Euro Jnl Phil Sci (2016) 6:377–400
DOI 10.1007/s13194-016-0145-1

ORIGINAL PAPER IN PHILOSOPHY OF SCIENCE

Model templates within and between disciplines: from magnets to gases – and socio-economic systems

Tarja Knuuttila[1,2] · **Andrea Loettgers**[3,4]

Received: 9 September 2015 / Accepted: 29 March 2016 / Published online: 22 April 2016

Abstract One striking feature of the contemporary modelling practice is its interdisciplinary nature. The same equation forms, and mathematical and computational methods, are used across different disciplines, as well as within the same discipline. Are there, then, differences between intra- and interdisciplinary transfer, and can the comparison between the two provide more insight on the challenges of interdisciplinary theoretical work? We will study the development and various uses of the Ising model within physics, contrasting them to its applications to socio-economic systems. While the renormalization group (RG) methods justify the transfer of the Ising model within physics – by ascribing them to the same universality class – its application to socio-economic phenomena has no such theoretical grounding. As a result, the insights gained by modelling socio-economic phenomena by the Ising model may remain limited.

Keywords Ising model · Physics · Socio-economic systems · Interdisciplinarity · Renormalization group theory

1 Introduction

New modelling and simulations methods abound, changing profoundly our understanding of science. What is striking about these methods is their thoroughly

✉ Tarja Knuuttila
knuuttil@mailbox.sc.edu; tarja.knuuttila@helsinki.fi

Andrea Loettgers
andrea.loettgers@unige.ch; andrea.loettgers@csh.unibe.ch

1 University of South Carolina, 901 Sumter St., Byrnes Suite, Columbia, SC 29208, USA

2 University of Helsinki, Helsinki, Finland

3 Université de Genève, Rue de Candolle 2, CH-1211 Genève 4, Switzerland

4 University of Bern, Bern, Switzerland

interdisciplinary nature. The same equation forms and mathematical and computational methods are used across different disciplines. Yet this interdisciplinary nature of modelling is anything but a new phenomenon. Both the mathematization of biology and economics, for example, relied on mathematical methods transferred notably from physics but also from physical chemistry (see Mirowski 1989; Kingsland 1985; Knuuttila and Loettgers 2012, In press). While the founding fathers of mathematical economics and biology were not talking about modelling, in contemporary terms it appears that this was precisely what they were engaged in. For example, Vito Volterra cast his theorizing on biological associations – the most famous example of which is the Lotka–Volterra model – in terms of "mathematical analogies" drawn from mechanics (Volterra 1901). Already the use of the word analogy hints at the theoretical transfer that was taking place and, on the other hand, analogies have been a recurrent theme in the philosophical discussion on modelling (see, e.g., Hesse 1966; Nersessian 2002; Knuuttila and Loettgers 2014b).[1]

To be sure, it is somewhat anachronistic to assume that the founders of mathematical economics or biology understood their endeavour in modern terms even though the fruits of their work are nowadays considered as models. Indeed, historian of science Israel (1993) locates the emergence of what he calls a "modelling approach" in the early 20th century, during which a new kind of idea of the relationship of mathematics to reality was born. The classical idea of the uniqueness of mathematical representation gave way to the modelling approach utilizing the same abstract mathematical representations across a multitude of domains. Such a modelling activity revolves around "formal structures capable of representing a large number of isomorphic phenomena" (1993, 478).

Those similar patterns and dynamical behaviours to which the same models are applied do not just lie in different fields covered by different disciplines. Model transfer takes place also *within* the disciplines. This raises the question of whether, from the perspective of modelling, intra- and interdisciplinary transfer are the same kinds of processes; and, if not, can the comparison of the intra- and interdisciplinary theoretical transfer tell us something revealing about the challenges of interdisciplinary theoretical work? With such questions in mind we will study the development and various uses of the Ising model within physics, contrasting them to a couple of its applications to socio-economic systems. Our choice of a physics model, the Ising model, is partly due to the fact that physics is the discipline that has provided most mathematical methods and templates for interdisciplinary transfer.

Another reason for our choice of the Ising model is due to its enormous versatility within physics and other disciplines. The Ising model is one of the most simplified and successful models in physics that is "used to gain insight into phenomena associated with a diverse group of physical systems – so diverse, in fact, that the branch of physics that explores what is common to them all is called 'universal physics'" (Hughes 1999, 97–98). The model was originally developed for the study of phase transitions in ferromagnetism. Subsequently, the model left its original scientific context, being applied to phase transitions in various kinds of systems in physics, and extended to

[1] In the 19th and early 20th centuries, models were understood in concrete physical terms (Boltzmann 1902), and when it came to mathematical modelling and interdisciplinary transfer, it was often discussed in terms of mathematical analogies (Bailer-Jones 2009, Ch. 3).

the study of a wide variety of natural and social phenomena such as neural networks, protein folding, and opinion formation.

Such versatility in the application of this one model seems surprising given the backdrop of the philosophical discussion on scientific representation and idealization that has tended to take one model and its target system as the basic unit of analysis (e.g., French 2003; Giere 2010; Suárez 2010; Strevens 2008; Mäki 2009). Some recent work on both scientific representation and idealization has sought to loosen this tight coupling between models and their supposed real-world targets in terms of widening the scope to cover several related models (e.g., Weisberg 2013; Knuuttila 2009; Kennedy 2012). These accounts, however, discuss the use of families of models within one area of inquiry, but do not target the trajectory of one model within a multitude of fields. From a philosophical point of view such interdisciplinary modelling practice raises at least two questions: What is it that motivates such cross-disciplinary usage of the "same" models? And, how is interdisciplinary model transfer justified?

Within physics and philosophy of science these questions have gone under the banner of "universality", or "multiple realizability", having arisen from the observation that materially different systems can exhibit the same kind of macro-behaviour with respect to phase transitions, for example (e.g., Batterman 2000; Batterman and Rice 2014; Morrison 2014; Reutlinger 2014; Lange 2015). When it comes to the Ising model the explanation for this fact in physics makes use of the renormalization group (RG) theory, and there has been a lot of discussion among philosophers of science, and also physicists, on how to understand it (see Shech 2013). In particular philosophers have focused on the question: can one find a physical explanation for the universality of such phenomena? However, much of this discussion overlooks how the Ising model is applied in other disciplines, for example, in economics and biology. In these disciplines it is less convincing to claim that most of the micro-details do not matter, and neither it is often possible to use the RG methods to justify the utilization of the Ising model.

Is there, then, a more comprehensive way to understand the intra- and interdisciplinary transfer of the Ising model – or other general models like it? We will argue that the Ising model can be approached as a kind of general *model template*. The notion of a model template is inspired by Paul Humphreys' work on computational science, and especially his discussion on what he calls computational templates (Humphreys 2002, 2004).[2] While for a computational template its tractability is the most important feature, the notion of a model template stresses also the conceptual side of model transfer. The model template is an abstract conceptual idea associated with particular mathematical forms and computational methods (Knuuttila and Loettgers 2014a). In the case of the Ising model this abstract conceptual idea is comprised of the interpretation of the mathematical structure of the model in terms of a general cooperative mechanism leading to clustering and phase transitions. It is this general idea, combined with the mathematical structure that makes the Ising model so attractive to intra- and interdisciplinary transfer.

[2] The term computational template may be a bit misleading. Humphreys is not using the term of a template to refer to a fixed pattern that travels from one scientific context into another. Quite the contrary, computational templates are rather flexible objects, which are changed and adjusted to suit the particular context of application.

In the following we will first consider the recent philosophical discussion on the Ising model, in particular the question of universality and how RG explanations link to minimal modelling strategy, as discussed by Batterman (2000, 2010; Batterman and Rice 2014). Then we go over to the historical examination of the Ising model and the RG methods, followed by two applications of the Ising model to socio-economic phenomena. A discussion of the intra- and interdisciplinary use of model templates concludes. We will show that even though a model may function as a model template in both intra- and interdisciplinary transfer, in interdisciplinary transfer, in particular, it may lose its associated theoretical and methodological toolbox that provided its justification and empirical content in the original field of application. As a result the model may end up being a thin analogue model as shown by the two socio-economic models studied.

