:

$$\psi(x, t) = \int_{-\infty}^{\infty} A_k e^{i(kx - w_k t)} dk. \tag{1}$$

When the wavefunction $\psi(x, t)$ reaches the barrier, in general it splits into a reflected part and a transmitted part. An expression for quantum tunnelling time is meant to be an expression for the average time spent in the region $x_1 < x < x_2$ by the transmitted part —by particles that are eventually transmitted through the barrier. The stationary components of the wavepacket take the following form:

$$\psi(x, k) = \begin{cases} e^{ikx} + R_k e^{i\beta_k} e^{-ikx} & \text{if } x < x_1 \\ C_k e^{\kappa x} + D_k e^{-\kappa x} & \text{if } x_1 < x < x_2 \\ T_k e^{i\alpha_k} e^{ikx} & \text{if } x > x_2. \end{cases} \tag{2}$$

R_k is the reflection coefficient for energy component k and T_k is the transmission coefficient for energy component k. $|R_k|^2$ and $|T_k|^2$ are the proportion of particles described by $\psi(x, k)$ that will be reflected and transmitted, respectively.

Bringing time back into the picture, the tunnelling process can be broken into three steps (see Fig. 1):

1. *The particle is approaching the barrier*. The amplitude of the wavefunction is negligible except to the left of the barrier region.
2. *The particle is interacting with the barrier*. The amplitude of the wavefunction is non-negligible in the barrier region.
3. *The particle has completed its interaction with the barrier*. The wavefunction is once again negligible within the barrier region. The full wavefunction now has two peaks: one peak to the left of the barrier, moving to the left, and one peak to the right of the barrier, moving to the right:

$$\psi(x, t) = \begin{cases} \psi_R(x, t) = \int_{-\infty}^{\infty} R_k e^{i\beta_k} e^{-ikx} e^{-i\frac{\hbar k^2}{2m}t} dk & \text{if } x < x_1 \\ 0 & \text{if } x_1 < x < x_2 \quad (3) \\ \psi_T(x, t) = \int_{-\infty}^{\infty} T_k e^{i\alpha_k} e^{ikx} e^{-i\frac{\hbar k^2}{2m}t} dk & \text{if } x > x_2. \end{cases}$$

A good expression for tunnelling time should provide the average time spent in the region $x_1 < x < x_2$ by particles detected at $x > x_2$. But the question is: how can that kind of expression be extracted from the behaviour of $\psi(x, t)$?

3 A brief history & the state of the literature

3.1 The early years: 1931–1962

The duration of quantum tunnelling was first raised by Condon (1931, 9) as an interesting issue to explore. Condon himself, however, did not provide a proposed value for tunnelling time. In fact, he suggested from the very start that it might be an difficult value to define.

Soon after, in 1932, MacColl (1932) assessed the tunnelling problem and concluded: "the transmitted packet appears at the point $x = a$ [our $x = x_2$] at about the time at which the incident packet reaches the point $x = 0$ [our $x = x_1$], so that there is no appreciable delay in the transmission of the packet through the barrier" (621).

Thus, the first value ascribed to tunnelling time was *zero*. MacColl's work suggested that a tunnelling particle in general spends no time at all in the barrier region before emerging on the other side. The problem then lay low for several decades until technological advances brought a resurgence of interest in the early 1960s.[1]

---

[1] For more on the history of quantum tunnelling and the early development of quantum tunnelling time as a concept, see Perovic (2017) and Merzbacher (2002).
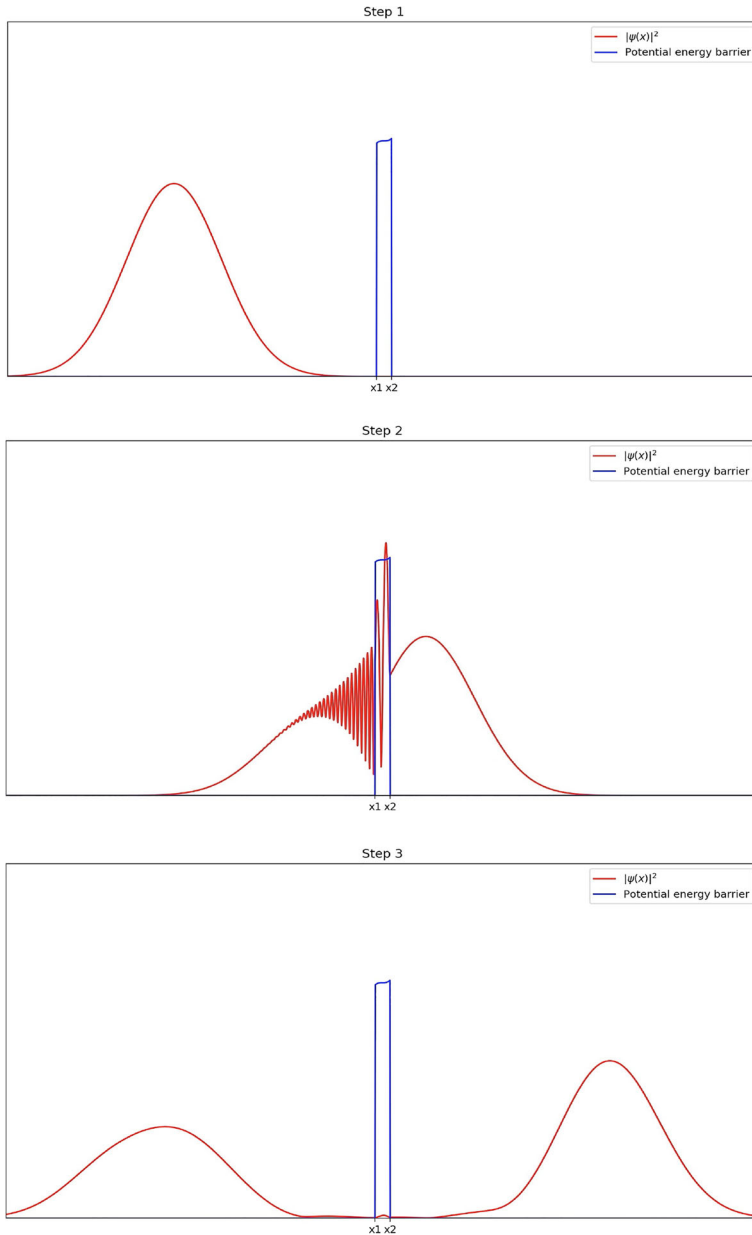
Fig. 1 The three steps of the tunnelling process

3.2 The variety of tunnelling times: 1962–Present

There have been three main developments in the literature on quantum tunnelling time since those early years. First, a time known as the *dwell time* has been

established as a clear, unique and uncontroversial expression for the average time spent in the barrier by incident particles of well-defined energy regardless of whether they are eventually transmitted or reflected (Section 3.2.1).

Second, multiple definitions of tunnelling time have been proposed by authors working within the 'standard' or 'orthodox' interpretation of quantum mechanics, on which particles exist and evolve as delocalized wavefunctions until they collapse indeterministically to some localized state on measurement (Section 3.2.2).[2] The vast majority of authors who assess quantum tunnelling time are working from this point of view.

Third, a minority of authors have assessed the tunnelling time problem from the perspective of de Broglie-Bohm theory, on which quantum particles follow entirely deterministic but epistemically inaccessible trajectories (Section 3.2.3).[3] Four themes have emerged alongside these developments (Section 3.2.4).

3.2.1 The dwell time

One well-established notion related to tunnelling time that does not depend on the difference between the orthodox interpretation and de Broglie-Bohm theory is the *dwell time* τ_d:

$$\tau_d(k) = \int_{x_1}^{x_2} |\psi(x,t)|^2 dx / \left(\frac{\hbar k}{m}\right). \tag{4}$$

(Hauge and Støvneng 1989, 920)

Introduced by Smith (1960), and adapted by Büttiker (1983) to the one-dimensional case we are considering here (Hauge and Støvneng 1989, 920), the dwell time is "defined within the context of a stationary-state scattering problem as the average number of particles within the barrier region divided by the average number entering (or leaving) the barrier per unit time" (Leavens & Aers 1989, 1202). It provides a measure of the average time spent in the barrier by an incident particle of well-defined energy regardless of whether it is eventually transmitted or reflected (Hauge & Støvneng 1989, 920).

Although well-established and uncontroversial, τ_d it is not a contender for tunnelling time, because it does not distinguish between transmitted and reflected particles. Leavens has argued on this basis that it is precisely the attempt to distinguish between particles that will be eventually transmitted and particles that will be eventually reflected that has led to so much confusion about tunnelling time within the orthodox interpretation. He writes, "Since expression (5) [i.e. our Eq. 4] for the mean dwell time is well established, it appears from the above considerations that it is the ambiguity (within 'conventional' quantum mechanics) in the decomposition into transmitted and reflected components that is at the heart of the 'tunneling time problem'" (Leavens 1996, 127).

Later on, in Section 4, I will argue that he is correct: within the orthodox interpretation, there is a fundamental problem with any attempt to decompose the wavefunction

---

[2] I will explain what I mean by 'orthodox interpretation' in more detail in Section 4.1.

[3] I will introduce the formalism at the basis of de Broglie-Bohm theory in more detail in Section 4.2.

into transmitted and reflected components, and this is what has led to the proliferation of definitons for tunnelling time in that interpretation. Those various definitions are what we will consider next.

3.2.2 Multiple 'orthodox' definitions

Many definitions of tunnelling time have been proposed by authors working in the orthodox interpretation. Here I will present the phase time, the Larmor clock time, the Salecker-Wigner-Peres clock time, Steinberg's weak measurement time, and—very briefly—a few other proposals.

