Eleventh Graders' Increasingly Elaborate Language Use for Disentangling Amount and Change: A Case Study on the Epistemic Role of Syntactic Language Complexity

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Abstract The syntactic dimension of academic language has often been studied with respect to students' difficulties with syntactic features in mathematical textbooks and test items, and these studies have contributed to understanding the communicative role of language. In contrast, the epistemic role of students' language use has mainly been explored in lexical and discourse dimensions. This research has shown that higher order cognitive demands require more elaborate language means. The aim of this article is to contribute to theorizing the epistemic role of syntactic language complexity by means of a topic-specific investigation using the mathematical topic of qualitative calculus, i.e., the informal meanings of amount and change. In order to do this, the learning process study presented in this article investigates 18 eleventh graders' conceptual pathways while dealing with challenging tasks on amount and change. The identification of different syntactic complexities in students' utterances provides an overview of the variance of possible phrase structures. Further, it shows that successive conceptual conciseness requires either increasing syntactic complexity or conceptual condensation. So increasing elaborateness in the lexical and syntactic dimensions seem to compensate each other.

Keywords Epistemic role of language · Syntactic complexity · Qualitative calculus · Conceptual compactness · Phrase analysis · Students' language use

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Zunehmend elaborierter Sprachgebrauch von Elftklässlerinnen und Elftklässlern beim Auffalten von Bestand und Änderung: Eine Fallstudie zur epistemischen Funktion von syntaktischer Sprachkomplexität

Zusammenfassung Die syntaktische Dimension der Bildungssprache wurde oft untersucht im Hinblick auf Lernenden-Schwierigkeiten mit syntaktischen Merkmalen in Schulbuchtexten und Testitems. Diese Studien haben beigetragen zum Verständnis der kommunikativen Funktion von Sprache. Im Gegensatz dazu wurde die epistemische Funktion des Sprachgebrauchs von Lernenden bislang vor allem in lexikalischer und diskursiver Dimension untersucht. Diese Forschung zeigte, dass kognitive anspruchsvollere Anforderungen elaboriertere Sprachmittel erfordern. Das Ziel dieses Artikels sind Beiträge zur Theoriebildung zur epistemischen Funktion der syntaktischen Sprachkomplexität durch gegenstandsspezifische Untersuchung zum Lerngegenstand qualitative Analysis, d.h. zu informellen Bedeutungen von Bestand und Änderung. Dazu berichtet der Artikel aus seiner Lernprozessstudie zu den konzeptuellen Lernwegen von 18 Elftklässlerinnen und Elftklässlern bei der Bearbeitung anspruchsvoller Aufgaben zu Bestand und Änderung. Die Identifizierung von unterschiedlichen Stufen syntaktischer Komplexität in den Lernendenäußerungen ermöglicht einen Überblick zur Varianz möglicher Phrasenstrukturen. Zudem zeigt es, dass die sukzessiv steigende konzeptuelle Prägnanz entweder eine erhöhte syntaktische Komplexität erfordert oder höhere konzeptuelle Verdichtung. Also scheinen sich Elaboriertheit in lexikalischer und syntaktischer Dimension gegenseitig zu kompensieren.

Schlüsselwörter Epistemische Funktion von Sprache · Syntaktische Komplexität · Qualitative Analysis · Konzeptuelle Verdichtung · Phrasenanalyse · Sprachgebrauch von Lernenden

Since the early works of Vygotsky (1934), language has been described in its *epistemic role*, which means its role as an important thinking tool in knowledge construction processes. In mathematics education, the role of language as thinking tool in processes of knowledge construction has also been investigated for 40 years (since Austin and Howson 1979; Pimm 1987). These studies contributed to explaining the empirical findings that students with high *academic* language proficiency outperform their peers that have lower language proficiency (Haag et al. 2013; Mullis et al. 2012) because of limitations of language as thinking tool: Higher-order cognitive thinking processes (such as developing conceptual understanding of mathematical concepts) are generally said to be connected to more elaborate language demands (Cummins 1979; Schleppegrell 2004). In spite of this wide consensus on this general phenomenon, there is a strong need for its further empirical elaboration before it can guide didactical actions: What exactly does “more elaborate” mean for a concrete mathematical topic such as amount and change in qualitative calculus, and how exactly does it connect to mathematical thinking?

For 30 years, research in mathematics education and linguistics has contributed to unpacking the connection between mathematical thinking and language, mainly with a focus on the *lexical dimension* (showing the relevance of extended vocabulary; see Pimm 1987; Schleppegrell 2007; Prediger and Zindel 2017) and the *discourse dimension* (identifying more complex discourse practices such as explaining and arguing; see Moschkovich 2015; Erath et al. 2018). In the *syntactic dimension*, however, academic language demands occurring in mathematical learning processes have mainly been investigated with respect to language reception: By analyzing students' comprehension processes for mathematical texts in textbooks (Bailey 2007; Österholm and Bergqvist 2013) and assessments (Abedi 2006; Haag et al. 2015; Leiss et al. 2017), detailed knowledge has been gathered about potential syntactic obstacles. Whereas these studies have focused on the *communicative role* of the academic language register, little exploration of its *epistemic role* has been done with respect to students' oral language *production* in mathematical learning processes.

Linguistic studies in other contexts suggest that increasing complexity of academic language is also connected to syntactic dimensions in *oral* language (Schleppegrell 2004; Kleinschmidt-Schinke 2018). As this became apparent also in our preliminary study on students' knowledge construction processes in qualitative calculus (Şahin-Gür and Prediger 2018), we decided to investigate the syntactic dimension of the academic language register in students' language production. Thus, this article investigates 11th graders' mathematical learning processes in acquiring conceptual understanding with respect to changing syntactic complexities by focusing on the mathematically challenging topic of qualitative calculus, specifically, the meanings underlying the first and second derivative and the relation of amount, change, and change of change. The learning process study investigates the following research question:

How can students' language use be characterized in the syntactic dimension while developing conceptual understanding of qualitative calculus?

For this analysis, we coordinate a concept analysis focusing on the mathematical meanings with a phrase analysis. The outcome of the inductive category-forming analytic procedures (Mayring 2015) are two topic-specific models in which increasing degrees of mathematical conciseness and syntactic complexity are integrated.

The article starts by summarizing the state of research on typical difficulties in the mathematical topic in view, qualitative calculus (Section 1). Section 2 describes the linguistic tools and relevant findings of existing research on language in mathematics learning. Section 3 presents the methodology of the learning process study and Section 4 the empirical results of the case studies.

1 Mathematical Topic in View: Counter-Directional Covariation as a Challenge in Qualitative Calculus

1.1 Conceptual Focus in Qualitative Calculus

Since Orton's (1983) pioneering work, many empirical studies have shown that most students tend to develop better procedural skills in calculus than conceptual understanding for its central concepts (Blum and Kirsch 1979; Steen 1988). The identified conceptual challenges that occur in all four main phases of the calculus learning trajectory:

- Phase 1 Recognize variation and co-variation in graphs (Clement 1989; Johnson and McClintock 2018)
- Phase 2 Distinction of amount and change (Nemirovsky and Rubin 1992; Hahn and Prediger 2008) and the interplay of their symbolic and graphical representations (Kinley 2016)
- Phase 3 Concept of rate of change as a quotient concept (Thompson and Thompson 1994)
- Phase 4 Dealing with limits and infinitesimal aspects of the derivative (Marx 2006; Mundy and Graham 1994)

In order to promote conceptual understanding of concepts in calculus, approaches in *qualitative calculus* (Thompson and Thompson 1994; Stroup 2002) suggest strengthening Phase 1 and 2 long before change is mathematized as average and instantaneous rate of change (Phase 3) and the derivatives and their procedural rules (Phase 4). Instructional approaches of qualitative calculus engage students in constructing meanings for the core concepts of amount, change, and change of change and in investigating their mutual relationship in different context situations and representations.

Rather than acknowledging only a preparatory function of qualitative calculus for the later formalizations in Phases 3 and 4, Stroup (2002) emphasizes that “understanding qualitative calculus is cognitively significant and ‘structural’ in its own right” (p. 170). The “own right” is justified by the relevance of qualitative concepts for out-of-school contexts such as newspaper headlines such as “Fewer child births. In recent years, the population growth has decreased”. Empirical evidence has been provided that many students and adults misinterpret this statement as reporting about declining populations (Nemirovsky and Rubin 1992; Hahn and Prediger 2008). But it is the *population growth* function, f' , that decreases, not the *population amount* function, f , which can still grow even if f' decreases, so that the growth just becomes slower. Distinguishing between a function f and its derivatives f' and f'' can provide the mathematical structure to interpret the newspaper headline (or in precise academic language without formal structure). One reason for many students' and adults' difficulties in interpreting the meaning of this headline is the necessity of precise language, to which not all students have automatic access. Additionally, in further phases of a calculus course, the same precise language is required to interpret meanings of symbols and notations in real-world situations or graphical representations.

1.2 Theoretical Framework for Disentangling Amount and Change in Situations with Counter-Directional Covariation

As Nemirovsky and Rubin (1992) have shown, the described misunderstanding occurs especially in context situations with a mathematical structure that Hahn and Prediger (2008) have called *counter-directional covariation*, i.e., when the covariation of f and f' have different directions: One increases and the other one decreases.

Hahn and Prediger (2008) suggested a framework for explaining the specific difficulties in understanding counter-directional covariation situations, the so-called level model (see Fig. 1). The level model builds upon Vollrath's (1989) distinction of two approaches for functions, the *correspondence* approach (asking for the value of f at x_1 and x_2) and the *covariation* approach (asking how f changes with x ; see also Confrey and Smith 1994).

The model visualizes necessary shifts of approaches when connecting levels (see Fig. 1): When changing from the level f to level f' , the covariation approach for f is turned into a correspondence approach for f' , as the dynamic perspective is objectified in a new correspondence. Analogously on the next level, the covariation approach for f' turns into a correspondence for f'' (Note that in the later formalizations of concepts in Phases 3 and 4, the shift between the levels includes the specific mathematization as a rate of change and the limit concept, but for the purely qualitative approach investigated in this paper, the rate and the limit is not yet relevant; see Stroup 2002).

The gray dashed arrows in Fig. 1 signify typical misconceptions. In particular, statements about the covariation of f' are often identified with statements about covariation of f , even when they have opposite directions. Nemirovsky and Rubin (1992) have first discovered students' "tendency to assume resemblances between the behavior [...] of the function and its derivative" (p. 4), for example, in tasks of graphical derivations.