2 Universality and minimal models

Robert Batterman has recently targeted the question of why various kinds of materially different systems such as magnets, fluids and gases having different kinds of micro-properties nevertheless exhibit the same kind of macro-behaviour when reaching their critical point, where they undergo a phase transition (Batterman 2000; Batterman and Rice 2014). For instance, in magnets one observes with an increase in temperature T a transition from the ferromagnetic into a paramagnetic phase near the critical temperature T_C, and in fluids a transition from liquid to gaseous state. Physicists approach this "sameness" in macro-behaviour in terms of a notion of universality, while Batterman also talks of "multiple realizability". In explaining the observed universality, Batterman refers to experimental findings, showing that the behaviour of a system at the critical point is characterized by so-called critical exponents. Such critical exponents characterize the nearly identical behaviour of various kinds of fluids and magnetic substances for example, as a function of temperature. From these kinds of examples Batterman extracts two general features of universality:

> First, the details of the microstructure of a given fluid are largely irrelevant for describing the behaviour of the particular system of interest. Second, many different systems (other fluids and even magnets) with distinct microstructures exhibit identical behaviour characterized by the same critical exponent. (Batterman 2000, 123)

Batterman claims that these two features – where the first feature is responsible for the second – are present also in many examples of multiple realizability discussed within philosophies of mind and other special sciences. For instance, he mentions pain as an example of an observed pattern that can be produced by physically distinct microstructures. What is crucial to notice about such phenomena, he argues, is that there is nothing mysterious about them. Instead, "there are *physical reasons why the details of the makeup of the individual realizers may be largely irrelevant for the upper level behaviour of the system*" (2000, 124). In order to argue for this he invokes the renormalization group (RG) methods that in the field of modern statistical physics show that certain macro-phenomena are largely insensitive to their microphysical base. We will discuss the specifics of RG methods in the following section in relation to the Ising model. For the moment, let us concentrate on the other big philosophical issue that is at stake: the question of representation.

Already in his 2010 article on the explanatory role of mathematics Batterman was wary of the "necessity of representation for explanation" (Batterman 2010, 10). More specifically, he argued against the idea that it is only the "static" relationship of a mapping between a mathematical model and the world that is explanatorily relevant, stressing instead the positive role of idealizations. Although idealizations are false in that nothing in the physical world involves them, the fact that they involve processes or limiting operations helps us to understand empirical regularities and the places where those regularities break down. Batterman (2010) makes use of RG methods to argue that it is not always necessary, or even possible, to de-idealize (in order to arrive at more accurate mappings). RG explanations are not representational in that "there are no structures (properties of entities) that are involved in the limiting mathematical explanations" (ibid., 19).

One way of saving the representationalist approach to the application of mathematics – and simultaneously accounting for how idealizations may positively contribute to explaining phenomena – uses the idea of minimal models/minimalist idealization. In his article on different kinds of idealization Weisberg (2007, 642) formulates the idea of minimalist idealization in the following way: "Minimalist idealization is the practice of constructing and studying theoretical models that include only the core causal factors which give rise to a phenomenon. Such a representation is often called a minimal model of the phenomenon." The idea of minimalist idealization has been cashed out in different ways by different philosophers such as Cartwright (e.g., 1989, 1999), Mäki (e.g., 1992, 2009) and Strevens (e.g., 2008). What all these accounts have in common is that they rely on the isolation of a few causal factors that either lie behind some causal capacities or make a difference in producing certain phenomena. Idealization makes a positive epistemic contribution in isolating these causal difference-makers or capacities.

Weisberg lumps into the group of proponents of minimalist idealization and minimal models also Batterman, and, moreover, uses the Ising model as an example of a minimal model. Yet Batterman's account of idealization does not really fit into the characterization of minimal models by Weisberg – and neither does the Ising model. Although Batterman speaks in favour of minimal models, his account of idealization does not make use of the isolation of a few core causal factors. Batterman and Rice (2014) clearly spell this out by challenging the isolationist and other representational approaches, which assume that models are explanatory in virtue of sharing some features with actual systems, e.g., the relevant causes in the case of isolationist accounts. Batterman and Rice call them "common features accounts" claiming that these accounts mistakenly expect that models are explanatory due to a "veridical representation of difference-making features within the model" (ibid., 355). Instead, they seek to show that "highly idealized models can play an explanatory role despite nearly total representational failure" (ibid.).

Their examples of such a combination of representational inaccuracy and epistemic efficiency consist of models that apply to a large class of diverse systems that nevertheless display the same macro scale phenomena. In such a case, they claim, the "story" of how models "latch onto the world" is not due to features that correspond. These models – like the minimal model for fluid flow, the Lattice Gas Automaton –

may share with real fluid such features as locality, conservation and symmetry, yet Batterman and Rice point out that "it stretches imagination to think of locality, conservation and symmetry as causal factors that make a difference to the occurrence of certain patterns of fluid flow" (ibid., 360). Rather, the fact that the different kinds of real fluids share these very general features with the model calls for an explanation instead of functioning as an explananda. Such an explanation, they suggest, is provided by a mathematical rescaling method such as RG. The method consists in eliminating the number of interacting components (or degrees of freedom) by some kind of averaging rule. Two systems belong to the same universality class if, as a result of repeated RG transformations, they flow to the same "fixed" point (see also Reutlinger 2014).[3] The Lattice Gas Automaton can be used to model different kinds of fluids because the RG methods show that it belongs to the same universality class as they do. The belonging to the same universality class, backed up by the RG methods, provides what Batterman and Rice call a "story"[4] that explains "patterns across extremely diverse systems and shows how minimal models can be used to understand real systems" (Batterman and Rice 2014, 349). This "story" applies also to the Ising model as The Lattice Gas Automaton is one variant of it. Indeed, the Ising model provides one of the prime examples of universality within physics. However, as we will show in the following, while the Ising model is applied across a variety of different disciplines, this story does not necessarily work for systems outside of physics and chemistry.

In order, then, to account for the transfer of the Ising model from physics and chemistry to other disciplines, we are putting forth a notion of a model template. It is a formal platform for minimal model construction coupled with very general conceptualization without yet any subject-specific interpretation or adjustment. It may be constructed in view of specific phenomena like the Ising model, becoming only subsequently generalized and detached from its original scientific context. It may or may not be backed up by methods like RG techniques. Before going into the notion of the model template we will first examine how the "story" of the independency of macro-scale behaviour developed from Ising's first attempts of modelling phase-transitions (1925), to the exact solution of the model by Onsager (1944), leading finally to the introduction of the renormalization group theory (RG) (Wilson 1969; Fisher 1975; Kadanoff 2013). RG provided an explanation as to why diverse systems show the same macro-scale behaviour.

3 The development of the Ising model and RG methods

The Ising model started out as a model for the investigation of phase transitions in ferromagnetic materials. We are all familiar with phase transitions from everyday life. If we boil water to make a cup of tea, the water will undergo a first order phase transition

[3] Lange (2015) argues that Batterman and Rice's account also makes use of common features, like flowing to the fixed point. We think that, at least with respect to the isolationist accounts, Batterman and Rice are on solid ground. Lange misses the point that the isolationist accounts do not understand the difference-making causal factors as general features of large classes of systems with different micro-ontologies. Moreover, RG explanations are neither causal while difference-making accounts are (see also Reutlinger 2014).

