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# Adjustment Processes Within Economic Evolution — Schumpeterian Approach

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## Abstract
This research is inspired by Schumpeter's theory of economic development and aims at analysing the outcomes of innovative and non-innovative changes implemented within economic evolution. A multiplicity of outcomes of economic processes leads to complexity of the structure of economic evolution. In order to examine innovative processes, the role of the circular flow, i.e. a form of the economy, which for Schumpeter was both the starting and ending points in the analysis of economic development, should be taken into account. In this context, we determine a simple model of economic evolution in line with Schumpeter's theory, using Hurwicz's concept of the adjustment process. This allows us to examine an impact of accessed information on diversification of economic processes. It should be added that in the model presented, non-innovative changes also play important roles, which is coherent with Schumpeter's theory. As a result, we prove, under some initial conditions coming from the mainstream of the Schumpeter's thought, that the economy under study can evolve in the direction of equilibrium and take a form of circular flow.

**Keywords** Economic evolution · Competitive economy · Innovation · Adjustment processes

**JEL Classification** D41 · D50 · O31

## Introduction

The aim of the paper, remaining in the core of the thought of Joseph Schumpeter, is to analyse some processes of a change of the economy with the special attention paid to the role of entrepreneurs-innovators within the economic evolution and the processes that can lead to equilibrium in economic systems. The last one

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was somewhat neglected in the economic growth literature. Innovative processes, which are considered as the driving factors for economic evolution, do not generally result in equilibrium. However, the economic processes leading to equilibrium are worth to be studied, because the economy could be in equilibrium for any time and in that state, economic agents can realise their potentials under the existed boundaries, among others, feasible technologies and the knowledge.

Modelling innovative changes in the economy, based on Schumpeter's concept of economic evolution, relies on mathematical description of agents' activities. We formalize the economy as a private ownership economy, also known in the economic literature as Arrow-Debreu economy (Arrow & Debreu, 1954; Mas-Colell et al., 1995). It is a static economic structure. To put the Arrow-Debreu economy "in motion," we apply a modification of the concept of adjustment processes (see Arrow & Intriligator, 1987, pp. 1441–1442). An adjustment process is a mathematical structure, defined by the use of difference equations, due to which the impact of information available on the market on agents' activities can be studied. It is represented by:

1. a message space consisting of information sent consciously or unconsciously by economic agents
2. response functions by the use of which links between messages and market activities in a considered environment are formalised
3. an outcome function which to a message assigns the result of analysing of this message

As a result, a mathematical model of economic evolution in which both innovative or non-innovative processes could be studied, is introduced, in which agents can acquire information by observation of other agents' market activities. It is assumed that producers and consumers have full access to knowledge about market, since nowadays information is transformed almost immediately. Innovation and imitation are endogenous, and firms are heterogeneous. Main outcomes, i.e., designing mechanisms which are generated by the structure of demand, are obtained by formal analysis. The designed mechanisms result in some technological changes and, in some cases, improve agents' economic positions whereas different kinds of mechanisms move an economic system into its innovative, imitative or destructive transformation.

For the model constructed, some theorems explaining the rules of economic evolution have been proven with the special attention paid to the processes which can lead to equilibrium (theorems 1 and 2). Due to the results of theorem 2, a process moving the economy toward the circular flow, which comes from the mainstream of the Schumpeter's thought (Andersen, 2009), can be analysed. The model presented could fill a gap existed in economic literature on transformations leading the economy to the circular flow. As an example, a group of economic processes which result in eco-changes (example 1) is presented. In those processes, a special role is assigned to attainable information as a main factor which influence on agents' decisions. In theorem 1, we show, under some natural assumptions typical for agents' rational behaviours, that the economy under study can be transformed into an economy being in equilibrium in which producers' profits, consumers' preferences and budget sets are not changed with respect to the initial economy. Following the Schumpeter's concept, an economic transformation (definition 4) starts precisely from an economy being in a form of the circular flow and is modelled as an adjustment process (definition 1). Due to the results of theorems 1 and 2, we show that an adjustment process could result in equilibrium in a transformation of the initial economy. If in that transformed economy equilibrium is destroyed, then a next adjustment process can start. In that case, we say about a composition of adjustment processes. In the considered model, the equilibria obtained within the economic transformation can be short or long life, whereas adjustment processes can be innovative or non-innovative since they can result into innovative or non-innovative changes in the economy, in other words, different kinds of processes move an economic system into its innovative or non-innovative transformation. Occurrence of various kinds of transformations of the economy is an equivalent of business cycles in the microeconomic model presented. Financing consumption also plays an important factor here, although the financial system is not considered *explicitly*. Conclusions are the results not only of the theoretical research but also the empirical analysis of some features of innovative processes as the theoretical results will be illustrated by an empirical study on Polish economy. However, the model presented is not just about a specific case of Polish economy.

The paper consists of seven parts. The second part deals with the literature review, the third part takes the short analysis of innovativeness of Polish economy in years 2005–2018 against the background of innovativeness of some European countries and the justification for building a model of economic evolution. In the fourth part, an adjustment process and a composition of adjustment processes are defined, whereas in the fifth part, the innovative and non-innovative changes in the model presented are analysed. The sixth part is devoted to discussion, and the seventh part takes conclusions.

## The Literature Review

Joseph Schumpeter, the founder of theory of economic development, distinguished two kinds of economic life: the circular flow and the economic development (Schumpeter, 1961).

In the economy in a form of the circular flow "the same products are produced every year in the same way. For every supply there waits somewhere in the economic system a corresponding demand, for every demand the corresponding supply. All goods are dealt in at determined prices with only insignificant oscillations, so that every unit of money may be considered as going the same way in every period. … There is no market for the bearers of the original productive services themselves (…) and there is also no price for them within the normal circular flow." (ibid., p. 61). Thus, the circular flow is based on routine behaviour of economic agents and governed by Walrasian tatonnement process to determine prices and quantities of goods in the market economy. Consequently, this process reminds of the blood circulation in a living organism (ibid., p. 61) and can be interpreted as numbness of the economic life.

The circular flow, according to Schumpeter, is the starting point for the economic development. The economic development is described as "a spontaneous and discontinuous change in the channel of the circular flow" (ibid. p. 64). According to Schumpeter (1961), during the economic development, two opposite processes in the production sphere of the economy are distinguished. The first involves the creation of innovations, the second the elimination of the existing products or organizational structures and their replacement by new ones. The coexistence of the above tendencies is called by Schumpeter the creative destruction principle (Schumpeter, 1950). On the basis of the creative destruction principle, it was clarified that the economic development usually causes the disturbance of equilibrium, which results in alteration and displacing the previously existing equilibrium state.

Schumpeter argues that (1961, pp. 81–94), the key role within the economic evolution is played by entrepreneurs-innovators, who to earn profits initiate and carry the process of economic development breaking the circular flow by introducing innovation in the economy. It should be noted that Schumpeter distinguishes entrepreneurship from managerships: a manager manages the affairs of an enterprise whereas the entrepreneur also takes high degree of risk. In the Schumpeter's vision of economic evolution, economic development is driven by supply side of economy while consumers play a passive role in this process. Entrepreneurs' innovative production plans determine structural changes of the whole economy whereas consumption plans reflect consumers' routine behaviour. There have been a number of papers in the last years in which the role of demand in innovative evolution of the economy is explored (Pyka & Andersen, 2012). The role of the entrepreneurs-innovators within economic evolution is summarized in Table 1.

In the current paper, remaining in the core of the Schumpeter's thought, the role of entrepreneurs within the economic development is the key; however, the processes under study are, mostly, demand-driven.

The neo-Schumpeterian approach was originated by Nelson and Winter (1982). They took the ideas expressed by Schumpeter and presented a concept of economic growth based on technological progress and competition between firms focused on dynamic changes. Hanusch and Pyka (2007) analysed the transition from the circular flow to the economic development in the neo-Schumpeterian paradigm. They emphasized that profit opportunities were seen not only by the market prices but also by the creativity and daringness of entrepreneurs which compensate scarcities in the economy. Day (2007) suggested that entrepreneurs-innovators drove the development of human creative intelligence, which he distinguished from rationality. At the same time, Shionoya (2007) wrote that, "Schumpeter emphasized that economic dynamics should be accompanied by, and based on, economic statics". Andersen (2009) dropped the concept of a single model of the circular flow as well as used Schumpeter's "the capitalist engine" to explain the process of creation of innovations. The driving force of the Schumpeter's capitalist engine was also studied by Freeman (1982), who, among others, distinguished at least its two versions which

**Table 1** The role of the entrepreneurs-innovators within economic evolution in the light of the literature

| Role of the entrepreneurs-innovators in economic change | Source |
|---|---|
| Earning profits | Schumpeter, 1912, 1961; Aghion & Howitt, 1992; Shelton et al., 2008; Foster, 2011; Assenza et al., 2015; Cantner, 2016; Almudi et al., 2019a; Freeman, 1982 |
| Breaking the circular flow | Schumpeter, 1912, 1961; Andersen, 2009; Foster, 2011 |
| Introducing innovation in the economy | Schumpeter, 1912, 1961; Freeman, 1982; Aghion & Howitt, 1992; Day, 2007; Andersen, 2009; Dosi et al., 2010; Foster, 2011; Mishra & Zachary, 2015; Assenza et al., 2015; Cantner, 2016; Almudi et al., 2019a; Boutillier, 2019; Nurmalia et al., 2020 |
| Initiating and carry the process of economic development | Schumpeter, 1912, 1961; Day, 2007; Andersen, 2009; Foster, 2011; Cantner, 2016; Nurmalia et al., 2020 |
| Determining structural changes of the whole economy | Schumpeter, 1912, 1961; Day, 2007; Andersen, 2009; Dosi et al., 2010; Foster, 2011; Assenza et al., 2015; Mishra & Zachary, 2015; Cantner, 2016; Nurmalia et al., 2020 |
| Creating radically new lines of production in a given economic system | Schumpeter, 1912, 1961; Mishra & Zachary, 2015; Cantner, 2016; Almudi et al., 2019a; Nurmalia et al., 2020 |
| Generating and adopting new ideas as a well as of economizing on them via innovation, imitation and adoption | Schumpeter, 1912, 1961; Andersen, 2009; Foster, 2011; Cantner, 2016 |
| Entering the market with new combinations | Schumpeter, 1912, 1961; Freeman, 1982; Day, 2007; Andersen, 2009; Foster, 2011; Assenza et al., 2015; Mishra & Zachary, 2015; Cantner, 2016; Almudi et al., 2019a; Nurmalia et al., 2020 |
| Creating sustained value | Mishra & Zachary, 2015 |
| Making do with the resources available | Mishra & Zachary, 2015 |
| Generating innovations in a systematic way | Aghion & Howitt, 1992; Schwab & Zhang, 2018 |
| Creating jobs | Boutillier, 2019; Fölster, 2000 |
| Supporting balanced regional development | Fischer & Nijkamp, 2009; Nurmalia et al., 2020 |
| Growing in GDP an income per capita | Shelton et. al, 2008; Assenza et al., 2015 |
| Increasing in the standard of living | Mishra & Zachary, 2015; Nurmalia et al., 2020 |
| Export development | Hessels & van Stel, 2011; Parwez, 2017 |
| Community development | Lyons, 2015; Parwez, 2017 |

have been widely expanded by Malerba and Orsenigo (1995, 1997) and Malerba (2005).

A review of the main ideas on the determinants of economic growth is presented in Foster (2011, pp. 9–14). Foster notes that initially, it was believed that economic growth was caused by processes of invention, entrepreneurship, technological and organizational innovation, education, training and experience (learning by doing), whereas institutional change played the main role in governing the growth. Later, the roles of other factors influencing the process of economic growth, such as the aggregate balance of saving, investment and population change (Harrod, 1948) as well as the creative destruction (Schumpeter, 1912, 1961), were indicated. The classical approach to economic growth was developed in Cambridge (Kaldor, 1985; Pasinetti, 1993; Robinson, 1956). In that approach, the growth rate, determined by the savings rate, the capital-output ratio, the natural growth rate, determined by the population growth rate and the rate of productivity growth, is viewed as the main factors of economic growth. In the endogenous growth theory knowledge, education, R&D and increasing returns due to the low cost of the transfer of knowledge play the key role in growth processes (for example Romer, 2012).