The phase time Hartman reevaluated the duration of quantum tunnelling in 1962 in light of technological developments that allowed for tunnelling across thin insulating layers. His interest stemmed from a desire to understand "The transmission times for metal-insulator-metal thin film sandwiches" (Hartman 1962, 3427).

Using the stationary phase approximation—and working, as with all the authors in this Subsection, within the orthodox interpretation—he came up with an expression for quantum tunnelling time now known as the *phase time*.

The phase time $\tau_T^\phi(x_a, x_b, k_0)$ associates tunnelling time with the phase that the eventually transmitted peak $\psi_T(x, t)$ seems to have picked up when it emerges on the right hand side of the barrier in Step 3 of the tunnelling process (see Fig. 1 in Section 2).

Given $\psi_T(x, t) = \int_{-\infty}^{\infty} T_k e^{i\alpha_k} e^{ikx} e^{-i\frac{\hbar k^2}{2m}t} dk$ (see Section 2), the phase time is defined by

$$\tau_T^\phi(x_a, x_b, k_0) := \frac{m}{\hbar k_0}\left[x_b - x_a + \frac{d\alpha_k}{dk}|_{k=k_0}\right], \tag{5}$$

where x_a is some point to the left of the barrier, x_b is some point to the right of the barrier, and $\tau_T^\phi(x_a, x_b, k_0)$ is meant to give the time taken by a tunnelling particle to travel betweent those two points. Its values are in general positive and nonzero (Hartman 1962, 3427).

Although appealing in its apparent simplicity, this expression for tunnelling time suffers from significant problems. First of all, and most importantly, the phase time is defined in terms of the characteristics of $\psi_T(x, t)$, which only becomes well-defined in what I have labelled Step 3 of the tunnelling process (see Section 2). In this sense, the phase time can only be derived *asymptotically* (Hauge & Støvneng 1989, 924)—with respect to the state of the system *after* the actual tunnelling event has taken place.

However, we have assumed that we are dealing with a wavepacket which is narrow in k-space, and will therefore necessarily be wide in x-space. So an appropriately asymptotic treatment of τ_T^ϕ can only be derived for $x_a \ll x_1$ and $x_b \gg x_2$ (924).

It might nevertheless be tempting to extrapolate back from $\tau_T^\phi(x_a, x_b, k_0)$ to a time spent between x_1 and x_2 by subtracting from τ_T^ϕ the time that a free particle travelling with momentum k_0 would take to traverse the distance $(x_1 - x_a) + (x_b - x_2)$. This would, however, be ill-advised, as Hauge & Støvneng (1989, 924–926) point out. On

approach to the barrier, the peak of the incoming wavepacket interferes with front-end components that have already been reflected (see Step 2 in Fig. 1); in Hauge and Støvneng's words, "free particle motion cannot be assumed all the way up to the barrier" (924).

τ_T^{ϕ} is therefore unable to provide an accurate value for the time spent *in the barrier region* by eventually transmitted particles; it can only provide an accurate value for the time spent in a much wider region that includes the barrier.

In addition to this serious failure to answer the question at hand, the phase time comes along with a strange and problematic feature: it predicts that tunnelling time will approach a constant value as the width of the barrier increases towards infinity, implying faster-than-light tunnelling for particles traversing a wide barrier (Hartman, 1962). This effect, now known as the 'Hartman effect', has been the source of considerable unease in the literature (see e.g. Lunardi et al. 2011; Calçada et al. 2009; Winful 2003, 2006 and Leavens 1990).

The Larmor clock time The *Larmor clock time*, introduced by Baz' (1967) and Rybachenko (1967), uses spin rather than phase as the marker of duration. A weak magnetic field is applied in the direction perpendicular to the polarisation of the particle's spin, throughout some region (x_a, x_b) including the tunnelling barrier. The idea is that the particle will then precess at a constant rate in the plane perpendicular to the applied field as it moves between x_a and x_b.

For example, if the magnetic field is applied in the z direction, and the particle is initially polarized in the x direction, its spin will precess in the x-y plane between x_a and x_b.

In the limit where the characteristic frequency of the field ω_L goes to zero, and considering only one momentum component k (i.e. the stationary case), the expectation value of the spin acquired by the eventually transmitted particle, $\langle S_y \rangle_T$, defines a time (Hauge and Støvneng 1989, 921; Landauer and Martin 1994, 225):[4]

$$\tau_{y,T}^L(x_a, x_b, k) := \lim_{\omega_L \to 0} \frac{\langle S_y \rangle_T}{-\frac{1}{2}\hbar\omega_L} \ . \tag{6}$$

Supporters of this Larmor clock time emphasize that it makes tunnelling time accessible to experiment, not just pen and paper analysis (Landauer and Martin 1994, 217, 218). Landauer and Martin (1994, 217, 218) write of clock times in general, "The strength of the clock approach: It is not only a path to analysis, but also tells us how to do experiments".

However, Falck and Hauge (1988) have convincingly argued that the Larmor clock is only reliable when applied to a wavepacket; stationary treatment, whether theoretical or experimental, is not good enough. In their words, "[i]t is not sufficient to study the stationary case, theoretically or experimentally, point to a plausible quantity with the dimension of time, and thus identify the time taken by the process in question. Not that the result of such a simplified procedure is necessarily wrong. The problem is

---

[4] $\omega_L = \frac{gqB_0}{2m}$, where g is the gyromagnetic ratio, q is the charge of the particle, m is the mass of the particle, and B_0 is the magnitude of the applied magnetic field.

that it is not necessarily right" (3294); "[s]tationary calculations of Larmor times correspond to coherent superpositions of widely different scattering events, and cannot therefore be interpreted as tunneling times" (3295).

In particular, this means that the Larmor clock time, like the phase time, must be treated asymptotically (i.e. with $x_a \ll x_1$ and $x_b \gg x_2$). This is necessary in order "[t]o prevent interference between the processes of entering and leaving the field region on the one hand, and the tunneling process on the other" (3288). Entering and leaving the magnetic field will affect the behaviour of the wavepacket, and if the three steps of the tunnelling process are not all contained within the magnetic field region, resulting values for traversal time will be contaminated by those affects.[5]

The only way to infer a *time spent in the barrier region* from the Larmor time is thus, as with the phase time, to extrapolate "based on the simple motion of free wave packets" (3294). But there is no reason why the problem that we found with the phase time should not also apply here—such extrapolation ignores the presence of interference effects on approach to the barrier.

In a serious way, the Larmor clock time therefore fails to answer the question it set out to answer. This perhaps should not be surprising, given that $\tau^L_{y,T}$ reduces to the phase time to first order when applied to a wide region (x_a, x_b) (3287, 3292).

Furthermore, when the tunnelling particle interacts with the applied magnetic field, there are really two relevant time scales. As explained above, $\tau^L_{y,T}$ is the time scale corresponding to the spin's precession in the plane perpendicular to the direction of the applied field. But there is another time scale $\tau_{z,T}$ corresponding to the spin's tendency to align with the applied field (Hauge and Støvneng 1989, 921):

$$\tau^L_{z,T}(x_a, x_b, k) = \lim_{w_L \to 0} \frac{\langle S_z \rangle_T}{-\frac{1}{2}\hbar w_L}.$$ (7)

The Larmor clock time completely fails to take this time scale into account.

Büttiker (1983, 6181) has proposed an alternative expression, the *Büttiker-Landauer time* τ^{BL}_T, which accounts for both $\tau^L_{z,T}$ and $\tau^L_{y,T}$:[6]

$$(\tau^{BL}_T)^2 = (\tau^L_{y,T})^2 + (\tau^L_{z,T})^2.$$ (8)

But it is fundamentally unclear which of τ^{BL}_T and $\tau^L_{y,T}$ more closely corresponds to the concept we aim to capture (see e.g. Hauge and Støvneng (1989), 921)); both τ^{BL}_T and $\tau^L_{y,T}$ are plausible markers of duration.

The Salecker-Wigner-Peres clock time The Salecker-Wigner-Peres (SWP) clock, like the Larmor clock, couples the particle of interest with an external observable. In fact, the SWP clock is equivalent to the Larmor clock in the limit of large spin (Sokolovski

---

[5]The recent University of Toronto weak measurement experiments, reported in Ramos et al. (2020), use a magnetic field confined to the barrier region in spite of this problem.

[6] τ^{BL}_T can be derived by invoking an oscillating barrier and associating tunnelling time with a transition between low-modulation-frequency and high-modulation-frequency behaviour (Büttiker, 1983; Hauge & Støvneng, 1989).

and Connor 1993, 4679–4680; Lunardi et al. 2011).[7] But where the Larmor clock applies a small magnetic field in the region of interest and tracks the resulting spin precession, the SWP clock applies a small x-independent addition of potential energy V_{SWP} in the region of interest and tracks the resulting change in phase.

First introduced by Salecker and Wigner (1958) and Peres (1980), the SWP clock has been applied to the tunnelling problem by Leavens (1993), Leavens and McKinnon (1994), Calçada et al. (2009), and Lunardi et al. (2011). The additional potential energy introduced by the clock adjusts the tunnelling particle's phase, and the average change in phase due to the presence of the SWP potential for particles detected on the right hand side of the tunnelling barrier is read off as the SWP clock reading for traversal time (Peres, 1980; Calçada et al., 2009; Lunardi et al., 2011).

This *Salecker-Wigner-Peres clock time*, conceptually just another version of the Larmor clock time, suffers from the same central problem. It can only be considered reliable when applied to the time-dependent problem, for the reasons raised by Falck and Hauge (1988) in the context of the Larmor clock; time-dependent treatment requires asymptotic treatment, due to the wavepacket's breadth in x-space; asymptotic treatment can only provide a traversal time for a region much wider than the barrier region; and, finally, we cannot extrapolate a tunnelling time from that longer traversal time due to interference effects on approach to the barrier.