[4] The use of the word "story" implies that Batterman and Rice are cautious as to any realistic/representational interpretations given to RG methods and their application to a wide variety of systems.

when the water starts boiling. The situation is quite similar in ferromagnetic materials. When heat is transferred into a ferromagnetic system and the system reaches its critical temperature T_C it undergoes a second order phase transition. It is important to notice that there is a difference between a first- and second-order phase transition. In the case of a first-order phase transition one has a continuous transition, such as in the transition from liquid to gaseous. In the second-order phase transition, on the other hand, the transition takes place at a critical point accompanied by a diverging order parameter such as the magnetization in the case of ferromagnetic materials. The order parameter indicates the order in a system below and above the critical temperature. Approaching T_C from $T < T_C$ the ferromagnetic material loses at T_C its property of being ferromagnetic ($M > 0$) and becomes paramagnetic ($M = 0$). In this unordered state the magnetic moments point in all possible directions. Approaching T_C from the other direction $T > T_C$ the paramagnetic material ($M = 0$) becomes ferromagnetic ($M > 0$). This includes a transition from the unordered into an ordered phase in which the magnetic moments have a preferred direction.[5] This may not seem very spectacular. But to find an explanation for this behaviour on the microscopic level turns out to be very challenging. The phase transition, which is accompanied by a change in the magnetization M of the system, is macro-scale behaviour according to Batterman and Rice. The question sums up to this: what are the elements and interactions necessary on the micro-level in order for this macro-scale behaviour to emerge?

The first attempts to provide such an account already took place in the 1920s when Ernst Ising introduced a model, which was subsequently named the Ising model (Ising 1925). In the standard 2-dimensional version of the Ising model the magnetic moments are arranged on a square lattice. They are denoted by S_i, corresponding to the two possible orientations up and down of the magnetic moments in the model. Accordingly, these spin magnetic moments[6] can only take values +1 (up) and −1 (down). Furthermore, the magnetic moments only interact with their nearest neighbours in the model. In the case of a ferromagnet the interaction J is constant and positive. From the description of the model one gets a good sense of its abstract character. In real ferromagnetic materials, magnetic moments, as well as the couplings between the magnetic moments, are complex objects. But the critical assumption is the quantization into a two-state system. The spatial quantization of magnetic moments was a hotly debated non-classical feature of the Bohr–Sommerfeld quantum theory of the time.

The complexity of the physical system is largely reduced in the construction of the Ising model. As a result, perhaps quite astonishingly, the Ising model qualifies as a model of not only the ferromagnetic materials but, as already mentioned, of a large number of other physical and chemical systems. Batterman and Rice's characterization of minimal model explanations summarizes this property of the Ising model in the following way: "[…] the details that distinguish the model system and the various real systems are irrelevant. In these minimal model explanations, the key connection between the model and the diverse real-world systems is that they are in the same universality class", meaning that they "exhibit the same patterns of behaviour at much higher scale" (Batterman and Rice 2014, 350). While we acknowledge this point, we

[5] Each material undergoing a phase transition has its own characteristic order parameter. For example, in the case of superconducting materials, the order parameter is given by the density of the Cooper pairs.

[6] Spin magnetic moments are induced by the spin of elementary particles.

suggest that one should also pay attention to some crucial aspects of the actual construction process of the Ising model, and the question of how such universality classes have been found, and, finally, what the existence of such classes mean for the application of the Ising model. By asking how the Ising model and the universality classes came into being we will bring to the fore the development of the Ising model as a model template and the entanglement of its computational and conceptual dimensions.

3.1 The mathematical structure of the Ising model

Suppose the model system consists of N magnetic moments, each of them taking a value +1 or −1, resulting in all 2^N possible different states. Depending on the size of the system, the number of possible states easily becomes very large. This creates serious numerical challenges. In the following we denote a state consisting of N magnetic moments by $\{S_i\}$. The energy of the system, depending on the state and the interaction between the magnetic moments, is given by:

$$E\{S_i\} = -\frac{J}{2}\sum_{i,j,j\neq i}^{N} S_i S_j - h_{ex}\sum_{j}^{N} S_j, \tag{1}$$

$J_{ij} = J$ is the coupling between the magnetic moments and h_{ex} an external magnetic field. Such an external field can be used to align the magnetic moments in case $T > T_c$. If the external field $h_{ex} = 0$, the energy of the system becomes:

$$E\{S_i\} = -\frac{J}{2}\sum_{i,j,j\neq i}^{N} S_i S_j. \tag{2}$$

In this case the energy takes a minimum if all the magnetic moments point in the same direction and a maximum in case half of the magnetic moments point in one and the other half in the opposite direction.

In addition, it is assumed that the magnetic spin system thus defined is a thermodynamic system. The model is embedded into statistical mechanics and probabilities are assigned according to the Gibbs canonical ensemble where a temperature T is realized in terms of a heat bath with which the Ising system is in thermodynamic contact. This embedding associates a probability of realization to each configuration. Depending on the temperature T and the energy $E\{S_i\}$, each of the 2^N states has a different probability. For example, take a state $\{S_i\}$ which is linked to a high energy state.[7] The probability that the system will be in this state will be low at low temperatures, where the system is in its ferromagnetic state. The situation starts to change with an increase in temperature. By transporting heat into the system the magnetic moments start to fluctuate and point in different directions. The heat leads to an increase in energy and high energy states become more probable. For states with

[7] The energy states $E\{S_i\}$ differ from each other by the orientation of the magnetic moments. At low energy all the magnetic moments point in the same direction. This changes when energy in the form of heat is applied to the system and the spins start to fluctuate (see below).

low energies the situation is just the other way around. The probability of such energy states increase with decreasing temperature *T.* These probabilities are described by the Boltzmann probability:

$$p\{S_i\} = Z^{-1}\exp\left(-E\{S_i\}\big/kT\right), \tag{3}$$

with k as the Boltzmann constant and Z the partition function, which is given by:

$$Z = \sum_{\{S_i\}} \exp\left(-E\{S_i\}\big/kT\right), \tag{4}$$

where the sum runs over all possible configurations.

In the Boltzmann distribution, Z functions as a normalization constant but in addition to this, the partition function also plays a crucial role in calculating thermodynamic properties of the system, such as the specific heat C. Via Z the macroscopic thermodynamic properties are related to microscopic details of the system. The numerical challenge, which has been mentioned before, comes to the fore when the partition function is calculated. As the Eq. (4) shows, the sum in the partition function runs over 2^N possible states of system.

The Ising model was not a success story right from the beginning. After constructing the model in 1923/24 Ising attempted to demonstrate the existence of a phase transition for the 1-dimensional case but was not able to find one. This result, which is true for the 1-dimensional case, is not true for higher dimensional cases. In these cases phase transitions do occur. But Ising generalized from his result in one dimension to higher dimensions concluding that also in the 2- and 3-dimensional cases no phase transition would occur. The next 20 years did not see much progress. It was only in 1943 that Lars Onsager succeeded in finding an analytical solution for the 2-dimensional Ising model without an external magnetic field h_{ex} (Onsager 1944). Onsager found a way to calculate the partition function and could show that below $T_C = 2.269\,J/k$ a spontaneous magnetization would be observed in the model. Furthermore, close to the critical temperature T_C the specific heat C diverges. We will come back to these properties of the Ising model because they play an essential role in the analysis of the general properties of the Ising-like systems. Before that we take a look at the actual construction process of the Ising model showing how, among other things, Ising chose the particular structure of the model on the basis of the research of his PhD advisor Wilhelm Lenz.

3.2 Construction of the Ising model

In the philosophy of science the importance of model construction has only recently started to gain more general interest. Therefore, it has been customary to introduce the Ising model in its final form. For example, in his detailed and informative article on critical phenomena and the Ising model R.I.G. Hughes introduces the model in the following way: "An Ising model is an abstract model with a very simple structure. It consists of a regular array of points, or sites, in geometrical space" (1999, 99). From the present perspective, indeed, the Ising model counts as one of the most successful models in physics that stands out by its simple and elegant structure. However, taking a closer look into its development shows how its construction was constrained by various factors. These factors include the theoretical knowledge on magnetism at the beginning of the 20th century, the empirical results from experiments on magnetic properties of different materials, the structures and compositions of these materials, and last but not least, the available computational tools.

