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Formal Modeling of Innovative Competition in a Production System — an Evolutionary Approach

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Abstract
Relationships between innovations and competition are the main bases of an evolutionary approach to economic development. Innovation is recognized as a major force to achieve success in an intensively competitive environment, and competition is an essential element of the coordination mechanism required for economic changes to be successfully brought about. One of the first who well explore these relationships was Schumpeter. The idea that innovative competition may improve the positions of some groups of economic agents involved in the evolutionary processes is rooted within the neo-Schumpeterian research program. It suggests that the price mechanism typical for the routine behavior of agents should be replaced by a qualitative one to take into account the structural changes of an economy based on innovative and competitive processes as drivers of economic evolution. In this context, the main aim of this paper is to give a new setting of the phenomenon of innovative competition. This problem relates to the classification of different kinds of innovations and diversification among innovators. Moreover, two major concepts of competition are studied: the classical concept in which competition is viewed as a dynamic process and the neoclassical one in which competition is an end state of the evolutionary processes.

Keywords Competition · Innovative evolution · Production system

JEL Classification D11 · D50 · C6 · O10 · O30 · O31

Introduction

One of the central subjects of evolutionary economics, especially in the line of thought of Schumpeter's theory of economic development, is the analysis of entrepreneurial innovation, evolutionary changes in a production sphere, and

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competition. Innovation is recognized as a major force to achieve the success of organizations in an intensively competitive environment. Competition is one of the essential elements of the coordination mechanism required for economic changes (e.g., Zhao 2021). The relationship between competition and innovation has been discussed for many years. One of the first who well explore this relationship was Schumpeter (1912, 1942, 1964). He also mentioned that there is an important impact of competition among producers on innovative development. However, in a large part of mainstream formalizations of the neo-Schumpeterian theory of economic development (i.e., Nelson and Winter 1982, 2002) and also in the research program on axiomatic analysis of Schumpeter's theory of economic development, realized from the 1990s (e.g., Ciałowicz 2015, Ciałowicz and Malawski 2011, 2016, Innovative Economy 2013), the role of the phenomenon of innovative competition in economic development is rarely formalized.

This program of the axiomatization of Schumpeter's theory belongs to the theoretical core of evolutionary economics and includes modeling of the two fundamental forms of economic life distinguished by Schumpeter, i.e., circular flow and economic development, presented in a form of specific extensions of the production system as part of the Arrow-Debreu model (Debreu 1959) and analysis takes static as well as dynamic forms. Research results on both the theoretical and applied levels of analysis, which are more or less correctly considered Schumpeterian (cf. Hodgson 1993). This approach is different from the mainstream of modern modeling of Schumpeterian evolution, and the difference can be seen in the mathematical setting based on the set-theoretical and topological apparatus borrowed from modern general equilibrium theory. The motivation for using this static framework is based on the fact that Schumpeter's theory was strongly inspired by Walrasian thinking (cf. Hodgson 1993). Moreover, the dynamic version of the static model is introduced by applying a qualitative theory of the dynamical system, where a quasi-semidynamical system is understood as a semigroup of multivalued transformations of a metric space. As a consequence, the development of an economic system is described as a continuing process in time and space which is a segment of the global evolutionary process of the universe.

In this context, the main aim of the research project is to develop the research program of axiomatic analysis of Schumpeter's theory and deepen the previous results in new directions covering the following thematic areas:

1. Classification of different kinds of innovative changes and diversification in a set of producers-innovators.
2. Axiomatic analysis of the phenomenon of innovative competition defined in the static and dynamic models of a production system concerning the neo-Schumpeterian approach (cf. Andersen 2009, Foster 2011). Making the ideas analytically manageable requires the application of dynamic modeling, which was not developed by Schumpeter: he only pointed to the need to consider Walras' general equilibrium model.

3. An axiomatic analysis of the impact of competition on the intensity of innovative evolution. In particular, sufficient conditions to guarantee the intensification of innovative changes in a competitive environment are given.

Hence, the aim of this article was to create a new, original research work in the field of mathematical economics undertaken primarily to acquire new knowledge and to deepen existing one without prejudice to direct practical application or use.

Moreover, two major conceptions of competition will be briefly recalled: classical conception in which competition is viewed as a dynamic process and neoclassical one, according to which competition is an end state of the dynamic processes.

The proposed research project belongs to theoretical economics, where the axiomatic method plays the primary role. Hence, research results take the form of mathematical theorems interpreted in an economic category, and each argument has a form based on formal deduction. This ensures compliance with the criteria of scientific rationality adopted in modern economics, such as inter alia, accuracy, generality, and simplicity of the set-up, as well as some of its formal elegance.

In this paper, we revisited Schumpeter's idea of competition within an analytical framework based on a formal model taken from the modern general equilibrium theory (Walras 1874, Debreu 1959). It is necessary to distinguish two distinctive types of innovative competition: competition between two production systems (inter-sector competition) in various time spans (i.e., Khyareh and Rostami, 2021) and competition between firms (intra-sector competition) (i.e., Bereznoy, 2019, Carayannis and Wang 2012b). Thus to analyze the first kind, it is necessary to introduce a specific classification of technological and non-technological modifications in elements of a characteristic of the given model. The second is based on diversification in a set of producers concerning their shares in the market of the given commodity.

However, it should be remembered about cooperation in innovation based on knowledge sharing and diffusion. This phenomenon, opposite to competition, has many positive effects on innovative evolution (Freire and Gonçalves, 2021).

Classical, Neoclassical and Schumpeter's Concepts of Competition

Competition, competitors and competitiveness, their sources, and consequences have been the objectives of investigation in many social sciences. Competition is "a situation in which someone is trying to win something or be more successful than someone else" (Cambridge English Dictionary, 2019); it is a condition of striving to gain or win something by defeating or establishing superiority over others. A competitor is "a person, team, or company that is competing with others" (Cambridge English Dictionary, 2019); it is any person or entity which is a rival against another. Competitiveness is the fact of being able to compete successfully with other companies, countries, and organizations. In the Global Competitiveness Report 2019, national competitiveness is measured by the Global Competitiveness Index 4.0 (GCI 4.0) defined as the set of institutions, policies, and factors that determine the level of productivity.

In business, a competitor is a company in the same industry or a similar industry which offers a similar product or service. The presence of one or more competitors can reduce the prices of goods and services as the companies attempt to gain a larger market share. The competition requires firms to be more efficient in order to reduce costs. Fast-food restaurants McDonald's, Burger King, and KFC are competitors, as are Coca-Cola and Pepsi, Apple, Microsoft, Dell, Samsung, Lenovo, HP, Sony, ASUS, Google, Huawei, and Philips are competitors, and many others.

The perception of competition based on economic sciences evolved in time, so it is important to outline the key phases in the development of the concept of competition. First-time reflections on the essence of the competition were raised on the basis of classical economics. Classical economists like Adam Smith, David Ricardo, Thomas Maltus, and John Stuart Mill promoted the idea of the self-adapting nature of the free market in which competition was a specific ordering force for the activity of participants of market exchange processes. According to them (e.g., Harris, 1988), competition is the mechanism that coordinates the conflicting activity of individuals acting independently on market. It directs them to the achievement of a state of equilibrium in a dynamic sense (Smith, 1776-1976). There are endless processes of elimination of any excess profits or losses and tendencies to natural prices on the market, independent of people's will. Furthermore, competition forces producers (firms) to innovate because innovative firms have better competitive positions.