The study on behaviour of economic agents can be found, among others, in Nelson (2016), while Cantner (2016) presented the survey on characterization of Schumpeter's entrepreneurs.

It is necessary to add that the modern theory of entrepreneurship has been also dynamically developing, the example of which are the papers by Mishra and Zachary (2015), Martinez et al. (2011), Schwab and Zhang (2018) and Shelton et al. (2008).

The comprehensive analysis of the nature of innovations as the result of the creative destructive principle, using the methods of the differential calculus the reader can find in Aghion and Howitt (1992). In this paper, which gave the beginning of the theory of endogenous economic growth, the authors saw the source of economic development in the effectiveness of the activities of the R&D sector, which through the mechanism of the creative destruction, here understood as producing commodities of higher quality, generated the economic growth (see also Aghion & Howitt, 1998), but not in the accumulation of capital, as was in the case of the Solow's neoclassical theory of economic growth (Romer, 2012).

During the last decade, there have appeared new formal models in the Schumpeterian evolutionary strand which aim at analysing general coordination mechanisms fully different with respect to the model presented in the current paper (e.g. Almudi et al., 2019a, b; Assenza et al., 2015; Dawid et al., 2019; Dosi et al., 2010). In Dosi et al. (2010), a past-Walrasian (see Colander, 2006) macroeconomic agent-based model with endogenous innovation that could bridge Keynesian theories of demand generation and Schumpeterian theory of technology bringing about economic growth is defined and analysed. The main results, obtained through simulations, show the existence of complementarities between factors influencing aggregate demand and drivers of technological change that affect both short-run fluctuations and long-term growth patterns. In Assenza et al. (2015), a macroeconomic agent-based model with capital and credit is determined and discussed. In that model, the firms do not have perfect knowledge about the demand function, and the agents do not know the behaviour of the others and use past information to decide on prices and quantities. Moreover, the quantities of production of consumption goods depend above all on the financial system. As a result, importance of the production sector in shaping the business cycle is shown. The definition of the macroeconomic model of economic dynamic presented in Dawid et al. (2019) is based on decisions of economic agents and assumes that each firm proceeds through a given sequence of economic activities. Each firm is financed by a bank sector and faces a production planning problem because of stochastic demand and stock-out cost. Firms may be restructured or disappear from the market. The rules for matching the firms and banks in the credit market are defined on the basis of a completely decentralized algorithm. The properties of the model are explored through simulations. In Almudi et al. (2019a), a co-evolutionary computational two-sector model with heterogeneous firms is presented. The paper aims at designing of national innovation policy taking into account the importance of "intersectoral absorptive capacity constraints in innovation linkages between sectors" (*ibid.*). In Almudi et al. (2019b), an evolutionary growth model in which the roles of a bank and financial structures of firms within innovative competition of firms are analysed is determined. The main results are the effects of simulations. In the paper, it is shown that the bank and the financial structures of firms may have a huge impact on competitiveness of firms and indication causes of emergence of crisis with long-run effects.

As we can see, modern studies on Schumpeterian evolution can be decomposed into two groups: studies within neo-Schumpeterian research program (Hanusch & Pyka, 2007; Day, 2007; Andersen, 2009; Foster, 2011; Freeman, 1982; Malerba & Orsenigo, 1995, 1997; Nelson, 2016; Witt, 2017) and research papers on Schumpeterian endogenous growth theory (Aghion & Howitt, 1992; Dosi et al., 2010, Assenza et al., 2015; Dawid et al., 2019; Almudi et al., 2019a, b). The models of economic growth presented above study innovations in economic systems characterized by far-from-equilibrium dynamics and patterns of agent interaction leading to topological features different from the topology of classical theory of general equilibrium (Debreu, 1959). However, we share the opinion (Hodgson, 1993) that the invocation of Schumpeter's name in some works labelled as neo-Schumpeterian is misleading, false and based on illusions: from Schumpeter's point of view; there is no analysis of dynamics without considering static. Additionally, Schumpeter's vision of economic evolution was strongly inspired by Walrasian thinking (see also Andersen, 2009). However, it should be emphasized that the idea of general equilibrium has been only a starting point for Schumpeter's study on economic development which runs far beyond equilibrium schemata (Malawski, 2013).

It is obvious (see for example Foster, 2011) that the contemporary factors of economic growth should be based on knowledge. It is essential for sustainable economic policy to create knowledge societies and knowledge-based economies, in which innovations, entrepreneurship and competitiveness lead to the improvement and protection of the environment as well as living conditions of societies. A knowledge-based economy is "an economy where knowledge is created, acquired, transmitted and used effectively by businesses, organizations, individuals and communities" (see Skrodzka, 2016) while knowledge society can be understood as a society which has potentials to create scientific and technological knowledge. Below, we present some examples of the papers exploring the economic growth of knowledge-based economies. In Abdouli and Hammami (2020), the investigation of the relationships between economic growth, foreign direct investment inflows environment duality and financial development is presented. Boutillier (2019) analyses the capacity of the small entrepreneur to introducing innovations with the special respect paid to eco-innovations. Nunes et al. (2019) examine the relationship between networking intensity and the innovation process in order to analyse the effect of these networks on firm performance whereas Omidi et al. (2020) try to verify selected theories (Romer, 1990 and Weitzman, 1998; Schmookler, 1966; Acemoglu & Robinson, 2000) on drivers of innovation in developing countries. As we can see, the literature on knowledge-based economies deals with many aspects of economic growth.

The study on the possibility of existence of equilibrium in an economy is in the core of interest of general equilibrium theory (Arrow & Debreu, 1954). Equilibrium in the competitive economy is a state in which every economic agent can maximize his profit or utility under existing boundary and conditions.

Let us recall that Gerard Debreu and Kenneth Arrow proved that under some mathematical assumptions on producers' and consumers' characteristics, there is a price vector at which aggregate supply is equal aggregate demand (Arrow & Debreu, 1954), which means that this economy is in equilibrium in Walras sense (see also Debreu, 1959). The Sonnenschein-Mantel-Debreu theorems expanded the Arrow and Debreu result for a market with agents behaving rationally and maximizing utilities (Debreu, 1974; Mantel, 1974; Sonnenschein, 1972, 1973). Modifications and generalizations of Arrow-Debreu model were also constructed: the competitive economy with infinitely many traders (Aumann, 1962, 1966), the rational expectation model (Radner, 1972), the multi-period economies (Magill & Quinzii, 2002) or the economy with infinitely many commodities (Aliprantis, 1996) can provide us some examples.

Some results on mathematical modelling of innovative development in the language of general equilibrium theory (Arrow & Debreu, 1954; Debreu, 1959) can be found, for instance, in Ciałowicz and Malawski (2011) and Lipieta and Malawski (2016a). Ciałowicz and Malawski analysed the role of banks within the Schumpeterian evolution; in Lipieta and Malawski (2016a), the basis for modelling the mechanisms of Schumpeterian evolution by the use of the Hurwicz's apparatus of the theory of mechanisms design was made. The examples of the processes that, under given boundaries, can lead to equilibrium not making the agents' economic positions worse were depicted in Lipieta (2015) whereas the results of analysis on the mechanisms resulting in changes which are friendly to the environment are presented in Lipieta and Malawski (2020).

## Some Notes on Innovativeness of Polish Economy

According to the definition of the innovation, obligatory in the European Union and OECD (see OECD, 2015), the innovation is introducing a new or significantly improved product that is new to the market or introducing a new or substantially improved technology, method of production, organizational method of production, marketing method, organization of a workplace or environments into a firm's activity. The above definition covers all five internal changes characterized by Schumpeter as an innovation (Schumpeter, 1961, p. 66), i.e.

1. the introduction of a new good,
2. the introduction of a new method of production,
3. the opening of a new market,
4. the conquest of a new source of supply of raw materials,
5. the carrying out of a new organization of any industry.

Within the above, innovations on real markets can be considered in the category of innovations in Schumpeter's meaning. The innovative activity is the whole activities of firms whose aim is to introduce innovations. The firms, which introduce innovations, are called the innovative firms or enterprises.

As it was mentioned, introducing of innovations is a typical feature of the Schumpeter's economic development, which is understood as "spontaneous and discontinuous change in the channels of the flow, disturbance of equilibrium, which forever alters and displaces the equilibrium state previously existing" (Schumpeter, 1961). We will analyse the values of four variables for Poland in years 2005–2017:

1. the share of sold production of new or significantly improved industry goods in industrial enterprises in total amount of sold goods,
2. the share of innovative enterprises in total enterprises,
3. the number of newly registered firms recorded in *REGON* register,
4. the number of firms removed from the *REGON* register.

The first and the second variables demonstrate innovativeness of the economy, while all of them are the symptoms of the creative destruction.

In Poland, within 2006–2017, the share of sold production of new or significantly improved industrial products in industrial enterprises compared to the total amount of sold goods in Poland was not less than 8.8% (see Fig. 1). Hence, in that period, innovations were introduced on the market and played a significant role in

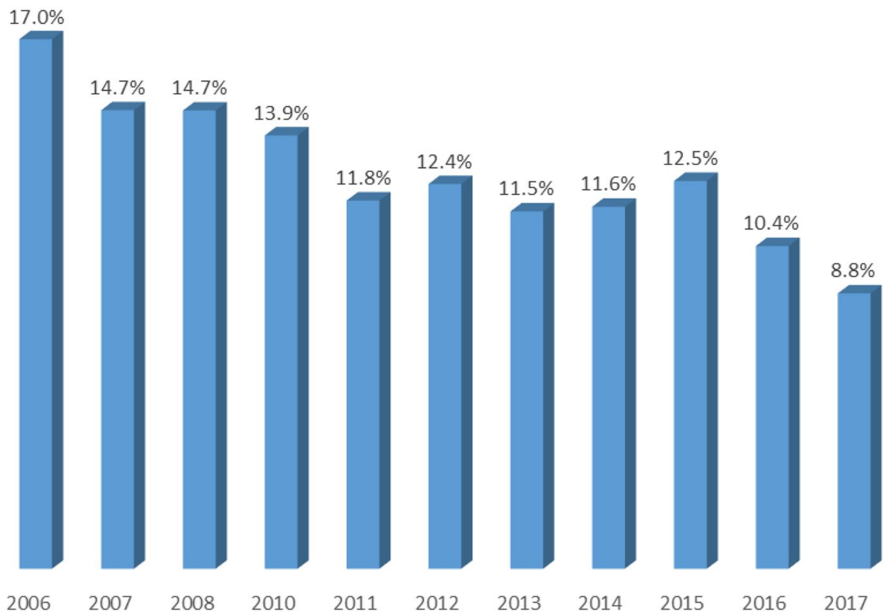


**Fig. 1** Share of sold production of new or significantly improved industry goods in industrial enterprises in total amount of sold goods in Poland in years 2006–2017. Data from Local Data Bank, https://bdl.stat.gov.pl/. Access: 6th August 2019. 2009 — no information

Polish economy. Taking into consideration the amount of innovative commodities in total Polish sales, it can be said that the Polish economy in 2006–2017 was in the form of the Schumpeter's economic development.

According to Schumpeter, within the economic development, as a result of the creative destruction, some old economic structures are destroyed and some new appear on the market (Schumpeter 1961). In 2005–2018 in Poland, the number of newly founded firms (namely newly registered) as well as the number of firms finishing their economic activity (removed from the *REGON* register) is systematically increasing (see Table 2), which is also the confirmation of Schumpeter's ideas. The reader can check the details, in Table 2.

Let us also notice that in 2006–2018 in Poland in the ranking (see Fig. 2), the share of inno-vative firms within the total number of firms is not constant, but its level is not less than 16.1%, in case of industrial enterprises, and not less than 9.8%, in case of service enterprises. As both shares were decreasing in 2006–2011, some firms stopped their innovative activities or some of them exited the markets. The highest decrease 3.3% (see Fig. 2) coincides with the world financial crisis in 2008. To sum up, in Polish economy in 2005–2017, we could not observe on the basis of statistical data an interval in the innovative activities or even though its significant decrease, which would be interpreted as the Schumpeter's circular flow.