Existing studies have revealed the first indications that the language might play a crucial role. Whereas Hahn and Prediger (2008) only described very generally that students' experienced the limitations of their everyday language, Nemirovsky

Fig. 1 Level model for complex relationships of amount and change (Hahn and Prediger 2008, p. 176)

and Rubin (1992) identified specific problematic linguistic cues: Expressions such as “more and more” or “less and less”, can pose challenges:

We can use the comparative words “more” and “less” in comparing two parallel events or two successive values of a single event. [...] However, a student may not be aware of this distinction when he says, “the faster the car, the more distance”, and he may thus confuse change over time with the comparison of two parallel events. This confusion may lead to an incorrect assumption that decreasing velocity implies decreasing distance. (p. 18).

These early hints to the possible relevance of the language for students’ disentangling of amount and change resonates with Thompson and Thompson’s (1994) brief remark that computational language as a “language of mathematical symbolism and operations” (p. 20) is not enough for conceptually distinguishing amount and change, and they request a “conceptual language”. However, they do not yet specify how it should look.

1.3 Conceptual Challenges While Disentangling Amount and Change

In many instructional approaches (not only in qualitative calculus), developing conceptual understanding is fostered by the design principle of *connecting multiple representations* (Duval 2006; Stroup 2002) and *language registers*, i.e. connecting students’ everyday language with the academic and technical register (Prediger et al. 2016). To do this, tasks such as those in Fig. 2 can provide opportunities for produc-

Task 1. Match the newspaper headlines with Graphs 1 - 6 shown below. Add fitting axis labels.

Fewer childbirths In recent years, the population growth has decreased.	[Fourth headline and three other graphs not printed]
Volcanic island arises The fast growth will soon bring the island above sea level.	
Orangutans threatened Palm-oil plantations entail dramatic decrease of population.	

Task 2. Match the formal conditions A - H shown on the cards with the graphs and headlines from Task 1. Justify your decisions.

A	B	C	D	E	F	G	H
$f(x) > 0$	$f(x) > 0$	$f(x) < 0$	$f(x) < 0$	$f(x) > 0$	$f(x) < 0$	$f(x) > 0$	$f(x) < 0$
$f'(x) > 0$	$f'(x) < 0$	$f'(x) > 0$	$f'(x) < 0$	$f'(x) > 0$	$f'(x) < 0$	$f'(x) < 0$	$f'(x) > 0$

Task 3. Match the third conditions as well.

$f''(x) < 0$	$f''(x) < 0$	$f''(x) < 0$	$f''(x) > 0$	$f''(x) > 0$	$f''(x) > 0$	$f''(x) < 0$	$f''(x) > 0$

Fig. 2 Activity for connecting multiple representations and registers in situations with counter-directional and equi-directional covariation (from Şahin-Gür et al. 2019)

Fig. 3 Conceptual compactness and syntactic complexity on inverse scales (illustrated by typical students' statements)

tive struggle in order to develop students' concepts and language in more concise and formal ways (Şahin-Gür and Prediger 2018).

As with all practices involving dealing with mathematically rich concepts, being able to match the headlines to the graphs and formal conditions involves the conceptual demand to unfold the compacted mathematical concepts (as shown in Prediger and Zindel 2017). This applies for the highly condensed academic language in the newspaper headlines and even more for the condensed technical language underlying the formal conditions. As the example in Fig. 3 indicates, the process of conceptually unfolding can go along with longer sentences, usually with a higher syntactic complexity. The example in Fig. 3 provides a first idea of the interplay between increasing *complexity* and a second characteristic of academic language that has been described as *integration* by Kleinschmidt-Schinke (2018). Integration in her sense can be considered a linguistic counterpart of conceptual compactness, but has to be further explored empirically. This requires a linguistic operationalization of what exactly is meant by syntactic complexity.

Whereas the utterances in Fig. 3 are only prototypical examples of potential unfoldings, no empirical study has so far investigated how students deal with this possible syntactic complexity in their cognitive processes of unfolding. That is why the focus of this research paper is on students' *language use* in the syntactic dimension.

2 Theoretical Framework for the Epistemic Role of Syntactic Complexity

2.1 Epistemic Role of the Academic Language Register for Developing Conceptual Understanding in Mathematics

Large-scale assessments have repeatedly revealed language gaps in mathematics achievement (e.g., Haag et al. 2013; Mullis et al. 2012; Prediger et al. 2018) and in learning gains over time (Paetsch et al. 2016) for both multilingual and monolingual learners. As several studies have shown, the language disparities seem to be larger for conceptual understanding than for procedural skills (Prediger et al. 2018) and also occur in items without reading obstacles (Ufer et al. 2013; Pöhler et al. 2017).

Current explanations for language disparities in school achievement mainly rely on the sociolinguistic distinction between the everyday register and the school academic register (Cummins 1979; Snow and Uccelli 2009; Schleppegrell 2004). The academic register is distinct from the everyday register in all three dimensions (Bailey 2007; Chamot and O'Malley 1994):

- in the *lexical dimension* (e.g., by specialized vocabulary, composite or unfamiliar words, and specific connectors);
- in the *syntactic dimension* (e.g., long and syntactically complex sentences, passive voice constructions, and nested phrase structures; see Imo 2016; van Gelderen 2010);
- in the *discourse dimension* (characterized by specific discourse practices, e.g., arguing and explaining meanings; Erath et al. 2018; Moschkovich 2015).

The language demands of the academic register have often been studied with respect to the *communicative role* of language, in other words, language in its role as a medium for interpersonal communication in written or oral form, for example, by identifying language obstacles in textbooks and test items (see Section 2.2).

However, the second crucial role of the academic language register is its *epistemic role* as an intrapersonal thinking tool in students' epistemic processes, that is, in the processes of knowledge construction (Vygotsky 1934; Lampert and Cobb 2003; Pimm 1987). This role is described as follows: "Language is 'used by teachers and students for the purpose of acquiring new knowledge and skills ..., imparting new information, describing abstract ideas, and developing students' conceptual understanding'" (Chamot and O'Malley 1994, p. 40). Although the role of language as thinking tool has been described in general over the past decades (Vygotsky 1934; Pimm 1987), Moschkovich (2002) still emphasizes the research need for investigating students' processes of developing conceptual understanding in order to identify "the mathematical knowledge, ideas, and learning that learners are constructing in, through, and with language" (p. 12).

According to the research overview of Solano-Flores (2010), most studies which have investigated the epistemic role of the academic language for students' processes of conceptual understanding have focused on the discourse and the lexical dimensions.

- In the *lexical dimension*, mathematical higher order thinking has been identified as requiring vocabulary that belongs not only to the everyday register, but also to the academic and technical register. Thus, "more elaborate" lexical demands refer to an extended, less familiar vocabulary, sometimes with more complex word compositions (Pimm 1987; Schleppegrell 2007; Prediger and Zindel 2017; Smit et al. 2013).
- In the *discourse dimension*, the "more elaborate" discursive demands refer to more complex discourse practices, which have been identified as necessary for mathematical higher-order thinking: not only narrating and telling, but also reporting, explaining, arguing, and others (Moschkovich 2015; Erath et al. 2018; Prediger and Zindel 2017).

In this article, we extend the existing approaches to the *syntactic dimension* in order to explore in which way the potential complexities presented in Fig. 3 really occur in students' language use during their processes of developing conceptual understanding.

2.2 Current Research on the Syntactic Dimension of Academic Language

So far, the syntactic dimension of language in mathematics has mainly been studied with respect to syntactic obstacles in *language reception*. By analyzing students' understanding of mathematical texts in textbooks (Bailey 2007; Österholm and Bergqvist 2013) and assessments (Abedi 2006; Haag et al. 2015; Leiss et al. 2017), some syntactic features have repeatedly been identified as potentially difficult in mathematics-related reading (see Abedi (2006) for an extensive research overview):

- active vs. passive voice,
- length of nominals (in compound nouns or complex nominal phrases),
- comparatives structure,
- complex prepositional phrases, and
- sub-clauses.

These identified features provided the first motivation to focus on syntactic complexities of phrase structures in the current study (see Section 2.3) as they are known for their potential to hinder reading. However, little is currently known about whether these syntactic complexities also occur in students' (and teachers') language production.

One interesting exception is the study of Chesnais (2018): In her classroom studies, she empirically shows that some teachers articulate relations of symmetry only as *unary* predicates ("The two figures are symmetric"), which are highly underdetermined, whereas others used *binary* relations ("Figure A is symmetric to Figure B"). The most conceptually concise articulations draw upon *ternary* relations (Figure A is symmetric to Figure B with respect to Line d). She emphasizes that the syntactically less complex articulations might hinder a concise conceptual understanding of symmetry. In the current study, the difference between unary and binary predicates will also turn out to play a prominent role, namely in comparative phrases (see Section 4.3).

With this article, we want to follow Chesnais (2018) and transfer the research questions on the epistemic role also to the syntactic dimension in order to overcome the gap in research on the syntactic dimension in students' processes of developing conceptual understanding that Solano-Flores (2010) outlined.

2.3 Framework for Capturing Syntactic Complexity in Students' Language Production: Phrase Analysis

In order to capture the syntactic complexity in students' language production, we apply phrase analysis, which has been well established in structural linguistics (Müller 2016). Across different approaches, phrases are identified as the key structural enti-

Fig. 4 Phrase structures of two sentences on counter-directional covariation (from Fig. 3) in a tree with branches connecting (parts of) phrases and abbreviated indexed bracket notation

ties that combine sequences of words into units of meanings (Müller 2016). Their combination is realized by grammatical means such as declinations and conjugations (depending on the language being considered).

In the German and the English languages, a sentence without subordinate clauses usually consists of a nominal phrase and a verb phrase that contains all other constituents. Phrases can consist of a single word or several words and are hierarchically structured, so they are often represented by being nested in a tree format (see Fig. 4) or by bracket notations.

Among different existing systems for categorizing types of phrases, we chose the system of Imo (2016), as it fits both the English and German languages. Imo distinguishes nominal phrases (NP), verb phrases (VP), infinitive phrases (InfP), adjective phrases (AP), adverbial phrases (AdvP), prepositional phrases (PP), sentences (S), subordinate clauses (SSub), and relative clauses (SRel).

A phrase analysis reveals an operationalization of syntactic complexity by studying the nesting structure of the phrases, which can be determined by successively extending elementary phrases. The result can be written in indexed bracket notation (von Gelderen 2010, p. 37), as in the following examples. Possible versions of:

NP: [Increase]_{NP} → [The increase]_{NP} → [The [slow]_{AP} increase]_{NP}
 AP: [slow]_{AP} → [[ever]_{AdvP} slower]_{AP} → [[faster and faster]_{AdvP} slower]_{AP}
 VP: [increases]_{VP} → [will become more]_{AP}VP → [increases [more and more]_{AdvP}]_{VP}

The example in Fig. 4 reveals the phrase structure for two of the four sentences from Fig. 3. Whereas the first sentence is represented in a three-level tree for the successive nestings, the second sentence is represented in a five-level tree, so it has a higher syntactic complexity. The later empirical analysis will show why it is worth analyzing these details.