Steinberg's weak measurement time   Steinberg, in 1995, introduced a new definition of tunnelling time based on the theory of weak measurement developed by Aharonov et al. (1988) and Aharonov and Vaidman (1990).

Weak measurement was developed precisely to deal with questions about subensembles: questions about the average state of some operator for particles prepared in a state $|i>$ which are later found in a state $|f>$. Aharonov et al. show that such an average can be theoretically defined and in principle measured, without causing collapse of the wavefunction, by combining *minimally invasive measurement* with *postselection of the final state*.

For tunnelling time, we want to ask: on average, how long did a particle spend in the barrier region, *given* that it ended up being detected on the right hand side of the barrier? Steinberg suggests that we can use weak measurement to answer this question, by applying imprecise individual measurements, postselecting for eventually transmitted states, and averaging over the measurement outcomes (Steinberg 1995a, b).

The operator of interest, in this case, is the projection operator which measures whether a particle is in the barrier region (Steinberg 1995b, 2405, p. 2405). The state $|i>$ is defined as a stationary state with no left-moving components to the right of the barrier, and the state $|f>$ is defined as a stationary state with no left-moving components to the left of the barrier (2407). A superposition of several states of the first kind corresponds to a wavepacket incident on the barrier from the left hand side.

---

[7]Note that this implies equivalence between the SWP clock time and the phase time for large spin *and* (x_a, x_b) much wider than (x_1, x_2), based on the equivalence between the Larmor clock time and the phase time for (x_a, x_b) much wider than (x_1, x_2).

A superposition of several states of the second kind corresponds to a wavepacket that has traversed the barrier, having approached it from the left hand side.

The resulting value for tunnelling time τ_R^{Weak} is, in effect, a weak measurement of the dwell time postselected for eventually transmitted states (Steinberg 1995a, 34–35). In general it is complex; its real part coincides with the in-plane Larmor time $\tau_{y,T}^L$, and its imaginary part coincides with the out-of-plane Larmor time $\tau_{z,T}^L$ (Steinberg 1995b, 2407).

Indeed one of the main benefits of this weak measurement approach, from Steinberg's perspective, is that it can be applied to the Larmor clock and thereby render tunnelling time accessible to experiment.[8] The complex components of τ_R^{Weak}, Steinberg argues, can be easily interpreted as soon as a physical measurement apparatus is taken into account. The real part, he argues, corresponds to the movement of the 'pointer' or 'clock' being used, and should therefore be read off as the tunnelling time (2406). The imaginary component corresponds to the backreaction of the measurement apparatus on the tunnelling particle, and therefore—although physically significant—should not be interpreted as a measure of tunnelling time (2406).

Steinberg describes his approach as "a sensible way to define conditional probabilities in quantum mechanics, assuming only Bayes's theorem and standard quantum theory" (Steinberg 1995a, 32). But others beg to differ. Sokolovski and Akhmatskaya (2021), Rivlin et al. (2021) and Chuprikov (2014, 13–14) are among those who have directly criticized Steinberg's proposal.

Sokolovski and Akhmatskaya (2021)'s critique is especially severe. They emphasize that weak measurement of the Larmor clock time probes the *probability amplitudes* associated with different durations, not the *probabilities* associated with those durations. Unlike probabilities, which are in general real and non-negative, probability amplitudes are complex. They are only constrained by the requirement that they must add up to unity when summed over all possible pathways. But, as Sokolovski and Akhmatskaya (2021, 3) show, this is not an especially stringent requirement; one can find probability amplitudes which satisfy it for *any chosen resulting value of the Larmor clock time*, including completely unreasonable values.

In their words,

> "[i]f you consider all possible experiments of this type, some of them will give seemingly reasonable outcomes, whereas other 'times' would be negative, too short, too long, etc. This is necessary, and is possible because such 'times' can be expressed as the combinations of probability amplitudes which, unlike probabilities, have few restrictions on their signs and magnitudes. Though your result of 0.61 *ms* does look plausible you cannot recommend using it the way you would use a classical time scale just because of this." (4)[9]

We should therefore be cautious in how we interpret the physical meaningfulness of τ_R^{Weak}.

---

[8]This is exactly what Ramos et al. (2020) accomplish in their recently reported experimental results.

[9]The reference to "0.61 *ms*" is an acknowledgement of the experimental results reported in Ramos et al. (2020). I will say more about those results in Section 3.2.4.

Sokolovski's critique of the weak measurement approach to tunnelling time fits with his stance on weak measurement in general; Sokolovski (2015, 2016a, b) argue that any weak measurement procedure will produce unreasonable values, including values outside the spectrum of the operator in question, given an appropriate choice of initial and final states. This all boils down to the distinction between *probabilities* and *probability amplitudes*, and the fact that weak measurement probes the latter not the former.[10]

Other proposals The expressions described above are just four of the main contenders for tunnelling time on the orthodox interpretation—there are many more. A time based on Feynman path integrals is supported by Sokolovski and Baskin (1987, 4611), among others, as "the natural generalization of the classical traversal time", but yields genuinely complex times—not just complex times whose real and imaginary parts need to be appropriately interpreted (Sokolovski and Baskin 1987; Hauge & Støvneng 1989, 922; Pollak & Miller 1984; Pollak, 1985). The Büttiker-Landauer time τ_T^{BL} (related to $\tau_{y,T}^L$ and $\tau_{z,T}^L$ by Eq. 8) has been disputed, as we saw above. The Stevens procedure (Stevens, 1980) measures tunnelling time by tracking the movement of a discontinuity in the wavepacket, but its accuracy has been undermined by later work (Hauge & Støvneng 1989, 920). Overall, no proposal has been accepted as the 'correct' expression for tunnelling time on the orthodox interpretation, except in its authors' own eyes.

3.2.3 de Broglie-Bohm treatment of the problem

Alongside the development of this varied landscape in the orthodox literature, a handful of authors have assessed the quantum tunnelling time problem from the perspective of de Broglie-Bohm theory.

By reinterpreting the wavefunction and postulating the existence of underlying point particles, de Broglie-Bohm theory is able to maintain the mathematical formalism of the standard interpretation but simultaneously adopt a completely deterministic underlying dynamics.[11]

The wavefunction is reinterpreted as a field that guides the time-evolution of underlying localised quantum particles, but simultaneously conceals information about which trajectory any particle is actually following. Only a measurement can reveal that information; and the collapse that we see during measurement is thereby demoted from a genuine indeterministic change in the state of the system to something that only seems indeterministic because it reveals information about the system that we previously could not access.

---

[10]For more on the interpretation of weak measurement, see e.g. Svensson (2013a, b).

[11]Throughout I restrict myself to the traditional version of de Broglie-Bohm theory that I describe here—other proposals for the ontology underlying the pilot wave program, although fascinating in their own right, do not bear on the conceptual points I aim to make in this paper.

The tunnelling time problem, when viewed through this lens, becomes much more tractable. For a given wavefunction $\psi(x, t)$, tunnelling time can be clearly and simply defined as the average time spent within the barrier region for de Broglie-Bohm trajectories that eventually continue past the barrier (Leavens 1990, 924–925).

Following Leavens (1990, 924–925) and Leavens and Aers (1996, 112–114), a de Broglie-Bohm particle initially at position x_0 will later spend a time in the region (x_a, x_b) given by:

$$t(x_0, x_a, x_b) = \int_0^\infty dt \Theta[\xi(x_0, t) - x_a]\Theta[x_b - \xi(x_0, t)] \tag{9}$$

where Θ is the unit step function, i.e.

$$\Theta(y) = \begin{cases} 1 \text{ if } y > 0 \\ 0 \text{ if } y < 0, \end{cases} \tag{10}$$

and $\xi(x_0, t)$ is the position of the particle as a function of time t and its initial position x_0.

This can be used to construct an expression for tunnelling time, simply by setting $x_a = x_1$ and $x_b = x_2$ and taking the average value $<>_T$ of $t(x_0, x_1, x_2)$ only over the subensemble of initial positions x_0 that lead to transmission through the barrier:[12]

$$\tau_T^{dBB} := < t(x_0, x_1, x_2) >_T . \tag{11}$$

This expression is unique and uncontroversial within de Broglie-Bohm theory; it has even been called "trivial" (Muga et al. 2008, 9). It was first introduced by Leavens and Aers (1989), and has since been assessed by Leavens (1990, 1996), Leavens and Aers (1996), Muga et al. (2008) and Cushing (1994, 1995), among others.

The de Broglie-Bohm interpretation, in its traditional form, therefore provides a clear and unique answer to a question that the standard interpretation has not been able to answer clearly or uniquely. Many existing reviews of the history of quantum tunnelling time do not even consider this perspective—including, for example, Winful (2006) and Hauge and Støvneng (1989). But, I suggest, it is an important and informative perspective to consider.

3.2.4 Four emerging themes

Four themes, which I label (1) **Problem of time**, (2) **Meaninglessness**, (3) **Crucial test**, and (4) **Weak measurement**, have emerged alongside and within these developments.