In neoclassical economics (cf. Wohlgemuth, 1995), the concept of competition is descriptive, characterized by a particularly ideal situation, opposite to monopoly. In this static theory, one of the basic requirements is an assumption of perfect competition, which means that competition is directly related to the number of participants. In opposition to neoclassical economics, classical economists described the competition as an equilibrating process, not as a final state, but, in general, they were not particularly clear how the number of participants affected competitive behavior.

Both classical and neoclassical economists accepted an assumption about static price in a short period; it follows that in this case, competition refers to rivalry among producers. By contrast, the attainment of natural prices requires longer periods, as capital flows in and out of industries, and this type of competition is between firms.

According to Schumpeter's ideas, any economic change involves two entirely distinct phenomena: growth and development. In his theory, growth is described as a purely quantitative phenomena and development is a process of spontaneous and discontinuous qualitative changes. It means that development is a necessary condition of growth. In this process, producers-innovators play the main role "to force the economic system into new channels". Therefore, the efficiency and creativity of innovators lead to two forms of competition between producers: firms compete by producing the same commodity but do so more efficiently than competitors or firms compete by creating a completely new product or service. It is worth noting that in the second kind of competition, producer-innovator tries to achieve a temporary monopoly and thus to avoid competition (Carayannis and Wang, 2012a, 2012b).

Joseph Schumpeter's vision of competition (Schumpeter, 1942) is connected with an endless destructive process in which effort, assets, and older technologies are continuously destroyed by innovations and displaced by another. This process led to economic growth far greater than more stable, conservative alternatives and cannot be reduced to an equilibrium path. Schumpeter's vision was striking in sharp contrast with the conventional neoclassical model of competitive markets (e.g. Hovenkamp, 2008), where the focus was on changes in output and price. His analysis was dismissive of the idea of the existence of perfect competition: "An entirely imaginary golden age of perfect competition that at some time somehow metamorphosed itself into the monopolistic age, whereas it is quite clear that perfect competition has at no time been more real than it is at present" (Schumpeter, 1942, p.81).

Schumpeter's analysis was critical of the static conception of competition and was focused on the nature of competition as an incessant process of discovery in which entrepreneurs seek new profit opportunities in a world of constant changes. Making these ideas analytically manageable requires the application of a dynamic method, which was not developed by Schumpeter.

In this context, the formal modeling of innovative competition can be reconstructed within the research program on modeling Schumpeterian innovative evolution in the modern dynamic general equilibrium theory (e.g., Cialowicz and Malawski, 2011, 2016; Cialowicz, 2015; Innovative Economy, 2013) for which this framework seems to be an effective and convenient toolkit. Indeed, within this approach, economic development in Schumpeter's sense is modeled by innovative extension of the production system as a component of the formal model of economy, and this setting can serve now as the basis for studying competitive processes. Consequently, our current studies will be reduced to the analysis of the internal structure of the production system in static and dynamic settings.

A Static Model of a Production System

Schumpeter in his works pointed to the need to consider a model that would take account of the characteristics of a process involving the breaking down of circular flow, that is, of Debreu's general equilibrium model (Debreu, 1959). In this model, there are two categories of economic agents, consumers and producers (firms). Consumers buy and sell goods with the ultimate goal of consuming those goods. Producers buy goods that they transform into other commodities that they later sell. So each market has two sides: demand and supply. The basic unit of activity on the production side of the market is the firm. The objective of the firm (in the neoclassical model) is to maximize profits. Due to the aim of this research, only the supply side of the economy is under consideration in a form of a formal model of the production system.

In our model commodity is a physical good or service (non-physical) for which there is demand and it satisfies human wants or needs. Each commodity is defined by its physical, temporal, and spatial characteristics. It means that it is specified by its physical characteristics, location, and date at which it is available (availability location). The traditional theory usually assumes that there exists a finite number ℓ of commodities (markets).

A commodity bundle, i.e., a list of real numbers $y_k \in \mathbb{R}$ indicating the quantity of commodity k for $k = 1, 2, \ldots, \ell$, can be described therefore as an ℓ-dimensional vector $(y_1, y_2, \ldots, y_\ell)$, where $y_k \in \mathbb{R}$ and as a point in ℓ-dimensional Euclidean (vector) space $\mathbb{R}^\ell$, and the commodity space: $\mathbb{R}^\ell = \{(y_1, y_2, \ldots, y_\ell) : y_k \in \mathbb{R}\ \mathit{for}\ k = 1, 2, \ldots, \ell\}$.

A vector $y \in \mathbb{R}^\ell \Leftrightarrow y = (y_1, y_2, \ldots, y_\ell)$ consists of y_1 units of the first good, y_2 units of the second good, up to y_ℓ units of the ℓ-th good and is called commodity bundle or plan of action of economic agent (consumption plan or production plan). We assume that each commodity has its unit (physical units or pieces); there is "perfect" or infinite divisibility of commodities (any real number is possible as a quantity for each commodity) and quantities of each commodity are measured by real numbers positive or negative. To justify negative real numbers, the following convention is introduced. Each commodity bundle is assumed to be an input-output vector in $\mathbb{R}^\ell$, where what is made available to an economic agent is called an input for him (negative coordinates in a production plan) and what is made available by an economic agent is called an output for him (positive coordinates in a production plan).

Let be given a finite set of the producers B. So there are n producers acting in the system. Each producer is characterized by its technological capacity to transform commodities.

Let $Y^b \subset \mathbb{R}^\ell$ is a set of production plans feasible (possible) for producer b (technology set) representing the producer's feasible production technology (a nonempty subset of the commodity space). Moreover, the set of all possible plans of total production $Y \subset \mathbb{R}^\ell$ is an algebraic sum of all productions sets of all producers acting in the given system, that is to say:

$$Y = Y^1 + \cdots + Y^n = \left\{ y \in \mathbb{R}^\ell \ : \ y = y^1 + \cdots + y^n, y^b \in Y^b \text{ for } b \in B \ \right\} \subset \mathbb{R}^\ell.$$

This set describes the whole system's feasible production technology and is called the aggregate production set.

Given a price system $p = (p_1, \ldots, p_\ell) \in \mathbb{R}^\ell$ and a production plan $y^b \in Y^b$, the profit of the producer $b \in B$ is defined by: $p \circ y^b = p_1 \cdot y_1 + \ldots + p_\ell \cdot y_\ell \in \mathbb{R}$.