The data concern only year periods. It could happen that in the meantime, Polish economy was in a form of the circular flow but it cannot be confirmed by empirical data. We can only suppose that, if the Polish economy took a form of the circular flow within 2005–2018, it was short-lived as innovative activities did not disappear in year periods.

Polish economy is less than an average economy with respect to innovativeness, what shows Fig. 3. That is why it might seem that not only the economic development but also the circular flow, interpreted as the stagnation of economic life, would be reflected in statistical data.

Reasoning in the same way as earlier, we could say that the economies of the countries, in which the share of innovative enterprises in total enterprises was higher than in Poland in the analysed period, were also in a form of the economic development in that period. However, again, the data do not show how these economies evolved within periods of time less than 1 year and whether they have ever taken a form of the circular flow.

A method of measure innovativeness of the economy is essential in the determining the level of innovativeness. There are many ways to define the so-called index of the innovativeness of the economy. *Bloomberg Innovation Index* or *Global Innovation Index* (https://www.bloomberg.com/news/articles/2020-01-18/germany-breaks-korea-s-six-year-streak-as-most-innovative-nation; https://www.ifpma.org/resource-centre/the-global-innovation-index-2019-report/) can serve us the examples. Additional barrier for detailed analysis, especially in case of weak developed countries, is collecting complete data. In *Visual Capitalist* ranking published in July 2019, finally, 95 countries were analysed although initially more than 200 countries were under consideration (https://www.visualcapitalist.com/the-10-most-innovative-economies-in-2019/). Similarly, of the more than 200 countries and sovereigns were evaluated in 2015 *Bloomberg Innovation Index*, only 69 had complete data (https://www.bloomberg.com/graphics/2015-innovative-countries/).

**Table 2** Entities of the national economy in Poland in 2005–2018

| Year | **2005** | **2006** | **2007** | **2008** | **2009** | **2010** | **2011** |
|---|---|---|---|---|---|---|---|
| Newly registered, recorded in the *REGON* register | 261,507 | 297,302 | 295,033 | 317,954 | 349,656 | 402,005 | 346,087 |
| Removed from the *REGON* register | 214,778 | 271,090 | 242,790 | 244,965 | 357,530 | 237,693 | 383,617 |

| Year | **2012** | **2013** | **2014** | **2015** | **2016** | **2017** | **2018** |
|---|---|---|---|---|---|---|---|
| Newly registered recorded in the *REGON* register | 358 367 | 365 487 | 357.35 m | 359,973 | 349,298 | 361,143 | 392,659 |
| Removed from the *REGON* register | 252,313 | 269,904 | 304.68 m | 292,358 | 293,997 | 286,833 | 331,648 |

Source: Local Data Bank (CSO), https://bdl.stat.gov.pl/. Access: 6th August 2019

*m* methodological changes

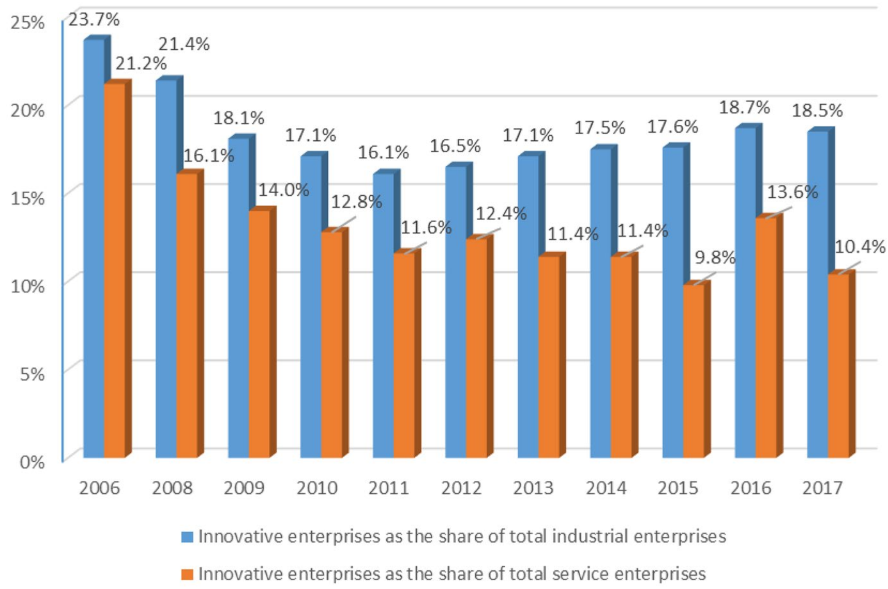


**Fig. 2** Innovative enterprises in Poland in 2006–2017 as the share of total enterprises. Source: Local Data Bank (CSO), https://bdl.stat.gov.pl/. Access: 6th August 2019. 2007 — no information

To see the changes of innovativeness of Polish economy in the last years, we focus on *Bloomberg Innovation Index* (see also Wolniak & Grebski, 2018). A place of an economy in the *Bloomberg* ranking is scored on the basis of seven equally weighted metrics, namely R&D intensity, patent activity, tertiary efficiency, manufacturing value-added, productivity, high-tech density, researcher concentration (the detailed definitions can be found on the website https://www.bloomberg.com/graphics/2015-innovative-countries/). Their scores are combined to provide an overall score for each country from zero to 100. The ranking in the area of each mentioned variable is also presented in every year (https://www.bloomberg.com/news/articles/2020-01-18/germany-breaks-korea-s-six-year-streak-as-most-innovative-nation). It shows what should be supported in a given country to improve its innovativeness in the *Bloomberg* sense. We present *Bloomberg Innovation Index* for selected European countries in years 2020 and 2019 in Table 3 and in Table 4.

As we can see, the place of Poland in the ranking published by *Bloomberg* fell: in 2020, Polish economy has 25th place in the ranking, while in 2019 22nd place (https://www.ifpma.org/resource-centre/the-global-innovation-index-2019-report/). The falls of Poland and Romania in the Bloomberg ranking are linked to the high falls in the ranking of at least one variable. The COVID-19 pandemic also may have caused those falls.

To sum up: statistical data on firms' activities, usually announced with regard to year periods, do not reflect the whole diversity of economic processes as well as their sensitivity to a change of technologies or prices. Because of the barriers, statistical data do not show whether the economy took or could take a form of the

**Table 3** 2020 Bloomberg Innovation Index in selected countries

| Country | 2020 rank | R&D intensity | Manufacturing value-added | Productivity | High-tech density | Tertiary efficiency | Researcher concentration | Patent activity |
|---|---|---|---|---|---|---|---|---|
| France | 10 | 13 | 39 | 16 | 2 | 20 | 17 | 8 |
| Germany | 1 | 8 | 4 | 18 | 3 | 26 | 11 | 3 |
| Italy | 19 | 24 | 23 | 21 | 16 | 33 | 25 | 20 |
| Poland | 25 | 35 | 17 | 39 | 22 | 19 | 38 | 29 |
| Romania | 32 | 56 | 19 | 32 | 23 | 27 | 47 | 30 |
| Spain | 33 | 31 | 30 | 34 | 44 | 18 | 26 | 35 |
| UK | 18 | 21 | 44 | 27 | 15 | 6 | 19 | 21 |

Source: https://www.bloomberg.com/news/articles/2020-01-18/germany-breaks-korea-s-six-year-streak-as-most-innovative-nation. Access: 24th August 2020

**Table 4** 2019 Bloomberg Innovation Index in selected countries

| Country | 2020 rank | R&D intensity | Manufacturing value-added | Productivity | High-tech density | Tertiary efficiency | Researcher concentration | Patent activity |
|---|---|---|---|---|---|---|---|---|
| France | 10 | 12 | 41 | 13 | 2 | 11 | 20 | 15 |
| Germany | 2 | 7 | 3 | 24 | 3 | 14 | 11 | 7 |
| Italy | 21 | 24 | 22 | 20 | 19 | 29 | 29 | 26 |
| Poland | 22 | 36 | 20 | 40 | 18 | 16 | 38 | 37 |
| Romania | 29 | 55 | 18 | 32 | 23 | 24 | 47 | 24 |
| Spain | 30 | 31 | 27 | 30 | 43 | 12 | 26 | 44 |
| UK | 18 | 20 | 45 | 26 | 14 | 5 | 21 | 19 |

Source: https://www.bloombergquint.com/global-economics/germany-nearly-catches-korea-as-innovation-champ-u-s-rebounds. Access: 24th August 2020

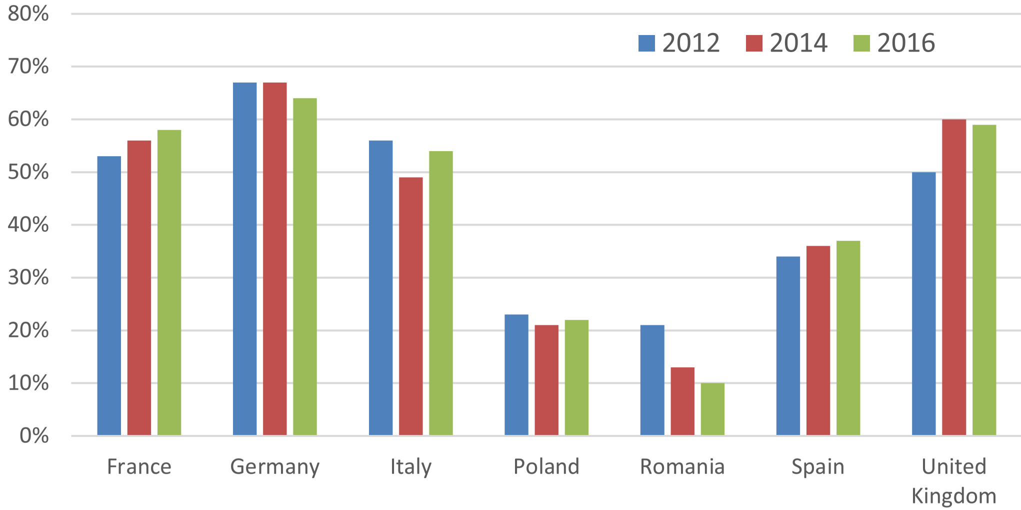


**Fig. 3** Percentage of enterprises that have either introduced an innovation or have any kind of innovation activity (including enterprises with abandoned/suspended or on-going innovation activities). Source: https://rio.jrc.ec.europa.eu/en/stats/innovative-enterprises-total-enterprises-size-class-and-type-innovation. Access: 6th August 2019

circular flow as well as do not help with examination of processes that could lead to the circular flow. That is why, to more explore the structure of innovative and non-innovative processes, we determine a model of economic evolution taking into account both Schumpeter's theory and results of the short review of statistical data on Polish economy against the background of innovativeness of some countries presented in the third section.

## Adjustment Processes in the Private Ownership Economy

In this part, we focus on modelling economic processes to determine a model of economic evolution. According to Schumpeter, the economy in the form of the circular flow is similar to the blood circulation in a living organism. It means, among others, that the same products are produced in the same way every year, for every supply (demand) waits somewhere corresponding demand (supply), all goods are traded at the determined prices with only insignificant oscillations, every unit of money may be considered as going in the same way in every period, and there is no market for inventors (Schumpeter, 1961). In the circular flow, an economic system tends to equilibrium to determine prices and quantities of goods. The changes in activities of economic agents, if they occur, are so small that they do not entail the changes in economic processes, and the economic positions of firms and consumers remain the same (Lipieta & Malawski, 2016a). The above lead us to modelling the economy in the form of the circular flow by the use of the same apparatus which was used by Arrow and Debreu (Mas-Colell et al., 1995). However, the assumptions considered in the current paper are not so strong as these applied by Arrow and Debreu (1954). To model economic processes, we apply the modification of the concept of the adjustment process presented in Arrow and Intriligator (1987, pp. 1441–1442).