1. **Problem of time**

    Various authors have blamed the apparent ambiguity of quantum tunnelling time on the fact that time enters quantum mechanics as a parameter, not a self-adjoint operator. Abolhasani and Golshani (2000, 1) write, for example: "In

---

[12]Since this expectation value is taken only over the eventually transmitted components of the original wavefunction, it needs to be renormalized by a factor inversely proportional to the probability of transmission $|T|^2$. This renormalization factor is absorbed into my notation for the restricted expectation value, $<>_T$.

quantum mechanics, time enters as a parameter rather than an observable (to which an operator can be assigned). Thus, there is no direct way to calculate tunneling times." Challinor et al. (1997) write, "There have been many attempts to define a physical time for quantum mechanical tunnelling processes... Orthodox quantum theory is unable to give a definite answer to this problem since time is not an observable (in the sense of being represented by an Hermitian operator) in the conventional formulation." This list is far from exhaustive; almost every paper on tunnelling time mentions time's status as a parameter as at least part of the reason for the existing controversy.[13]

2. **Meaninglessness**

   The proliferation of mutually exclusive definitions highlighted in Section 3.2 has led some authors to dismiss tunnelling time as meaningless within the orthodox interpretation. Dumont and Marchioro (1993, 85) write that the question, "How much time does a tunneling particle spend under the barrier?... has no meaning as it requires the simultaneous measurement of incompatible observables". Leavens and Aers (1996, 137) write: "There is no consensus among proponents of conventional interpretations on whether or not transmission and reflection times are meaningful concepts. The point of view that they are not meaningful appears to be a consistent one *within conventional interpretations*". Leavens expresses the same view in Leavens (1993, 781). But not everyone agrees. Sokolovski (2017, 11) writes, "we disagree with the final conclusion of [37] [i.e. Leavens (1993)] that 'it is only the dwell time, which does not distinguish between transmitted and reflected particles, that is a meaningful concept in conventional interpretations of quantum mechanics.' The dwell time is, we argue, just a special case of the complex time and is no more, and no less, meaningful than the tunneling and reflection times in Eqs. (62) and (63)".

3. **Crucial test**

   The existence of a unique and clear expression for quantum tunnelling time in de Broglie-Bohm theory, contrasted with the total lack of consensus on the definition of tunnelling time in the orthodox interpretation, has led to speculation about whether tunnelling time could provide evidence for de Broglie-Bohm theory. Cushing (1995, 274–277) made the most substantial such suggestion in his paper 'Quantum Tunneling Times: A Crucial Test for the Causal Program?'. There he argues that since tunnelling time is well-defined in de Broglie-Bohm theory, an experimental test of tunnelling time could in principle function as a test of whether de Broglie-Bohm's predictions are empirically adequate.[14] He even proposes a rough sketch of an experimental setup: a detector placed to the left of the barrier, designed to 'click' when the incident particle is released towards

---

[13]See e.g. Challinor et al. (1997, 143), Leavens (1990, 923), Lunardi et al. (2011, 415), and Peres (1980, 552).

[14]Cushing entertains the same possibility, although in less detail, in Chapter 4 of his 1994 book on the historical dominance of standard quantum mechanics over de Broglie-Bohm theory (see pages 54–55 and 72–75 of Cushing (1994) in particular).

the barrier, and a detector placed to the right of the barrier, designed to 'click' when hit by an eventually transmitted particle. Although he eventually drops this suggestion in the face of an objection by Bedard (1997), various authors have expressed similar views, albeit without proposing an experimental set-up for a possible test (see e.g. Acuña 2019, 21 and Abolhasani and Golshani 2000, 1). Others, without going quite so far as to suggest that tunnelling time could provide a crucial test of the Bohmian program, have argued that "reasonable" features of the Bohmian definition for tunnelling time should stand in contrast to unreasonable features of the various orthodox definitions (including, for example, the Hartman effect) as "concrete evidence in support of Bohm's interpretation of quantum mechanics" (Leavens 1990, 927).

4. **Weak measurement**

Recent experimental realisations of Steinberg's proposal for probing tunnelling time have raised the idea that weak measurement might be able to resolve the long-standing controversy and provide empirical measurements of quantum tunnelling time that come along with a clear interpretation. In a 2020 Nature paper, Ramos et al. (2020, 529) claim to have measured a traversal time of "0.61(7) milliseconds at the lowest energy for which tunnelling is observable" for "Bose-condensed ^{87}Rb atoms tunnelling through a 1.3-micrometre-thick optical barrier". They apply a weak magnetic field within the barrier region and postselect for eventually transmitted particles, using the Larmor clock to read off the time spent in the barrier region by particles that are eventually detected on its right hand side. The authors write, "This experiment lays the groundwork for addressing fundamental questions about history in quantum mechanics: for instance, what we can learn about where a particle was at earlier times by observing where it is now" (529). However, the physical meaning of these experimental results has been directly questioned by Sokolovski and Akhmatskaya (2021), as we saw in Section 3.2.2. In addition to the arguments provided by Sokolovski and Akhmatskaya (2021), which challenge the fruitfulness of trying to measure the Larmor time at all, Rivlin et al. (2021, 1) claim to "show that these experiments cannot reveal the Larmor time, due to finite energy width of the incident particles".

The existing literature therefore leaves us with four unanswered questions. First, does the confusion about tunnelling time really stem from the more general "problem of time" in quantum mechanics—namely, the fact that time lacks an operator? Second, is tunnelling time really a meaningless concept in the standard interpretation of quantum mechanics? If so, why, and in what sense? Third, is it possible to use quantum tunnelling time to gain concrete evidence for the de Broglie-Bohm interpretation? And finally, are weak measurement techniques—and experimental realisations thereof—able to resolve the decades-long debate and provide clear, unique and experimentally verifiable values for tunnelling time?

In the next Section I will attempt to provide the groundwork for answering these questions by looking more closely at how tunnelling time features in the orthodox interpretation (Section 4.1) and in de Broglie-Bohm theory (Section 4.2).

4 Where was the particle right before measurement?

The difference between the orthodox intepretation and de Broglie-Bohm theory that will weigh most heavily on the issues just raised is revealed by the question: *where was the particle right before a position measurement?* The orthodox interpretation answers that it was not at the measured position, or any position at all. It genuinely existed as a spread-out probability amplitude over various position states —as $\psi(x, t)$. In Griffith's words, on the standard textbook interpretation "[a] particle simply does not have a precise position prior to measurement, any more than the ripples on a pond do; it is the measurement process that insists on one particular number, and thereby in a sense creates the specific result, limited only by the statistical weighting imposed by the wave function" (Griffiths 1995, 4). The Bohmian interpretation answers instead that the particle was exactly in the measured position, and that it had been following a well-defined, localised path at every time leading up to the time of measurement.

These answers clearly do not permit the same analysis of a tunnelling particle's behaviour before detection. The orthodox interpretation must claim that immediately before a particle corresponding to Step 3 of the tunnelling system is detected either to the right of the barrier or to the left of the barrier (see Fig. 1), it did not exist on either side: it was spread out over both sides of the barrier, in every location with nonzero $\psi(x, t)$. In Section 4.1, I will argue that this makes the question at the basis of the tunnelling time problem ill-posed within the orthodox interpretation. De Broglie-Bohm theory, by contrast, can assert that the particle was exactly in the measured position immediately before measurement, and furthermore that it followed a well-defined and deterministic trajectory to get there. In Section 4.2, I will argue that this is exactly why de Broglie-Bohm theory is able to provide a clear and unique expression for tunnelling time.

4.1 Tunnelling time in the orthodox interpretation

By the 'orthodox interpretation', I mean the interpretation—broadly construed—on which quantum particles do not have well-defined trajectories and collapse indeterministically to seemingly determinate values only when they are measured.

In particular, I refer to any interpretation in which a normalised quantum state

$$\psi(x, t) = \sum_j c_{Q_j}(t) \psi_{Q_j}(x) \tag{12}$$

is taken to evolve deterministically according to the Schrödinger equation in the absence of measurement, and then collapse at time t_0 to $\psi(x, t_0) = \psi_{Q_{j'}}(x)$ with probability

$$Prob(\text{result } q_{j'}) = |c_{Q_{j'}}(t_0)|^2 \tag{13}$$

when disturbed by a measurement process corresponding to operator Q, where the ψ_{Q_j}'s are Q's eigenstates.

As reviewed in Section 3.2.2, several definitions of tunnelling time have been put forward by authors who subscribe to this intepretation. No single option has been

unanimously accepted as 'correct'; in general they contradict each other and are reconciled only in certain limits. And although many authors have suggested that the problem might not admit a unique solution from the orthodox perspective, some still write as if such a solution should be sought.[15] Calçada et al. (2009, 1), for example, claim that "[a]n unambiguous definition of tunneling time is an important problem in quantum mechanics, due to both its applications and its relevance to the foundations of the theory".

4.1.1 An ill-posed question

If we are working within the orthodox interpretation, however, then we have no choice but to take the state of a quantum particle to be identical to the amplitude of its wavefunction; it evolves deterministically when not being measured but collapses indeterministically on measurement to any of the observable states for which the previous amplitude was nonzero. For the tunnelling time system, this means that the state of an incident particle evolves as outlined in Fig. 2 below. The particle interacts with the barrier as a complex probability amplitude, and then evolves into a state with two peaks: one to the left of the barrier and one to the right of the barrier. On detection, this state collapses to a sharp peak at some point where the previous state had nonzero amplitude. If the point of collapse is to the left of the barrier, we appear to have detected a particle that reflected from the barrier.

If the point of collapse is instead to the right of the barrier, we appear to have detected a particle that tunnelled through the barrier. But immediately before detection the particle had not reflected or tunnelled—rather it existed as a complex probability amplitude including a reflected peak and a transmitted peak. It is therefore in contradiction with the basic tenets of the orthodox interpretation to attempt to extract an expression for the average duration-behaviour-in-the-barrier of eventually transmitted particles, because according to those basic tenets, any property corresponding to a particle detected on the right hand side of the barrier was attained not by a particle destined to be detected as such, but by the probability distribution in the 'During interaction with barrier' step of Fig. 2.