What do we know about the actual construction of the model? The Ising model originated from the research on paramagnetism by Lenz (1920). When Lenz started his work on paramagnetism, the dominant theory was Pierre Weiss' theory of magnetism, which had been introduced in 1907 (Weiss 1907). In studying Weiss' theory of magnetism, Lenz found some inconsistencies. According to Lenz new experiments had shown that in paramagnetic salts and gases the susceptibility shows the same temperature dependency as described by the Curie law. From this result Lenz inferred that the Curie law

$$\chi \cdot T = const, \tag{5}$$

with χ as the magnetic susceptibility, constitutes a fundamental principle providing a key for understanding the phenomenon of paramagnetism.

Weiss had performed two derivations of the Curie law. In both cases he made an assumption which, according to Lenz, was not compatible with the structure of crystals. The first derivation was based on the assumption that the molecules in crystals rotate freely such as in gases. In the second derivation Weiss assumed that elementary magnets are only allowed to perform oscillations around equilibrium positions. Lenz's solution to the problem of this inconsistency was to allow for turnovers of the magnetic moments, constrained by a given crystal symmetry. For example a magnetic moment could change its direction by a turnover of 90°. The spatial quantization of the magnetic moment, which is expressed by the variable S_i in the Ising model, results from determining the possible orientations of magnetic moments depending on the symmetries of crystals. By introducing a quantization of the spatial orientation of magnetic moments, Lenz introduced an element that became central for the structure of the Ising model.

A further crucial question concerned how to model the interaction between the magnetic moments. Here Lenz drew an analogy to electric fields by replacing the magnetic moments by electric dipoles. By doing so, Lenz was able to show that in the case of electric dipoles the internal electric field resulting from the ensemble of all electric dipoles in the crystal would be unreasonably big. Lenz replaced the assumption of an internal magnetic field by the assumption that only nearest neighbour magnetic moments interact with each other.

It turns out, then, that the construction process of the Ising model was mainly based on taking care of the inconsistencies in the earlier theoretical accounts of paramagnetism such as introducing a spatial quantization in the orientation of the magnetic moments guided by computational feasibility and making use of the rather controversial quantum theoretical speculation of the Bohr–Sommerfeld space quantization. A proponent of the isolationist account of minimal models might be inclined to think that the model had been constructed by systematically taking away details from an actual magnetic system. But as our discussion shows, the knowledge of magnetic phenomena as well as the composition of magnetic systems was incomplete at the time the model was constructed. The insight that one can show similar macro-scale behaviour independently of the details of different systems became an integral part of the process in which the Ising model was developed in conjunction with RG methods.

3.3 The emergence of a complex but successful relationship: The Ising model and the renormalization group theory

Measurements on ferromagnetic materials show that the three thermodynamic quantities C (specific heat), $\chi = \partial M / \partial h_{ex}$ (magnetic susceptibility) and ξ (correlation length) and the order parameter M (magnetization), change dramatically when the T approaches T_C. Usually one describes the variations of the order parameter and the thermodynamic quantities with the temperature not by the temperature T itself, but by the reduced temperature t:

$$t = \frac{(T - T_C)}{T_C}, \tag{6}$$

with $t = -1$ at $T = 0$ and $t = 0$ at $T = T_C$.

The behaviour of the order parameter and the thermodynamic quantities M, C, χ and ξ near the critical temperature T_C are governed by power laws of the following form:

$$M \approx |t|^{\beta}, C \approx |t|^{-\alpha}, \chi \approx |t|^{-\gamma}, \xi \approx |t|^{-\nu}. \tag{7}$$

The exponents α, β, γ and ν are the critical exponents and their respective values can be measured in experiments or simulations or calculated by RG techniques (Wilson 1969), asymptotic expansions or other numerical methods. In the case of the 2-dimensional Ising model one gets:

$$\alpha = 0, \beta = 1/8, \gamma = 7/4, \nu = 1. \tag{8}$$

Approaching T_C from $T < T_C$, the magnetization M becomes 0 at the critical point. The other three quantities C, χ and ξ on the other hand show a singularity at T_C. The four parameters and their critical exponents display the existence of a phase transition. Among the four parameters, the correlation length ξ plays a particular role in being linked to *cooperative phenomena*. Cooperative phenomena are of central importance when it comes to emergent properties. In the case of ferromagnets, cooperative behaviour becomes apparent when the system approaches its critical temperature T_C. At this point the spins within the system *do not just interact with their nearest neighbours but start to be correlated with each other at larger distance*s. This leads to the establishment of so-called *long-range order*. Such cooperative phenomena are not restricted to ferromagnetic systems. They are part of all systems showing second-order phase-transitions, such as, for instance, superconductors (Liu 1999).

The calculation of the critical exponents posed a tremendous challenge. A major breakthrough in the context was, as already earlier mentioned, Onsanger's exact solution of the 2-dimensional Ising model without an external magnetic field h_{ex} (Onsager 1944). The calculation, which is considered as a veritable tour de force, allowed for a solution of the partition function:

$$Z = \sum_{\{S_i\}} \exp\left(-E\{S_i\} \big/ kT\right). \tag{9}$$

From the partition function Onsager calculated the specific heat C and its critical exponent β. The calculations show that the specific heat C tend to infinity when the system reaches T_C. This result was in agreement with measurements. In Onsager's own words: "The partition function of a two-dimensional "ferromagnetic" with scalar "spins" (Ising model) is computed rigorously for the case of vanishing field. [...] The two-way infinite crystal has an order–disorder transition at a temperature $T = T_C$ [...]. The energy is a continuous function of T; but the specific heat becomes infinite as $-\log|T - T_C|$." (Onsager 1944, 117.) Onsager aimed at exploring mathematically the behaviour of the Ising system at the critical temperature. This, on the other hand, would only be possible by an analytical solution of Z. Taking these points together Onsager introduced mathematical methods and techniques to solve Z in order to gain important insights into the properties of the Ising system. At this point perhaps the most puzzling result was the agreement between the results produced by this very abstract model and the experimental results gained from different magnetic systems.[8]

Comparing the characteristic behaviours of different systems at T_C provides the basis for the identification of universalities. One of those characteristic behaviours is the change of the correlation length ξ with the temperature. The correlation length is defined via the expectation value of the spin product S_iS_j:

$$< S_iS_j > \approx \exp\left(|i-j| \big/ \xi(T)\right), \tag{10}$$

for spins at different points on the grid. The expectation value shows how strongly the spins are correlated depending on their distance to each other, the correlation length and the temperature. For $T > T_C$ the correlation length vanishes exponentially with increasing distance between the spins. As the critical exponent shows, at $T = T_C$, the correlation length ξ diverges.

The behaviour of the ferromagnetic material in dependence of ξ is analysed by block spin transformations. The main idea consists of examining the system at different length scales by changing the scale of the lattice by performing a so-called block-spin transformation. For example in such a transformation 9 spins at a time are grouped together. The orientation of the resulting spin is given by the majority of the orientations of the 9 spins. Each time such a block-spin transformation is performed, the scale of the lattice is changed by a factor of 3. Below T_C one observes long-range order because of small thermal fluctuations. Accordingly the cluster sizes are respectively large. By performing block-spin transformation the size of the cells of the block-spins

[8] In his calculations Onsager introduced an infinite system consisting of an infinite number of spins ($N \rightarrow \infty$). This is in contradiction with the measurements, which are carried out in finite systems. There has been a discussion in the philosophy of science on this point (see Shech 2013).

become larger and larger and therefore the fluctuations smaller. Those fluctuations, which are smaller than the lattice spacing, even disappear. In the end when $T=0$ the correlation length has a large but finite value. The spins are strongly correlated. Above T_C no long range order exists. Under each transformation the correlation length decreases. The clusters become smaller and smaller. This procedure can be best described as coarse graining. At $T=T_C$ clusters of all correlation lengths exist, one of them with "infinite" correlation length. By performing further transformations, the "infinite" correlation remains infinite; in addition, the distribution of cluster sizes remains unchanged. The system has reached a fixed point.