Now, let us go into details. The number of economic agents operating on the market and the number of commodities are finite. According to Schumpeter's theory, those numbers can be changed during the processes of evolution, what can be also observed in real economies. These changes are reflected in the idea of considering inactive agents (Lipieta & Malawski, 2016b) and future goods in the modelling of economic processes (see also Lipieta, 2017). Two kinds of economic agents: producers and consumers are considered. It is denoted:

- $A = (a_i)_{i \in \mathbb{N}}$ — a countable set of consumers
- $B = (b_j)_{j \in \mathbb{N}}$ — a countable set of producers
- $t = 0, 1, 2, \dots$ — a point of time

An inactive agent at time $t$ is the agent whose activity is reduced to the zero plan at given $t$. Every inactive agent can be interpreted as a potential future agent, who waits for the proper time to enter the market. By the above, we assume that, for every $t$, there exist numbers $m_t, n_t \in \{1, 2, \dots\}$ such that, for every $i > m_t$, and, for every $j > n_t$, every consumer $a_i$ and every producer $b_j$ are inactive. The sets of active consumers and producers at time $t$ are denoted by $A_t$ and $B_t$, adequately. Taking the previous arrangements into consideration, we denote.

$$A_t = \left(a_1, \dots, a_{m_t}\right), B_t = \left(b_1, \dots, b_{n_t}\right).$$

Moreover, at time $t' > t$, new consumers or producers can enter the market, what means that $A_t \subset A_{t'} \subset A$, $B_t \subset B_{t'} \subset B$ and consequently $m_{t'} \geq m_t$ and $n_{t'} \geq n_t$.

Let $\ell_t \in \mathbb{N}_+ \overset{\text{def}}{=} \{1, 2, \dots\}$ be the number of the commodities, which are produced and consumed in the economy at time $t$ or which were produced and consumed earlier. Define

$$\mathcal{R}^{\ell_t} \overset{\text{def}}{=} \mathbb{R}^{\ell_t} \times \{0\} \times \{0\} \times \dots \tag{1}$$

Space $\mathcal{R}^{\ell_t}$ is interpreted as the commodity-price space in the analysed economy at time $t$. Especially, every coordinate $l \in \left\{\ell_t + 1, \ell_t + 2, \dots\right\}$ is interpreted as the number of a future good. Hence,

$\ell_t \leq \ell_{t'}$ for $t < t'$ and $\mathcal{R}^{\ell_t} \subset \mathcal{R}^{\ell_{t'}}$.

Such approach simplifies the description of processes in which the set of commodities can be changed in time.

Below, activities and characteristics of economic agents are formalized. At a given $t$, activities of a producer $b$ in space $\mathcal{R}^{\ell_t}$ with respect to feasible technologies are demonstrated by his production plans, which form the so called production set $Y^b(t) \subset \mathcal{R}^{\ell_t}$ of producer $b$ at time $t$. Moreover,

$$\exists n_t \in \{1, 2, \dots\} \forall j > n_t : Y^{b_j}(t) \overset{\text{def}}{=} \{0\}.$$

The above illustrates the assumption that a finite number of producers operate on the market at given $t$, while every producer $b_j$, for $j > n_t$, is an inactive producer at moment $t$. Due to such set-up, it is underlined that an unknown number of producers might enter or exit the market in the future, and an unknown number of new commodities can be produced. Additionally, it is assumed that at time $t$, there is a producer $b \in B_t$, for whom $\ell_t$-th commodity is the input or the output, which means that at least one production plan $y^b(t) \in Y^b(t)$ has $\ell_t$-th coordinate different from zero. Denote by:

- $X^a(t) \subset \mathcal{R}^{\ell_t}$ — the consumption set of consumer $a$ at time $t$; moreover

$$\exists m_t \in \{1, 2, \dots\} \forall i > m_t : X^{a_i}(t) \overset{\text{def}}{=} \{0\};$$

the above means that for $i > m_t$, every consumer $a_i$ is the inactive consumer

- $\Xi_t$ — set of preference relations in space $\mathcal{R}^{\ell_t}$
- $\preccurlyeq^a_t \subset X^a(t) \times X^a(t)$ — the preference relation of consumer $a$ at time $t$, $\preccurlyeq^a_t \in \Xi_t$
- $\omega^a(t) \in X^a(t)$ — the initial endowment of consumer $a$ at time $t$
- $\omega(t) = \sum_{a \in A_t} \omega^a(t) \in \mathcal{R}^{\ell_t}$ — the total endowment of the economy at time $t$,
- $\theta_t : A_t \times B_t \to [0, 1]$ — the share mapping, where

- for $a \in A_t$ and $b \in B_t$, number $\theta_t(a,b)$, means the share of consumer $a$ in the profit of producer $b$
- $\forall b \in B_t : \sum_{a \in A_t} \theta_t(a,b) = 1$
- $\theta_t(a,b) = 0$ if $a \notin A_t$ or $b \notin B_t$

On the basis of the above notation, economic environment $e^k(t)$ of every agent $k \in K \overset{\text{def}}{=} A \cup B$ (see Hurwicz & Reiter, 2006, p. 25) at time $t$ is defined. Namely,

$$e^k(t) = \left( Y^k(t), X^k(t), \omega^k(t), \preccurlyeq^k_{\ell_t}, \Theta_t(k, \cdot) \right) \tag{2}$$

where:

- $Y^k(t) = \{0\}$ for $k \in K \backslash B_t$,
- $X^k(t) = \{0\}$ for $k \in K \backslash A_t$,
- $\omega^k(t) = 0$ for $k \in K \backslash A_t$,
- $\preccurlyeq^k_t = \{\emptyset\}$ for $k \in K \backslash A_t$.
- The mapping $\Theta_t : K_t \times K_t \to [0, 1]$, $K_t \overset{\text{def}}{=} A_t \cup B_t$, satisfies
- $\Theta_t(k, \cdot) \equiv 0$ for $k \in K_t \backslash A_t$, $\Theta_t(\cdot, k) \equiv 0$ for$k \in K_t \backslash B_t$,
- $\Theta_t(a,b) = \theta_t(a,b)$ for $a \in A_t$ and $b \in B_t$.

Set $E^k(t)$ stands for the set of all feasible economic environments of agent $k$ at time $t$, set $E(t)$ stands for the set of economic environments at time $t$ (see (2)). It is defined as Cartesian product of agents' environments at time $t$:

$$E(t) \overset{\text{def}}{=} E^{k_1}(t) \times E^{k_2}(t) \times \dots$$

Vector $e(t) = (e^{k_1}(t), e^{k_2}(t), \dots) \in E(t)$ is called the economic environment at time $t$.

Let us notice that components of economic environment $e(t)$ (see (2)) form a private ownership economy with space $\mathcal{R}^{\ell_t}$ (see (1)) as the commodity-price space (compare to Arrow & Debreu, 1954; Debreu, 1959; Mas-Colell et al., 1995). Recall that the private ownership economy at time $t$ is a mathematical structure:

$$\epsilon(t) = \left( \mathcal{R}^{\ell_t}, P(t), C(t), \Theta_t, \omega(t) \right) \tag{3}$$

in which:

- $p(t) \in \mathcal{R}^{\ell_t}$ is the price vector
- structure $P(t) = \left( B_t, \mathcal{R}^{\ell_t}; \left( Y^b(t) \right)_{b \in B_t}, p(t) \right)$ is the quasi-production system
- structure $C(t) = (A_t, \mathcal{R}^{\ell_t}, \Xi_t; (X^a(t))_{a \in A_t}, \left( \preccurlyeq^a_t \right)_{a \in A_t}, (\omega^a(t))_{a \in A_t}, p(t))$ is the quasi-consumption system

(see Lipieta & Malawski, 2016a, b). Moreover,

- If every producer $b \in B_t$ can maximize the profit, which means that set

$$\eta_t^b(p(t)) \stackrel{\text{def}}{=} \left\{ y^{b*}(t) \in Y^b(t) \, : \, p(t) \circ y^{b*}(t) = \max \left\{ p(t) \circ y^b(t) \, : \, y^b(t) \in Y^b(t) \right\} \right\},$$

is not empty, then quasi-production system $P(t)$ is the production system.

- If every consumer $a \in A_t$ has non empty budget set
$$\beta_t^a(p(t)) \stackrel{\text{def}}{=} \left\{ x^a(t) \in X^a(t) \, : \, p(t) \circ x^a(t) \leq p \circ \omega^a(t) + \sum_{b \in B_t} \theta_t(a,b) \cdot p(t) \circ y^{k*}(t) \right\}$$

and can maximize his preference on set $\beta_t^a(p(t))$, which means that set

$$\varphi_t^a(p(t)) \stackrel{\text{def}}{=} \left\{ x^{a*}(t) \in \beta_t^a(p(t)) \, : \, \forall x^a(t) \in \beta_t^a(p(t)) x^a(t) \preccurlyeq^a x^{a*}(t) \right\},$$

is not empty, then quasi-consumption system $C(t)$ is the consumption system (compare to Lipieta & Malawski, 2016a).

Let $\kappa(t)$ be the number of elements of set $K_t$, $\kappa(t) \leq m_t + n_t$. If in economy $\epsilon(t)$ there exists a sequence

$$(x^*(t), y^*(t), p(t)),$$

where $x^*(t) = \left( x^{k_1*}(t), x^{k_2*}(t), \ldots, x^{k_{\kappa(t)}*}(t) \right)$, $y^*(t) = \left( y^{k_1*}(t), y^{k_2*}(t), \ldots, y^{k_{\kappa(t)}*}(t) \right)$ such that

- $y^{k*}(t)$ maximizes the profit of producer $k \in B_t$, what means that $y^{k*}(t) \in \eta_t^b(p(t))$; $y^{k*}(t) = 0$, if $k \notin B_t$,
- $x^{k*}(t)$ maximizes the preferences of consumer $k$ on budget set $\beta_t^a(p(t))$, if $k \in A_t$, what means that then $x^{k*}(t) \in \varphi_t^a(p(t))$; $x^{k*}(t) = 0$, if $k \notin A_t$,
- $\sum_{k \in K_t} x^{k*}(t) - \sum_{k \in K_t} y^{k*}(t) = \omega(t)$,

then it is called the state of equilibrium in economy $\epsilon(t)$ (Lipieta, 2010; Mas-Colell et al., 1995). In the private ownership economy, agents do not always behave rationally (Lipieta & Malawski, 2016a, b). We assume that producers in economy $\epsilon(t)$ compete, but in difference to the perfect rationality assumption (Simon, 1947, 1957), the aim of producers-innovators is introducing innovations to maximize the profit now or in the future, while the aim of producers-non-innovators is to maximize profit. The above is coherent with Schumpeter's theory. That is why, we distinguish systems from quasi-systems. In economic systems, agents behave rationally: producers maximize profits, and consumers maximize preferences on budget sets. A quasi-production system is a sphere, where innovative activities can be modelled. In a quasi-consumption system, a budget set can be empty or unbounded. However, in case of consumers, we assume that in both: in the consumption system and in the quasi-consumption system, consumers tend to maximise their preferences. In an economic system in equilibrium, economic agents realize their optimal plans of actions, at given prices and economic environments, so they do not have incentives to change their activities. Hence, an economy in equilibrium is in the form of the circular flow.