This research report in English about phrase structures in German data is possible as English phrase structures largely resonate with German phrase structures (Müller

2016). However, two differences between German and the English grammars are relevant for this article:

- (1) German has more *Kasus* (grammatical cases) than English for marking the role of nominal phrases: Nominal phrases in case *nominative* case take the role of subject in a sentence and nominal phrases in *dative* or *accusative case* take the role of objects and are part of the verb phrase. The *genitive case* indicates a relation of possession, which cannot be directly translated to English:

[Steigung]_{NP-Nom} and [Steigung [der Steigung]_{NP-Gen}]_{NP-Nom} is translated into [Increase]_{NP} and [Increase [of the Increase]_{PP}]_{NP}, thus into a prepositional phrase.

- (2) Whereas English grammar demands a formal distinction between adverbs and adjectives (e.g., quickly vs. quick), the same German word form can sometimes be an adverb or adjective. Both differences will be indicated in the translation of transcripts.

2.4 Summary and Refined Research Questions

The state of the research presented in Sections 1 and 2 allows the following line of argumentation about the research questions:

- As qualitative calculus (especially the relationship of amount, change, and change of change in contextual situations with counter-directional covariation) poses challenges for many students, it can count as an example of “higher order thinking” (Cummins 1979).
- Making sense of highly compacted mathematical concepts (such as the increase of the increase) requires conceptual processes of unfolding. The unfolded sentences can be syntactically complex, and earlier data analysis gave the first hints that students seem to struggle with these complexities in processes of conceptual development (Şahin-Gür and Prediger 2018). However, the earlier studies lacked a systematic operationalization of syntactic complexities.
- Linguists have developed grammatical theories that provide operationalizations for the syntactic complexity of phrases so that this complexity can be analyzed in detail (Müller 2016; van Gelderen 2010; Imo 2016).
- These linguistic frameworks provide the analytic tool to study how students’ thinking pathways are shaped by different degrees of syntactic complexity and which epistemic role they play.

This calls for refining the research questions as follows, splitting the former general research question into three sub-questions that will guide the later qualitative analysis:

- RQ1   Which pathways do students take through the level model while trying to make sense of contextual situations with counter-directional covariation? (auxiliary question)

- RQ2 Which degrees of syntactic complexity do students' nominal and verb phrases reveal while trying to make sense of situations with counter-directional covariation?
- RQ3 What is the epistemic role of increasing degrees of syntactic complexity, and how are they related to different aspects of increasing conceptual conciseness?

3 Methodological Framework of the Learning Process Study

This research is embedded in the larger project MuM-Calculus (e.g., Şahin-Gür and Prediger 2018), which follows a topic-specific Design Research methodology (Gravemeijer and Cobb 2006; Prediger and Zwetschler 2013) in five design experiment cycles. In this paper, we focus on Cycle 3 and present a learning process study.

3.1 Design Experiments for Data Collection

In order to be able to investigate the epistemic role of syntactic complexity in students' learning processes, an interventionist method of data collection was required (Gravemeijer and Cobb 2006; Prediger and Zindel 2017), because these processes can rarely be captured in uninfluenced classrooms.

For the current learning process study, we drew upon design experiments conducted in laboratory settings. In this study there were nine pairs of 11th graders, and the second author was the design experiment leader (in the following referred to as "tutor"). The students were sampled with respect to a varying mathematics achievement (teachers' evaluation) and academic language proficiency (assessed by a cloze test; Grotjahn et al. 2002).

Two sessions of 45–60 minutes each were completely video-recorded and partly transcribed for the nine pairs of students (in total about 1000 minutes of video material).

The analysis concentrates on students' processes while working on the focus tasks in Fig. 2, as these tasks resonates best with the research question (for the complete teaching learning arrangement, see Şahin-Gür et al. 2019). The manual for the design experiment regulated how the tutor elicited and questioned students' utterances and gave prompts for further precision without providing answers or language support.

The data analysis presented in this paper starts with a focus case of two 16-year-old boys, Neo and Simon, in the 11th grade of the pre-university track (*Gymnasium*) with rather strong mathematics achievement and above-average academic language proficiency. They were selected as focus pair due to their contrasting profiles in the video data in mastering the concept and language demands and because their process reveals several relevant phenomena found for many students who were also weaker in these areas. The data from the 16 other students in the study (also from

comprehensive schools with pre-university track) is accounted for in the inventory of phrase structures and comparison of cases.

3.2 Methods for Qualitative Data Analysis

In order to analyze the connection between students' conceptual learning pathways and the syntactic complexity of their language use, the qualitative data analysis coordinates two *deductive* and one *inductive* analytic procedures (Mayring 2015): Two theory-driven *deductive codings* were coordinated to separately capture the conceptual and syntactic sides (Steps 1 and 2). In Step 3, the conceptual and syntactic codings were integrated inductively to develop topic-specific degrees for conceptual conciseness and syntactic complexity.

Step 1. Concept Analysis as a Deductive Coding Process for Capturing Students' Conceptual Pathways To pursue the auxiliary research question RQ1, the concept analysis in Step 1 was methodologically founded on Vergnaud's (1996) epistemological constructs of concepts- and theorems-in-action and relied on Hahn and Prediger's (2008) level model (Fig. 1) for coding students' conceptual thinking about the amount and change function. Vergnaud (1996) defines a theorem-in-action as a "proposition that is held to be true by the individual subject for a certain range of situation variables" (p. 225), which are shaped by concepts-in-action, defined as "categories ... that enable the subject to cut the world into distinct ... aspects and pick up the most adequate selection of information" (ibid.).

Having extracted the individual's concepts- and theorems-in-action in each turn, all students' concepts- and theorems-in-action were coded. The mental activities of assigning and interpreting graphs or functions were coded as ASSIGN/INTERPRET f in order to describe the context for each utterance. The other codes stem from the level model in Fig. 1: COV f (or COR f) was assigned if students referred to a covariation approach in the level of the amount function f (or the correspondence approach in the level of the change function f). For the notation, codes in capital letters such as COV f and COR f signify that students' concepts- or theorems-in-actions were adequate, while lowercase letters such as cor f or cov f' symbolize an incorrect reference. This coding has proved insightful in capturing students' conceptual development in similar tasks (Hahn and Prediger 2008).

Step 2. Phrase Analysis as a Deductive Coding Process for Capturing Syntactic Complexity To pursue research question RQ2, the phrase analysis in Step 2 was methodologically founded on Imo's (2016) and van Gelderen's (2010) grammar and relied on their phrase-structure analysis for coding. Students' utterances were deconstructed in nominal phrases and verb phrases and coded according to the phrase structure into nominal phrases (NP), verb phrases (VP), adjective phrases (AP), adverbial phrases (AdvP), prepositional phrases (PP), and subordinate clauses (SSub). Fig. 4 gives an example of the identified tree structure.

For notation, in order to save space, the analysis in Section 4 was written in the indexed bracket notation (van Gelderen 2010), in which each branch of the tree

is symbolized by a pair of brackets. To enhance readability, we used the bracket-reduction convention where brackets of single-word phrases are omitted.

The translation of the German transcripts into English was conducted only after the concept and phrase analyses were completed. The original German transcripts are available in the Appendix.

Step 3. Inductive Category-Formation Process for Developing Integrated Degrees of Syntactic Complexity and Conceptual Conciseness To treat research question RQ3, in Step 3 the conceptual and syntactic codings of the two separate analyses were systematized and successively integrated in an inductive category-formation process (Mayring 2015). The outcome of this step, which will be presented in Section 4.3, consists of two topic-specific double scales for nominal phrases and comparing verb phrases, which show how the syntactic complexity is connected to different aspects of conceptual conciseness. The double scales were identified by iteratively systematizing the inventories of all 18 analyzed cases until a theoretical saturation was reached.

4 Empirical Analysis on Students' Language Use While Constructing Meanings for the Second Derivative

4.1 Episode 1: Neo's and Simon's Increasing Explicitness of Language Use While Rediscovering Counter-Directional Covariation

The transcript of Neo and Simon's dialogue shows excerpts of the boys' learning pathway when working on the focus task (Fig. 2), with the second column presenting the results of the concept analysis and phrase analysis for each turn.

Neo and Simon had worked on similar headlines some weeks before. In the meantime, they had learned to formalize the derivative as a rate of change and to determine it procedurally by the derivative rules. In the current task, they rediscovered the phenomenon of counter-directional covariation while trying to match the headline to one of the graphs. In their first attempt, they correctly chose Graph G3 for the newspaper headline "Fewer child births: In recent years, the population growth has decreased". Simon describes the covariation of the graph (first wrong in Turn #58a, symbolized by **cov graph**, then correct in #58b, symbolized by **COV GRAPH**):

58a	Simon	Okay, here [<i>points to G3</i>] with the births again, um, growth has gone up even further and then goes down and so—		cov graph: growth _{NP} [[has gone up [even further] _{AdvP}]VP and then _{AdvP} [goes down] _{VP}]VP
58b	Simon	Here [<i>points to the lower half of G3</i>], the graph of the increase, how can you say, the graph stays constant and no longer goes up or down.	interpret graph as of f': [the graph [of the increase] _{NP}]NP	
...		Growth is here on the y-side and time on the x-axis.	COV GRAPH: [the graph [of the increase] _{NP}]NP [...]] [[stays constant] _{AP}]VP and [[no longer] _{AdvP} goes [up or down] _{AdvP}]VP]VP	
79	Neo	[...] yes, growth is the y-axis and you can see that growth has really decreased over the years. Um, but it is still there, it just stays at the level and simply does not go up anymore.	interpret graph as of f': growth _{NP} [is [the y-axis] _{NP}]VP COV GRAPH: growth _{NP} [has really _{AdvP} decreased [over the years] _{PP}]VP COR GRAPH: it _{NP} [is [still _{AdvP} there] _{AdvP}]VP COV GRAPH: it _{NP} [just _{AdvP} stays [at the level] _{PP}]VP COV GRAPH: it _{NP} [simply _{AdvP} does not go up _{AdvP} anymore _{AdvP}]VP	

Although Simon correctly describes the covariation of the graph in #58b (“growth_{NP} [has gone up [even further]_{AdvP}]VP”), he uses the wrong term, “growth” (in this case population growth) when referring to the graph. By confounding the terms “population amount” and “population growth”, he articulates the increase in the wrong level but is, however, correct for the uninterpreted graph (“[the graph [of the increase]_{NP}]NP [[stays constant_{AP}]VP and [[no longer]_{AdvP} goes [up or down]_{AdvP}]VP]VP” in #58b instead of graph of the amount). The complexity of the phrase structure shows his early efforts to be concise.