Kadanoff writes about the meaning of this fixed point: "The act of renormalization is a sort of focusing in which many different irrelevant couplings fade away and we end up at a single fixed point representing a whole multi-dimensional continuum of different possible Hamiltonians. These Hamiltonians form what is called a universality class. Each Hamiltonian in its class has exactly the same critical point behavior, with not only the same critical indices, but also the same long-ranged correlation functions, and the same singular part of the free energy function" (Kadanoff 2013, 151). What Kadanoff describes is the result of the repeated transformation. The Hamiltonians belonging to different systems lose their microscopic details in the course of these RG transformations so that in the end they are all represented by a Hamiltonian that is freed from the details. Interestingly, before RG methods it was already empirically established that one could group systems into universality classes at their critical temperatures (Stanley 1999). RG methods provided an understanding of why systems could be grouped into universality classes while offering at the same time a mathematical tool to obtain critical exponents.

3.4 Intradisciplinary transfer of the Ising model

So far we have discussed the Ising model in the context of ferromagnetism developing an understanding of how the Ising model can be applied in the study of phase transitions in ferromagnetic materials and how, by the use of RG methods, ferromagnetic systems can be assigned to different universality classes. But as implied by the concept of minimal models, and anticipating our discussion of model templates, the Ising model is not tied to ferromagnetic materials. An example of an intradisciplinary transfer of the Ising model is the Lattice-Gas model, already mentioned in relation to the work of Batterman and Rice (2014). The Lattice-Gas model is used in modelling gas–liquid phase transitions. The model consists of a 2-dimensional grid. Each site of the grid can either be occupied by an atom or stay empty. In this case S_i takes the value +1 in case the site is occupied and −1 when it is empty. Like in the Ising model the interaction only takes place between the nearest neighbours, but the coupling constant J changes to an interaction energy $d_{12}=-d$ if $S_1=S_2=1$ otherwise $d_{12}=0$. The energy and partition function are given by:

$$E=-\frac{d}{8}\sum_{i,j}(S_i+1)\left(S_j+1\right), \tag{11}$$

and

$$Z = \sum_{\{S_i\}} \exp\left(-E\{S_i\} \Big/ kT\right). \tag{12}$$

The model is designed in such a way that the atoms can move around on the grid. How fast they move around depends on the temperature T. Analogously to the case of magnetic moments, where at high temperatures ($T > T_C$) the magnetic moments fluctuate between the two possible orientations, the atoms fluctuate between different empty spaces. Here the fluid is in its gaseous phase. With a decreasing temperature T that is approaching T_C, the correlation length starts to increase. The atoms start to form clusters. With a further decrease in temperature ($T < T_C$) there are almost no more fluctuations meaning that the system reaches its fluid phase. The analogy between the Ising model and the Lattice-Gas model can hardly be overlooked. The basic structure of the model is preserved. Only the meaning of the binary variables and interaction has changed. As in the case of ferromagnets the calculated thermodynamic properties and behaviour of the model system correspond to experimental findings.[9]

4 Socio-economic applications of the 2-dimensional Ising model

The Ising model has been applied to a multitude of systems studied in diverse disciplines. Perhaps the most astonishing examples of these model transfers can be found in the social sciences. In this section we will discuss the application of the Ising model in modelling urban segregation (the so-called Schelling model, Schelling 1969) and opinion formation processes concerning the future economic developments. As we will show, the Ising model is able to reproduce the observed segregation in urban neighbourhoods as well as the structures exhibited by the data surveys of the opinion-forming process. The question is what kind of conclusions can be drawn from such results concerning the processes of racial segregation or decision-making by individuals. How do we understand the interaction between individuals, and their peer groups, or other individuals?

4.1 Case 1: urban segregation

On the initiative of a small group of physicists, the field of sociophysics started to develop during the late 1970s and early 1980s (see Galam 2012). As an interdisciplinary field of research, sociophysics seeks to apply the methods and tools from statistical physics to the exploration of social, psychological, political and economic phenomena. Sociophysics has neither received a broad, positive reception in physics, nor within the sociology community. Most physicists and sociologists have remained sceptical of the idea of applying methods and models of statistical mechanics to socio-economic phenomena. A common concern of both sociologists as well as physicists has been that details may be important when it comes to social systems. This concern has been

[9] There are very detailed studies on the analogy between the two models. See for example Baxter (2007).

frequently expressed by pointing out that humans do not behave like atoms (Galam 2012; Stauffer and Solomon 2009). As humans are obviously not atoms, what has been the motivation behind this interdisciplinary transfer of methods and models from physics into sociology?

According to the economist Steven Durlauf, sociophysics has offered novel insights into the causal factors determining individual decision making, showing the importance of peer groups, role models, and social networks in decision making ranging from hairstyle, smoking or non-smoking, to life determining choices. This development, Durlauf argues, implies a change in how to model individual decision making. Deviating from the classical approach in which individuals make purposeful choices, which could be described in terms of maximizing a utility function under given constraints, the *interaction* between the individual and its peer group or other individuals became the main modelling objective (Durlauf 2001). For physicists the Ising model seemed to be the appropriate framework for modelling these interactions.

Schelling's segregation model serves as an illustrative example. While Schelling himself did not develop his segregation model on the basis of the Ising model, physicists working on socio-economic phenomena recognised a "mathematical link" or "physical analogue" between the Schelling model and the physics of clustering (see, e.g., Vinković and Kirman 2006; Stauffer and Solomon 2007). Looking at urban neighbourhoods where individuals with two different ethnic backgrounds (A and B) live together, Schelling observed a tendency of forming clusters with a majority of A and B people. This seems to imply that people prefer neighbours with the same ethnic background. Yet the most surprising result of the model was that assuming that the two groups had only a very mild preference for the neighbours of the same ethnic background, strongly segregated patterns occur.[10] The classical approach of maximizing the utility function is taken up by the model of Vinković and Kirman (2006). According to this model the Schelling model can be understood as forming a solid structure. The agents are allowed to move only if their utility increases, that is, energy decreases. The whole system comes to a halt after all agents have maximized their utility. As Vinković and Kirman put it: "In the context of the physical analogue … the system is frozen into a solid" (2006, 19263).

Looking at this sociological phenomenon of segregation, described by the Schelling model, from the perspective of a physicist, the Ising model seemed to provide an appropriate modelling framework. In the model of Stauffer and Solomon (Stauffer 2008; Stauffer and Solomon 2007), an explicit link between the Ising model and Schellings's model is drawn. The Schelling model is transformed into a 2-dimensional Ising model, members of A and B groups are placed at the sites of a square lattice with A people corresponding to up spins and B people corresponding to down spins. Each one of the A and B individuals is surrounded by four neighbours. The probability that an individual belonging to one of the two groups will move to another site, depends exponentially on the ratio $(n_A\text{-}n_B)/T_o$ in which n_A and n_B are the number of neighbours from each group and T_o the tolerance. The lower the tolerance, the higher the probability for a move and the higher the tolerance the lower the probability to move. This means that in the case of low tolerance strongly segregated neighbourhoods will develop. Such behaviour is analogous to what has been observed in the Ising

[10] For philosophical discussions of the Schelling model, see, e.g., Sugden 2002 and Weisberg 2013.

model of ferromagnetism at temperatures different from the critical temperature T_C. As a further analogue to the Ising model, a critical tolerance exists so that if $T_o < T_{Co}$ the two ethnic groups will completely segregate.

Obviously, the Ising model served as a model template for the Schelling model in the most basic way. Modelling the segregation phenomena by the Ising model resulted in no surprises, although the Ising models succeeded in modelling the already familiar phenomena more successfully than Schelling's original model. The Schelling-type Ising model also addressed large-scale segregation in not being limited to small clusters like the original Schelling model – while remaining at the same time simpler in structure. Moreover, the introduction of the temperature T (standing for tolerance) to the Schelling model makes the behaviour of the model more interesting. According to Stauffer and Solomon "it ensures that in the presence of additional random factors the segregation effect can disappear in a quite abrupt way. The cities or neighbourhoods that are currently strongly polarized may be transformed into a uniformly mixed area by tiny changes in the environment: school integration, financial rewards, citizen campaigns, sport centres, common activities etc." (2007, 478). How seriously is this prediction supposed to be taken? It seems fair to say that the Schelling-type Ising model merely reproduced the assumed herding behaviour.