A correspondence $\eta : B \to P_0(\mathbb{R}^\ell)$ which to every producer $b \in B$ assigns a set of the production plans maximizing his profit in a given price system $\eta(b) \overset{\text{def}}{=} \eta^b(p) \subset Y^b$ is called a correspondence of supply; that is to say: for each $b \in B$

$$\eta(b) := \eta^b(p) \overset{\text{def}}{=} \left\{ y^{b*} \in Y^b \ : \ p \circ y^{b*} = \max_{y^b \in Y^b} \{ p \circ y^b \} \right\},$$

where y^{b*} is called an equilibrium production or producer's optimal choice or profit-maximizing production plan.

$\eta(p) = \sum_{b=1}^{n} \eta^b(p)$ (algebraic sum of sets) is a set of total production plans maximizing the profit of the whole system and is called an aggregate supply.

A mapping $\pi : B \to \mathbb{R}$, which for every producer $b \in B$ measures the maximum profit value $\pi(b) \overset{\text{def}}{=} \pi^b(p)$ in the set of plans $\eta(b)$ in a given price system p is called maximum profit function; i.e., for each $b \in B : \pi(b) := \pi^b(p) := \max_{y^b \in Y^b} \{ p \circ y^b \} = p \circ y^{b*}$ for $y^{b*} \in \eta^b(p)$.

$\pi(p) = \sum_{b=1}^{n} \pi^b(p)$ (sum of numbers) is called a maximum total profit function.

Let be given the formal model of a production system $P=(B,\mathbb{R}^{\ell};p,\eta,\pi)$ (Cialowicz and Malawski, 2011; Innovative Economy, 2013) in the form of a multi-range relational system.

In this system, each producer $b \in B$, operating on an ℓ-dimensional commodity-price space $\mathbb{R}^{\ell}$, tries to choose the production plans maximizing his profit in a given price system p. The resulting action is called an equilibrium production of the producer b relative to p. Moreover, the assumption of perfectly competitive markets requires that no producers' (or consumers') actions have any effect on the market price.

This general multi-range relational model gives the opportunity to analyze not only the activity of producers in the given economy but also the monopoly (when $B=\{1\}$) or market of one commodity (when $\ell=1$).

Innovative Competition Between Production Systems

The relationship between competition and innovation has motivated numerous studies, both theoretical and empirical (e.g., Hart, 1980; Aghion et al., 2000; Aghion et al., 2005; Aghion and Griffith, 2006; Gilbert, 2006; Vives, 2008; Schmutzler, 2009; Carayannis and Wang, 2012a, 2012b, Ciocanela and Pavelescua, 2015).

The following table shows data on the Global Competitiveness Index (GCI) 4.0 (2019, p. xiii) of 15 the most competitive countries and rank of 10 the most innovative countries in the world in the year 2019 (Global Innovation Index, GII).

The results from Table 1 enable us to make the following (wildly known in economics) conclusions: first, there is a strong relationship between competitiveness and innovativeness of the economy, and second, the most innovative countries are among the most competitive.

Conceptually, the question of whether there is a relationship between competition and innovation falls into two parts. First, does more innovation lead to more competition? Second, is more competition desirable for innovative evolution? It seems to be obvious that the path to the competitiveness of firms and whole economies goes through innovation. But on the other side, competition effects an innovative activity of market participants. Sometimes these effects are positive (*the escape-competition effect*) and sometimes negative (*the Schumpeterian effect*). Moreover, an understanding of the relationship between competition and innovation is one of the main issues in many policies.

Conclusions resulting from empirical studies show these relationships more precisely, e.g.:

1) There is the linear correlation between innovation (Innovation Union Scoreboard) and competitiveness (IMD Word Competitiveness Scoreboard) of European countries and the improving of innovation performance leads to the increasing of national competitiveness (e.g., Ciocanela and Pavelescua, 2015).

Table 1 Ranking of competitive and innovative countries (the most innovative countries are written in bold print)

		GCI	GII
1	**Singapore**	84.8	**8**
2	**USA**	83.7	**3**
3	Hong Kong	83.1	
4	**Netherlands**	82.4	**4**
5	**Switzerland**	82.3	**1**
6	Japan	82.3	
7	**Germany**	81.8	**9**
8	**Sweden**	81.2	**2**
9	**UK**	81.2	**5**
10	Denmark	81.2	**7**
11	Finland	80.2	**6**
12	Taiwan, China	80.2	
13	South Korea	79.6	
14	Canada	79.6	
15	France	78.8	
16	Australia	78.7	
17	Norway	78.1	
18	Luxembourg	77.0	
19	New Zealand	76.7	
20	**Israel**	76.7	**10**

Source: http://www.weforum.org/ (access date: 15.12.2019)

2) Competition is conducive to innovation because intensive competition on market forces entrepreneurs to introduce innovations like new technologies or new products in order to avoid loss of profits (Aghion et al., 2000).
3) Innovations have an important impact on competition position of firms in developing countries (Carayannis and Wang, 2012a, 2012b).
4) The impact of technological innovation on competitiveness in many countries started the process of their transition in 1990s (e.g., Grbic et al., 2012).
5) Increased competition leads to a significant increase in R&D investments and affects industry composition which is consistent with the predictions of innovation models (e.g., Aghion et al., 2014, Kerber, 2006).

In this context, the formal modeling of competition in the conceptual apparatus of the modern theory of general equilibrium, introduced in this paper, allows us to analyze the twin-track relationship between competitiveness and innovativeness of the production systems as one of the most essential elements of economic development. On the one hand, it is obvious that the presence of competitiveness among producers or firms accelerates or sometimes decelerates the process of innovative evolution. Furthermore, the insensitivity of innovative changes depends on the level of competition. On the other hand, innovations are important factors that impact competitive intensity and rivalry within industries. Thus the main aim of this section is to prove a positive correlation between competitiveness and innovativeness of the production system.

To perform this task classical analysis of competition as a dynamic process gradually was translated into static terms, where the number of producers and type of product may characterize the form of competition. Moreover, crucial factors for the innovative competition are technological changes in the whole system. As a consequence, a basis of the theoretical setting of the phenomenon of innovative competition concerning the neo-Schumpeterian approach (cf. Aghion, 2002, Aghion and Harris et al., 2000; Gaffard, 2008; Elgar companion, 2007; Hanush and Pyka, 2006, 2007a, 2007b) is the classification of different kinds of innovative changes in the production system.

Let two production systems: P and $P^{'}$ are given.

Definition 4.1 (Cialowicz, 2015) A production system $P^{'}$ is called an innovative extension of a production system P, shortly $P \subset_i P^{'}$, if:

1) $\ell \leq \ell^{'}$, 2) $\exists b^{'} \in B^{'} \forall b \in B$

(2.1) $\mathrm{proj}_{\mathbb{R}^{\ell}}\left(Y^{\prime}_{b\prime}\right) \not\subset Y_b$ (2.2) $\mathrm{proj}_{\mathbb{R}^{\ell}}\left(\eta^{\prime}_{b^{\prime}}\left(p^{\prime}\right)\right) \not\subset \eta_b(p)$ (2.3) $\pi_b(p) < \pi^{\prime}_{b^{\prime}}\left(p^{\prime}\right)$.