Any proposal for extracting a tunnelling time from the orthodox interpretation will encounter this problem.[16] The phase time, for example, associates time spent in the barrier region by eventually transmitted particles with the average extra phase picked up by particles that are detected as eventually transmitted. But according to the reasoning outlined above, a particle detected as eventually transmitted did not pick up its extra phase as an eventually transmitted particle. The Larmor clock time associates tunnelling time with the average spin picked up by particles detected as eventually transmitted. But a particle detected as eventually transmitted did not pick up its spin as an eventually transmitted particle.

---

[15]For authors who argue that we should not expect to find a unique expression within the standard interpretation, see e.g. Hauge and Støvneng (1989, 917), Muga et al. (2008, 21), and Sokolovski and Baskin (1987, 4604).

[16]Definitions based on weak measurement might at first sight seem to present an exception; this possibility will be discussed in Section 5.4.
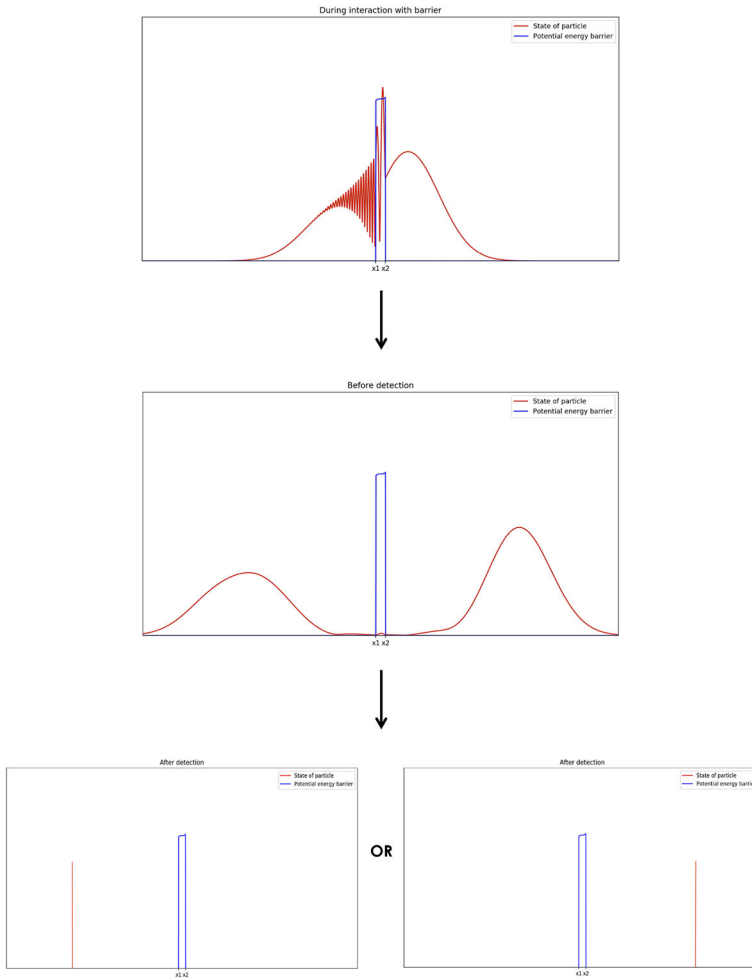
Fig. 2 The orthodox account of the state of a particle described by the tunnelling time problem, in three stages: during interaction with the barrier, after interaction with the barrier but before detection, and after detection

Within the orthodox interpretation, the question "*how long did a particle spend in the barrier region, given that it was eventually transmitted?*" is therefore ill-posed in an important sense. It requires us to access information about how a particle was behaving before detection based on where it ended up being detected. But the orthodox interpretation is not set up to provide that kind of information.

4.1.2 A double slit analogy

A loose analogy with the familiar double slit experiment illustrates the force of this point. For the double slit experiment, we want to ask: given many individual runs

that produce an interference-pattern-shaped probability distribution on the detection screen, which of the two slits did each individual particle go through? For the tunnelling time problem, we want to ask: given that a particle is detected on the right hand side of the barrier, on average how much time did it spend in the barrier region? Both are questions about how to infer information about a quantum particle's previous behaviour given information about the state to which it collapsed on measurement.

But the orthodox interpretation explicitly prohibits this kind of inference.[17] It claims that knowledge of the collapsed state of a quantum particle does not give us any extra information about which component of its previous superposition state was "veridical", because all of the components of its previous superposition state were veridical. The particle's previous state was no more and no less than the amplitude of its wavefunction, and cannot be separated into 'eventually transmitted' and 'eventually reflected' behaviour for the tunnelling time problem, any more than it can be separated into 'went through left slit' and 'went through right slit' behaviour for the double slit experiment.

4.2 Tunnelling time in de Broglie-Bohm theory

There are two sides to the underlying dynamics of de Broglie-Bohm theory, both of which can be derived by mathematical manipulation of the orthodox formalism: the wave dynamics, and the particle dynamics. In three spatial dimensions, they are given by:

1. **The wave dynamics**

   The wavefunction $\psi(x, t)$ evolves according to the Schrödinger equation, as in the standard interpretation:

   $$i\hbar\frac{\partial \psi(\boldsymbol{x}, t)}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2 \psi(\boldsymbol{x}, t) + V(x)\psi(\boldsymbol{x}, t). \tag{14}$$

2. **The particle dynamics**

   Subject to the assumption that the probability distribution of underlying particles matches the density of the wavefunction[18]—i.e. $\rho(\boldsymbol{x}, t)dx = |\psi(\boldsymbol{x}, t)|^2 dx$— each underlying quantum particle follows a trajectory $\boldsymbol{\xi}(t)$:[19]

   $$\frac{d\boldsymbol{\xi}}{dt} = \frac{\hbar}{m}\left[Im\left(\frac{\nabla\psi(\boldsymbol{x}, t)}{\psi(\boldsymbol{x}, t)}\right)\right]\big|_{\boldsymbol{x}=\boldsymbol{\xi}(t)}. \tag{15}$$

   For $\psi(\boldsymbol{x}, t)$ of the form $\psi(\boldsymbol{x}, t) = A(\boldsymbol{x}, t)e^{iS(\boldsymbol{x},t)/\hbar}$, this reduces to:

   $$\frac{d\boldsymbol{\xi}}{dt} = \frac{1}{m}[\nabla S(\boldsymbol{x}, t)]|_{\boldsymbol{x}=\boldsymbol{\xi}(t)}. \tag{16}$$

---

[17] The correspondence between the tunnelling and double slit scenarios is not exact, but it is not intended to be; the analogy is enough to demonstrate this key point.

[18] It is disputed whether this assumption needs a dynamical justification or should just be taken as a matter of postulate. For a recent review, see Valentini (2019).

[19] See e.g. Bohm (1961, 169–172).

The wave $\psi(\boldsymbol{x}, t)$ guides the motion of the particle, since $\frac{d\xi}{dt}$ depends on $\psi(\boldsymbol{x}, t)$. However, a given $\psi(\boldsymbol{x}, t)$ permits various particle trajectories $\xi(t)$, upholding the uncertainty principle and allowing de Broglie-Bohm theory to reproduce standard results.

Each underlying particle *is* in reality following a perfectly well-defined and deterministic trajectory, on this view; but we cannot come to know which one without gravely disturbing the outcome of our measurement.

The interpretation as a whole can be criticized on various grounds: among other issues, it is strongly nonlocal, and relies on the controversial assumption that the probability distribution of underlying particles matches the density of the wavefunction. But, as introduced in Section 3.2.3, de Broglie-Bohm theory provides an expression for tunnelling time that is clear, unique and uncontroversial—not in general, but from a Bohmian perspective. That expression, τ_T^{dBB}, is the average time spent in the barrier region by eventually transmitted de Broglie-Bohm trajectories.

4.2.1 A well-posed question

Within this interpretation of quantum mechanics, each tunnelling particle travelled through the barrier in a state that was destined to end up being transmitted, or in a state that was destined to end up being reflected. See Fig. 3, for example, for a space-time diagram showing the distribution of de Broglie-Bohm trajectories for a particle incident on a rectangular classically forbidden barrier.

It is the uncertainty principle that blocks us from being able to see, by looking at a particle's behaviour in the barrier region, which of these trajectories it is following: we only have access to the behaviour of the wavefunction, which permits both eventually transmitted and eventually reflected trajectories.

Thus, whereas in the orthodox interpretation, a tunnelling particle interacts with the barrier in a superposition state that cannot be broken into eventually transmitted and eventually reflected components, in de Broglie-Bohm theory it makes sense to speak of a particle interacting with the barrier in a state that is destined to be eventually transmitted.

This makes the question at the core of the tunnelling time debate —*"how long did a particle spend in the barrier region, given that it was eventually transmitted?"*— well-posed within the Bohmian framework.

4.2.2 A double slit analogy

The same double slit analogy can be introduced here. On the Bohmian view, each particle incident on a double slit goes through either the left slit or the right slit on a localized, deterministic trajectory. It is the uncertainty principle that blocks us from being able to see, by tracking a particle's behaviour at the double slit, which slit it is going through: we only have access to the behaviour of the wavefunction, which permits both left-slit and right-slit trajectories.