4.2 Case 2: opinion formation

Another example of the application of the Ising model in the context of socio-economics is provided by the opinion-formation processes concerning future economic developments. Here the 2-dimensional Ising model is used to probe the assumption that the opinions and expectations of individuals are not solely based on economic facts, but also on herding between individuals. This seems to be suggested by the data collected in monthly surveys among individuals. In these surveys individuals are asked about their opinions and expectations about future economic development. They can choose between three different answers: positive, negative and neutral. The opinions show fluctuations, which can be interpreted to result from the tendency of economic agents to align themselves with the prevalent expectations of peers. In modelling this social-economic phenomenon scientists use results from social psychology in drawing an analogy to the Ising model. Social psychologists argue that "an individual is the more likely to conform with the judgment of others the less the individual, who has to form the judgment (in our case an expectation about economic prospects), is in a position to do so in a rational and informed manner" (Stauffer 2013, 12). This insight forms the basis for the assumption that there is a strong interaction between individuals that then leads to herding.

Such herding behaviour is analogous to the next neighbour coupling in the Ising model. In the model, A denotes the number of individuals, which translates into the number of magnetic moments in the case of the Ising model. An individual can take three different values $a = \{+1, 0, -1\}$. $a = -1$ denotes a negative expectation, $a = 0$ a neutral expectation, and $a = +1$ a positive expectation. Differing from the Ising model each of the elements have three instead of two possible values. The model is therefore an extended Ising model (i.e., a 3-state Potts model that is a generalization of the Ising model in statistical mechanics). In the economic model the space of the possible states has 3-dimensions instead of 2-dimensions as in the Ising model. The interaction between the components takes place the same way as in the Ising model, that is, between the nearest neighbours. In this the two models agree.

For the socio-economic model there exists in all A^3 different states $\{a_i\}$. The probability for each of these different states is like in the Ising model given by a Boltzmann distribution of the form:

$$p\{a_i\} = Z^{-1}\exp(-E\{a_i\}), \tag{13}$$

with the partition function:

$$Z = \sum_{\{a_i\}} \exp(-E\{a_i\}), \tag{14}$$

and the energy function:

$$E\{a_i\} = J\sum_{ij} (a_i - a_j)^2. \tag{15}$$

The more similar the expectations of the neighbouring managers are, the higher the probability of a given state is, because a_i and a_j agree on their value (−1,0 or 1) and therefore each such pair of neighbouring managers contribute 0 to the sum. Neighbouring managers holding different expectations contribute J or $4J$ to the sum. The partition function is calculated by numerical methods. As a result one gets a critical interaction energy J=1.68 instead of a critical temperature. For this particular coupling strength the model is able to reproduce the fluctuations observed in the data survey.

The two examples discussed show how the insights gained in the context of physics have been transferred via the Ising model into the socio-economic context for modelling herding behaviour. Underlying this transfer is the assumption that, given the observed behaviour and empirical data, the herding phenomena could be modelled with the Ising model since herding behaviour is similar to the ferromagnetic case in which only the next neighbour interaction has been taken into account. The parameters have been chosen in accordance with the parameters in the Ising model. The primary criteria for model transfer, it seems to us, has been the probing of the applicability of the Ising model to the herding behaviour. The results are not very striking and the modelling exercises tend to remain at the level of theoretical speculation. Indeed, as Sobkowicz (2009) points out in his review of modelling opinion formation with physics tools, such sociophysics models tend to concentrate more on the models themselves than on the social phenomena. Moreover, "in choosing which results are interesting, sociophysicists are often guided by folk psychology, and select as interesting those that agree with their general expectations" (1.6).

Last but not least, in the socio-economic cases applying RG methods to ground the applicability of the model to the phenomenon to be modelled comes with its very own limitations. For example, how to identify in the socio-economic system the order parameter and the analogues to the thermodynamic quantities of specific heat, susceptibility and correlation length? Socio-economic systems and physical systems are so different from each other that the identification of these quantities in a meaningful way would be difficult and most probably remain questionable. But in addition to this problem, the socio-economic systems are also empirically less assessable. The contrast to physics applications of the Ising model is striking as in physics the model really succeeded in unifying various kinds of systems/phenomena in an empirically successful manner. Although the Ising model functions as a model template in both intra- and interdisciplinary transfer, the computational aspects are clearly more important in the case of its applications within physics than to socio-economic systems. It seems fair to conclude, then, that even though the Ising model is applied outside of physics, its journey into these other disciplines has not always been an easy one. In the following section we will discuss the notions of a computational template and model template in relation to the Ising model and its intra- and interdisciplinary applications.

5 Intra- and interdisciplinary transfers of the Ising model

Writing about the computational science Humphreys (2004, 53) suggests that "one must be willing to … switch the attention from problems of representation to problems of computation". He launches the notion of a computational template to account for the fact that a relatively restricted amount of mathematical forms and methods are used across the disciplines to model a wide variety of phenomena. Among such computational templates are different computational methods used, for example, in individual-based modelling, and widely used equations of which the Ising model provides one of the most famous examples. The most important feature of computational templates, according to Humphreys, is their tractability and usability across different disciplines. Batterman and Rice's analysis accords with Humphreys' view that the representational approach to modelling does not capture the success of such models as the Ising model (see also Batterman 2000, 2010). According to them, as we have seen, the almost identical macro-behaviour of a wide variety of systems with different material basis cannot be explained by invoking some common features among them without considerably stretching one's imagination and what a "common causal factor" is supposed to refer to. A different explanation is provided by the RG methods showing that the systems in question belong to the same universality class. What RG methods accomplish is transforming the Ising model into an effective computational template.[11] Let us explain this in more detail.

In our discussion of the Ising model we noted that it was originally introduced in 1925 to model phase transitions in ferromagnetic materials. By that time phase-transitions had been observed in experiments and it was known that under a phase-transition some thermodynamic quantities $C=0$ (specific heat), M (magnetization), χ (susceptibility) and ξ (correlation length) change dramatically. It took two decades before the first analytical solution of the Ising model was worked out (Onsager 1944). Onsager's solution was in agreement with the experiments, but it was mathematically challenging. RG methods provided a mathematical tool, which allowed for the calculation of the critical exponents of the order parameter M and thermodynamic quantities C, and ξ at the critical point. The critical exponents determine the behaviour of the thermodynamic quantities at this point. RG was not only a mathematical tool, it also

[11] The RG methods themselves can also be approached as computational templates.

functioned as an explanation as to why the Ising model, by leaving aside microscopic details, was able to model the behaviour observed in experiments and, moreover, why systems differing in their microscopic detail show universal behaviour and belong to the same universality class.

The Ising model provided the test case for the RG techniques as these techniques were developed by making use of the Ising model and related experimental data. The RG methods provide a justification for applying the Ising model to the other systems of the same universality class. This also explains the astonishing observation that Onsager's result was in agreement with empirical data. As such the RG methods underpinned a new kind of account of minimal modelling, one that is not based on sharing the same relevant causal factors. In fact, RG explanations are not causal at all, Batterman claims, because they do not link the detailed mechanisms on the micro-level to the phenomena observed on the macro-level (Batterman 2000, 2010; Batterman and Rice 2014). Reutlinger (2014) does not agree with this reasoning, although he does agree with Batterman's conclusions, offering a different argument for why RG does not lead to causal explanations. According to him RG explanations are mathematical explanations. They consist of operations such as spatial contraction and the renormalization of parameters which entail a transformation in the length scale – a coarse graining. The recurrent transformations map the Hamiltonians to a fixed point, where further transformations have no more effect (Reutlinger 2014, 1168). Moreover, RG explanations do not relate tokens or types of events, relating instead entire Hamiltonians in the space of the possible Hamiltonians (ibid.). As a result, Reutlinger concludes, "RG are noncausal explanations because their explanatory power is due to mathematical operations that do not serve the purpose of representing causal relations" (Reutlinger 2014, 1160). Reutlinger's account clearly, then, stresses the importance of the computational tractability of RG methods.