Time is understood as the discrete variable, $\tau \in \{1, 2, \ldots\}$. Numbers $t_0, t_1, \ldots, t_\tau \in \{0, 1, 2, \ldots\}$, where $t_0 < t_1 < \cdots < t_\tau$, are interpreted as successive points at the time axis. For $t \in \{t_0, t_1, \dots, t_\tau\}$ vector $p(t) \in \mathcal{R}^{\ell_t}$ (see (1)) denotes a price system on the market at time $t$. Let $s \in \{1, \dots, \tau\}$, $t = t_{(s-1)}$ and $t' = t_s$. Consider economy $\epsilon(t)$ (see (3)). Suppose that at time $t$, in the spirit of the creative destruction principle, consumers are not interested in the consumption of at least one commodity manufactured by some producers (a producers' outcome). Denote that commodity by $l_1 (l_1 \in \{1, 2, \dots, \ell_t\})$. Due to the last assumption, all consumers' plans and endowments are contained in subspace

$$V \overset{\text{def}}{=} \left\{ x \in \mathcal{R}^{\ell_t} : \tilde{g}(x) = 0 \right\} \tag{4}$$

where

$$\tilde{g} : \mathcal{R}^{\ell_t} \ni \left( x_1, \dots, x_{\ell_t}, 0, \dots \right) \to x_{l_1} \in \mathbb{R} \tag{5}$$

and consequently

$$\forall a \in A : X^a(t) \subset V \tag{6}$$

Let $p \in \mathcal{R}^{\ell_t}$, $p = p(t)$ or $p \neq p(t)$. Now, we prove the following:

**Theorem 1**. Suppose that in economy $\epsilon(t)$ :

1. for every $k \in B_t$, vector $y^{k*}(t)$ maximizes at price vector $p$ the profit of producer $k$ on set $Y^k(t)$; $y^{k*}(t) = 0$, if $k \notin B_t$,
2. for every $k \in A_t$, vector $x^{k*}(t)$ maximizes the preferences of consumer $k$ at price vector $p$ on budget set $\beta_t^a(p)$; $x^{k*}(t) = 0$, if $k \notin A_t$,
3. $\zeta(t) = \sum_{k \in K_t} x^{k*}(t) - \sum_{k \in K_t} y^{k*}(t) - \omega(t) \neq 0$,
4. $p \circ \zeta(t) = 0$,
5. assumption (6) is satisfied with subspace $V$ of the form (4),

then there exists an economy $\epsilon(t')$, which differs from economy $\epsilon(t)$ only in producers' activities, and in which there is a state of equilibrium at price vector $p\left(t'\right) = p$.

**Proof**. See Appendix.

Assumption 1 and 2 by theorem 1 mean that sets $\eta_t^b(p)$ and $\varphi_t^a(p)$, for every $b \in B_t$ and for every $a \in A_t$, adequately, are not empty. Hence, if $p(t) = p$, then in economy $\epsilon(t)$ considered in theorem 1, structure $P(t)$ is the production system, and structure $C(t)$ is the consumption system. Assumption 4 is the Walras Law.

In theorem 1, we prove that under some assumptions, economic system $\epsilon(t)$ (see (3)) can evolve into economy $\epsilon(t')$ in which there exists equilibrium. Moreover, commodity $l_1$, unwanted by consumers, is eliminated from production processes, i.e. at time $t'$, every production plan has $\ell_1$-th coordinate equals zero. Theorem 1 can be easily generalized for a case when more than one commodity is excluded from the market or when some technologies are eliminated from the producers' processes (see Lipieta, 2015).

In competitive models, agents acquire information on other agent characteristics by observing their activities on the market. Hence, the message of every agent $k \in K_t$ at time $t$ is understood as the triple:

$$m^k(t) \stackrel{\text{def}}{=} (p(t), \check{y}^k(t), \check{x}^k(t)) \tag{7}$$

where

- $\check{x}^k(t) \in X^k(t)$ for $k \in A_t$ and $\check{x}^k(t) = 0 \in \mathcal{R}^{\ell_t}$ for $k \notin A_t$
- $\check{y}^k(t) \in Y^k(t)$ for $k \in B_t$ and $\check{y}^k(t) = 0 \in \mathcal{R}^{\ell_t}$ for $k \notin B_t$

Vector $\check{x}^a(t)$ is a plan of action of consumer $a$ at time $t$, while vector $\check{y}^b(t)$ is producer's $b$ plan of action at time $t$. The set of all messages of the form (7) is contained in set $\mathcal{R}^{\ell_t} \times \mathcal{R}^{\ell_t} \times \mathcal{R}^{\ell_t}$ and is denoted by $M^k(t)$. Vector

$$m(t) \stackrel{\text{def}}{=} \left(m^{k_1}(t), m^{k_2}(t), \dots, m^{\kappa(t)}(t)\right) \in M^{k_1}(t) \times M^{k_2}(t) \times \cdots \times M^{\kappa(t)}(t)$$

is called the message at time $t$. Assume that, for $s = 0, \dots, \tau$,
$M(t_s) \subset M^{k_1}(t_s) \times M^{k_2}(t_s) \times \cdots \times M^{\kappa(t)}\left(t_s\right)$ and $M(t_s) \neq \emptyset$.

The process of exchanging messages is defined as a system of equations of the form:

$$m^k\left(t_s\right) \stackrel{\text{def}}{=} f^k_{t_s}\left(m_{t_{(s-1)}}, e_{t_{(s-1)}}\right) \tag{8}$$

where function

$$f^k_{t_s} : M(t_{(s-1)}) \times E(t_{(s-1)}) \to M^k(t_s)$$

is the agent $k$'s response function at time $t_s$. Message $m^k\left(t_s\right)$ can be interpreted as a reply to agent $k$ to prices $p(t_s)$ at time $t_s$, and it determines $\check{x}^k(t_s)$ or $\check{y}^k(t_s)$ — his plans of action as a consumer and as a producer at time $t_s$. The function

$$f_{t_s} = \left(f^{k_1}_{t_s}, f^{k_2}_{t_s}, \dots\right) : M(t_{(s-1)}) \times E(t_{(s-1)}) \to M(t_s)$$

is called the response function at time $t_s$.

Nonempty set $Z(t)$, for $t = t_0, \dots, t_\tau$, means a set of outcomes at time $t$, while by

$$h_t : M(t) \to Z(t) \tag{9}$$

an outcome function at time $t$ is denoted. Outcome function $h_t$ to every message $m(t) = (m^{k_1}(t), m^{k_2}(t), \dots, m^{\kappa(t)}(t)) \in M(t)$ assigns an allocation which is the result of retrieving and the analysis of message $m(t)$ by economic agents. Function.

$$h = h_{t_0} \times \cdots \times h_{t_\tau}, h(m) = \left(h_{t_0}\left(m\left(t_0\right)\right), \dots, h_{t_\tau}\left(m\left(t_\tau\right)\right)\right),$$

which to sequence of messages $m \stackrel{\text{def}}{=} \left(m\left(t_0\right), \dots, m\left(t_\tau\right)\right)$, where $m(t_s) \in M(t_s)$ for $s = 0, 1, \dots, \tau$, assigns the sequence of outcomes is called the outcome function. Now, we put the following definition:

**Definition 1** (compare to Arrow & Intriligator, 1987, pp. 1441–1442). The sequence

$$(M, f, h)$$

where $M = M_{t_0} \times \cdots \times M_{t_\tau}$, $f = f_{t_1} \times \cdots \times f_{t_\tau}$, $h = h_{t_0} \times \cdots \times h_{t_\tau}$ as well as there exists $s \in \{0, 1, \ldots, \tau - 1\}$ such that $m(t_s) = \cdots = m(t_\tau)$ is called the adjustment process.

Message $m(t_s)$ is called the stationary message.

The point of time $t = t_0$ is called the starting point of the adjustment process, while point $t = t_\tau$ is its ending point. Number $\tau$ is called the number of steps of adjustment process $(M, f, h)$.

Below, we prove, under the assumptions considered in theorem 1, the existence of an adjustment process in the sense of definition 1. Assume that points of time $t = t_0, \ldots, t_\tau$ are determined by the occurrence of any change on the market. Their existence is the consequence of the density of the set of real numbers. Hence, within every time interval $[t_{(s-1)}, t_s)$, $s = 1, \ldots, \tau$, agents' plans of action, characteristics of economic agents and prices of commodities are constant. By the above, every point of time $t_s$ can be identified with time interval $[t_{(s-1)}, t_s)$. Let $\tau \in \{2, 3, \ldots\}$, $t = t_{\tau-1}$, $t' = t_\tau$.

**Theorem 2**. If assumptions of theorem 1 are satisfied as well as condition

$$0 \in Y^{b_1}(t) + \cdots + Y^{b_{n_t}}(t) \tag{10}$$

is fulfilled, then there exists an adjustment process leading to equilibrium in economy $\epsilon(t')$, which differs from economy $\epsilon(t)$ in producers' activities, and in which there is a state of equilibrium at price vector $p(t') = p$.

**Proof**. See Appendix.

It can be noticed that in the adjustment process defined in the proof of theorem 2 (see Appendix) information plays a key role. This is due to the fact that on the basis of the knowledge of the messages sent by agents in a given period, every producer and consumer chooses his plan of action to be realized in the next period. As a result, messages determine the sets of outcomes and that is why they cause diversification of adjustment processes. Additionally, in the process defined in the proof of theorem 2:

- every production set $Y^b(t)$ which is not contain in space $V$ of the form (4) ($Y^b(t) \not\subset V$) evolves into set $Y^b(t')$, which means that technological abilities of producer $b$ also evolved,
- at least one unwanted, and therefore in consumers' opinion worse than others, commodity is eliminated from the market (the result of the creative destruction),
- some firms can be eliminated from the market (if $Y^b(t) \neq \{0\}$ and $Y^b(t') = \{0\}$),
- if $p \neq p(t)$, then producers' maximal profits and consumers' maximal preferences can be changed within that process; especially, if $p(t) = \alpha \cdot p$, for some $\alpha \in (0, 1)$ and the maximal profit of a producer at price system $p(t)$ is positive, then its maximal profit at time $t'$ is greater than at time $t$.

If a competitive economy is in equilibrium, then economic agents do not have motivation to change their plans of action, and the economy is in the form of the circular flow. However, according to Schumpeter (1961), in that form of the economy, mild alterations in the activities of economic agents, which preserve equilibrium, can be feasible. If an innovation (a new product, a technology etc.) is introduced on the market in a following point of time and, as a consequence, equilibrium is destroyed, then another adjustment process is initiated. To model such a situation, we define a composition of adjustment processes.

Let two adjustment processes $\left(M_1, f_1, h_1\right)$ and $\left(M_2, f_2, h_2\right)$ be given. The points of time in which successive changes appear in the economy are denoted by $t_{0_1}, \dots, t_{\tau_1}$, adequate points of time in the second adjustment process — by $t_{0_2}, \dots, t_{\tau_2}$. As earlier, every point of time $t_{s_r}$, for $r = 1, 2$ and $s = 1, \dots, \tau$, is identified with time interval $[t_{(s-1)_r}, t_{s_r})$.

**Definition 2**. If $t_{\tau_1} = t_{0_2}$, $e\left(t_{\tau_1}\right) = e(t_{0_2})$, $m\left(t_{\tau_1}\right) = m(t_{0_2})$, $h_{t_{\tau_1}} = h_{t_{0_2}}$, then sequence $\left(\left(M_1, f_1, h_1\right), \left(M_2, f_2, h_2\right)\right)$ is called the composition of two adjustment processes. The number of steps of the composition $\left(\left(M_1, f_1, h_1\right), \left(M_2, f_2, h_2\right)\right)$ is defined as $\tau_1 + \tau_2$.

In the composition of two adjustment processes, the points of time in which successive changes appear in the economy are denoted by $t_{0_1}, t_{1_1}, t_{2_1}, \dots, t_{\tau_1}, t_{1_2}, t_{2_2}, \dots, t_{\tau_2}$.

On the basis of definition 2, we define the composition of finite or countable number of adjustment processes. Let adjustment processes $\left(M_1, f_1, h_1\right), \dots, \left(M_r, f_r, h_r\right)$, where $r \in \{2, 3, \dots\}$ be given.

**Definition 3**. If, for every $s \in \{2, .., r\}$, pair $\left(\left(M_{s-1}, f_{s-1}, h_{s-1}\right), \left(M_s, f_s, h_s\right)\right)$ is a composition of two adjustment processes, then sequence

- $\left(\left(M_1, f_1, h_1\right), \dots, \left(M_r, f_r, h_r\right)\right)$ is called the composition of $r$ adjustment processes
- $\left(\left(M_1, f_1, h_1\right), \left(M_2, f_2, h_2\right), \dots\right)$ is called a composition of countable number of adjustment processes.