Any attempt to try to measure which slit a given particle is going through destroys the interference pattern on the screen, because the measurement apparatus interacts
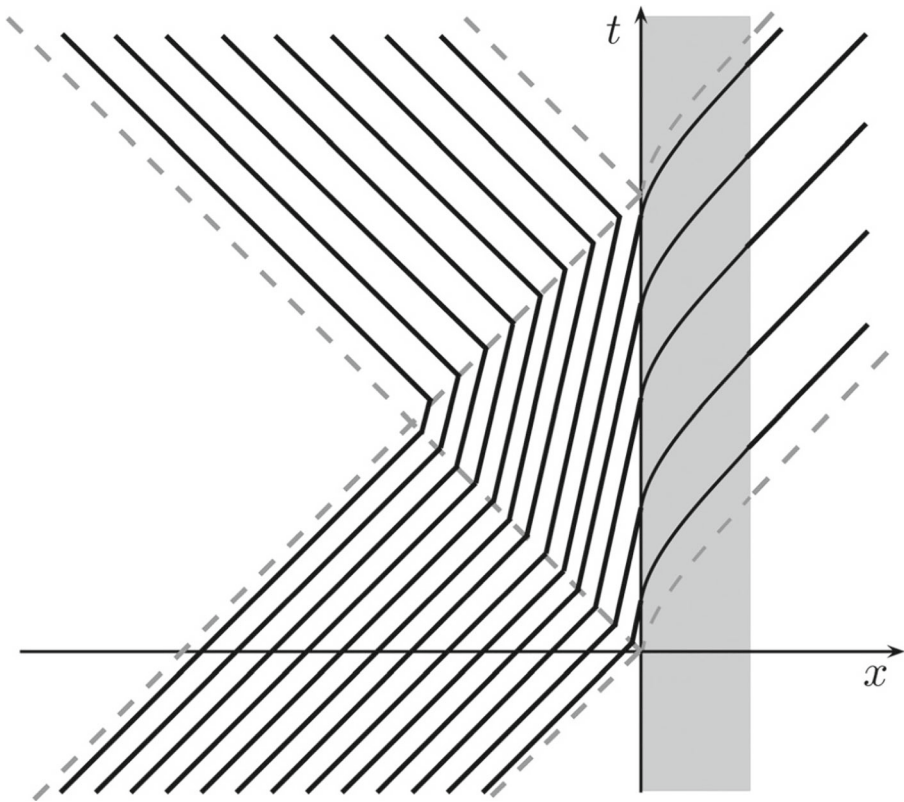
Fig. 3 Space-time diagram showing a representative sample of possible particle trajectories for the case of a plane-wave packet incident from the left on a rectangular potential barrier. Image and caption reproduced with permission from Norsen (2013, 264).

with the underlying wave field $\psi(x, t)$, and the resulting change in the evolution of the wave field changes the evolution of the particle trajectories.[20]

So we cannot know whether a given particle went through the left or right slit just by subscribing to de Broglie-Bohm theory; but at least it makes sense to ask the question.

5 Morals

Tunnelling time, I have argued, is therefore an inherently unclear and ambiguous concept on the orthodox interpretation, because it stems from a question about particle histories that the orthodox interpretation is not set up to answer. It is an inherently

---

[20]For more details on the double slit experiment in de Broglie-Bohm theory, see e.g. Bricmont (2016, 134–137) and Gondran and Gondran (2014, Section 3).

clear and unambiguous concept in de Broglie-Bohm theory, because de Broglie-Bohm theory has the trajectory infrastructure to be able to answer a wide range of questions about particle histories.

These conclusions shed light on the four issues raised in Section 3.2.4: on whether the tunnelling time controversy stems from the more general problem of time in quantum mechanics, on whether tunnelling time is meaningful in the orthodox interpretation, on whether tunnelling time can provide evidence for de Broglie-Bohm theory, and on whether weak measurement can offer a resolution to the long-standing tunnelling time debate. I will elaborate on the implications for each of these themes in Sections 5.1, 5.2, 5.3 and 5.4, respectively.

5.1 Why the trouble is not with time

As reviewed in Section 3.2.4, many authors have suggested that tunnelling time is difficult to define on the standard interpretation precisely because time is difficult to define in quantum mechanics in general. It is a parameter, rather than an self-adjoint operator.

Even before considering the conclusions of the previous section, this view is called into question by the fact that quantum clock observables can be defined: they just cannot be represented by PVMs (Pashby 2014, Chapter 6, Chapter 7, Chapter 8). And as we saw in Section 3.2.2, various quantum clocks have been applied to the tunnelling time problem, but within the orthodox interpretation, controversy persists.

Furthermore, quantum theory has no problem providing a clear and unique expression for other duration-based concepts, including, for example, the time of flight of a free particle. In most cases we can simply exploit functional relationships between time and other quantities, like position and momentum, that can be represented by self-adjoint operators.

The conclusions established in Section 4 confirm that the confusion over quantum tunnelling time should not be attributed to the fact that time lacks a self-adjoint operator. Rather, it should be attributed to the status of superposition in the orthodox interpretation, which forbids us from asking about how a particle was behaving *then* based on where we observe it *now*.

Tellingly, the time spent within the barrier region is not ambiguous even for tunnelling, provided no attempt is made to distinguish between eventually reflected and eventually transmitted particles. As shown in Section 3.2.1, the dwell time is an uncontested expression for the average time spent in the barrier by an incident particle regardless of whether it is eventually reflected or transmitted. Ambiguity arises only when an attempt is made, within the orthodox interpretation, to specify the earlier behaviour of a tunnelling particle based on the state in which it is eventually measured.

The confusion surrounding quantum tunnelling time *can* therefore be traced back to problems about distinguishing between eventually reflected and eventually transmitted particles, vindicating several authors' claims to that effect (see e.g. Leavens and Aers 1996, 107; Sokolovski 2017; Steinberg 1995a, 32 and Leavens 1996, 127).

5.2 Tunnelling time: meaningful or not?

As discussed in Section 3.2.4, several authors have dismissed tunnelling time as "meaningless" in the orthodox interpretation—but are they correct, and if so, what can they mean by "meaningless"?

Meaningless is perhaps too strong a term. What we can assert with confidence is that tunnelling time requests an answer to a question that the orthodox interpretation is unable to provide, having committed itself to the state of a particle *just being* its wavefunction, even if that wavefunction is a superposition with respect to the quantity of interest.

Within the de Broglie-Bohm interpretation, tunnelling time is not problematic in this sense, simply because the de Broglie-Bohm interpretation does not commit itself to the wavefunction being a full and complete description of a particle's state. Rather, it posits an underlying dynamics that includes deterministic and localised trajectories.

I suggest that we should, nevertheless, hesitate to call tunnelling time "meaningless" in the orthodox interpretation. Various definitions have been proposed, and some are even experimentally measurable. Even if we admit that no unique 'correct' definition will ever be established, we can weigh the existing expressions on pragmatic grounds, as authors have indeed done for decades.

Authors have assessed proposed orthodox expressions based on their generality, experimental accessibility, convergence on classically expected results, on whether they involve stationary or time-dependent treatment, asymptotic or local treatment, the absence or presence of the Hartman effect, the satisfaction or violation of a weighted average identity that reduces to the dwell time, and on whether the values they yield are real or complex. Each of these features might be considered desirable or undesirable by authors with different aims.[21]

In a sense, the proliferation of different orthodox definitions for tunnelling time is similar to the proliferation of interpretations of quantum mechanics as a whole. They have provoked the same kind of debate: a debate that is largely considered to be unresolvable, but which is not thereby rendered meaningless. We can identify desirable and undesirable criteria for orthodox definitions of tunnelling time, just like we can identify desirable and undesirable criteria for different interpretations of quantum mechanics.

---

[21] For a consideration of generality, see e.g. Sokolovski and Baskin (1987). For experimental accessibility, see e.g. Potnis (2015), Steinberg (1995b), and Landauer and Martin (1994). For convergence on classically expected results, see e.g. Davies (2005) and Sokolovski and Baskin (1987), and for stationary vs. time-dependent treatment, see e.g. Lunardi et al. (2011), Winful (2006), Falck and Hauge (1988), and Hauge et al. (1987). The importance of asymptotic vs. local treatment is considered by e.g. Lunardi et al. (2011), Davies (2005), Leavens and McKinnon (1994), Leavens (1993), Hauge and Støvneng (1989), Falck and Hauge (1988), and Büttiker (1983), and the Hartman effect is considered by e.g. Lunardi et al. (2011), Calçada et al. (2009), Winful (2006), and Leavens (1990). A weighted average identity that reduces to the dwell time is discussed by various authors, including e.g. Lunardi et al. (2011), Calçada et al. (2009), Steinberg (1995b), Hauge and Støvneng (1989), and Leavens and Aers (1989). The merits of real vs. complex values are discussed by e.g. Lunardi et al. (2011) and Hauge and Støvneng (1989).

5.3 No crucial test

Various authors have suggested that tunnelling time could be used to provide experimental evidence for de Broglie-Bohm theory (Section 3.2.4). Their suggestions are inspired by the clarity, naturalness, and uniqueness of the expression for tunnelling time within the de Broglie-Bohm interpretation, and the proliferation of mutually inconsistent and controversial candidate expressions for tunnelling time within the orthodox interpretation.

Two versions of this idea have been raised: first, the idea that tunnelling time could be used to set up a crucial test of the Bohmian program, capable of confirming or disconfirming its predictive accuracy, and second, the somewhat weaker idea that desirable features of tunnelling time could or should be used to provide evidence in support of de Broglie-Bohm theory.

These suggestions, especially of the first kind, have been controversial—even Cushing, the first to provide a detailed proposal for a crucial experimental test based on tunnelling time, eventually gave up the idea. But some speculation remains—and besides, where authors have dismissed the possibility they have not always based their arguments on the same reasoning.

Bedard (1997), for example—in a direct response to Cushing—argues that even if his experimental proposal could measure de Broglie-Bohm tunnelling time, its outcome would not be able to falsify de Broglie-Bohm theory without simultaneously falsifying the standard interpretation. This is because, she argues, the two interpretations "(for all practical purposes) make the same predictions"; they only differ in their interpretation of experimental results (Bedard 1997, 186). Thus, although quantum tunnelling time in the standard interpretation and quantum tunnelling time in de Broglie-Bohm theory are "two different properties which are not coextensive and are perhaps measurable in different ways" (186), an experiment designed to measure tunnelling time in one is still a well-defined experiment in the other. If it produces an unexpected outcome, it has provided a falsification of both interpretations.