According to the definition, the production system $P^{'}$ is an innovative extension of the system P if at least one new product or commodity may appear in $P^{'}$ (condition 1). Those new products can be interpreted as a better way of meeting the needs present earlier in the system P and are introduced by brand new firms or by the ones already existing. Moreover, in the production system $P^{'}$, there is at least one producer $b^{'}$ whose technological abilities go beyond the abilities of all producers acting within the production system P (condition 2.1). Hence, the optimal (i.e., maximizing the profit) production plans of the producer $b^{'}$ cannot be reduced to the analogous plans being realized by the producers in the production system P (condition 2.2). Moreover, the fixed producer's maximum profit is greater than the one any of the producers in the system P can make (condition 2.3).

The innovative extension $P \subset_i P^{'}$ is called innovative extension in scale micro.

The scale micro concerns changes for individual producers only (not for the whole system).

If $\ell = \ell^{'}$, then this extension is called weak. If $\ell < \ell^{'}$, then this extension is called strong, and it means changes in the dimension of a commodity space.

Definition 4.2 A production system $P^{'}$ is called an innovative extension in scale macro of a production system P, shortly $P \subset_I P^{'}$, if $P^{'}$ is an innovative extension in scale micro of a production system P and (1) $\mathrm{proj}_{\mathbb{R}^{\ell}}\left(Y^{\prime}\right) \not\subset Y$, (2) $\mathrm{proj}_{\mathbb{R}^{\ell}}\left(\eta^{\prime}\left(p^{\prime}\right)\right) \not\subset \eta(p)$, (3) $\pi(p) < \pi^{'}(p^{'})$.

In this extension, not only new innovators may appear but changes in the characteristic of the whole system are observed. It means that in an innovative extension in scale macro new products are introduced on market but also creative destruction in a space of commodities and in the set of producers is possible. Moreover, there are new technological feasible production plans in an aggregate production set (condition 1), and at the same time, there are changes in an aggregate supply (condition 2). As a result, the effectiveness of the whole system, measured by a maximum total profit function increases (condition 3).

Remark If $\ell = \ell^{'}$, then Definitions 4.1 and 4.2 define a week innovative extensions in scale micro ($P \subset_{is} P^{'}$) and in scale macro ($P \subset_{Is} P^{'}$).

Notice, that according to Definition 4.2 if $P \subset_I P^{'}$ then $P \subset_i P^{'}$.

Definitions 4.1 and 4.2 describe only two kinds of innovative changes distinguish concerning changes in commodity space, effectiveness visible in the measurable results (increasing profits) of producer-innovator activity, and improvement of the condition of the whole system. Let us now define another two kinds of innovative changes taking into consideration that some innovators are producers acting in the basic model but sometimes innovators are new market agents.

Definition 4.3 A production system $P^{'}$ is called a technological innovative extension in scale micro (macro) of a production system P, shortly $P \subset_{it} P^{'} (P \subset_{It} P^{'})$, if $P^{'}$ is an innovative extension in scale micro (macro) of a production system P, $\ell = \ell^{'}$ and $B = B^{'}$.

According to the above definition in a technological innovative extension, innovators are producers already acting in the production system, but by introducing new methods of production, they became innovators.

Definition 4.4 A production system $P^{'}$ is called a concentrating innovative extension in scale micro (macro) of a production system P, shortly $P \subset_{ic} P^{'}$ $(P \subset_{Ic} P^{'})$, if $P^{'}$ is an innovative extension in scale micro (macro) of a production system P and $B^{'} \subsetneq B$.

Definition 4.4 describes concentrating on production by eliminating non-effective firms from the market. At the same time, other entrepreneurs extend their technological abilities and increase their maximum profits. This kind of change leads to oligopoly or, in a specific case, to monopoly.

Notice that if $cardB = 1$, the production system describes a strict monopolistic market; if $cardB = 2$, this is a duopoly, and for $cardB > 2$ but "small enough," this is an oligopoly.

Definition 4.5 A production system $P^{'}$ is called an atomizing innovative extension in scale micro (macro) of a production system P, shortly $P \subset_{ia} P^{'}$ $(P \subset_{Ia} P^{'})$, if $P^{'}$ is an innovative extension in scale micro (macro) of a production system P and $B \subsetneq B^{'}$.

In an atomizing innovative extension, new firms are acting on the market. In particular, producer-innovator can be a new market agent.

According to Schumpeter's ideas, it is impossible to produce new products with old productive capacity so any innovative change requires the construction of new production processes and the destruction of the old. Moreover, innovations may cause the elimination of firms from the market.

Definition 4.6 A production system $P^{'}$ is called creative destruction of a production system P, shortly $P \subset_d P^{'}$, if: 1) $P \subset_i P^{'}$ 2) $\exists k \in \{1, 2, \dots, \ell\} \forall b' \in B' \; y^{b'} = \left(y_1^{b'}, \dots, y_k^{b'}, \dots, y_\ell^{b'}\right) \in Y^{b'} \Rightarrow y_k^{b'} = 0$ or $\exists b^{'} \in B^{'}$ $Y^{b'} = \varnothing$.

Creative destruction described in Schumpeter's theory is a result of competition between new and old products, technologies, or producers, This phenomenon is observed between industries and inside industries. Generally, competition related to innovative changes can be called innovative (cf. Winter, 1984; Futia, 1980; Schumpeterian Perspective, 2009).

It is worth remembering that following Schumpeter's intuition, innovative competition as a process is always connected with monopolistic practices in the early phase of innovation. In particular in the case when firms compete with others by creating a completely new product, they try to achieve a temporary monopoly and thus avoid competition. Hence, creative destruction is mainly related to concentrating innovative extension (Definition 4.4).

To analyze competition between innovative extensions of the given production system, spaces of different kinds of innovative extensions will be introduced:

$\boldsymbol{P_i} = \{P_i : P \subset_i P_i\}$ is a space of all innovative extensions in scale micro of a production system P.

$\boldsymbol{P_I} = \{P_i : P \subset_I P_i\}$ is a space of all innovative extensions in scale macro of a production system P.

$\boldsymbol{P_{it}} = \{P_i : P \subset_{it} P_i\} \subset \boldsymbol{P_i}$ is a space of all technological innovative extension in scale micro of a production system P.

$\boldsymbol{P_{ic}} = \{P_i : P \subset_{ic} P_i\} \subset \boldsymbol{P_i}$ is a space of all concentrating innovative extension in scale micro of a production system P.

$\boldsymbol{P_{ia}} = \{P_i : P \subset_{ia} P_i\} \subset \boldsymbol{P_i}$ is a space of all atomizing innovative extension in scale micro of a production system P.

Notice, that: $\boldsymbol{P_I} \subset \boldsymbol{P_i}$, $\boldsymbol{P_{it}} \cap \boldsymbol{P_{ic}} = \emptyset$, $\boldsymbol{P_{it}} \cap \boldsymbol{P_{ia}} = \emptyset$, $\boldsymbol{P_{ia}} \cap \boldsymbol{P_{ic}} = \emptyset$.

Let two innovative extensions of the production system $P^1, P^2 \in \boldsymbol{P_i}$ are given.