The confounding of an unquestioned term is even more explicit in Neo’s utterance in #79 when he first interprets the graph as showing the growth (“growth_{NP} [is the y-axis]_{VP}”) and describes the covariation of this graph with the same nominal phrase in the next verb phrase, thus uses growth for f and f' in one sentence. In contrast, his description of the covariation in #79 is already very elaborate, juxtaposing four verb phrases to refer to the whole graph course. All of these verb phrases have high syntactic complexity (e.g., “growth_{NP} [has really_{AdvP} decreased [over the years]_{PP}]VP”).

This snapshot of the boys’ first approach shows that two challenges have to be mastered in order to interpret graphs and headlines with counter-directional covariation:

1. finding the adequate level and referring to it explicitly (not yet mastered here) and
2. comparing values of the function to describe the covariation (mastered here with a variety of different phrases).

When the tutor invites them to focus on the amount of population in #80, Simon and Neo start to distinguish both levels in the context (population versus births; deaths are not taken into account here) and to increasingly specify their interplay more exactly:

80	Tutor	And what about the population number?	
81	Simon	Yeah, it goes up even further, but not that, that much.	COV f: it _{NP} [goes up [even further] _{AdvP} , but [not [that, that] _{AdvP} much] _{AP}] _{VP}
85	Neo	It doesn't go up as much as before [...] But, basically, some are still born.	COV GRAPH: It _{NP} [doesn't go up [as much as before] _{AdvP}] _{AP}] _{VP} COV f: some _{NP} [are still _{AdvP} born] _{VP}
...			
87	Neo	It's simply that not more and more people are born each year, but fewer.	COV f: [[[not [more and more] _{AdvP}] _{AP} but fewer] _{AP}] _{AP} people] _{NP} are born [each year] _{AdvP}] _{VP}
95a	Simon	If there are fewer births, then the number of the population will also increase less	COR f' → COV f: [if there _{AdvP} are _{VP} [fewer] _{AP} births] _{NP}] _{SSub} , [then [the number [of the population] _{NP}] _{NP} [will also _{AdvP} increase less] _{AP}] _{VP}] _{SSub}
95b	Simon	and some time - without any births - the population will remain constant.	COR f' → COV f: [without [any births] _{NP}] _{PP} , [the population] _{NP} [will remain constant] _{AP}] _{VP}

Neo describes the change of the population in comparative ways (“It_{NP} [doesn't go up [as much as before]_{AdvP}]_{AP}]_{VP}” in #85 and “[not [more and more]_{AdvP}]_{AP} but fewer_{AP}]_{AP} people]_{NP}” in #87). Although the nominal phrase is underdetermined by “it”, he makes the levels *f* and *f'* more exact by his narrative on people being born. This narrative prompts Simon (in #95) to condense the process of change of the amount into a characteristic of population growth: He uses [fewer_{AP} births]_{NP} as a binary comparative adjective phrase (“fewer than”) in the nominal phrase, even if the compared quantities have not yet been addressed directly. After the concept of “growth” has been unfolded into a complete narrative by Neo in #87, Simon can use it as a meaningful compact term in #95a. This conceptual compaction goes along with a syntactic simplification and enables Simon to consider both levels at the same time and to explicitly address *f* and *f'* in order to relate them and their change processes conditionally (“[if there_{AdvP} are_{VP} [fewer]_{AP} births]_{NP}]_{SSub}, [then [the population]_{NP} [will also_{AdvP} increase less]_{AP}]_{VP}]_{SSub}” in #95a). This illustrates nicely how the analytic framework of phrase analysis helps to distinguish the complexities of the students' utterances and their interplay with increasing conceptual compactness.

Based on this achieved understanding, the students start to resort all assignments of Task 1 and see quickly that it is necessary to change the y-axis label of G3. In the end, they reject the term “population growth” and decide on “population”:

137	Neo	Or it simply stands for all the people [points at G3], how many people there are.		INTERPRET GRAPH AS f: [[how many] _{AdvP} people] _{NP} [there _{AdvP} are] _{VP}
138	Simon	But this is not a clear graph for growth. This would just be for the whole population. [...]	ASSIGN f: this _{NP} [is not [a clear _{AP} graph [for growth] _{PP}] _{NP}] _{VP} ASSIGN f: [the whole _{AP} population] _{NP}	
151	Neo	Yes, population.	ASSIGN f: population _{NP}	

By unpacking the meaning of decreasing growth of the population (in #87 and #95), the students were finally able to clearly distinguish the terms “growth” and “population” (in #138 and #151). Between #58 and #138, they mainly speak about *processes* of change in covariation approaches using comparative verb phrases, but rarely condense them into nouns. In these turns, both boys inappropriately assign the

addressed level and the expressed change process. Adequately labeling the y-axis using suitable nouns (starting in #95 and completed in #151) allows the students to make their references explicit and concise. It is only in #95 that Simon has appropriately condensed all information in the covariation phrases “If f is less, then f increases less” on two levels without going back to correspondence approaches (see Fig. 1).

The analysis of the Episode 1 shows that the students have sufficient *lexical language resources* to express processes of change, for instance, through verbs (“decrease_{VP}/increase_{VP}”, in #58 and #79, and “[goes up/down]_{VP}”, in #58 and #79) and comparative structures (“[as much as]_{AP}”, in #85, and “fewer_{AP}/less_{AP}”, in #95a) and are even able to support using a qualifier (“[more and more_{AdvP}]_{AP}”, in #81).

In contrast, their challenges involve the *syntactic and discursive demands* to make explicit the levels to which the change processes refer and expressing their mutual relationships with successive conciseness, even if the tutor wants to promote the explication again and again by her moves. As linguistically explained by Schleppegrell (2007, p. 146), making something explicit requires sentences with explicit references (instead of the implicit deictics “this_{NP}” and “it_{NP}”). This in turn requires a condensation of processes into nouns, which function as lexical markers for objectified concepts: The long sentence in #87, “[not [more and more_{AdvP}]_{AP} but fewer_{AP}]_{AP} people]_{NP} [are born [each year]_{AdvP}]_{VP}”, is later condensed into a nominalization that allows combination with a verb and even an adjective “population_{NP} [increases less_{AP}]_{VP}” (in #95a) and which leads to a syntactically less complex comparative structure because the comparison then expresses a single binary relation, even if the object of comparison is still indirectly addressed.

4.2 Episode 2: Neo’s and Simon’s Increasing Syntactic Complexity While Constructing Meanings for the Second Derivative

Despite these initial difficulties, Neo and Simon quickly succeeded in the further course in matching the headlines to the graphs and in assigning the formal conditions about f and f' to the graphs and headlines as requested in Tasks 1 and 2 shown in Fig. 2. Episode 2 starts 18min after Episode 1. Their new struggle begins with Task 3, which demands matching the formal conditions $f''(x) > 0$ or $f''(x) < 0$:

238	Neo	Oh God, what does the second derivative stand for again? I’ve forgotten.
239	Simon	Hmm. Now that’s a good question.
...		
324	Neo	I just could derive it from maximum and minimum but what, what the second derivative really means, I don’t know.

With f'' , Neo in particular only associates procedural aspects of calculating extreme points and continues to try until #324, 90 turns later, to infer the meaning of the second derivative from the formal conditions for extreme points. When the tutor invites them to compare the change processes of the increase for G3 and G5 (#412), they start to make sense of f'' by describing the change of change:

- 412 Tutor What is the specific difference between these two graphs?
- 414 Simon Well, they're also kind of opposite.
- 416a Simon [...] at first strong here, then flat [*points to the beginning and end of G3*],
- 416b Simon and here [*points to G5*] flat and then strong.
- 417 Neo That is the biggest difference, otherwise they are very similar, considering, um, you can also see that in the other two conditions above [*points to the formal conditions on f and f' for G3 and G5*].
- 420 Tutor What does that generally mean for the second derivative; what can the second derivative indicate?
- 424a Simon Perhaps also the increase? Because here [*points to G3*] the increase is..., well, it becomes less
- 424b Simon and here [*points to G5*] the increase becomes more.

COV GRAPH for G3: [[[at first]_{AdvP} strong (gesture)]_{AP}, [then_{AdvP} flat (gesture)]_{AP}]_{AP}

COV GRAPH for G5: [[[here (gesture)]_{AdvP} flat]_{AP} and [then_{AdvP} strong]_{AP}]_{AP}

A	E
$f(x) > 0$	$f(x) > 0$
$f'(x) > 0$	$f'(x) > 0$
$f''(x) < 0$	$f''(x) > 0$

COV f' for G3: [the increase]_{NP} [becomes less_{AP} [here (gesture)]_{AdvP}]_{VP}

COV f' for G5: [the increase]_{NP} [becomes more_{AP} [here (gesture)]_{AdvP}]_{VP}

The tutor successfully elicits attention to the change of change by comparing G3 and G5. Simon starts with a tentative conceptualization of change of change by using contrasting adjectives to express the change process of the graph (“[[[at first]_{AdvP} strong]_{AP}], [then_{AdvP} flat]_{AP}]_{AP}”, in #416a). In #424, he turns his unary comparison into a more precise description through nominalization and a more elegant binary comparison structure by using the comparative (“increase_{NP} [becomes less_{AP}/more_{AP}]_{VP}”), still as an indirect comparison by omitting the “than”. This condensation prepares Simon’s connection between the formal conditions about f'' (correspondence approach for f'') and the covariational approach for level f' (in #478).

All in all, treating the increase as an object with attributes allows them to develop their conceptual pathway towards the “new” formal element of the second derivative f'' . However, the need to unfold the meaning of the formal condition poses further syntactic challenges:

- 440 Simon [...] here [*points to the beginning of G3*] it is relatively much [*gesture*], he increases more or much and then the increase – that increases still,
- but the increase goes then more and more in direction zero.
- 443 Tutor So what does the second derivative mean?
- 458 Neo Here it goes toward zero [*points to G3*], [...], the increase [...]
- 459 Tutor So, what does it describe?