Belousek (2005, 680) seems to agree:

"Regarding the question of whether the 'transit' or 'tunneling' times in Bohmian mechanics constitute excess empirical content over 'orthodox' quantum mechanics (cf. Ref. 31), I am of the view that, while the ontology of particles following definite trajectories does constitute surplus physical content, this does not generate any excess empirical content in the sense of novel predictions. Instead of novel prediction, Bohmian mechanics allows a more detailed interpretation, and perhaps a more satisfactory explanation, of the measurement outcomes of certain experiments in terms of the dynamical quantities definable within its own theoretical framework. What one has here is a case not of excess empirical content but rather of the well-known 'theory-ladenness of observation'."

Cushing, in a response published as a postcript to Bedard's paper, agrees that his original proposal is unviable, but for a slightly different reason. The problem, in his view, lies not only with how to interpret a successful observation of de Broglie-Bohm tunnelling time, but with whether de Broglie-Bohm tunnelling time is observable at all (Bedard 1997, 186).

I suggest, based on the conclusions of Section 4, that there is a simple explanation for why we should not expect to be able to construct a crucial test using tunnelling time. The possibility of a tunnelling-based experimental test of de Broglie-Bohm theory is in principle precluded by exactly the same feature that preserves its empirical equivalence with the orthodox interpretation in other contexts: namely, the fact that we only have access to the indeterministic behaviour of the wavefunction, not the underlying deterministic dynamics (see Section 4.2).

The double slit, once again, provides a useful analogy. De Broglie-Bohm theory tells us whether a particle went through the upper or lower slit based on where it appears on the screen far beyond the slits. But it is still not possible to put a detection device at the slits, and measure which slit each individual particle is going through, without changing the particle trajectories and destroying the interference pattern on the screen. Even though the theory distinguishes left-slit from right-slit trajectories, we cannot experimentally isolate either set of trajectories without running into exactly the same problems that we would run into in the standard interpretation.

The situation is similar for tunnelling time. De Broglie-Bohm theory tells us, based on whether a particle ends up being detected as eventually transmitted or eventually reflected, how long on average it spent in the barrier region. But it is still not possible to use a measurement device to pick out the eventually transmitted particles before they have been transmitted, and track their duration behaviour, without changing the particle trajectories themselves. Even though the theory distinguishes between eventually transmitted and eventually reflected trajectories, we cannot experimentally isolate either set of trajectories and keep the underlying dynamics intact.

This aligns closely with Cushing's own last word on the topic. He writes: "for Bohm all measurements are ultimately position measurements and the distribution of these results must be given by $|\psi|^2$, both in Bohm and in Copenhagen. Hence, this does not appear to be a promising avenue for the resolution of our underdetermination" (Cushing and Bowman 1999, 92–93).

On the weaker question of whether we should take the reasonableness of tunnelling time in de Broglie-Bohm theory as evidence for the Bohmian program, I suggest that we should not even go that far, because we should not expect the orthodox interpretation to provide a reasonable answer to a question it is not well-posed to answer. We should not be surprised that de Broglie-Bohm theory offers a clear and unique definition for tunnelling time while the orthodox interpretation does not: its ability to provide such a definition stems from its ability to provide particle trajectories. Its ability to provide particle trajectories, in itself, can be seen as a desirable feature of de Broglie-Bohm theory. But the clear tunnelling time that stems from those trajectories should not be interpreted as additional evidence for the Bohmian program.

5.4 Weak measurement as a perspective, not a resolution

Experiments like the one reported in Ramos et al. (2020) use the Larmor clock to perform weak measurement combined with postselection, and might therefore seem to be able to evade the fundamental issue established in Section 4: namely, the orthodox interpretation's inability to answer questions about the histories of subensembles.

I suggest, however, that they do not, as long as their results are still interpreted from an orthodox perspective. A weak measurement approach to tunnelling time does make progress, in some sense, precisely because it uses postselection. At least it leaves the quantum tunnelling time question well-posed: we can reasonably ask an experiment based on weak measurement to provide a time spent in the barrier *specific to transmitted particles*.

The problem lies in the interpretation of those results, because it is not clear what measurements like those in Ramos et al. (2020) represent within a framework that does not allow for well defined particle trajectories. We already saw in Section 3.2.2 that Sokolovski warns against overinterpretation of the values provided by weak measurement, including those reported in Ramos et al. (2020), on the grounds that they probe *probability amplitudes* not *probabilities* (Sokolovski 2015, 2016a, b; Sokolovski and Akhmatskaya 2021). Other authors like Svensson (2013a, b), and Bhati (2022) urge similar caution.

This is a general problem for weak measurement, and it is the reason why weak measurement is not taken to "resolve" the double slit which-way question, despite having revealed average trajectories for single photons in a double slit interferometer.[22] It is simply unclear what such results mean physically, if we are committed to an interpretation on which localised particle trajectories do not exist.

The point stands quite aside from concerns about whether the Larmor clock is a reliable clock for measuring tunnelling, and about whether the implementation of the Larmor clock reported in Ramos et al. (2020) is optimal. Falck and Hauge (1988) and Hauge & Støvneng (1989, 930–932), for example, argue that the Larmor clock is only reliable when applied to a field much wider than the barrier region. This would discredit the experimental setup reported in Ramos et al. (2020), which applies a magnetic field confined to the barrier region.

Thus weak measurement offers a new perspective on the tunnelling time problem, not a resolution. It would, nevertheless, be interesting to see a weak measurement experiment extract average trajectories for particles incident on a tunnelling barrier— and to see whether they coincide with the Bohmian trajectories, as was shown to be the case for the double slit experiment (see Kocsis et al. 2011, 1173). In a recent paper, Spierings and Steinberg hint that they might be undertaking such a project in the near future. They write, "[f]uture work will study the distinct histories of transmitted and reflected particles, not only due to the role of atomic interactions, but more fundamentally because of the differing forbidden regions these subensembles probe" (Spierings and Steinberg 2021, 5).

6 Conclusion

The orthodox interpretation does not deny us the information we rely on to calculate expectation values for position, or momentum. In general, it does not even deny us the information we need to track duration. But it does deny us exactly

---

[22]For the paper which reports these average trajectories, see Kocsis et al. (2011).

the information we need to distinguish between eventually transmitted and eventually reflected behaviour of a tunnelling particle within the barrier region, just like it denies us the information we need to distinguish between went-through-left-slit and went-through-right-slit behaviour in the double slit experiment.

On this basis, I have argued for answers to four controversial questions about quantum tunnelling time. Motivated by the links that have been drawn in the literature between the tunnelling time problem and the more general "problem of time" in quantum mechanics, I asked whether the confusion and ambiguity surrounding tunnelling time on the orthodox interpretation can really be traced back to time's status as a parameter rather than an operator. I argued 'No': the lack of clarity stems from the status of superposition in the orthodox interpretation, not the fact that time cannot be represented by a self-adjoint operator. In fact it does not have much to do with time at all.

Motivated by claims in the literature about the meaninglessness of tunnelling time on the orthodox interpretation, I asked whether tunnelling time is in fact meaningless on the orthodox view, and if so, in what sense. I argued that the question at the basis of the tunnelling time problem is one which the orthodox interpretation is ill-posed to answer. But I suggested that this does not make tunnelling time meaningless in an absolute sense; the various competing orthodox definitions for tunnelling time might all be useful in different contexts.

Several authors have suggested that tunnelling time could be used to extract evidence for the Bohmian program, and I asked whether this is something we can or should aim to do. I answered that we cannot, and should not, in two senses. First, any experimental test of tunnelling time will be predicted to provide the same results on both de Broglie-Bohm theory and the standard interpretation, and therefore we cannot hope to be able to construct a crucial test of the Bohmian program using tunnelling time as the deciding factor. Second, pragmatically desirable features of tunnelling time in the de Broglie-Bohm interpretation (including, for example, its simplicity and clarity) should not be interpreted as outright evidence for that interpretation, because we should not expect the orthodox interpretation to provide a reasonable value for a concept that it is ill-posed to define.

Finally, I asked whether recent experiments based on weak measurement offer a solution to the orthodox interpretation's inability to track duration behaviour specific to eventually transmitted particles. I argued that they do not, because the meaning of weak measurement results —within a framework of quantum mechanics that does not endorse the veridicality of particle trajectories—is bound to remain unclear.

These answers will, I hope, bring some clarity on a topic that remains genuinely controversial amongst physicists, and largely unknown within philosophy of physics.

Sciences and Humanities Research Council of Canada. None of these funding bodies played any role in the writing of the article or in the decision to submit the article for publication.

Declarations

Ethics approval N/A

Consent to participate N/A

Consent for publication N/A

Conflicts of interest There are no conflicts of interest/competing interests to declare.

References

Abolhasani, M., & Golshani, M. (2000). Tunneling times in the Copenhagen interpretation of quantum mechanics. *Physical Review A*, *62*(012106), 1–7.

Acuña, P. (2019). Charting the landscape of interpretation, theory rivalry, and underdetermination in quantum mechanics. *Synthese*, 1–30. https://doi.org/10.1007/s11229-019-02159-z.

Aharonov, Y., & Vaidman, L. (1990). Properties of a quantum system during the time interval between two measurements. *Physical Review A*, *41*(1), 11–20.

Aharonov, Y., Albert, D. Z., & Vaidman, L. (1988). How the result of a measurement of a component of the spin of a spin-1/2 particle can turn out to be 100. *Physical Review Letters*, *60*(14), 1351–1354.