Definition 4.7 A production system P^2 is more (innovatively) competitive than a system P^1, in short: $P^1 \lhd_{ic} P^2$ if:

1) $\mathrm{card}B_i^1 < \mathrm{card}B_i^2$ or
2) $\mathrm{card}B_i^1 = \mathrm{card}B_i^2$ and $\ell^1 < \ell^2$ or
3) $\mathrm{card}B_i^1 = \mathrm{card}B_i^2$ and $\ell^1 = \ell^2$ and $\pi^1 < \pi^2$.

From the above, it follows that the intensity of innovative competitiveness of the production system is directly proportional to the number of producers-innovators, the structure of an industry, and its effectiveness. In this definition, competition is defined as a static state but also as a factor of the efficiency of the production system. The set of producers-innovators plays a role of the most important competitive potential and maximal profit is a competitive performance.

Moreover, this definition introduced specific order (partition) in a space of innovative extensions of the given model:

1) The most competitive is an atomizing innovative extension (Definition 4.5) because increasing of number of firms and, at the same time, reducing the size of temporary monopolies intensifies competition.
2) The second best kind is a strong innovative extension because the density of the product's population is proportional to the intensity of competition.
3) The least competitive is concentrating innovative extension (Definition 4.4), because its final result is a monopoly which has no incentives to innovate and, hence, to create new methods or new products.

To grasp a core of a process of economic development and preserve the principles of scientific rationality dominating today's economic theory, such as rigor, generality, and analytical simplicity, the given formal model of a production system seems to be an insufficient tool, because of its static character so it needs to be dynamized by introducing a (quasi)-semidynamical system, which emphasizes the role of change in modeling economic systems, and to construct a dynamic model of the Schumpeterian process of competition via technological innovations.

To this aim, let us consider the space of all production systems: $\boldsymbol{P} \coloneqq \{P : P$ is a production system$\}$.

A dynamic analysis of the process of economic development is possible now by the application of a mathematical idea of a (quasi)-semidynamical system.

Definition 4.8 (cf. Sibirskij and Szube, 1987) A mapping $f_P : \boldsymbol{P} \times \mathbb{R}_+ \to P(\boldsymbol{P})$ is a (quasi)-semidynamical production system if:

1) $f_P(P, 0) = \{P\}$, 2) $f_P(f_P(P, t_1), t_2) = f_P(P, t_1 + t_2)\ \forall t_1, t_2 \in \mathbb{R}_+$.

The further analysis of a quasi-semidynamical production system f_P is based on the general premise that it is possible to decompose it into component systems in the following way: $f_P(P, t) = P^t = (f_B(B, t); f_\ell(\mathbb{R}^\ell, t); f_p(p, t); f_\eta(\eta, t); f_\pi(\pi, t))$.

Definition 4.9

1) A (quasi)-semidynamical system $f_P : \boldsymbol{P} \times \mathbb{R}_+ \to P(\boldsymbol{P})$ is called single-valued if all the values of the system f_P are one-element sets so that we may think of a function $f_P : \boldsymbol{P} \times \mathbb{R}_+ \to \boldsymbol{P}$.

2) A single-valued (quasi)-semidynamical system $f_P : \boldsymbol{P} \times \mathbb{R}_+ \to \boldsymbol{P}$ is called:

a) innovative if $f_P(P, t_1) \subset_i f_P(P, t_2)$,

b) process of creative destruction if $f_P(P, t_1) \subset_d f_P(P, t_2)$,

for any $t_1, t_2 \in \mathbb{R}_+$, $t_1 < t_2$.

Each process of qualitative changes has its life cycle and is characterized by age structure, which means the path of the consecutive phases. In the given framework this path is called a trajectory.

Definition 4.10

A set $\tau_+(P) \coloneqq \{f_P(P, t) : t \in \mathbb{R}_+\ i\ f_P(P, 0) = P\}$ is called a positive semi-trajectory of (quasi)-semidynamical production system P.

A positive semi-trajectory $\tau_+(P)$ is called innovative evolution of production system P if for any $t_1, t_2 \in \mathbb{R}_+ f_P(P, t_1) \subset_i f_P(P, t_2)$.

It is important to point out that competitive innovative evolution is a process based on continuous interaction among economic agents and reflecting choices the results of which are known in the future (i.e., Sum, Jessop 2013). It is a process of dynamic disequilibrium that can be observed within a specified period.

Definition 4.11 An innovative evolution $\tau'_+(P)$ is more competitive than an innovative evolution $\tau_+(P)$ over a defined period of time $[0, T]$ if $f_P(P, T) \lhd_{ic} f'_P(P, T)$.

In the analysis of innovation processes, one of the most important, and at the same time, the most difficult problem is to evaluate the innovativeness of the given production system because the measurement of innovation includes various dimensions and varies according to firms and their life-cycle phases. Thus innovativeness of the whole system and its performance can be measured in many ways. To compare innovative extensions of the defined system, I suggest applying an innovative metric taking into account qualitative changes in specific elements of the characteristic of the given model, which are important for its innovativeness.

Definition 4.12

An innovative metric in a space of production systems is a mapping $\rho_i \colon \boldsymbol{P} \times \boldsymbol{P} \to \mathbb{R}$ such that $\forall P^1, P^2 \in \boldsymbol{P}$ $\rho_i\left(P^1, P^2\right) = \begin{cases} 0 & if \quad P^1 = P^2 \\ \rho_{\Delta \ell} + \rho_{\Delta B} + \rho_{\Delta \pi} & if \quad P^1 \neq P^2 \end{cases}$

where $\rho_{\Delta \ell} = \frac{\left|\ell^1 - \ell^2\right|}{\max\left\{\ell^1, \ell^2\right\}}, \rho_{\Delta B} = \frac{\left|\mathrm{card}\left(B_i^1\right) - \mathrm{card}\left(B_i^2\right)\right|}{\max\left\{\mathrm{card}\left(B_i^1\right), \mathrm{card}\left(B_i^2\right)\right\}}, \rho_{\Delta \pi} = \frac{\left|\max\limits_{b^1 \in B^1} \pi_b^1 - \max\limits_{b^2 \in B^2} \pi_b^2\right|}{\max\left\{\max\limits_{b^1 \in B^1} \pi_b^1, \max\limits_{b^2 \in B^2} \pi_b^2\right\}}$.

The metric structure in a space of economic systems allows us to compare innovative extensions of the given production system.

Let be given a production system P and its two innovative extensions P^1, P^2, such that $P \subset_i P^1$, $P \subset_i P^2$.

Definition 4.13 A production system P^2 is at least as an innovative extension of the system P as system P^1, in short: $P^1 \underline{\angle}_i P^2$ iff $\rho_i(P^1, P) \leq \rho_i(P^2, P)$.

The above definition says that more innovative is this extension for which the distance from the given model is greater. Based on this definition, it is possible to demonstrate that more innovative development of the given production system gives a more competitive extension of the model.

Theorem 4.1

Let a production system P and its two innovative evolutions in a form of positive semi-trajectories $\tau_+(P)$ and $\overline{\tau}_+(P)$ be given. If:

1) $f_P(P, T) \underline{\angle}_i \overline{f}_P(P, T)$ in a time $[0, T]$,

2) $B_i \subset B_i^T$ and $B_i \subset \overline{B}_i^T$ then $f_P(P, T) \lhd_{ic} \overline{f}_P(P, T)$.