COR f' : it_{NP} [is [relatively_{AdvP} much]_{AP} here (gesture)]_{VP}

COV f or $f' + \text{COV } f$: he_{NP} [increases [more_{AP} or much_{AP}]_{AP}]_{VP} and then_{AdvP} that_{NP} [increases still_{AdvP}]_{VP}

COV f' : [the increase]_{NP} [goes then_{AdvP} [more and more_{AdvP}]_{AP} [in direction zero]_{PP}]_{VP}

COV f' for G3: it_{NP} [goes [toward zero]_{PP}] [here (gesture)]_{AdvP}]_{VP}

460	Neo	The course of the increase.	INTERPRET f'' : [the course [of the increase] _{NP}] _{NP}
476	Simon	How it progresses. Whether it goes like this or like that [<i>draws a faster increasing and then a slower increasing graph with his hand in the air</i>].	COV f' : it _{NP} [goes [[like this] _{AdvP} or [like that] _{AdvP}] _{AdvP}] _{VP}
477	Tutor	What does “like this or like that” mean?	
478a	Simon	First little increase and then more increase,	COR f' for G5 + COV f' for G5 : [First _{AdvP} [little _{AP} increase] _{NP} and then _{AdvP} [more _{AP} increase] _{NP}] _{NP}
478b	Simon	and the other one [<i>points to G3</i>] was just a lot of increase and then less increase at the end.	COR f' for G3 + COV f' for G3 : [the other one] _{NP} [was just _{AdvP} [[a lot of] _{AP} increase] _{NP} and then _{AdvP} [less _{AP} increase [at the end] _{AdvP}] _{NP}] _{VP}

Again, the main language demands are not on the lexical level, but on the syntactic and discourse levels: In these turns, the nominal phrase, which refers to the subject of increase or decrease, is of particular interest. The high number of *underdetermined deictic expressions* (“it”) for the levels being addressed shows the need for more explicitness, because every underdetermined expression causes a risk of confounding the levels, f , f' , and f'' . Explicit navigation through the levels in Fig. 1 (Section 1.2) is only possible with explicitly articulated references to the levels. The tutor’s prompts in #443, #459, and #477 were aimed at supporting the students to make their references explicit. And, indeed, both students were able to work with these prompts in #460 and #478 while confronting different parts of the graph: Neo, in rather underdetermined language (“it_{NP} [goes [toward zero]_{PP} ...]_{VP}” in #458), and Simon very explicitly and precisely (“[less_{AP} increase [at the end]_{AdvP}]_{NP}” in #478b). This shows that shifting to the level of f'' (change of change) requires more elaborate language means in which the consideration of the increase as an object and the resulting nominalization are indispensable parts.

At the same time, change processes of f' must be captured correctly and therefore described exactly in order to construct the meaning of the formal conditions $f''(x) < 0$ and $f''(x) > 0$ in the case of an increasing graph ($f'(x) > 0$). Shifting back and forth between the covariation approach for f' and the correspondence approach for f'' substantially supports the appropriate development of concepts for the second derivative.

In the next sequence, the tutor intends to offer a further condensation, assuming that the students should be ready to unpack the meaning of f'' by their previous ideas:

482	Tutor	We also like to say [for f''] the increase of the increase.	
483	Simon	Ah, the increase?	
490	Neo	Yes, that makes yes—no, that makes sense now, because that's the derivative of the first function; it describes the increase. If you now, for example, if you just cross it off [points to the first condition of A: $f(x) > 0$], then this [points to the second condition of A: $f'(x) > 0$] would be considered a normal function. Then yes—the derivative would describe the increase [points first to the third ($f''(x) < 0$), then to the second condition of A: $f'(x) > 0$], so it only makes sense. OK.	ASSIGN f': [the derivative [of the first_{AP} function]_{NP}]_{NP} INTERPRET f' as f': it_{NP} [describes [the increase]_{NP}]_{VP} INTERPRET f'' AS DERIVATIVE OF f': [the derivative]_{NP} [would describe [the increase]_{NP}]_{VP}
520	Simon	That just the increase of the increase, no, the increase of the increase runs differently.	ASSIGN f'' : [the increase [of the increase] _{NP}] _{NP}

This shows that students were still struggling hard to construct the meaning of the highly condensed expression “[the increase [of the increase]_{NP}]_{NP}”. Even if Neo and Simon had already made their first attempts (from #459 until #478) to apply the concept of growth to any increase other than the amount, they at first could not cope with the double nominalization, of the genitive attribute in a complex nominal phrase. After a short time, in #490, Neo first unpacked the nominalization (see Fig. 5, Step 1), as well as the underlying interplay of two neighbor levels (f' and f'' in Step 2 and then f and f' in Step 3): A function (amount/change) and its derivative (change/change of change) as a representation of the increase of the amount or change function.

This refined view of the relationship between the levels together with the explicit references to the levels prepared the nearly final interpretation of f'' as “population_{NP} [increases [less and less_{AdvP}]_{AP}]_{VP}” in #966a, which can be considered an unpacking of the level of amount (see Fig. 5). Neo finally completely unpacked the meaning of $f'(x) < 0$ for the change processes of f (see Steps 4 and 5) by describing the

Fig. 5 Neo's shift of levels

development of the population more and more precisely through binary comparisons with directly addressed compared objects:

966a	Neo	We see an increasing population which does not increase that much at the end. But which – which slowly, simply slowly increases less and less	COV f : [an increasing _{AP} population, [which [does not increase [that _{AdvP} much] _{AP} [at the end] _{AdvP}] _{VP}] _{SRel} but [which [slowly ...] _{AdvP} , [increases [less and less _{AdvP}] _{AP}] _{VB}] _{SRel}] _{NP}
966b	Neo	and then here [<i>points to the maximum of G3</i>] it is almost arrived by zero, that it no longer increases at all.	COV f' → COV f : it _{NP} [is almost _{AdvP} arrived [by zero] _{PP}] _{VP} , [that it _{NP} [[no longer] _{AdvP} increases [at all] _{AdvP}] _{VP}] _{SSub}
1003	Tutor	[...] What does the increase of population growth now mean for the amount of the population?	
1004	Simon	That the population will become less in the long run.	COV f : [the population] _{NP} [will become less _{AP} [in the long run] _{AdvP}] _{VP}
1010a	Neo	The population will continue, at least the increase of the population, this [<i>points to f'</i>] simply shows that the increase of the population [<i>points to f'</i>] continues into negative values.	COV f : [the population] _{NP} [will continue] _{VP} INTERPRET COR f' AS COV OF f' : [this (<i>gesture</i>)] _{NP} shows _{VP} , [that [the increase [of the population] _{NP}] _{NP} [continues further _{AdvP} [into negative values] _{PP}] _{VP}] _S
1010b	Neo	And that's why the whole population decreases [...] in the further course.	→ COV f : [the whole _{AP} population] _{NP} [decreases ... [in the further course] _{PP}] _{VP}

Overall, this last episode shows that Neo and Simon finally cracked the meaning of the formal condition $f''(x) < 0$, with clear reference to what it means for f and f' , making even more determined references by explicitly naming the levels and forming less syntactically complex descriptions by networking/connecting and switching the levels.

Summing up, the concept analysis and the phrase analysis of the case of Neo and Simon shows phenomena that we also found in the analysis of the other 16 students:

- students' possible productive struggle in situations with counter-directional co-variation and making sense of the second derivative;
- an amazingly high syntactic complexity in students' utterances (Neo's and Simon's being higher some other students'): In all 18 students there was a large range of phrase structures for expressing similar phenomena and a variety in precision;
- a first indication for a possible pattern for how increasing syntactic complexity might go along with successive mathematical conciseness; and
- an indication that students' can indeed profit from conceptual compaction, as it can reduce syntactic complexity: In the specific topic in view, this often goes along with shifting levels, but this must be analyzed more systematically.

4.3 Building Integrated Categories for Conceptual Conciseness and Syntactic Complexity for Nominal Phrases and Comparative Verb Phrases

In order to further explore the identified first pattern of connection between syntactic complexity and successive mathematical conciseness, Step 3 of the analysis aimed

at an inductive category formation procedure. The analysis of the individual cases showed that on the *syntactic side*, the complexity occurs in

- (S1) the nominal phrases and
- (S2) the verb phrases by which comparisons are usually expressed (abbreviated comparative verb phrases).

On the *conceptual side*, the challenges can be described in more detail now as referring to

- (C1) making explicit to which level a statement refers and
- (C2) exactly describing the covariance on different levels and most preferably at the same time.

In an inductive process of category formation of all analyzed utterances with respect to the conceptual and syntactic side, we were able to arrange both in integrated ways: C1 with S1 and C2 with S2.

4.3.1 (*S1 & C1*) Degrees for Nominal Phrases Expressing Increasingly Explicit References

After the sequential analysis of the transcripts that was done as presented in Sections 4.1 and 4.2, all nominal phrases from further students were collected and categorized according to their syntactic complexity, from minimum nominal phrases consisting of one underdetermined word to complex nominal phrases with subordinate nominal phrases (mostly in the genitive case in German, which is translated to English as a prepositional phrase).

We first inventory (a) the identified connection between explicitness and syntactic complexity. The inventory can then be explored with respect to (b) variance, (c) theoretical saturation, and (d) chronology.

(a) Identified Connection Between Explicitness and Syntactic Complexity of Nominal Phrases The inductive category-forming procedure revealed that this scale developed on the syntactic side can also be characterized on the conceptual side, namely as different degrees of explicitness, from only vague and implicit ways of addressing the levels to explicitly addressing and explaining them. Many students start with Degree 1: vague and implicit reference. References that were still implicit but could be identified by students' gestures, deictic means (e.g., "here"), or other resources were classified as Degree 2. On the syntactic side, both degrees have a similar structure. In contrast, Degree 3 does not use implicit references but makes them explicit using nouns. Degrees 4 and 5 qualify the reference further, for example, by descriptions or explanations. As a consequence, Fig. 6 depicts the five identified degrees for all nominal phrases in our data that addressed the levels of amount, change, and change of change (examples are provided in Table 1).

This double scale provides a topic-specific explanation for the epistemic role of syntactic complexity showing *how* an increasing syntactic complexity can go along with successive mathematical conciseness for the specific topic: The increasing

Conceptual side: Explicitness of reference to levels	Vague and im- plicit reference	Implicit but identi- fiable reference	Explicit reference	Explicitly described reference	Explicitly explained reference
	Degree 1	Degree 2	Degree 3	Degree 4	Degree 5
Syntactic side: Complexity of no- minal phrases (NP)	Minimum NP without reference	Minimum NP with reference	Simple NP: (article +) noun	Complex NP with AP or AdvP	Complex NP with PP / Genitive case

Fig. 6 Five degrees of explicitness (on the conceptual side) and complexity (on the syntactic side) for nominal phrases

complexity of nominal phrases seems to serve in particular to *increase explicitness of references*.

Table 1 shows the concrete inventories for the different cases: The first section shows Neo's, the second shows Simon's, and the third shows the rest of the 16 students'.

(b) Variance of Phrases The inventory lists 29 different nominal phrases by which Neo referred to amount and change, and 22 different nominal phrases for Simon. They span all five degrees of explicitness and complexity. This inventory in Table 1 of the nominal phrases the students used shows the huge variety of the ways in which they were able to express their ideas. This structured list can help teachers and tutors to formatively assess students' degrees of explicitness. Looking at the diverse use of the pronouns and their identified multiple references in Degree 2 clearly highlights the need to address the levels in a conceptually oriented explicit way, as, for example, in Degree 5.

(c) Theoretical Saturation As the third section of Table 1 shows, the inventories of the two focus boys' nominal phrases alone do not yet cover the whole range of possible nominal phrases. In order to increase theoretical saturation, the third section lists all 74 additional nominal phrases that occurred in the other 16 analyzed cases. Again, the many different versions, especially of underdetermined, implicit references, show the need to challenge students to elaborate the explicitness of references.