Both Batterman's and Reutlinger's accounts conform with the historical development of the Ising model that led from the puzzlement over why magnetic systems with different microscopic structures show the same behaviour at a critical point to the development of a mathematical method for the identification of universal behaviour among different systems. With RG methods the Ising model became a fully-fledged computational template, which allows for the examination of the various kinds of systems at their critical points. Besides temperatures, the critical points can be, for example, pressure or density. Yet, the Ising model is also more than a computational template that becomes especially visible when one considers the socio-economic applications of it.

In the context of its socio-economic applications, one cannot justify the use of the Ising model in the same way as one does in physics. How can one, then, understand its application to the modelling of herding? Let us first note that some aspects of the Ising model are of such general character that the components of the two valued variables S_i, and the coupling J can be assigned to atoms, humans, or molecules, that is, to everything that interacts. It means that one can use it as a very general model of interaction. Other parts of it are very specific, so it is not at all clear how the notion of a heat bath, temperature, and partition function transfers to non-thermodynamic systems. On the other hand, it is important to keep in mind that not all very general models are applicable to socio-economic systems.

The question, then, is why is one able to use the Ising model to study socio-economic systems? The answer to this question, we suggest, is fourfold: Firstly, it is based on some empirically observed patterns such as clustering. Secondly, the Ising model offers a well-understood mathematical and computational method to model such phenomena. Yet, these two points alone are not yet enough to enable the model transfer from physics to economics. Something else is still needed. The third element that needs to be in place is a very *general conceptual idea* of the kind of structure or interaction that the model exhibits, that is, the model is able to exhibit clustering and phase transitions. And, finally, these kinds of very general phenomena can be given a new interpretation in social psychological terms that gives initial plausibility to the model. We call a *model template a mathematical structure that is coupled with a general conceptual idea that is capable of taking on various kinds of interpretations in view of empirically observed patterns in materially different systems*.[12] For the transferability of a template its computational characteristic, tractability, is not enough. It is the conceptual side of the model template that mediates between the mathematical/computational forms and their various empirical interpretations.

All this may still seem very general. What is at stake? The notion of a model template highlights that it is a combination of a mathematical structure *and* some general theoretical concepts that, taken together, specify a general mechanism that can be applied to a subject or field displaying particular patterns of interaction. In the case of the Ising model, the conceptual core of the model consists of such notions as cooperative phenomena, phase transition and long-term order that are embodied in the mathematical structure of it. These theoretical ideas got their first mathematically concise formulation in the simple model of ferromagnetism, the Ising model. Subsequently, as these conceptual ideas matured theoretically and computationally, they were ready to be detached from their original context and transferred to the study of other kinds of systems seemingly displaying the same kinds of patterns.

The notions of cooperative phenomena and phase-transitions are due to the establishment of a long-range order in the system indicated by an infinite correlation length ξ at $T = T_C$. The establishment of long-range order is the characteristic feature of the cooperative phenomena. Ferromagnetism is a prominent example for cooperative phenomena and the Ising model has been indispensable tool for studying this phenomenon, and other phenomena displaying phase transitions. In the application of the Ising model to socio-economic phenomena the temperature T changes its meaning and becomes in the model of urban segregation (Schelling) the tolerance T_o and in the herding case the coupling energy J that can be interpreted as the strength of interaction between individuals. But in these socio-economic applications there is no order parameter analogue to the magnetization M in ferromagnetic materials. For both physics and socio-economic applications a critical value can be calculated, in which the correlation length becomes infinite and a phase-transition occurs. At this point the correlation is no longer restricted to next neighbours/individuals but takes place between all the neighbours/individuals in the system. The question is how to understand such a phase transition in socio-economic applications. There is some empirical evidence that something like this happens when some social phenomenon suddenly goes viral – as Salganik, Dodds and Watts have empirically shown in their study of the

[12] For an earlier, less developed account of the notion of a model template, see Knuuttila and Loettgers 2014a.

inequality and unpredictability of cultural markets (2006). But how general are these kinds of socio-economic phenomena?

The socio-economic applications show that the Ising model can be applied outside of physics due to its features as a model template – even though RG methods do not relate to such applications. However, the insights gained by RG methods within physics give more theoretical grounding to the Ising model as a model template explaining the universality of phenomena such as phase-transitions. This amounts to accounting for which of the many details are necessary and which are not for the phenomena to occur. In the case of applying the Ising model to socio-economic phenomena there is no basis for such decisions. Therefore, we suggest that the results gained by applying the Ising model in socio-economic contexts may remain rather limited. It is able to reproduce the phenomena thereby showing, for example, what happens if neighbourhood segregation or opinion formation are understood as some kind of herding. This is not of course an uninteresting result given that much of economic modelling can be understood in terms of conceptual exploration (e.g., Hausman 1992). But within socio-economic contexts the model does not have the same predictive value, and empirical and theoretical grounding as in physics.

6 Conclusion

We have argued that the various applications of the Ising model show that its intra- and interdisciplinary reach is not just based on its computational aspects. The conceptual side of the Ising model is equally important, as it guides model transfer enabling researchers to conceive of the system to be modelled as a system of a certain kind, involving specific kinds of interactions. Moreover, we have shown that the use of a model template, like the Ising model, may be justified in different ways in different disciplines. While the extant philosophical discussion on the Ising model has been focused on its universal character in physics, and in particular the role of RG methods in establishing such universality, these kinds of considerations do not apply in its socio-economic uses. Because of this, such general physics models like the Ising model are more contested in cases of interdisciplinary, rather than intradisciplinary, transfer. It is not just a question of humans not being atoms etc., but more crucially, we suggest, of the fact that transferred models may lack the accompanying computational techniques and justification in the new context. The RG methods and their centrality in establishing the universality of the Ising model serves as case in point.

Especially the transfer of the methods for modelling complex systems from physics to economics has created some controversy. The conviction of physicists that micro-ontology does not often matter for the macro-scale features has aroused considerable uneasiness especially within economics where the requirement of micro-foundations for macro-economic phenomena is so deeply rooted. Marchionni (2013) discusses such a clash of perspectives in relation to network science. While for physicists the system-independence of the network properties constitutes a major achievement, economists tend to find these models wanting since they only address "how" and not "why". From their perspective explaining "why" would amount to giving micro-level causal details.

Yet such micro-level causal interpretation is precisely what these methods are not even designed to give as they target the collective larger-scale phenomena emerging in complex systems. Through the discussion of the construction of the Ising model we have shown how the idea that micro-ontology does not matter is accounted for by methods like the Renormalization Group (RG) theories that provide inherently non-causal explanations for universality. Because of this asking for a causal micro-level justification for macro-scale phenomena is beside the point. RG methods have a precise meaning and established uses in physics that do not, however, apply when the Ising model is used to model socio-economic phenomena. Consequently, we suggest that in interdisciplinary transfer one should pay attention to the discipline-specific justification of the model templates applied, including, importantly, the associated theoretical and methodological tools. When models of, for example, complex systems are transferred from one discipline to another, they may lose large parts of their original theoretical and mathematical grounding, thereby turning into thin analogue models. This does not mean, however, that they would not be able to acquire new content in the new disciplinary context (see Knuuttila and Loettgers 2014a). In particular, it is worth considering whether mainstream economics would not do better by contemplating seriously the possibility of genuine macro-phenomena and letting go some of its insistence on micro-level, often individualistic, behavioural assumptions.

References

Bailer-Jones, D. M. (2009). *Scientific models in philosophy of science*. Pittsburgh: University of Pittsburgh Press.

Batterman, R. W. (2000). Multiple realizability and universality. *The British Journal for the Philosophy of Science, 51*(1), 115–145.

Batterman, R. W. (2010). On the explanatory role of mathematics in empirical science. *The British Journal for the Philosophy of Science, 61*(1), 1–25.

Batterman, R. W., & Rice, C. C. (2014). Minimal model explanation. *Philosophy of Science, 81*(3), 349–376.