Proof: If $f_P(P, T) \underline{\angle}_i \overline{f}_P(P, T)$ then $\rho_i\left(f_P(P, T), P\right) \leq \rho_i\left(\overline{f}_P(P, T), P\right)$.

It means $\rho_{\Delta \ell} + \rho_{\Delta B} + \rho_{\Delta \pi} < \rho_{\Delta \overline{\ell}} + \rho_{\Delta \overline{B}} + \rho_{\Delta \overline{\pi}}$ and, according to Assumption 2:

$$\frac{\left|\ell - \ell^T\right|}{\max\left\{\ell, \ell^T\right\}} + \frac{\left|\mathrm{card}\left(B_i\right) - \mathrm{card}\left(B_i^T\right)\right|}{\max\left\{\mathrm{card}\left(B_i\right), \mathrm{card}\left(B_i^T\right)\right\}} + \frac{\left|\max\limits_{b \in B} \pi_b - \max\limits_{b \in B^T} \pi_b^T\right|}{\max\left\{\max\limits_{b \in B} \pi_b, \max\limits_{b \in B^T} \pi_b^T\right\}} = \frac{\left|\ell - \ell^T\right|}{\ell^T}$$

$$+ \frac{\left|\mathrm{card}\left(B_i\right) - \mathrm{card}\left(B_i^T\right)\right|}{\mathrm{card}\left(B_i^T\right)} + \frac{\left|\max\limits_{b \in B} \pi_b - \max\limits_{b \in B^T} \pi_b^T\right|}{\max\limits_{b \in B^T} \pi_b^T} < \frac{\left|\ell - \overline{\ell}^T\right|}{\max\left\{\ell, \overline{\ell}^T\right\}}$$

$$+ \frac{\left|\mathrm{card}\left(B_i\right) - \mathrm{card}\left(\overline{B}_i^T\right)\right|}{\max\left\{\mathrm{card}\left(B_i\right), \mathrm{card}\left(\overline{B}_i^T\right)\right\}} + \frac{\left|\max\limits_{b \in B} \pi_b - \max\limits_{b \in B^T} \overline{\pi}_b^T\right|}{\max\left\{\max\limits_{b \in B} \pi_b, \max\limits_{b \in B^T} \overline{\pi}_b^T\right\}} = \frac{\left|\ell - \overline{\ell}^T\right|}{\overline{\ell}^T}$$

$$+ \frac{\left|\mathrm{card}\left(B_i\right) - \mathrm{card}\left(\overline{B}_i^T\right)\right|}{\mathrm{card}\left(\overline{B}_i^T\right)} + \frac{\left|\max\limits_{b \in B} \pi_b - \max\limits_{b \in B^T} \overline{\pi}_b^T\right|}{\max\limits_{b \in B^T} \overline{\pi}_b^T}.$$

It follows that $\frac{\left|\ell - \ell^T\right|}{\ell^T} < \frac{\left|\ell - \overline{\ell}^T\right|}{\overline{\ell}^T}$ or $\frac{\left|\mathrm{card}\left(B_i\right) - \mathrm{card}\left(B_i^T\right)\right|}{\mathrm{card}\left(B_i^T\right)} < \frac{\left|\mathrm{card}\left(B_i\right) - \mathrm{card}\left(\overline{B}_i^T\right)\right|}{\mathrm{card}\left(\overline{B}_i^T\right)}$ or $\frac{\left|\max\limits_{b \in B} \pi_b - \max\limits_{b \in B^T} \pi_b^T\right|}{\max\limits_{b \in B^T} \pi_b^T} < \frac{\left|\max\limits_{b \in B} \pi_b - \max\limits_{b \in B^T} \overline{\pi}_b^T\right|}{\max\limits_{b \in B^T} \overline{\pi}_b^T}$.

If $\frac{|\mathrm{card}(B_i)-\mathrm{card}(B_i^T)|}{\mathrm{card}(B_i^T)} = 1 - \frac{\mathrm{card}(B_i)}{\mathrm{card}(B_i^T)} < \frac{\left|\mathrm{card}(B_i)-\mathrm{card}\left(\overline{B}_i^T\right)\right|}{\mathrm{card}\left(\overline{B}_i^T\right)} = 1 - \frac{\mathrm{card}(B_i)}{\mathrm{card}\left(\overline{B}_i^T\right)}$ then $\frac{\mathrm{card}(B_i)}{\mathrm{card}(B_i^T)} > \frac{\mathrm{card}(B_i)}{\mathrm{card}\left(\overline{B}_i^T\right)}$ and $card(B_i^T) < \mathrm{card}\left(\overline{B}_i^T\right)$. Similarly, if $\frac{|\ell-\ell^T|}{\ell^T} = 1 - \frac{\ell}{\ell^T} < \frac{\left|\ell-\overline{\ell}^T\right|}{\overline{\ell}^T} = 1 - \frac{\ell}{\overline{\ell}^T}$ then $\frac{\ell}{\ell^T} > \frac{\ell}{\overline{\ell}^T}$ and $\ell^T < \overline{\ell}^T$. Finally, if $\frac{\left|\max_{b\in B}\pi_b - \max_{b\in B^T}\pi_b^T\right|}{\max_{b\in B^T}\pi_b^T} = 1 - \frac{\max_{b\in B}\pi_b}{\max_{b\in B^T}\pi_b^T} < \frac{\left|\max_{b\in B}\pi_b - \max_{b\in B^T}\overline{\pi}_b^T\right|}{\max_{b\in B^T}\overline{\pi}_b^T} = 1 - \frac{\max_{b\in B}\pi_b}{\max_{b\in \overline{B}^T}\overline{\pi}_b^T}$ then $\frac{\max_{b\in B}\pi_b}{\max_{b\in B^T}\pi_b^T} > \frac{\max_{b\in B}\pi_b}{\max_{b\in \overline{B}^T}\overline{\pi}_b^T}$ and $\max_{b\in B^T}\pi_b^T < \max_{b\in \overline{B}^T}\overline{\pi}_b^T$.

As a result $f_P(P,T) \lhd_{ic} \overline{f}_P(P,T)$ (Definition 4.7).

According to this theorem, when we compare two trajectories of development of the given production system, after specific period of time, the evolution which is more innovative (more innovators or more innovative products or more new markets) results in more competitive production system. It means that more innovative evolution gives more competitive results.

In the next theorem, opposite relationship is proved. It means that under special assumption, the competition encourages firms to innovate.

Theorem 4.2

Let a production system P and its two innovative evolutions in a form of positive semi-trajectories $\tau_+(P)$ and $\overline{\tau}_+(P)$ be given. If:

1) $f_P(P,T) \lhd_{ic} \overline{f}_P(P,T)$ in a time $[0,T]$,
2) $\ell \le \ell^T = \overline{\ell}^T$
3) $B_i \subset B_i^T$ and $B_i \subset \overline{B}_i^T$
4) $\max_{b\in B^T}\pi_b^T \le \max_{b\in \overline{B}^T}\overline{\pi}_b^T$

then $f_P(P,T) \angle_i \overline{f}_P(P,T)$.