The number of steps of $r$ adjustment processes is defined as $\sum_{s=1}^{r} \tau_s$.

Applying of the concept of composition of adjustment processes enables us to model the activities of economic agents in the competitive economy under study on a bounded or unbounded time interval. If some conditions are fulfilled at a stage of a process of transformation of the analysed economic system, it can reach equilibrium as a consequence of elimination of some harmful or outdated commodities from the market. So, we observe the results of the creative destruction in the model, what is coherent with Schumpeter's theory. Due to the last statement, the composition of adjustment processes can be regarded as evolutionary (Metcalf, 2001, p. 6). On the basis of the above, a process of transformation of a private ownership economy can be identified with composition of adjustment processes. Hence, we introduce a formal definition:

**Definition 4**. Finite or countable composition of adjustment processes is called the transformation process of economy $\epsilon(t_{0_1})$. Economy $\epsilon(t)$ is said to be the transformation of economy $\epsilon(t_{0_1})$, which is shortly denoted by $\epsilon(t_{0_1}) \subset \epsilon(t)$, if there is a transformation processes of economy $\epsilon(t_{0_1})$ in which $t = t_{s_r}$, for some $r \in \{1, 2, \dots\}$ and $s \in \{1, \dots, \tau\}$.

The concept of the transformation process of a private ownership economy will be used in the next section for modelling the Schumpeterian vision of economic development on finite or on infinite time interval.

## Innovation and Imitation Within a Transformation Process of a Private Ownership Economy

In this part of the paper, we concentrate on modelling and analysing changes appearing within a transformation process of an economy defined in the second part of the paper, considered on finite or countable set of time periods $T$ (see Remark 1 and definition 4).

Changes in the economy with regard to a given moment can be observed by comparing plans of action of producers or consumers from the initial economy with the adequate plans of action from a transformation of that economy at the given time. Consider a transformation process of economy $\epsilon(t_{0_1})$ (see (3) and Def. 4). Let $\epsilon(t) \subset \epsilon(t')$, for every $t, t' \in T$, $t < t'$. Now, we put the following definition:

**Definition 5**. It said that there is an innovation in the economy $\epsilon(t^{'})$, if

$$\exists b_0 \in B \exists y^{b_0}\left(t'\right) : \left(\forall t \in T\left(t < t' \Rightarrow \forall b \in B : y^{b_0}\left(t'\right) \notin Y^b(t)\right)\right). \tag{11}$$

A producer $b_0$ satisfying (11) is called the innovator. Any change in activities of producers on the market, which results in occurrence of an innovation, is called an innovative change.

Plan $y^b(t^{'})$ satisfying (11) is called the innovative plan. If, for $b \in B_t$, vector $y^b(t^{'})$ does not satisfy condition (11), then it is called the imitative production plan.

If, for $t, t^{'} \in T, t < t^{'}$

$$\exists b_0 \in B \exists y^{b_0}\left(t^{'}\right) \forall b \in B : y^{b_0}\left(t^{'}\right) \notin Y^b(t), \tag{12}$$

then economy $\epsilon(t^{'})$ is called the innovative transformation of the economy $\epsilon(t)$, which is shortly denoted by $\epsilon(t) \subset_{it} \epsilon(t^{'})$. If condition (12) is not satisfied, then $\epsilon(t^{'})$ is called the imitative transformation of the economy $\epsilon(t)$, which is shortly denoted by $\epsilon(t) \subset_{imt} \epsilon(t^{'})$.

If (12) is satisfied, then we say that an innovative change appear in the economy $\epsilon(t^{'})$ in comparison to economy $\epsilon(t)$. If $\ell_t = \ell_{t'}$, then condition (12) means that there are no new commodities in the economy $\epsilon(t^{'})$ with regard to economy $\epsilon(t)$, and at least one new technology is revealed in the plans of action of producer $b_0$ in economy $\epsilon(t^{'})$ in comparison to economy $\epsilon(t)$. If $\ell_t < \ell_{t'}$, then (12) means that at least one new product and a new technology appear in economy $\epsilon(t^{'})$ with regard to economy $\epsilon(t)$.

The cumulative changes are mild non-innovative changes in the structure of the given economy in the framework of the circular flow. They do not influence the characteristics of economic agents and their economic activities, and they do not imply that economic agents are worse off under the criteria of profits and preferences maximization. Precisely,

**Definition 6**. If $\epsilon(t) \subset \epsilon(t')$ as well as

1. $\ell_t = \ell_{t'}$,
2. $\forall k \in K_t : Y^k(t) \subset Y^k(t')$,
3. $\forall k \in K_t \forall y^k(t) \in Y^k(t) \exists y^k\left(t'\right) \in Y^k(t') : p(t) \circ y^k(t) \leq p(t') \circ y^k(t')$,
4. $\forall k \in K_t : X^k(t) \subset X^k(t')$,
5. $\forall k \in K_t : \omega^k(t) \leq \omega^k\left(t'\right)$,
6. $\forall k \in K_t : \preccurlyeq_t^k \subset \preccurlyeq_{t'}^k$,
7. $\forall k \in K_t : \beta_t^k(p(t)) \subset \beta_{t'}^k(p(t'))$,
8. $\forall k \in K_t \forall x^k(t) \in X^k(t) \exists x^k(t') \in X^k(t') : x^k(t) \in X^k(t') \wedge x^k(t) \preccurlyeq_{t'}^k x^k(t')$,

then it is said that a cumulative change occurs in economy $\epsilon(t')$ compared to economy $\epsilon(t)$.

If conditions 1–8 are satisfied, then economy $\epsilon(t')$ is called the cumulative transformation of economy $\epsilon(t)$, which is shortly denoted by $\epsilon(t) \subset_{ct} \epsilon(t')$.

In a cumulative transformation of an economy, the same commodities as earlier can be produced and consumed (condition 1), producers' and consumers' plans from the initial system are still feasible (conditions 2 and 4), profits and consumers' endowments are not decreased (conditions 3, 5), and budget sets are not less (condition 7). Moreover, the preference relation of every consumer is the extension of the preference relation from the initial consumption system (condition 6), which does not result in worse plans (condition 8). Let $t, t'' \in T$ and $t < t''$. If

$$\forall t' \in T : (t < t' \leq t' \Rightarrow \epsilon(t) \subset_{ct} \epsilon(t')), \tag{13}$$

then we can say that economy is in the form of the circular flow in period $[t, t'']$.

**Definition 7**. If $\epsilon(t) \subset \epsilon(t')$ as well as economy $\epsilon(t')$ is neither innovative nor cumulative transformation of economy $\epsilon(t')$, then economy $\epsilon(t')$ is called the regressive transformation of economy $\epsilon(t)$, which is shortly denoted by $\epsilon(t) \subset_{rt} \epsilon(t')$.

If $\epsilon(t) \subset_{rt} \epsilon(t')$, then condition 1 by definition 6 is satisfied, and every production plan $y^b(t')$, for every $b \in B_t$, is an imitative production plan with respect to economy $\epsilon(t)$. It means that every producer at time $t'$ can realize only production plans of action feasible at a time $t$. Moreover, at least one of conditions 2–8 by definition 6 is not satisfied, which means that only non-innovative changes can be introduced by producers in economy $\epsilon(t')$. To sum up, in the regressive transformation of the economy:

1. set of commodities is not changed, which means that there are no new commodities,
2. only imitative changes in the production sector are feasible,
3. some consumers' plans which were feasible earlier may not be feasible any more,
4. the profits or the consumers' endowments may be decreased,
5. some budget sets may be lesser or empty,
6. a consumer's preference relation may not be the extension of the preference relation from the initial consumption system,
7. the consumers plans of action at time $t'$ may be worse than at time $t$.

The regressive transformation of real economies can be easily recognized. Polish economy in 1989 can be presented as an example. In that time in Poland, there were not enough commodities in shops, the organizational structures were destroyed or transformed, earnings were too low to satisfy the workers, innovative activities practically did not exist etc.

Conditions 4–7 formulated above mean that the position of an economic agent (compare to Lipieta & Malawski 2016a) at time $t'$ is worse than at time $t$. If condition 3 is satisfied, then, among others, some of commodities may not be wished by consumers, or consumers may want lesser amount of a commodity than they did earlier. If those commodities are the output, then producers can limit or finish their production. So, the occurrence of the regressive transformation of the economy should lead to changes of prices of commodities or to changes in activities of economic agents. As a result, the economy evolves, what may lead to introducing innovative changes.

Below, we present an example in which it is shown how to model cumulative, innovative and imitative transformations of an initial economy $\epsilon(t)$. Let $t = t_{(s-1)_r}$, for some $s \in \{1, \dots, \tau\}, r \in \{1, 2, \dots\}$ and $t' = t_{s_r}$.

**Example 1**. Assume that in economy $\epsilon(t)$ the following are satisfied:

1. $p(t) \circ \zeta(t) = 0$,
2. $0 \in Y^{b_1}(t) + \dots + Y^{b_{n_t}}(t)$,
3. for every $k \in B_t$, vector $y^{k*}(t)$ maximizes at price vector $p(t)$ the profit of producer $k$ on set $Y^k(t)$; $y^{k*}(t) = 0$, if $k \notin B_t$,
4. for every $k \in A$, vector $x^{k*}(t)$ maximizes the preferences of consumer $k$ at price vector $p(t)$ on budget set $\beta_t^a(p(t))$; $x^{k*}(t) = 0$, if $k \notin A$.

Let $\zeta(t) = \sum_{k \in K_t} x^{k*}(t) - \sum_{k \in K_t} y^{k*}(t) - \omega(t)$.

We show that under the above assumptions, there is equilibrium in such economy $\epsilon(t')$ in which the consumption system is the same as in economy $\epsilon(t)$, while the production system is a transformation of production system from economy $\epsilon(t)$.

**Solution**. Within economic evolution, in the spirit of the creative destruction principle, obsolete or harmful commodities (e.g. plastic bags or carbon dioxide) are removed from the market. Let those commodities be denoted by $l_1, \dots, l_f (l_1, \dots, l_f \in \{1, 2, \dots, \ell_t\}, l_1 < \dots < l_f, f \in \{1, \dots, \ell_t - 1\})$. Hence, consumption sets satisfy the following:

$$\forall a \in A : X^a(t) \subset V \overset{\text{def}}{=} \bigcap_{l=1}^{f} ker \tilde{g}_l \tag{14}$$

where, for $l = l_1, \dots, l_f$,

$$\tilde{g}_l : \mathcal{R}^{\ell_t} \ni (x_1, \dots, x_{\ell_t}, \dots) \rightarrow x_l \in \mathbb{R}.$$

Under the above assumptions, an adjustment process leading to equilibrium, in such a transformation of economy $\epsilon(t)$ in which commodities $l_1, \dots, l_f$ are eliminated from production processes, is determined.

Two cases are considered.

1. Economy $\epsilon(t)$ is in equilibrium. Then, $\zeta(t) = 0$. In that case, a slight modification of the procedure presented in theorem 2 can be applied.

Let vectors $q^1, \dots, q^f \in \mathbb{R}^{\ell_t}$ be a solution of system of equations:

$$\begin{cases} \tilde{g}^s(q^r) = \delta^{sr} \\ p \circ q^r = 0 \end{cases} \text{for } s, r \in \{1, \dots, f\} \tag{15}$$

and

$$Q(x) = x - \sum_{s=1}^{f} \tilde{g}^s(x) \cdot q^s.$$

Some notes on the existence as well as on the number of solutions of (15), the reader can find in Lipieta (2010), theorem 4.2 and Remark 4.3. The rest of the construction of the adjustment process goes in the same way as in the proof of theorem 2. Namely,

$$f_t^k(m(t), e(t)) = \left(p\left(t'\right), \check{y}^k\left(t'\right), \check{x}^k\left(t'\right)\right)$$

where

$$\check{y}^k\left(t'\right) = Q\left(\check{y}^k(t)\right), \check{x}^k\left(t'\right) = Q\left(\check{x}^k(t)\right) = \check{x}^k(t)$$

$$\omega^k\left(t'\right) = Q\left(\omega^k(t)\right) = \omega^k(t), p\left(t'\right) = p(t).$$

Put $x(t) = \left(x^{k_1}(t), x^{k_2}(t), \dots, x^{k_{\kappa(t)}}(t)\right)$, $y(t) = \left(y^{k_1}(t), y^{k_2}(t), \dots, y^{k_{\kappa(t)}}(t)\right)$. Set

$$Z(t) = \left\{(x(t), y(t)) : \sum_{k \in K_t} x^k(t) - \sum_{k \in K_t} y^k(t) = \sum_{k \in K_t} \omega^k(t)\right\}$$

is not empty since $0 \in Y^{b_1}(t) + \dots + Y^{b_{n_{t_s}}}(t)$. Let $p\left(t'\right) = p(t)$.