Baxter, R. J. (2007). *Exactly solved models in statistical mechanics*. London: Academia Press.

Boltzmann, L. (1902/1911). Models. *Encyclopaedia Britannica* (11th ed.), 638–640. Cambridge: Cambridge University Press.

Cartwright, N. (1989). *Nature's capacities and their measurement*. Oxford: Clarendon.

Cartwright, N. (1999). The vanity of rigour in economics: Theoretical models and Galilean experiments. Centre for Philosophy of Natural and Social Science. Discussion paper series 43/99.

Durlauf, S. N. (2001). A famework for the study of individual behavior and social interactions. *Sociological Methodology, 31*(1), 47–87.

Fisher, M. E. (1975). The renormalization group in the theory of critical behaviour. *Review of Modern Physics, 47*(2), 543–556.

French, S. (2003). A model-theoretic account of representation (Or, I don't know much about art…but I know it involves isomorphism). *Philosophy of Science, 70*(5), 1472–1483.

Galam, S. (2012). *Sociophysics. A physicist's modeling of psycho-political phenomena*. New York: Springer.

Giere, R. N. (2010). An agent-based conception of models and scientific representation. *Synthese, 172*(2), 269–281.

Hausman, D. M. (1992). *The inexact and separate science of economics*. Cambridge: Cambridge University Press.

Hesse, M. B. (1966). *Models and analogies in science*. Notre Dame: Notre Dame University Press.

Hughes, R. I. G. (1999). The Ising model, computer simulation, and universal physics. In M. S. Morgan & M. Morrison (Eds.), *Model as mediators. Perspectives on natural and social sciences* (pp. 97–145). Cambridge: Cambridge University Press.

Humphreys, P. (2002). Computational models. *Philosophy of Science, 69*(S3), S1–S11.

Humphreys, P. (2004). *Extending ourselves: Computational science, empiricism, and scientific method*. Oxford: Oxford University Press.

Ising, E. (1925). Beiträge zur Theorie des Ferromagnetismus. *Zeitschrift für Physik, 31*(1), 253–258.

Israel, G. (1993). The emergence of biomathematics and the case of population dynamics: A revival of mechanical reductionism and Darwinism. *Science in Context, 6*(2), 469–509.

Kadanoff, L. P. (2013). Theories of matter: Infinities and renormalization. In R. Batterman (Ed.), *The Oxford handbook of philosophy of physics* (pp. 141–189). Oxford: Oxford University Press.

Kennedy, A. G. (2012). A non representationalist view of model explanation. *Studies in History and Philosophy of Science, 43*(2), 233–240.

Kingsland, S. E. (1985). *Modeling nature*. Chicago: The University of Chicago Press.

Knuuttila, T. (2009). Isolating representations versus credible constructions? Economic modelling in theory and practice. *Erkenntnis, 70*(1), 59–80.

Knuuttila, T., & Loettgers, A. (2012). The productive tension: Mechanisms vs. templates in modeling the phenomena. In P. Humphreys & C. Imbert (Eds.), *Representations, models, and simulations* (pp. 3–24). New York: Routledge.

Knuuttila, T., & Loettgers, A. (2014a). Magnets, spins, and neurons: the dissemination of model templates across disciplines. *The Monist, 97*(3), 280–300.

Knuuttila, T., & Loettgers, A. (2014b). Varieties of noise: analogical reasoning in synthetic biology. *Studies in History and Philosophy of Science, 48*, 76–88.

Knuuttila, T. & Loettgers, A. (In press). Modeling as indirect representation? The Lotka–Volterra model revisited. *The British Journal for the Philosophy of Science*.

Lange, M. (2015). On minimal model explanations: a reply to Batterman and Rice. *Philosophy of Science, 82*(2), 292–305.

Lenz, W. (1920). Beiträge zum Verständnis der magnetischen Eigenschaften in festen Körpern. *Physikalische Zeitschrift, 21*, 613–615.

Liu, C. (1999). Explaining the emergence of cooperative phenomena. *Philosophy of Science, 66*, S92–S106.

Mäki, U. (1992). On the method of isolation in economics. *Poznan Studies in the Philosophy of Science and Humanities, 26*, 316–351.

Mäki, U. (2009). MISSing the world. Models as isolations and credible surrogate systems. *Erkenntnis, 70*(1), 29–43.

Marchionni, C. (2013). Playing with networks: how economists explain. *European Journal for Philosophy of Science, 3*(3), 331–352.

Mirowski, P. (1989). *More heat than light: Economics as social physics, physics as nature's economics*. Cambridge: Cambridge University Press.

Morrison, M. (2014). Complex systems and renormalization group explanations. *Philosophy of Science, 81*(5), 1144–1156.

Nersessian, N. J. (2002). Maxwell and "the method of physical analogy": Model-based reasoning, generic abstraction, and conceptual change. In D. Malament (Ed.), *Essays in the history and philosophy of science and mathematics* (pp. 129–166). Lasalle: Open Court.

Onsager, L. (1944). Crystal statistics. I. A two-dimensional model with an order disorder transition. *Physical Review, 65*(3–4), 117–149.

Reutlinger, A. (2014). Why is there universal macrobehavior? Renormalization group explanation as non-causal explanation. *Philosophy of Science, 81*(5), 1157–1170.

Salganik, M. J., Dodds, P. S., & Watts, D. J. (2006). Experimental study of inequality and unpredictability in an artificial cultural market. *Science, 311*(5762), 854–856.

Schelling, T. C. (1969). Models of segregation. *American Economic Review, 59*(2), 488–493.

Shech, E. (2013). What is the 'Paradox of phase transitions'? *Philosophy of Science, 80*(5), 1170–1181.

Sobkowicz, P. (2009). Modelling opinion formation with physics tools: call for closer link with reality. *Journal of Artificial Societies and Social Simulation, 12*(1), 11.

Stanley, H. E. (1999). Scaling, universality, and renormalization: three pillars of modern critical phenomena. *Reviews of Modern Physics, 71*(2), S358–S366.

Stauffer, D. (2008). Social applications of two-dimensional Ising models. *American Journal of Physics, 76*, 470–473.

Stauffer, D. (2013). A biased review on sociophysics. *Journal of Statistical Physics, 151*(1), 9–20.

Stauffer, D., & Solomon, S. (2007). Ising, Schelling and self-organising segregation. *European Physical Journal B, 57*(4), 437–479.

Stauffer, D., & Solomon, S. (2009). Physics and mathematics applications in social sciences. In R. A. Meyers (Ed.), *Encyclopedia of complexity and systems science* (pp. 6804–6810). New York: Springer.

Strevens, M. (2008). Depth: An account of scientific explanation. Harvard University Press.

Suárez, M. (2010). Scientific representation. *Blackwell's Philosophy Compass, 5*(1), 91–101.

Sugden, R. (2002). Credible worlds: The status of the theoretical models in economics. In U. Mäki (Ed.), *Fact and fiction in economics: Models, realism, and social construction* (pp. 107–136). Cambridge: Cambridge University Press.

Vinković, D., & Kirman, A. (2006). A physical analogue of the schelling model. *Proceedings of the National Academy of Sciences of the United States of America, 103*(51), 19261–19265.

Volterra, V. (1901). On the attempts to apply mathematics to the biological and social sciences. In J. R. Goodstein (Ed.), The Volterra chronicles: Life and times of an extraordinary Mathematician 1860–1940 (pp. 247–260). History of mathematics, vol. 31. American Mathematical Society, 2007.

Weisberg, M. (2007). Three kinds of idealization. *Journal of Philosophy, 104*(12), 639–659.

Weisberg, M. (2013). Simulation and similarity. Using models to understand the world. Oxford University Press.

Weiss, P. (1907). L'hypothèse du Champ Moléculaire et la Propriété Ferromagnétique. *Journal de Physique Théorique et Appliquée, 6*(1), 661–690.

Wilson, K. G. (1969). Non-Lagrangian models of current algebra. *Physical Review, 179*(5), 1499–1512.