(d) Chronology The succession of turn numbers for the focus cases of Neo and Simon in the first and second sections reveal first insights into the chronology of explication. Even if the distribution of #58–#1010 over the columns of Table 1 indicates no deterministic pattern, there appears to be a tendency for higher degrees of syntactic complexity to occur more often in later moments of the learning process. The explicitly explained phrases (Degree 5) in particular require a condensation in nominalization that starts only in #460. In contrast, underdetermined, that is, vague and implicit, utterances (Degree 1) are normal for oral language at all times, and this is reflected by the fact that they also occur late in the process (during high turn numbers).

Table 1 Inventory of Neo's and Simon's nominal phrases referring to amount and change, ordered according to degrees of explicitness and syntactic complexity (complemented by those used by the other 16 students)

Degree 1	Degree 2	Degree 3	Degree 4	Degree 5
Vague implicit reference	Implicit but identifiable reference	Explicit reference	Explicitly described reference	Explicitly explained reference
(Minimum NP, no ref.)	(Minimum NP with reference)	(Simple NP: article + noun)	(Complex NP with added AP or AdvP)	(Complex NP with PP/genitive case)
Inventory of Neo's nominal phrases				
#85: some	#79: it (<i>growth</i>) #85/137: it (<i>G3</i>) #324: it (<i>f'</i>) #417: they (<i>graphs</i>) #458/966b: it (<i>the increase</i>) #490: it (<i>formal condition</i> $f(x) > 0$) #490: this (<i>formal condition</i> $f'(x) > 0$) #966a: which (<i>increasing population</i>) #1010a: this (<i>f'</i>)	#79: growth #79: the y-axis #151: population #417: the conditions #458/490: the increase #490: the derivative #1010a: the population	#87: [not [more and more AdvP] AP but fewer AP] AP people #137: all AdvP the people #137: [how many] AdvP people #238/324: the second AP derivative #490: a normal AP function (<i>the first derivative</i>) #966a: increasing AP population #1010b: the whole AdvP population	#460: the course [of the increase] NP #490: the derivative [of the first AP function] NP #996a: increase [of the increase] NP #996b: the increase [of population growth] NP #1010a: the increase [of the population] NP
Inventory of Simon's nominal phrases				
#440: he	#81: It (<i>population number</i>) #138: this (<i>G3</i>) #414: they (<i>both graphs</i>) #424a/440/476: it (<i>the increase</i>) #440: that (<i>G3</i>)	#58a, b: growth #58a: the births #95b/1004: the population #424a, b/440/483: the increase #478b: the other one (<i>G3</i>)	#95a: fewer AP births #95b: any AdvP births #138: the whole AP population #478a: little AP increase #478a: more AP increase #478b: [a lot of] AP increase #478b: less AP increase	#58b: the graph [of the increase] NP #95a: the number [of the population] NP #138: a clear AP graph [for growth] PP #520: the increase [of the increase] NP
Additional phrases articulated by the 16 other students that were not in Neo's and Simon's inventory				
He she it this that	He/she/it (<i>f</i>) he/she/it/that (<i>f'</i>) he/that (<i>f'</i>) it (<i>people</i>) it (<i>growth of increase</i>) he/this (<i>the increase</i>) he/she/it/that (<i>population growth</i>) he/she (<i>G3</i>) she/that (<i>pop. amount</i>) she (<i>f'</i> ; <i>the increase</i>) she/that (<i>increase of increase</i>) that (<i>births</i>) that (<i>headline</i>) that (<i>x-axis of G3</i>) that (<i>formal conditions</i>) this (<i>formal condition</i> $f'(x) > 0$) many (<i>people</i>) many (<i>births</i>) some (<i>people</i>)	<i>f</i> prime <i>x</i> <i>f</i> double prime people children the population number the population growth the growth a/the function the normal the derivative function the both the first the second the graph the curve three <i>x</i> thing	fewer AP people more AP people no AdvP people most AP births most AP increase the highest AP increase without AdvP growth which AP increase population growth [per year] AdvP this AP graph the first AP derivative the normal AP increase these local AP change rates the third AP condition	<i>f</i> [of <i>x</i>] NP the number [of people/population] NP quantity [of births] NP the change [of the population growth] NP the change [of the increase] NP the derivative [from $f(x)$] PP the derivative [of $f'(x)$ /the first AP derivative] NP the growth [of the increase] NP the growth [[of what] NP [of the function] NP] NP the increase [of this one] NP the increase [from this point] PP the increase [of the graph] NP the increase [of the derivative] NP the increase [of the increase] NP increase [of the real AdvP population growth] NP the population growth [of the population growth] NP

4.3.2 (S2 & C2) Degrees for Comparative Verb Phrases for Increasingly Precise Comparisons

In the next step, all verb phrases that were uttered to express covariance were inventoried and categorized according to the preciseness of their comparisons (following Chesnaïs 2018) and syntactic complexity in an inductive category-forming procedure. Fig. 7 shows the resulting category system with five degrees, which allowed integration of two scales. Table 2 provides examples for each category. Again, these can be analyzed with respect to four aspects: (a) identified connection between explicitness and syntactic complexity of nominal phrases, (b) variance, (c) theoretical saturation, and (d) connection to levels of derivatives.

(a) Identified Connection Between Preciseness of Comparisons and Syntactic Complexity of Verb Phrases *On the conceptual side*, the degrees are characterized by an increasing preciseness of the description of the comparison, from

- (D1) comparisons that are unary and address the object of comparison only indirectly (“it_{NP} [is much_{AP}]_{VP}”); to
- (D2) direct unary comparisons (“[[at first]_{AdvP} strong]_{AP}, [then_{AdvP} flat]_{AP}”); via
- (D3) binary comparisons with indirect comparison objects (“[the increase]_{NP} [becomes less_{AP} here_{AdvP}]_{VP}”, without “than there”); to
- (D4) direct naming of both comparison objects (“[becomes [less_{AP} here_{AdvP} than there_{AdvP}]_{AP}]_{VP}”); and finally
- (D5) binary comparisons in which comparison objects (mostly intervals within a graph segment) are addressed indirectly, but the comparing adjective phrase is even qualified (“[slowly_{AdvP} increases [less and less_{AdvP}]_{AP}]_{VP}”).

Conceptual side: Preciseness of comparisons	Indirect unary comparison	Direct unary comparison	Indirect binary comparison	Direct binary comparison	Indirect binary qualified comp.
	Degree 1	Degree 2	Degree 3	Degree 4	Degree 5
Syntactic side: Complexity of verb phrases (VP) operationalized by					
(a) count of phrase types (PT) involved	At least 1 PT, max. 1 edge	At least 2 PTs, two single-edge branches in the same nodes	At least 2/3 PTs, 1 single or double-edge branch with AP/AdvP	At least 3 PTs, 1 double-edge branch, binary complex AP	At least 3 PTs, 1 double-edge branch AP and AdvP
(b) count of edges in the trees i.e., nestings					
Prototypical examples:					
Example in brackets:	increases still _{AdvP}	increases strong _{AP} and then flat _{AP}	increases [less [at the end] _{AdvP}] _{AP}	increases [less than before _{AdvP}] _{AP}	increases [less and less _{AdvP}] _{AP}

Fig. 7 Topic-specific double scale for comparative verb phrases: Degrees on the conceptual and syntactic side

Table 2 Inventory for Neo's, Simon's and the 16 other students' comparative verb phrases ordered according to degree of preciseness and syntactic complexity

Degree 1	Degree 2	Degree 3	Degree 4	Degree 5
Indirect unary comparison (At least 1 PT, max. 1 edge)	Direct unary comparison (At least 2 PTs, two single-edge branches in the same nodes)	Indirect binary comparison (At least 2/3 PTs, 1 single or double-edge branch with AP/AdvP)	Direct binary comparison (At least 3 PTs, 1 double-edge branch, binary complex AP)	Indirect binary qualified comparison (At least 3 PTs, 1 double-edge branch AP and AdvP)
Inventory of Neo's comparative verb phrases for level <i>f</i>				
#79: stays [at the level]PP #79: does not go up AdvP any more AdvP #85: are still AdvP born #966b: [no longer] AdvP increases [at all] AdvP #1010a: will continue #1010b: decreases [in the further course] PP	–	#966a: does not increase [that AdvP much] AP [at the end] AdvP	#85: doesn't go up [as much as before AdvP] AP	#87: [not [more and more AdvP] AP but fewer AP] AP #966a: increases [less and less AdvP] AP
Inventory of Neo's comparative verb phrases for level <i>f'</i>				
#79: has really AdvP decreased [over the years] PP #458: goes [towards zero] PP #966b: is almost AdvP arrived [by zero] PP #1010a: continues further AdvP [into negative values] PP	–	–	–	–
Inventory of Neo's comparative verb phrases for level <i>f''</i>				
//				
Inventory of Simon's comparative verb phrases for level <i>f</i>				
#58b: stays constant AP #58b: [no longer] AdvP goes [up or down] AdvP #95b: will remain constant AP #440: increases still AdvP	#58a: has gone up [even further] AdvP and then AdvP goes down	#81: goes up [even further] AdvP, but [not [that, that] AdvP much] AP #95a: will also AdvP increase less AP #1004: will become less AP [in the long run] AdvP	–	–
Inventory of Simon's comparative verb phrases for level <i>f'</i>				
#440: is [relatively AdvP much] AP #478b: was just AdvP [[a lot of] AP increase] NP	#416a: [[at first] AdvP strong] AP, [then AdvP flat] AP #476: goes [like this] AdvP or [like that] AdvP	#95a: are VP [fewer AP births] NP #424a: becomes less AP #478b: less AP increase [at the end] AdvP	–	#440: goes then AdvP [more and more AdvP] AP [in direction 0] PP

Table 2 (Continued)

Degree 1	Degree 2	Degree 3	Degree 4	Degree 5
Inventory of Simon's comparative verb phrases for level f''				
//				
Additional phrase structures articulated by the 16 other students on all levels that were not covered by Neo and Simon				
[AdvP + AP]VP [[AdvP + AdvP]NP]VP [NP + AdvP]VP [[AP]NP]VP	[[AP]NP + [AP]NP]VP [AP]VP + [AP]VP VP + VP	[AP]VP + [AP]VP [[AP]AP]VP [AP + AP]VP [[AP + AP + AP]AP]VP	[[AP]AP]VP [[AdvP + AP]AP]VP [[[AP]NP + AdvP]AP]VP [[AP]AP]VP	[[[AdvP]AP]NP]VP
Additional phrase structures articulated by the 16 other students for level f that were not covered by Neo and Simon				
[AdvP + AP]VP [AP]VP [[AdvP + AdvP]NP]VP [NP + AdvP]VP [AdvP + PP]VP	[[AP]NP + [AP]NP]VP	[[AP]AP]VP [[AP]NP]VP	[[AdvP + AP]AP]VP [[[AP]NP + AdvP]AP]VP [[AP + PP]AP]VP	[[[AdvP]AP + AdvP]AP]VP [[AdvP]AP]VP [[[AdvP]AP]NP]VP
Additional phrase structures articulated by the 16 other students for level f' that were not covered by Neo and Simon				
VP [[AP]NP]VP	[AP]VP + [AP]VP VP + VP	[AP]VP + [AP]VP [AP + AP]VP [[AdvP]AP]VP [[AP + AP + AP]AP]VP	[[AP]AP]VP	[[[AdvP]AP + AdvP]AP]VP [AdvP[AdvP]AP]VP
Additional phrase structures articulated by the 16 other students for level f'' that were not covered by Neo and Simon				
VP [AP]VP [AdvP]VP	[AP + AP]VP [[AdvP]AP + [AdvP]AP]VP	[AP]VP [[AdvP]AP]VP	[[AP]AP]VP	–

On the syntactic side, the syntactic complexity of verb phrases is operationalized by the increasing (a) count of different phrase types (abbreviated PT) involved in the verb phrase (e.g., VP, AP, AdvP, or PP) and (b) counts of edges in the phrase structure trees, in other words, the counts of nestings and branches (Degrees 3, 4, and 5 mainly differ on the conceptual side).