In the same way as in theorem 4.2 in Lipieta (2010), we get that set

$$Z\left(t'\right) \overset{\text{def}}{=} \left\{\left(x\left(t'\right), y\left(t'\right)\right) : \left(x\left(t'\right), y\left(t'\right), p\left(t'\right)\right) \text{is the state of equilibrium in economy } \epsilon\left(t'\right)\right\}$$

is not empty. Now, it is enough to define outcome functions $h_t$ and $h_{t'}$ as in the proof of theorem 2.

Let $\epsilon(t')$ be a transformation of economy $\epsilon(t)$, in which for every $k \in K_t, Y^k\left(t'\right) = Q\left(Y^k(t)\right)$ and consumption system is the same as in economy $\epsilon(t)$. By the above, sequence

$$\left(x^*\left(t'\right), y^*\left(t'\right), p\left(t'\right)\right)$$

where, for $k \in K_t, y^{k*}\left(t'\right) = Q\left(y^{k*}(t)\right)$, $x^{k*}\left(t'\right) = x^{k*}(t)$ and $p\left(t'\right) = p(t)$, is the state of equilibrium in economy $\epsilon(t')$.

2. Economy $\epsilon(t)$ is not in equilibrium. Then, $\zeta(t) \neq 0$. Now, one of the procedures presented in Lipieta (2015, pp. 196–202) can be applied. As a result, we get a

state of equilibrium in economy $\epsilon(t')$ — an adequate transformation of economy $\epsilon(t)$, in which commodities $l_1, \dots, l_f$ are eliminated from production processes.

If $p(t) = p$, then the constructed adjustment processes do not change the position of economic agents since the producers' profits and consumer plans maximizing preferences on budget sets in initial economy $\epsilon(t)$ are the same as in its transformation $\epsilon(t')$. Moreover, for economies $\epsilon(t)$ and $\epsilon(t')$, one of the following is satisfied:

1. $Y^b\left(t'\right) \subset Y^b(t)$ for every $b \in B$,
2. $Y^b\left(t'\right) \not\subset Y^b(t)$ for at least one producer $b$.

If (1) is satisfied, then $\epsilon(t) \subset_{imt} \epsilon(t')$; additionally, if $Y^b\left(t'\right) \neq Y^b(t)$ for a producer $b$, then, within the constructed adjustment process, production plans from set $Y^b(t) \backslash Y^b\left(t'\right)$ are not feasible at time $t'$. If (2) is valid, then $\epsilon(t) \subset_{it} \epsilon(t')$; production plans from set $Y^b\left(t'\right) \backslash Y^b(t)$ are innovative at time $t'$ with respect to time $t$. By (1)–(2), economy $\epsilon(t')$ is not the regressive transformation of economy $\epsilon(t)$.

The changes introduced into the production sphere, modelled in the above example, can be considered in the category of eco-innovations, since elimination of harmful or obsolete commodities is mostly friendly for the environment.

The above example also shows how economy $\epsilon(t)$ could evolve in the direction of equilibrium with keeping agents' economic positions. If there are no changes in agents' activities in period $[t, t')$, $p(t) = p$ as well as

- $Y^b(t) \subset Y^b\left(t'\right)$, for every $b \in B$, then conditions 1–8 by definition 6 are satisfied in economy $\epsilon(t')$ defined in example 1; it means that the economy is in the form of the circular flow in period $[t, t']$
- $Y^b\left(t'\right) \not\subset Y^b(t)$, for at least one producer (firm) $b$, then feasible plans of producer $b$ are transformed to the plans from set $Y^b\left(t'\right)$; consequently, the considered economy is in the form of the economic development in period $[t, t']$

At the end, some simple properties of cumulative and innovative transformations on a long period are presented. Consider a finite or countable transformation process of economy $\epsilon(t_{0_1})$. Let $t \in T$, $t', t'' \in T \backslash \{t_{0_1}\}$ and $t < t' < t''$. As immediate consequences of definitions 5 and 6, we get that

**Theorem 3**.

1. If $\epsilon(t) \subset_{ct} \epsilon(t')$ and $\epsilon(t') \subset_{ct} \epsilon(t'')$, then $\epsilon(t) \subset_{ct} \epsilon(t'')$.
2. If $\epsilon(t) \subset_{ct} \epsilon(t')$ and $\epsilon(t') \subset_{it} \epsilon(t'')$, then $\epsilon(t) \subset_{it} \epsilon(t'')$.
3. If $\epsilon(t) \subset_{it} \epsilon(t')$ and $\epsilon(t') \subset_{ct} \epsilon(t'')$, then $\epsilon(t) \subset_{it} \epsilon(t'')$.

Let us have a closer look at the results of theorem 3. If, within time intervals $[t, t')$ and $[t', t'']$, the economy takes the form of the circular flow, then, within time interval $\left[t, t''\right]$ as a whole, the economy also remains in that form (assertion 1). If, within time interval $[t, t')$, an economy takes the form of the circular flow and an innovation is introduced on the market in period $[t', t'']$, then it can be said that within time interval $[t', t'']$ as a whole, the economy is in the form of the economic development (assertion 2). Similarly, assertion 3 can be interpreted. If, in period $[t, t')$ an innovation is introduced on the market and within interval $[t', t'')$ the economy is in the form of the circular flow, then in period $[t, t'']$ as a whole, the economy is in the form of the economic development.

Theorem 3 provides the confirmation of the statements formulated at the end of part 2 of the paper. Nowadays, in developed countries, innovations are introduced on the market more frequently than statistical data are announced. In such cases, it is impossible to distinguish whether in the meantime, i.e. between subsequent announcements of annual data, the economy was in the form of the circular flow. The possibility of the appearance of the economy in the form of the circular flow as the result of the creative destruction, which is coherent with Schumpeter's theory, is proved in the model presented in the current paper (theorems 1 and 2, example 1).

Finally, a level of innovativeness $LI_{\epsilon(t)}$ of the private ownership economy $\epsilon(t)$ is defined. Under the previous assumptions and the notations, we put

$$LI_{\epsilon(t)} \overset{\text{def}}{=} \sum_{b_0 \in B_{in}} \left( \left| y^{b_0}_{\ell_T+1}(t) \right| + \cdots + \left| y^{b_0}_{\ell_T}(t) \right| \right) +$$

$$\sum_{b_0 \in B_{in}} inf \left\{ \| y^{b_0}(t) - y^{b}\left(t'\right) \| \,:\, t' < t, b \in B \right\}$$

where, for $x \in \mathcal{R}^{\ell_t}$, $\|x\| = \sum_{l=1}^{\ell_T} |x_t|$, a set $B_{in}$ means the set of innovators at period $t$ as well as the vector $y^b\left(t'\right)$ is a plan of action of producer $b$ at period $t'$.

In the first components of the number $LI_{\epsilon(t)}$, quantities of innovative commodities at time $t$ are taken into account; in the second, the infimum of distances between the realized at time $t$, plans of innovators and earlier realized plans of actions of all producers. The second sum means the level of technological innovativeness at time $t$. We can see that the level of innovativeness $LI_{\epsilon(t)}$ grows with increasing number of innovators, quantities of innovative goods and growing level of technological innovation at time $t$.

## Discussion

Statistical data on Polish economy confirm the existence of innovative processes. The necessary date is complete and enables to calculate *Bloomberg Innovation Index* (Tables 3 and 4). To analyse the changes in position in the ranking, it is worth to have a look at the ranking in the area of each variable by the use of which the index is created. The progress in the ranking is linked to the progress in the ranking of at least one variable. In case of Polish economy, it is R&D Intensity or Productivity. The best result could be obtained by improving of a country's places in the area of all these variables in which the country's positions are worse than the country's place in *Bloomberg Innovation Index*. If data are not complete, then applying the results of implications of the model, we can say that the increase of the numbers of both, innovators and quantities of innovative goods, and introducing new technologies could lead to the increase of the innovativeness level.

The statistical data show that Polish economy does not belong to the group of top-innovative European countries. Polish economy, however, has the properties pointed by Schumpeter as the features of the economic development what prevent Polish economy from moving toward a regressive transformation. They are the existence of innovative goods and commodities (Figs. 1 and 2) as well as appearance of new firms on the market (Table 2) at every period under study with the simultaneous disappearance of old obsolete goods and making-loss firms. The first observation means that, at every analysed period, there were producers whose plans of actions were innovative, which next mean the appearance of innovative processes. The second observation shows the features of the creative destruction. The processes moving Polish economy to its regressive transformation have also not been reflected in statistical data. This is probably due to the fact that there are many difficulties and barriers in taking statistical data on innovativeness which means that data often are incomplete. Moreover, statistical data on innovativeness in most countries are announced not more than once a year. Consequently, the analysed intervals are too long to discover the existence of short-life processes typical for the circular flow. That means in innovative countries, the circular flow (if appears) cannot be long-life.

The possibility of distinguishing and the ability to implement the processes that make the living conditions of members of a society remain at the same level is important. Especially, if some external or internal aspects, such as the COVID-19 pandemic, cause that many firms will have to fight to survive, and many people may become unemployed. Those can decline in innovation and the economic downturn. Knowledge about what non-innovative changes are worth to introduce to not decrease the supply and profits could help the firms endangered by a possible collapse to stay on the market. On the other hand, adversities or disasters may generate the emergence of new businesses and innovative activities of some firms resulting in development of new solutions: medicines and vaccines, which is consistent with Schumpeter's theory.

In the model presented, the economic agents take the decisions on their market activities taking into account their own feasibilities and the messages "provided" by other agents. Knowledge about agents' activities on the market now is the basis for determining the agents' choices in the future, especially the decision on the choice of innovative or imitative activity.

It is the role and aim of innovative processes to make the economic positions of members of societies better. It is feasible due to introducing new cheap technologies and innovative goods. It should be noted that, according to Schumpeter (1912, 1964), within some stages of economic evolution, innovative processes may cause the profits of some firms to be decreased. Consequently, the living conditions of some consumers also can degrade. The model presented in the paper shows that, in many cases, the circular flow could be obtained in any point of many economic processes, without making the position of economic agents' worse. What is more, such processes could be resulted in changes friendly to the environment and human beings. In some cases, those processes may lead to increase the profits and utilities.

However, the importance of the demand side of the economy within the economic development should not be neglected. The consumers in many cases may cause the innovative activities of producers. The examples are the processes which result in eco-changes. Eco-innovation can be analysed in the model presented, and it is incorporated in the model directed to improve sustainability. From this point of view, the model under study could be seen as a micro-foundation for an analysis of sustainable development.

The drivers of adjustment processes are not specified *explicitly* in the paper. The analysis of the model confirms that the supply side of the economy determines the economic development and consequently the economic growth. The role of innovators-entrepreneurs is to introduce changes into the economy and move the economy to the higher level of innovativeness. It is precisely the producers recognize initial conditions and they or an unspecified in the model decision maker can take decision on which changes are worth to be introduced. This is the role of innovators-entrepreneurs, to initiate the economic development by introducing innovations. The drivers of innovation process are innovators who meet consumers' needs and expectations introducing changes in their own, whereas a number of innovators, quantities of introduced innovations and the level of technological innovativeness are the determinants of innovation process.