Proof. Taking into consideration Assumption 2, we have $\frac{|\ell-\ell^T|}{\ell^T} = \frac{\left|\ell-\overline{\ell}^T\right|}{\overline{\ell}^T}$.

From Assumption 1, we have $f_P(P,T) \lhd_{ic} \overline{f}_P(P,T)$, what means

1) $\mathrm{card}B_i^T < \mathrm{card}\overline{B}_i^T$ or
2) $\mathrm{card}B_i^T = \mathrm{card}\overline{B}_i^T$ and $\ell^T < \overline{\ell}^T$ or
3) $\mathrm{card}B_i^T = \mathrm{card}\overline{B}_i^T$ and $\ell^T = \overline{\ell}^T$ and $\pi^T < \overline{\pi}^T$.

If $\mathrm{card}B_i^T < \mathrm{card}\overline{B}_i^T$ then $\frac{\mathrm{card}(B_i)}{\mathrm{card}(B_i^T)} > \frac{\mathrm{card}(B_i)}{\mathrm{card}\left(\overline{B}_i^T\right)}$ and $\frac{|\mathrm{card}(B_i)-\mathrm{card}(B_i^T)|}{\mathrm{card}(B_i^T)} = 1 - \frac{\mathrm{card}(B_i)}{\mathrm{card}(B_i^T)} < \frac{\left|\mathrm{card}(B_i)-\mathrm{card}\left(\overline{B}_i^T\right)\right|}{\mathrm{card}\left(\overline{B}_i^T\right)} = 1 - \frac{\mathrm{card}(B_i)}{\mathrm{card}\left(\overline{B}_i^T\right)}$.

If $\mathrm{card}B_i^T = \mathrm{card}\overline{B}_i^T$ and $\ell^T = \overline{\ell}^T$ and $\pi^T < \overline{\pi}^T$, then according to Assumption 4: $\frac{\max_{b\in B}\pi_b}{\max_{b\in B^T}\pi_b^T} > \frac{\max_{b\in B}\pi_b}{\max_{b\in \overline{B}^T}\overline{\pi}_b^T}$ and $\frac{\left|\max_{b\in B}\pi_b - \max_{b\in B^T}\pi_b^T\right|}{\max_{b\in B^T}\pi_b^T} = 1 - \frac{\max_{b\in B}\pi_b}{\max_{b\in B^T}\pi_b^T} < \frac{\left|\max_{b\in B}\pi_b - \max_{b\in B^T}\overline{\pi}_b^T\right|}{\max_{b\in B^T}\overline{\pi}_b^T} = 1 - \frac{\max_{b\in B}\pi_b}{\max_{b\in \overline{B}^T}\overline{\pi}_b^T}$.

$$\rho_i\left(f_P(P,T),P\right) = \rho_{\Delta\ell} + \rho_{\Delta B} + \rho_{\Delta\pi}$$
$$= \frac{|\ell-\ell^T|}{\ell^T} + \frac{\left|\mathrm{card}(B_i)-\mathrm{card}\left(B_i^T\right)\right|}{\mathrm{card}\left(B_i^T\right)} + \frac{\left|\max_{b\in B}\pi_b - \max_{b\in B^T}\pi_b^T\right|}{\max_{b\in B^T}\pi_b^T} < \frac{\left|\ell-\overline{\ell}^T\right|}{\overline{\ell}^T}$$

It gives $+ \frac{\left|\mathrm{card}(B_i)-\mathrm{card}\left(\overline{B}_i^T\right)\right|}{\mathrm{card}\left(\overline{B}_i^T\right)} + \frac{\left|\max_{b\in B}\pi_b - \max_{b\in B^T}\overline{\pi}_b^T\right|}{\max_{b\in B^T}\overline{\pi}_b^T} = \rho_{\Delta\overline{\ell}} + \rho_{\Delta\overline{B}} + \rho_{\Delta\overline{\pi}}$.

Hence, $\rho_i\left(f_P(P,T),P\right) < \rho_i\left(\overline{f}_P(P,T),P\right)$ so $f_P(P,T) \angle_i \overline{f}_P(P,T)$.

According to this theorem when there are two processes of innovative evolution, more competitive transformation of the given production system gives more intensive innovative changes.

Innovative Competition Within Industries

The analysis of competition as a state in the neo-classical theory requires the model of perfect competition, like the static model of the production system introduced before. This model describes the ideal conditions that must hold in the market to ensure the existence of the competitive behavior of producers. In this model, all market participants have perfect information about prices and the costs of each good. Moreover, there are a large number of small firms producing and selling commodities so producers are incapable of influencing the price of the product and so each of them chooses the levels of inputs and outputs consistent with maximization of its profit. This model is a kind of industry that consists of firms that use similar technology. The intensity of competition in this model is directly proportional to the number of producers and the structure of an industry. The larger the number of firms, the more competitive their behavior. By contrast, the smaller the number of firms, the more monopolistic or oligopolistic is the form of competition. In particular, for a monopolistic production system, the model is non-competitive. Similar conclusions are drawn from Walras's conception of attainment of equilibrium (Walras, 1874), where perfect competition is the main determinant of equilibrium prices.

To analyze competition between firms, the market share of individual producers will be introduced.

Let be given a production system $P=(B,\mathbb{R}^{\ell};y,p,\eta,\pi)$.

Remarks

For the given commodity $k\in\{1,2,\ldots,\ell\}$ and producer $b\in B$:

1) A number $\sum_{y^b\in Y^b} y_k^b = \overline{y}_k^b \geq 0$ is a producer b's level of production of commodity k, where $y^b = \left(y_1^b,\ldots,y_k^b,\ldots,y_\ell^b\right) \in Y^b$,
2) $y_k = \sum_{b\in B} \overline{y}_k^b = y_k$ is aggregate level of production of commodity k in the whole system,
3) $\frac{\overline{y}_k^b}{y_k} = S_k^b$ is a (percentage) share of producer b in the market of commodity k such that for each commodity $k : \sum_{b\in B} S_k^b = 1$,
4) $m_s : B\times\{1,2,\ldots,\ell\} \to [0,1]$ is a function of market shares, such that $m_s(b,k) = S_k^b \in [0,1]$.

It means that for the given commodity $k \in \{1, 2, \ldots, \ell\}$ (output in production plans), the market share is calculated by dividing the volume of goods produced (and sold) by a particular firm by the total number of units in the market.

Remarks

1) Producer b is a leader on the market of commodity k if $S_k^b > S_k^{b\prime}$ for each b' $\in B$, b' $\neq b$.

2) Producer b is a monopolist on the market of commodity k if $S_k^b = 1\left(S_k^{b\prime} = 0\right.$ for each b' $\in B$, b' $\neq b$).