This inductively developed double scale again provides a topic-specific instantiation for the epistemic role of syntactic complexity: Increasing preciseness of comparative descriptions of change processes seem to have a greater requirement for binary relations, which are syntactically more complex, than unary relations. The qualified binary relation (“slowly increases less and less”) is necessary for talking about two levels (f and f' or f' and f'') at the same time and are syntactically even more complex. Again, the possibility of integrating the conceptual preciseness and syntactic complexity scales provides further insights into the functioning of connections between syntactic complexity and successive mathematical conciseness.

(b) Variance of Phrases and Phrase Structures Table 2 shows the concrete inventories for the students: The first three sections list 14 different comparative verb phrases for Neo on the levels of f and f' and none on the level of f'' and the next three sections list 16 different comparative verb phrases for Simon on the levels of f and f' , none on the level of f'' . The inventories of the two boys' comparative verb phrases are not theoretically saturated: The 16 other students have provided many more phrases, which show a huge lexical variance. Because of this huge variance, the additional phrases are not inventoried in Table 2: Only the additional phrase structures are shown in the last four sections. These sections show that the 18 stu-

dents used an astonishing variety of comparative verb phrases, with variance not only in the wording in the lexical dimension but also in the syntactic dimension.

(c) Theoretical Saturation Although the inventory of all 18 students is not theoretically saturated in the lexical dimension, it reaches a theoretical saturation in the syntactic dimension: No further phrase structures were identified in each of the levels.

(d) Connection to Levels of Derivatives Whereas the analysis of the chronology of turn numbers is not very informative as students oscillate between the degrees, there are interestingly distinct patterns for the distribution of degrees over the levels f , f' , and f'' (see Fig. 8). In Neo's process, it seems that higher degrees of syntactic complexity more often occur when he expresses the relationship of at least two levels as a binary comparison in order to implicitly or explicitly unfold the meaning of the lower levels. The increasing syntactic complexity in Neo's statements about the change process of f , while he integrates the meaning of f'' more and more concisely

Fig. 8 Relationship between syntactic complexity and conceptual compactness for Neo's and Simon's comparative verb phrases

in his utterances (in #966a), provides a first qualitative insight for this assumption. For Simon, on the other hand, this pattern has a different representation (see Fig. 8). Since he cannot completely interpret the meaning of the second derivative either for the change level or for the change processes of the amount function, his utterances become syntactically more complex: For f' in particular he completely unfolded the meaning of f'' .

Remarkably, the other 16 students also articulated phrase structures for the second derivative. Once the concept is condensed on the level of the second derivative, the phrase structures can remain easier, mainly in Degrees 1–3.

In addition, the distribution over more than 900 turns (#58–#1010) indicates a tendency for the syntactic complexity of comparative verb phrases to not only be associated with the degree of precise comparisons, but also with the explicitness of references when shifting the level forward (e.g., COV $f' \rightarrow$ COR f''): Explicitly explained references through objectification of change (“[increase [of the population]_{NP}]_{NP}”; #1010a) may reduce the syntactic complexity of verb phrases (“[continues [into negative values]_{PP}]_{VP}”; #1010a), at least in Neo’s case, while less explicit references to the level (“[the increase]_{NP}”; #460) can lead to increasingly precise comparisons and syntactically more complex statements (“[goes then_{AdvP} [more and more_{AdvP}]_{AP} [in direction 0]_{PP}]_{VP}”; #440), as in Simon’s case.

Although far from being exhaustively explored, the two cases indicate that the combined consideration of increasing conceptual compactness and increasing syntactic complexity for nominal phrases and comparative verb phrases can provide further interesting insights.

5 Conclusion and Outlook

5.1 Main Results

What does it mean in the syntactic dimension to talk about amount and change in an increasingly elaborate way? And why is it epistemically required? The case study of Neo and Simon and the comparison with the 16 other students has provided insights into students’ pathways through productive struggle with the concepts of qualitative calculus (amount, change, and change of change) while trying to make sense of contextual situations with counter-directional covariation.

For the analysis of students’ learning processes, phrase analysis (van Gelderen 2010; Imo 2016) has proven to be a suitable analytic tool that can be transferred from analyzing mathematical texts (e.g., Solano-Flores 2010) to students’ language production. Phrase analysis enabled us to capture a first range of syntactic complexities, both for nominal phrases and for comparative verb phrases. As we have restricted the analysis of verb phrases to comparative verb phrases, this only provides a topic-specific lens and is not directly transferable to other topics.

The analysis shows that the calculus concepts change and change of change are highly compacted mathematics concepts, and their unfolding requires a high syntactic complexity in students’ speech. When articulating phenomena of counter-directional covariation (when, e.g., $f'' < 0$ but $f' > 0$) in particular, students require

either highly compacted expressions with reified concepts or syntactically complex phrases. What has already been discussed with respect to reification on the conceptual side in the level model (Hahn and Prediger 2008) here turns out to have an important impact on the syntactic side that must be taken into consideration by teachers and designers.

The main answer to research question RQ3 is the outcome of the inductive category-formation process (Mayring 2015) in the form of two double scales capturing syntactic complexity and conceptual conciseness at the same time. In these scales, conciseness has been topic-specifically differentiated into explicitness of level references and preciseness of comparisons (following Chesnais's (2018) focus on unary and binary relations).

The inventories in Tables 1 and 2 reveal a huge variance of nominal and verb phrases. The coordinated scales indicate that at least for the 18 investigated students, these categories occur simultaneously, meaning that increasing syntactic complexity (operationalized separately for nominal and verb phrases) here seems to go hand in hand with conceptual conciseness in explicitness of references or completeness of comparisons. Whereas the general pattern is well known in functional linguistics (Schleppegrell 2004), this article contributes to its topic-specific concretization for qualitative calculus and to show its epistemic role in students' thinking processes.

As discussed by Kleinschmidt-Schinke (2018) for a completely different context, syntactic complexity seems to hold an inverse connection to conceptual compactness: The process of conceptually unfolding requires longer sentences, usually with a higher syntactic complexity, mostly for the verb phrases. This corresponds to earlier results for the lexical dimension (Prediger and Zindel 2017) and is therefore worth being explored further in future studies.

In the current state of research, these results may contribute to sensitizing designers and teachers to the complexities of language: Longer texts are not necessarily more challenging for students; more challenging may be the more compacted texts that may require unpacking in classrooms before students can construct meanings for the compacted expressions. Unpacking specifically involves connecting different expressions in a flexible way (Prediger and Zindel 2017).

5.2 Limitations and Further Research Needs

Inevitably, the case study presented here has methodological limitations: Generalizability of the findings is restricted by the small sample size of only 18 students, by the topic specificity of the learning processes, and by the topic-specific tools for the qualitative data analysis, that is, the choices for the coding system (which neglected, for example, all verb phrases that were not comparative).

Future research should overcome the methodological limitations of this case study by expanding the analysis (1) to more students, (2) to other activities with amount and change and change of change, and (3) by transferring the research framework to other mathematical topics beyond amount and change and adapted coding systems.

Nevertheless, the study contributes to research and development efforts on content- and language-integrated learning by emphasizing the need to provide ample opportunities for developing increasing conciseness while unpacking compacted

concepts. This empirical insight can strengthen the previously discussed need to transfer the theoretical discourse about the epistemic role of academic language for mathematics learning into the syntactic dimension. These empirical insights show the potential for further research to overcome the gap that Solano-Flores (2010, p. 117) noted between functional and formal perspectives on language in order to further unpack the epistemic role of structural syntactic complexities.

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Appendix

Original Transcripts in German

Episode 1a

58a	Simon	Ja, dann hier [zeigt auf $G3$] bei den Geburten wieder, ähm, das Wachstum hat- ist ja wohl gestiegen noch und verringert sich dann und so		kov graph: [das Wachstum] _{NP} [[ist noch _{AdvP} gestiegen] _{VP} und [verringert sich] _{VP} dann _{AdvP}] _{VP}
58b	Simon	hier [zeigt auf die untere Hälfte des Graphen] der Graph von der Steigung, wie kann man sagen, der ist konstant und geht nicht mehr nach oben oder nach unten. Da ist halt hier Wachstum auf der y-Seite und Zeit auf der x-Achse.		interpretiert graph als f': [der Graph [von der Steigung] _{NP}] _{NP} KOV GRAPH: [der Graph [von [der Steigung] _{NP}] _{NP} [...] [[ist konstant _{AP}] _{VP} und [geht [nicht mehr] _{AdvP} [nach oben oder nach unten] _{AdvP}] _{VP}] _{VP}
...				
79	Neo	[...] ja das Wachstum ist halt die y-Achse und man sieht halt, dass das Wachstum, also hat sich halt sehr stark verringert über die Jahre. Ähm, aber es ist halt immer noch vorhanden, es bleibt halt nur auf der Höhe und steigt halt nur nicht mehr.		interpretiert graph als f': [das Wachstum] _{NP} [ist [die y-Achse] _{NP}] _{VP} KOV GRAPH: [das Wachstum] _{NP} [hat sich [sehr _{AdvP} stark] _{AP} verringert [über die Jahre] _{PP}] _{VP} ZUO GRAPH: _{esNP} [ist [immer noch] _{AdvP} vorhanden] _{VP} KOV GRAPH: _{esNP} [bleibt nur _{AdvP} [auf der Höhe] _{PP}] _{VP} KOV GRAPH: _{esNP} ... [steigt nur _{AdvP} [nicht mehr] _{AdvP}] _{VP}