However, some adjustment processes could be viewed as demand-driven, especially, if consumers indicate, by their market activities, the unwanted goods as well as the commodities manufactured by the use of technologies which are not acceptable for consumers. Similarly, imitators are responsible for imitative adjustment processes, namely, above all, diffusion of innovations and moving the economy toward a form of the circular flow. A regressive form of the economy appears when the creative destruction is reduced to the destruction: there is no innovation, and activities of imitators are reduced. The "drivers" of processes which lead to a regressive form of the economy are firms-imitators which cannot operate in the economy in a form of the circular flow. Therefore, in real economies, innovative activities should be supported, among others by governments, in order to prevent the economy from taking a regressive transformation.

To improve the level of innovativeness of the economy as well as prevent the economy from moving toward a regressive form, all activities (of agents and organisations) which lead to the increase of a number of innovators and competition between them should be amplified. Consequently, the development of selected economic sectors and specificity for a country are worth to be supported. In Polish economy, especially in a the context on COVID-19 pandemic, they are health care, developments in science and education leading to introducing cheap technologies, information technology, biotechnology and the computer games industry. The activity of the decision-makers should lead to introducing friendly legislation and effective financing. The activities leading to introducing eco-changes should not be neglected. They can improve the environment and, consequently, the living conditions, in some cases, without decreasing the agents' profits and utilities. Generally, innovation, due to its specificity, is associated with a high risk and often high costs not only of development research but also of entering the market. For this reason, it is important to develop and effectively implement a support system that would encourage companies and entrepreneurs to invest more in R&D.

## Conclusions

Studying the Schumpeter's researches in detail, a fundamental question arises: has an economy ever taken the form of the circular flow? We have not found a satisfactory answer to such a question in the referred papers belonging to the neo-Schumpeterian research program or the economic growth literature, discussed in the second section. These studies are focussed, above all, on the examination of innovative processes and their outcomes. The analysis based on the 2007–2018 data of chosen as example Polish economy, presented in the third section, has failed to provide any answer. Polish economy did not display, at analysed periods, the features that would be characteristic for the circular flow, although that economy in the analysed years did not belong to the group of the top innovative countries. Then, in search for the circular flow and processes leading to the circular flow, we focused on economic modelling. It should not be surprising that in modelling of the economic evolution in which Schumpeter's premises would be taken into account, we applied the tools usually used in theory of general equilibrium since the economy in equilibrium has all the properties pointed by Schumpeter as the features of an economy being in a form of the circular flow. As a result, we presented the model of economic evolution in which, firstly, innovative processes could be initiated at every period, irrespective of whether the economy was in a form of the circular flow or not, which appeared to be confirmed in Polish economy; secondly, under natural assumptions, the economic system could take a form of the circular flow, the existence of which was essential for Schumpeter theory.

In many evolutionary models labelled as Schumpeterian (e.g. Almudi et al., 2019a, b; Assenza et al., 2015; Dawid et al., 2019; Dosi et al., 2010), the role of economic statics within economic evolution is reduced to minimum. In contrast to many evolutionary studies, in the current paper, the role of economic static is not neglected. In the model considered, the economic development is initiated in the economy in a form of the circular flow, and, under some natural assumptions interpreted in economic theory, the economy could take a form of the circular flow in any period. Therefore, the current paper extends the research program of modelling the Schumpeterian vision of economics development.

In the model presented, due to the rigorous mathematical approach, besides the circular flow and the economic development, the third form of the economy, called in the paper a regressive transformation of the economy, is revealed. A regressive form of the economy can occur within a long or a short period and is a logical consequence of the definitions of the components of the model of economic evolution. That form of the economy could be seen in many countries, among others, in Poland in 1980s and in USA in 2020 as result of COVID-19 pandemic. Additionally, we noticed that the circular flow and the economic development did not complement each other in the model under consideration. As a consequence, we distinguished the third form which could be taken by the economy.

In the approach presented, evolving in time, economic systems in which agents' behaviour go beyond the perfect rationality framework are modelled. Using the classical topological apparatus coming from general equilibrium theory provides us the possibility of uniform description of equilibrated and disequilibrated processes. Generally, equilibrium is not the object of the study of growth models, and there is no long-run equilibrium model in theory of economic growth (see also Foster, 2011). As a consequence, there are a few numbers of papers analysing processes directing to the economy in the form of a circular flow. In the current paper, we modelled some processes in which entrepreneurs-innovators introducing a special kind of technological change would play the leading roles in processes leading to equilibrium (theorems 1 and 2, example 1). By the above, we have shown that some innovative processes resulting in a technological innovation could move to equilibrium in the framework of the sustainable development. Therefore, the model presented enables the analysis of both, economic dynamics and economic statics. The results we presented are new and could be useful in the economic theorizing to understand the nature of economic processes, especially if the access to empirical data is impossible or limited.

Although the model presented is a micro-economic model while most growth models are macro-economic ones, from a general theoretical view, all of them are the models of global general interactions within economic systems as a whole. Hence, from the meta-theoretical background, all the models are comparable.

Theoretical analysis based on mathematical models supported by Schumpeter's theory and the analysis of statistical data brought us to deeper exploring the nature of evolutionary processes in economics. Applying Hurwiczian apparatus revealed the significant role of signals sent and analysed by the economic agents in the processes of obtaining the desired goals and enabled us to diversify the processes within economic evolution.

In the model, the composition of a finite or infinite number of adjustment processes on a finite or infinite time interval adequately can be analysed. Due to that, the processes of moving the economy from the circular flow toward the economic development and vice versa within any long time period can be analysed. The presented results and the precise definitions delivered us the tools and the arguments for formulation relationships between Schumpeterian circular flow and the economic development.

During economic evolution, to every point of time, the economy in the form of the circular flow is assigned. Hence, the circular flow can be identified with the set of values of economic evolution but also with the domain of the economic development.

On the basis of the above set-up, from a broader perspective on evolution of the economy modelled in Hurwiczian approach, in a sufficiently long time interval, the process of change in an economy can be innovative or regressive, while in a short time interval, it can be innovative cumulative or regressive. The diversity of innovative and imitative changes introduced on the market led us to the specification od various kinds of adjustment processes within the evolution of the economy. In the approach presented, this economic structure can be seen as a component of the overriding structure—the transformation process of a private ownership economy. With the use of the criteria formulated in Lipieta and Malawski (2018), the qualitative properties of the above processes can be analysed.

In the considered model of economic evolution, at every point of time in the future, as the result of the creative destruction, the economy could achieve equilibrium. However, as a result of data analysis, we know that the circular flow in middle and highly developed economies could appear only in a short period of time, and innovative transformations of middle or highly developed economies are observed over a long period of time.

The innovative activities of entrepreneurs often cause the increase in wealth of some consumers as well as the increase in wealth of the whole economy. Hence, innovative activities should be supported by the coordinators of the economic life. On the basis of the above, the designing of innovative processes seems to be worth further elaborating. The study on characterization of adjustment processes which result in short- or long-life equilibrium also remains under our research perspectives.

The presented results do not aspire to the role of the best or most effective. They should be rather regarded as an attempt to adapt new concepts and methods for exploring the existing problems in order to more fully understand the phenomenon of economic evolution.

## Appendix. Remaining proofs

**Proof of theorem 1**. By assumptions 1, 2 and 3 sequence

$$(x^*(t), y^*(t), p(t))$$

is not the state of equilibrium in economy $\epsilon(t)$ and $\zeta_{l_1}(t) \neq 0$. Put $\psi = \frac{1}{\zeta_{l_1}(t)} \cdot \tilde{g}$. Then

$$\psi(\zeta) = 1$$

as well as the mapping $Q : \mathcal{R}^{\ell_t} \to V$,

$$Q(x) = x - \psi(x) \cdot \zeta \tag{16}$$

is the linear and continuous projection on subspace $V$ of the form (4) (see Cheney, 1966) satisfying

$$Q(\zeta) = 0.$$

For $k \in K_t$:

$$y^k(t') \overset{\text{def}}{=} Q(y^k(t)), x^k(t') \overset{\text{def}}{=} Q(x^k(t)) = x^k(t),$$

$$\omega^k(t') \overset{\text{def}}{=} Q(\omega^k(t)) = \omega^k(t), p(t') \overset{\text{def}}{=} p$$

$$Y^k(t') = Q(Y^k(t)), X^k(t') = Q(X^k(t)) = X^k(t).$$

Now, we immediately obtain that:

- $y^{k*}(t')$ maximizes at price $p$ the profit of producer $k$ on set $Y^k\left(t'\right)$ if $k \in B_t$; $y^{k*}\left(t'\right) = 0$, if $k \notin B_t$,
- $x^{k*}(t')$ maximizes at price $p$ the preferences of consumer $k$, if $k \in A_t$, on budget set

$$\beta^a_{t'}\left(p\left(t'\right)\right);x^{k*}\left(t'\right) = 0, \text{if } k \notin A_t,$$

- $\sum_{k\in K_t} x^{k*}\left(t'\right) - \sum_{k\in K_t} y^{k*}\left(t'\right) = \omega\left(t'\right).$

The above conditions mean that sequence

$$\left(x^*\left(t'\right), y^*\left(t'\right), p\left(t'\right)\right)$$

is a state of equilibrium in economy $\epsilon(t')$.

**Proof of theorem 2**. Let us notice that under assumption (10) set

$$Z(t) \overset{\text{def}}{=} \left\{(x(t), y(t)) : \sum_{k\in K_t} x^k(t) - \sum_{k\in K_t} y^k(t) = \omega(t)\right\}$$

is not empty. Put $p\left(t'\right) = p$. By theorem 1 set

$$Z(t') \overset{\text{def}}{=} \left\{ \begin{array}{c} \left(x^*\left(t'\right), y^*\left(t'\right)\right) : \left(x^*\left(t'\right), y^*\left(t'\right), p\left(t'\right)\right) \\ \text{is the state of equilibrium in economy } \epsilon(t') \end{array} \right\}$$

is also not empty. Let

$$f^k_{t'}(m(t), e(t)) = \left(p\left(t'\right), \check{y}^k\left(t'\right), \check{x}^k\left(t'\right)\right),$$

(see (7) and (8)) where

$$\check{y}^k\left(t'\right) = Q\left(\check{y}^k(t)\right), \check{y}^k\left(t'\right) = Q\left(\check{x}^k(t)\right) = x^k(t),$$

$$\omega^k\left(t'\right) = Q\left(\omega^k(t)\right) = \omega^k(t), p\left(t'\right) = p,$$

where $Q$ is the mapping of the form (16). By theorem 1, we get that

- $\check{y}^{k*}(t')$ maximizes at price $p$ the profit of producer $k$ on set $Y^k\left(t'\right) = Q\left(Y^k(t)\right)$, if $k \in B_t$;$\check{y}^{k*}\left(t'\right) = 0$, if $k \notin B_t$
- $\check{x}^{k*}\left(t'\right)$ maximizes at price $p$ the preferences of consumer $k$ on budget set $\beta^a_{t'}(p)$, if $k \in A_t$;$\check{x}^{k*}\left(t'\right) = 0$, if $k \notin A_t$,:
- $\sum_{k\in K_t} \check{x}^{k*}\left(t'\right) - \sum_{k\in K_t} \check{y}^{k*}\left(t'\right) = \omega(t'),$

which means that there is a state of equilibrium in economy $\varepsilon$ ($t'$). Now, it is enough to define the rest of components of the adjustment process. We put (see (9))

$$h_t\left(m^{k_1}(t), m^{k_2}(t), \dots\right) = \left(\left(x^{k_1}(t), x^{k_2}(t), \dots, x^{k_{\kappa(t)}}(t)\right), \left(y^{k_1}(t), y^{k_2}(t), \dots, y^{k_{\kappa(t)}}(t)\right)\right),$$

$$h_{t'} = h_t$$

Components $M_{t''}$, $h_{t''}$, $f^k_{t''}$, for every $k \in K$ and $t'' \in \{t_0, \ldots, t_{\tau-2}\}$, can be of any form which end the proof.

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## Declarations

**Competing Interests** The authors declare no competing interests.

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