It is important to point out that in the situation when a small number of gigantic firms (megacorps) possess power over the market forces so that they can fix their prices and thus manage to secure a higher than average (competitive) rate of profit. Monopolists dominate their relevant markets and leave only limited space for a relatively small competitive firm. It should be remembered also that innovations are successful only by reducing costs or increasing productivity which makes possible the undercutting of price and the elimination of competitors. Innovations leading to techniques with lower cost make possible the reduction of the selling price, thereby increasing the market share of innovators. Greater share in the market with the same prices causes greater profit on sales.

It should be remembered that competition among producers of the same or similar products is based on competition in prices. But in the given formal static model of the economy, described in a short period, we assumed perfect competition which requires that no producers (or consumers) actions have any effect on the market price. As a result, this kind of competition is based on the activity of producers which has an impact on their shares on market.

In this context, it is necessary to modify the basic production system into a system with market shares $P_s = (B, \mathbb{R}^{\ell}; y, p, \eta, \pi, m_s)$. Moreover, the above assumptions allow us to analyze the competitiveness of modified production systems.

Let two production systems with shares $P_s = (B, \mathbb{R}^{\ell}; y, p, \eta, \pi, m_s)$ and $P'_s = \left(B', \mathbb{R}^{\ell'}; y', p', \eta', \pi', m'_s\right)$ are given.

Definition 5.1

1) A production system P'_s is more competitive (on a market of commodity k) than a production system P_s if $\mathrm{card}\{b \in B : S_k^b \neq 0\} < \mathrm{card}\{b' \in B' : S_k^{b'} \neq 0\}$.
2) A production system P'_s is more innovatively competitive (on a market of commodity k) than a production system P_s if $\mathrm{card}\{b_i \in B : S_k^{bi} \neq 0\} < \mathrm{card}\left\{b'_i \in B' : S_k^{b'_i} \neq 0\right\}$, where b_i is a producer-innovator.

More competitive on a market of commodity k is a production system with a greater amount of producers with nonzero shares in this market. More innovatively competitive on a market of commodity k is a production system with more innovators with nonzero shares in this market.

Theorem 5.1 If $P_s \lhd_{ic}^k P'_s$, $B = B'$ and for each $b \in B \backslash B_i$ $S_k^b = S'^b_k$ then $P_s \lhd_c^k P'_s$.

Proof:

If $P_s \lhd_{ic}^k P'_s$ then $\mathrm{card}\{b_i \in B : S_k^{bi} \neq 0\} < \mathrm{card}\left\{b'_i \in B' : S_k^{b'_i} \neq 0\right\}$ (Definition 5.1b).

Moreover:

$$\mathrm{card}\left\{b \in B : S_k^b \neq 0\right\} = \mathrm{card}\left\{b \in B_i : S_k^b \neq 0\right\} + \mathrm{card}\left\{b \in B \backslash B_i : S_k^b \neq 0\right\}$$

I. If $B_i = \varnothing$ then $\mathrm{card}\left\{b \in B : S_k^b \neq 0\right\} = 0 + \mathrm{card}\{b \in B \backslash B_i : S_k^b \neq 0\} < \mathrm{card}\{b'_i \in B' : S_k^{b'_i} \neq 0\}$ $+\mathrm{card}\{b' \in B' \backslash B'_i : S_k^{b'} \neq 0\}$.

II. If $B_i \neq \varnothing$ then $\mathrm{card}\left\{b \in B : S_k^b \neq 0\right\} = \mathrm{card}\left\{b \in B_i : S_k^b \neq 0\right\} + \mathrm{card}\left\{b \in B \backslash B_i : S_k^b \neq 0\right\} <$ $\mathrm{card}\{b'_i \in B'_i : S_k^{b'_i} \neq 0\} + \mathrm{card}\{b' \in B' \backslash B'_i : S_k^{b'} \neq 0\} = \mathrm{card}\{b' \in B' : S^{b'_k} \neq 0\}$.

According to the above remark, a production system with market shares P'_s more innovatively competitive on market of commodity k is also more competitive if shares of producers-non-innovators in this market are the same in both systems.

Theorem 5.2 If $P \subset_I P'$ and $B \subset B'$ then $P \lhd_{ic} P'$.

Proof: $B \subset B'$ means that $cardB \leq cardB'$ and if $cardB = cardB'$ then $B = B'$

From Definition 4.2 if $P \subset_I P'$ then $\pi(p) < \pi'(p')$.

I. If $cardB < cardB'$ then $P \lhd_{ic} P'$ (Definition 4.7 case 1).
II. If $cardB = cardB'$ and $\ell < \ell'$ then $P \lhd_{ic} P'$ (Definition 4.7 case 2).
III. If $cardB = cardB'$ and $\ell = \ell'$ and $\pi(p) < \pi'(p')$ then $P \lhd_{ic} P'$ (Definition 4.7 case 3).

The above theorem shows that the innovative extension on scale macro of a production system is more competitive than the basic model. It means that innovators and innovations are the driving competitive force.

Discussion (the Aims and Scope of Further Analysis)

It should be remembered that competition is a very complex process full of interactions in time among economic agents — firms, consumers, and banks so it involves a great number of economic agents and several interrelated markets. So the results from this research can be generalized to the whole economy, where, for instance, in a consumption system, competition processes can be analyzed in a setting based on preference relations. Furthermore, the conceptual apparatus given in this paper is a starting point for the deeper analysis of the role of innovative competition in the process of innovative evolution of the whole system, and the conclusions drawn from the given analysis provide the ideas for a future study. In particular:

1. Analysis of the multi-level and twin-track relationship between competition in the supply side of the economy and the situation of consumers.
2. Study of the impact of the competitive environment on social transformation and allocation of recourses in the economic system.
3. Analysis of how quickly one product (or a firm) will displace another product in a competitive environment.
4. Study the role of cooperation and innovative imitation in competition.

5. Analysis of impacts of competition on a process of diffusion of innovation (positive and negative).

Conclusions

In this paper, two kinds of innovative competition are discussed: the first is competition between production systems described in a static model situation and generalized to a dynamic method, where different kinds of innovative changes are compared; the second is a dynamic conception of competition as a process of rivalry between firms (producers) in their incessant struggle to increase their market share leading to a gradual displacement and subsequent absorption or elimination of rival firms.

Hence, the main aim of this paper is to show that an adequately targeted extension of the conceptual apparatus of modern, dynamic general equilibrium theory enables us to include the competitive behavior of producers in evolutionary economics. In this framework, as in the neo-Schumpeterian analysis, the competitiveness of the producer depends on their innovative activities and their shares in the market. Research results will help to develop the theoretical basis of modern evolutionary economics, which is dominated by empirical studies. Moreover, the use of the given conceptual apparatus in analysis of evolutionary processes is a step toward the integration of approaches adopted in the mainstream of evolutionary economics, which contributes to the highly desired unification of the science of economics as a monoparadigmatic discipline.

All research results in this paper have the form of mathematical theorems interpreted from an economic perspective, and each argument is based on formal deduction. Hence, the aim of this article was to create a new, original research work in the field of mathematical economics undertaken primarily to acquire new knowledge and to deepen existing one without prejudice to direct practical application or use.

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