Episode 1b

80	Tutor	Und was ist mit der Bevölkerungsanzahl?	
81	Simon	Ja, die steigt weiter, aber nicht mehr so, so stark.	KOV f: es _{NP} [steigt weiter _{AdvP} , aber [[nicht mehr] _{AdvP} [so so] _{AdvP} stark] _{AP}] _{VP}
85	Neo	Also die steigt nicht mehr so wie vorher [...]. Aber prinzipiell werden immer noch welche geboren.	KOV GRAPH: die _{NP} [steigt [[nicht mehr] _{AdvP} so wie vorher _{AdvP}] _{AP}] _{VP} KOV f: welche _{NP} [werden [immer noch] _{AdvP} geboren] _{VP}
87	Neo	Es werden halt nur nicht von Jahr zu Jahr immer mehr Menschen geboren, sondern weniger.	KOV f: [[[nicht [immer _{AdvP} mehr] _{AP} sondern weniger _{AP}] _{AP} Menschen] _{NP} werden geboren [von Jahr zu Jahr] _{AdvP}] _{VP}
95a	Simon	Wenn es weniger Geburten gibt, dann steigt ja die Anzahl der Bevölkerung auch weniger	ZUO f' → ZUO f: [wenn _{AdvP} [es [weniger _{AP} Geburten] _{NP} gibt] _{VP}] _{SSub} , [dann [steigt _{VP} [die Anzahl [der Bevölkerung] _{NP}] _{NP} auch _{AdvP} weniger _{AP}] _{VP}] _{SSub}
95b	Simon	und irgendwann ohne Geburten wäre die Bevölkerung ja auch konstant.	ZUO f' → KOV f: [ohne Geburten] _{NP} PP, [bleibt [die Bevölkerung] _{NP} auch _{AdvP} konstant _{AP}] _{VP}

Episode 1c

137	Neo	Oder es steht einfach für die Gesamtheit [zeigt auf G3] der- ja, wie viele Menschen es gibt.		INTERPRETIERT GRAPH ALS f: [[wie viele] _{AdvP} Menschen] _{NP} [es gibt] _{VP}
138	Simon	Das ist aber kein klarer Graph für Wachstum, nur- es wäre nur dann nur für die gesamte Bevölkerung. [...]		ORDNET f ZU: das _{NP} [ist kein [klarer _{AP} Graph [für Wachstum] _{PP}] _{NP}] _{VP} ORDNET f ZU: [die gesamte _{AP} Bevölkerung] _{NP}
151	Neo	Ja, Bevölkerung.		ORDNET f ZU: Bevölkerung _{NP}

Episode 2a

238	Neo	Ach Gott, wozu wird die zweite Ableitung nochmal dargestellt? Irgendwie vergessen.
239	Simon	Hmm. Ist eine gute Frage jetzt.
...		
324	Neo	Ich könnte es mir halt nur mit Minimum und Maximum jetzt herleiten, aber was die zweite Ableitung wirklich bedeutet, das weiß ich nicht.

Episode 2b

- 412 Tutor Was ist jetzt der konkrete Unterschied zwischen diesen beiden Graphen?
- 414 Simon Das ist ein bisschen gegenteilig auch.
- 416a Simon [...] erst hier stark, dann flach [*zeigt auf den Anfang und das Ende von G3*],
- 416b Simon und hier [*zeigt auf G5*] flach und dann stark.
- 417 Neo Das ist halt der größte Unterschied daran, sonst sind die sehr ähnlich, wenn man sich äh, das sieht man auch in den anderen beiden Bedingungen darüber [*zeigt auf formale Bedingungen zu f und f' von G3 und 5*].
- 420 Tutor Was bedeutet das generell dafür, was die zweite Ableitung angeben kann?
- 424a Simon Vielleicht auch die Steigung? Weil hier [*zeigt auf G3*] ist die Steigung ja .. so gesehen wird ja weniger
- 424b und hier [*zeigt auf G5*] wird die Steigung ja mehr.

KOV GRAPH für G3: [[[erst]_{AdvP} stark (Geste)]_{AP}, [dann_{AdvP} flach (Geste)]_{AP}_{AP}

KOV GRAPH für G5: [[[hier (Geste)]_{AdvP} flach]_{AP} und [dann_{AdvP} stark]_{AP}]_{AP}

A	E
$f(x) > 0$	$f(x) > 0$
$f'(x) > 0$	$f'(x) > 0$
$f''(x) < 0$	$f''(x) > 0$

KOV f' für G3: [die Steigung]_{NP} [wird ja weniger]_{AP} [hier (Geste)]_{AdvP}_{VP}

KOV f' für G5: [die Steigung]_{NP} [wird ja mehr]_{AP} [hier (Geste)]_{AdvP}_{VP}

Episode 2c

- 440 Simon [...] hier [*zeigt auf den Anfang von G3*] ist es halt relativ viel, also er steigt mehr oder viel und dann wird die Steigung- das steigt ja immer noch, aber die Steigung wird dann halt immer mehr Richtung null.
- 443 Tutor Was bedeutet also die zweite Ableitung?
- 458 Neo Hier läuft es auf die Null zu [*zeigt auf G3*], [...], die Steigung [...]
- 459 Tutor Also, was beschreibt sie?
- 460 Neo Den Verlauf der Steigung.
- 476 Simon Wie das verläuft. Also ob sie jetzt so oder so läuft [*zeichnet mit der Hand in der Luft zunächst einen schneller steigenden und dann einen langsamer steigenden Graphen*].
- 477 Tutor Was bedeutet „so oder so“?
- 478a Simon Erst wenig Steigung und dann mehr Steigung,
- 478b Simon und das andere [*zeigt auf G3*] war halt viel Steigung und dann weniger Steigung am Ende.

ZUO f' : es_{NP} [ist [relativ_{AdvP} viel]_{AP} [hier (Geste)]_{AdvP}]_{VP}

KOV f oder $f' + KOV f$: et_{NP} [steigt [mehr_{AP} oder viel_{AP}]_{AP}]_{VP} und dann_{AdvP} das_{NP} [steigt [immer noch]_{AdvP}]_{VP}

KOV f' : [die Steigung]_{NP} [wird [immer_{AdvP} mehr]_{AP} [Richtung null]_{PP}]_{VP}

KOV f' für G3: es_{NP} [läuft [auf die Null]_{PP} zu [hier (Geste)]_{AdvP}]_{VP}

INTERPRETIERT f'' : [den Verlauf [der Steigung]_{NP}]_{NP}

KOV f' : sie_{NP} [läuft [so_{AdvP} oder so_{AdvP}]_{AdvP}]_{VP}

ZUO f' für G5 + KOV f' für G5: [Erst_{AdvP} [wenig_{AP} Steigung]_{NP} und dann_{AdvP} [mehr_{AP} Steigung]_{NP}]_{NP}

ZUO f' für G3 + KOV f' für G3: [das andere]_{NP} [war [viel_{AP} Steigung]_{NP} und dann_{AdvP} [weniger_{AP} Steigung [am Ende]_{AdvP}]_{NP}]_{VP}

Episode 2d

482	Tutor	Wir sagen auch gerne die Steigerung der Steigung [für f''].	
483	Simon	Ah, die Steigerung?	
490	Neo	Ja, das macht jetzt ja- nein, das macht jetzt auch Sinn, weil das ist ja die Ableitung von der ersten Funktion, es beschreibt ja quasi die Steigung. Wenn man jetzt, zum Beispiel, wenn man die einfach streichen würde [zeigt auf die erste Bedingung von A: $f(x)>0$], würde dann die [zeigt auf die zweite Bedingung von A $f'(x)>0$] als normale Funktion sehen. Dann beschreibt ja- würde ja die Ableitung die Steigung beschreiben [zeigt zuerst auf die dritte ($f''(x)<0$) und dann auf die zweite Bedingung von A: $f'(x)<0$], also macht es auch nur Sinn. Okay.	ORDNET f' ZU: [die Ableitung [von [der ersten_{AP} Funktion]_{NP}]_{PP}]_{NP} INTERPRETIERT f'' ALS f': e_{SNP} [beschreibt [die Steigung]_{NP}]_{VP} INTERPRETIERT f'' ALS ABLEITUNG VON f': [die Ableitung]_{NP} [würde [die Steigung]_{NP} beschreiben]_{VP}
520	Simon	Dass halt die Steigung der Steigerung- nee, die Steigung der Steigung anders verläuft.	ORDNET f'' ZU: [die Steigung [der Steigung] _{NP}] _{NP}

Episode 2e

966a	Neo	Wir sehen eine steigende Bevölkerung, die zum Schluss nicht mehr so viel steigt. Sondern die- die sich halt langsam immer- die langsam immer weniger steigt	KOV f: [eine steigende _{AP} Bevölkerung, [die [[zum Schluss] _{AdvP} [[nicht mehr] _{AdvP} so _{AdvP} viel] _{AP} steigt] _{VP}] _{SRel} sondern [die [langsam _{AdvP} [immer _{AdvP} weniger] _{AP} steigt] _{VB}] _{SRel}] _{NP}
966b	Neo	und dann hier [zeigt auf den Hochpunkt von G3] ist es ja quasi bei null angekommen, dass die gar nicht mehr steigt.	KOV $f' \rightarrow$ KOV f: e _{SNP} [ist quasi _{AdvP} [bei null] _{PP} angekommen] _{VP} , [dass di _{NP} [[gar nicht mehr] _{AdvP} steigt] _{VP}] _{SSub}
1003	Tutor	[...] Was bedeutet jetzt die Steigung des Bevölkerungswachstums für die Bevölkerungszahl?	
1004	Simon	Dass die Bevölkerung auf lange Sicht weniger wird.	KOV f: [die Bevölkerung] _{NP} [wird weniger _{AP} werden [auf lange Sicht] _{AdvP}] _{VP}
1010a	Neo	Die Bevölkerung wird immer weiter, also zumindest die Steigung der Bevölkerung, also das [zeigt auf die zweite Ableitung] zeigt, dass die Steigung der Bevölkerung [zeigt auf die erste Ableitung f'] weiter ins Negative verläuft.	KOV f: [die Bevölkerung]_{NP} [wird [immer_{AdvP} weiter]_{AdvP}]_{VP} INTERPRETIERT ZU f'' ALS KOV VON f': [das (<i>Geste</i>)_{NP} zeigt_{VP}, [dass [die Steigung [der Bevölkerung]_{NP}]_{NP} [weiter_{AdvP} [ins Negative]_{PP} verläuft]_{VP}]_S
1010b	Neo	Und dadurch sinkt auch die gesamte Bevölkerung [zeigt auf den Hochpunkt von G3] [...] im weiteren Verlauf.	$\rightarrow$ KOV f: [die gesamte _{AP} Bevölkerung] _{NP} [sinkt ... [im weiteren Verlauf] _{PP}] _{VP}

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