Baz', A. J. (1967). *Soviet Journal of Nuclear Physics*, *4*(182).

Bedard, K. (1997). Reply to "Quantum tunneling times: a crucial test for the causal program?". *Foundations of Physics Letters*, *10*(2), 183–187.

Belousek, D. W. (2005). Underdetermination, realism, and theory appraisal: an epistemological reflection on quantum mechanics. *Foundations of Physics*, *35*(4), 669–695.

Bhati, R. S. (2022). Do weak values capture the complete truth about the past of a quantum particle? *Physics Letters A*, *429*(127955).

Bohm, D. (1961). *Quantum theory*. Upper Saddle River: Prentice-Hall.

Bricmont, J. (2016). *Making sense of quantum mechanics*. Springer.

Büttiker, M. (1983). Larmor precession and the traversal time for tunneling. *Physical Review B*, *27*(10), 6178–6188.

Calçada, M., Lunardi, J. T., & Manzoni, L.A. (2009). Salecker-Wigner-Peres clock and double-barrier tunneling. *Physical Review A*, *79*.

Challinor, A., Lasenby, A., Somaroo, S., Doran, C., & Gull, S. (1997). Tunnelling times of electrons. *Physics Letters A*, *227*, 143–152.

Chuprikov, N. L. (2014). Tunneling time problem: at the intersection of quantum mechanics, classical probability theory and special relativity. March https://arxiv.org/abs/1303.6181.

Condon, E. U. (1931). Quantum mechanics of collision processes: scattering of particles in a definite force field. *Reviews of Modern Physics*, *3*(1), 43–77.

Cushing, J. T. (1994). *Quantum mechanics: historical contingency and the Copenhagen hegemony*. London: The University of Chicago Press.

Cushing, J. T. (1995). Quantum tunneling times: a crucial test for the causal program? *Foundations of Physics*, *25*(2), 269–280.

Cushing, J. T., & Bowman, G. E. (1999). Bohmian mechanics and chaos. In M. Redhead, J. Butterfield, & C. Pagonis (Eds.) *From physics to philosophy* (pp. 90–107). Cambridge University Press.

Davies, P. C. W. (2005). Quantum tunneling time. *American Journal of Physics*, *73*.

Dumont, R. S., & Marchioro, T. L. (1993). Tunneling-time probability distribution. *Physical Review A*, *47*(85-97).

Falck, J. P., & Hauge, E. H. (1988). Larmor clock reexamined. *Physical Review B*, *38*(5), 3287–3297.

Gondran, M., & Gondran, A. (2014). Measurement in the de Broglie-Bohm interpretation: double-slit, Stern-Gerlach, and EPR-B. *Physics Research International*, *2014*(605908).

Griffiths, D. J. (1995). *Introduction to quantum mechanics*. Upper Saddle River: Prentice Hall.

Hartman, T. E. (1962). Tunneling of a wave packet. *Journal of Applied Physics*, *33*(12).

Hauge, E. H., & Støvneng, J. A.. (1989). Tunneling times: a critical review. *Reviews of Modern Physics*, *61*(4), 917–936.

Hauge, E. H., Falck, J. P., & Fjeldly, T. A. (1987). Transmission and reflection times for scattering of wave packets off tunneling barriers. *Physical Review B*, *36*(8), 4203–4214.

Kocsis, S., Braverman, B., Ravets, S., Stevens, M. J., Mirin, R. P., Shalm, L. K., & Steinberg, A. M. (2011). Observing the average trajectories of single photons in a two-slit interferometer. *Science*, *332*(6034), 1170–1173.

Landauer, R., & Martin, T. h. (1994). Barrier interaction time in tunneling. *Reviews of Modern Physics*, *66*(1), 217–228.

Leavens, C. R. (1990). Transmission, reflection and dwell times within Bohm's causaul interpretation of quantum mechanics. *Solid State Communications*, *74*(9).

Leavens, C. R. (1993). Application of the quantum clock of Salecker and Wigner to the "Tunneling Time Problem". *Solid State Communications*, *86*(12), 781–788.

Leavens, C. R. (1996). The "Tunneling-time problem" for electrons. In J.T. Cushing, A. Fine, & S. Goldstein (Eds.) *Bohmian mechanics and quantum theory: an appraisal* (Vol. 184, pp. 111–130). Boston Studies in the Philosophy of Science. Springer.

Leavens, C. R., & Aers, G. C. (1989). Dwell time and phase times for transmission and reflection. *Physical Review B*, *39*(2), 1202–1206.

Leavens, C. R., & Aers, G. C. (1996). Bohm trajectories and the tunneling time problem. In R. Gomer (Ed.) *Springer series in surface sciences* (Vol. 29, pp. 105–140). Springer.

Leavens, C. R., & McKinnon, W. R. (1994). An exact determination of the mean dwell times based on the quantum clock of Salecker and Wigner. *Physics Letters A*, *194*, 12–20.

Lunardi, J. T., Manzoni, L. A., & Nystrom, A.T. (2011). Salecker-Wigner-Peres clock and average tunneling times. *Physics Letters A*, *375*, 415–421.

MacColl, L. A. (1932). Note on the transmission and reflection of wave packets by potential barriers. *Physical Review*, *40*, 621–626.

Merzbacher, E. (2002). The early history of quantum tunneling. *Physics Today*, *55*(8), 44–50.

In G. Muga, R. S. Mayato, & Í. Egusquiza (Eds.) (2008). *Time in quantum mechanics* (Vol. 1). Berlin: Springer.

Norsen, T. (2013). The pilot-wave perspective on quantum scattering and tunneling. *American Journal of Physics*, *81*(4), 258–266.

Pashby, T. (2014). Time and the foundations of quantum mechanics. PhD diss., University of Pittsburgh.

Peres, A. (1980). Measurement of time by quantum clocks. *American Journal of Physics*, *48*(7), 552–557.

Perovic, S. (2017). Niels Bohr's complementarity and quantum tunneling. A penultimate version of the paper published. In H. Folse, & J. Faye (Eds.) *Niels Bohr and the philosophy of physics: twenty-first century perspectives* (p. 2017). Bloomsbury. http://philsci-archive.pitt.edu/14031/.

Pollak, E. (1985). The significance of imaginary time in quantal reactive scattering. *The Journal of Chemical Physics*, *83*(3), 1111–1120.

Pollak, E., & Miller, W. H. (1984). New physical interpretation for time in scattering theory. *Physical Review Letters*, *53*(2), 115–118.

Potnis, S. K. (2015). Tunneling dynamics of a Bose-Einstein condensate. PhD diss., University of Toronto.

Ramos, R., Spierings, D., Racicot, I., & Steinberg, A.M. (2020). Measurement of the time spent by a tunnelling atom within the barrier region. *Nature*, *583*.

Rivlin, T., Pollak, E., & Dumont, R.S. (2021). Comparison of a direct measure of barrier crossing times with indirect measures such as the Larmor time. *New Journal of Physics*, *23*(063044).

Rybachenko, V. F. (1967). Soviet Journal of Nuclear Physics, *5*(635).

Salecker, H., & Wigner, E. P. (1958). Quantum limitations of the measurement of space-time distances. *Physical Review*, *109*(2), 571–577.

Smith, F. T. (1960). Lifetime matrix in collision theory. *Physical Review*, *118*(1).

Sokolovski, D. (2015). The meaning of "anomalous weak values" in quantum and classical theories. *Physics Letters A*, *379*, 1097–1101.

Sokolovski, D. (2016a). Weak measurements measure probability amplitudes (and very little else). *Physics Letters A*, *380*, 1593–1599.

Sokolovski, D. (2016b). Quantum measurements, stochastic networks, the uncertainty principle, and the not so strange "Weak values". *Mathematics*, *4*(56), 1–19.

Sokolovski, D. (2017). The Salecker-Wigner-Peres clock, Feynman paths, and a tunneling time that should not exist. *Physical Review A*, *96s*.

Sokolovski, D., & Akhmatskaya, E. (2021). Tunnelling times, Larmor clock, and the elephant in the room. *Scientific Reports*, *11*(10040).

Sokolovski, D., & Baskin, L. M. (1987). Traversal time in quantum scattering. *Physical Review A*, *36*(10).

Sokolovski, D., & Connor, J. N. L. (1993). Quantum interference and determination of the traversal time. *Physical Review A*, *47*(6).

Spierings, D. C., & Steinberg, A. M. (2021). Tunneling takes less time when it's less probable. https://arxiv.org/abs/2101.12309.

Steinberg, A. M. (1995a). Conditional probabilities in quantum theory and the tunneling-time controversy. *Physical Review A*, *52*(1), 32–42.

Steinberg, A. M. (1995b). How much time does a tunneling particle spend in the barrier region? *Physical Review Letters*, *74*(13).

Stevens, K. W. H. (1980). A note on quantum mechanical tunnelling. *European Journal of Physics*, *1*, 98–101.

Svensson, B. E. Y. (2013a). New wine in old bottles: quantum measurement—direct, indirect, weak—with some applications. https://arxiv.org/abs/1202.5148.

Svensson, B. E. Y. (2013b). What is a quantum-mechanical "weak value" the value of? *Foundations of Physics*, *43*, 1193–1205.

Valentini, A. (2019). Foundations of statistical mechanics and the status of the Born rule in de Broglie-Bohm pilot-wave theory. https://arxiv.org/abs/1906.10761.

Winful, H. G. (2003). Delay time and the hartman effect in quantum tunneling. *Physical Review Letters*, *91*(26).

Winful, H. G. (2006). Tunneling time, the Hartman effect, and superluminality: a proposed resolution of an old paradox. *Physics Reports*, *436*, 1–69.