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ORIGINAL PAPER

Economic analysis of the N-1 reliable unit commitment and transmission switching problem using duality concepts

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Abstract Currently, there is a national push for a smarter electric grid, one that is more controllable and flexible. Only limited control and flexibility of electric assets is currently built into electric network optimization models. Optimal transmission switching is a low cost way to leverage grid controllability: to make better use of the existing system and meet growing demand with existing infrastructure. Such control and flexibility can be categorized as a "smart grid application" where there is a co-optimization of both generators or loads and transmission topology. In this paper we form the dual problem and examine the multi-period N-1 reliable unit commitment and transmission switching problem with integer variables fixed to their optimal values. Results including LMPs and marginal cost distributions are presented for the IEEE RTS 96 test problem. The applications of this analysis in improving the efficiency of ISO and RTO markets are discussed.

Keywords Duality · Generation unit commitment · Mixed integer programming · Power generation dispatch · Power system economics · Power system reliability · Power transmission control · Power transmission economics

Nomenclature

Indices

c:	Operating state; $c = 0$ indicates the no contingency state (steady-state); $c > 0$ is a single contingency state, i.e. $c \in C$, $C = CT \cup CG$.
e:	Generator, load, or transmission element.

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g: Generator or load; for generators, $g \in G$; for load, $g \in D$.
$g(n)$: Set of generators or load at node n.
k: Transmission element (line or transformer).
$k(n, .)$: Set of transmission assets with n as the 'from' node.
$k(., n)$: Set of transmission assets with n as the 'to' node.
m, n: Nodes.
$n(g)$: Generator g located at bus n.
t: Time period; $t = 1, \dots, T$.

Parameters
B_k: Electrical susceptance of transmission element k.
c_g: Production cost for generator (or value of load) g; generally $c_g > 0$.
CG: Set of generator contingencies.
cr_g^+, cr_g^-: Ramp rate cost in the up and down direction for generator (or load) g.
CT: Set of transmission contingencies.
d_n: Real power load (fixed) at bus n.
$N1_{ec}$: Binary parameter that is 0 when element e is the contingency and is 1 otherwise.
P_{gc}^+, P_{gc}^-: Max and min capacity of generator (or load) g in state c; for $g \in D$, $P_{gc}^+ = P_{gc}^-$.
P_{kc}^+, P_{kc}^-: Max and min rating of transmission element k in state c; for lines $P_{kc}^+ = -P_{kc}^-$.
R_{gt}^+, R_{gt}^-: Max ramp rate in the up and down direction for generator (or load) g at node n in period t except in the startup period.
R_{gt}^{c+}, R_{gt}^{c-}: Max emergency (contingency) ramp rate in the up and down direction for generator (or load) g in period t.
R_g^s: Max ramp rate for the start up period in the up direction for generator (or load) g at node n.
SU_{gt}: Startup cost for generator (or load) g in period t; generally for $g \in G$, $SU_{gt} \geq 0$.
T: Number of periods.
UT_g, DT_g: Min up and down time for generator (or load) g.
θ^+, θ^-: Max and min voltage angle; $\theta^+ = -\theta^-$.
ρ_c: Contingency c indicator; $\rho_c = 1$ for $c = 0$; $\rho_c = 0$, otherwise.

Variables
P_{gct}: Real power supply from generator $g(>0)$ or demand from load $(<0)g$ (at node n), in state c and period t.
P_{kct}: Real power flow from node n to m for transmission element k, in state c and period t.
r_{gct}^+, r_{gct}^-: Ramp rate in the up and down direction for generator g, in state c and period t.
u_{gt}: Binary unit commitment variable for generator (or load) g in period t (0 down, 1 operational).
v_{gt}: Startup variable for generator (or load) g in period t (1 for startup, 0 otherwise).

w_{gt}: Shutdown variable for generator (or load) g in period t (1 for shutdown, 0 otherwise).
z_{kt}: Binary variable for transmission element k in period t (0 open/not in service, 1 closed/in service).
$\alpha_{nct}^{-}, \alpha_{nct}^{+}$: Marginal value of lowering (raising) the min (max) phase angle at node n, in state c and period t.
$\beta_{gct}^{-}, \beta_{gct}^{+}$: Marginal value of reducing (increasing) the min (max) level of generator (or load) g, in state c and period t.
γ_g: Uplift or additional profit for generator g.
δ_{kt}: Marginal value of switching transmission element k in period t.
$\eta_{kct}^{-}, \eta_{kct}^{+}$: Marginal value of reducing (increasing) the lower (upper) limit for transmission element k, in state c and period t.
θ_{nct}: Bus voltage angle at node n, in state c and period t.
λ_{nct}: Marginal value of a unit of generation or load at node n, in state c and period t.
μ_{kct}: Marginal susceptance value of transmission element k, in state c and period t.
σ_{gt}: Marginal value of enforcing the startup value for generator g in period t.
τ_{gt}: Marginal value of enforcing the relationship between startup, shutdown, and unit commitment variables for generator g in period t.
$\chi_{gct}^{+}, \chi_{gct}^{-}$: Marginal value of increasing the up (down) ramp rate for generator g, in state c and period t.
$\chi_{gct}^{c+}, \chi_{gct}^{c-}$: Marginal value of increasing the up (down) emergency ramp rate for generator g, in state c and period t.
ψ_{gt}: Marginal value of enforcing the shutdown value for generator g in period t.
$\omega_{gct}^{+}, \omega_{gct}^{-}$: Marginal value of increasing the up (down) ramp rate constraint for generator g, in state c and period t.

1 Introduction

There is a national goal to create a "smarter" electric grid, one that is more controllable and flexible. Currently, only limited approximations of electric asset control and flexibility are built into electric power system optimization models. System operators can and do change the network topology to improve system performance. Operators switch transmission elements to improve voltage profiles or increase transfer capacity. In PJM and other ISOs, Special Protection Schemes (SPSs) allow the operator to disconnect a line during normal operations but return it to service during a contingency; there are also SPSs that allow the operator to open a line during a contingency demonstrating that further grid modifications during a contingency can be beneficial. The operator makes these decisions under a set of prescribed rules rather than including this flexibility in the optimization formulation.

Transmission switching has been explored in the literature as a control method for problems such as over or under voltage situations, line overloading, loss and/or cost reduction, and system security [1–12]. Recent research suggests that the network topology should be co-optimized along with the generation [13–17].

New transmission infrastructure can be expensive and hard to site. Therefore, optimal use of the existing system and optimal expansion should be a priority. The US Energy Policy Act of 2005 includes a directive for FERC to "encourage...deployment of advanced transmission technologies," including "optimized transmission line configuration."[1] This research is also in line with FERC Order 890—to improve the economic operations of the electric transmission grid. It also addresses the items listed in Title 13 "Smart Grid" of the Energy Independence and Security Act of 2007: (1) "increased use of... controls technology to improve reliability, stability, and efficiency of the grid" and (2) "dynamic optimization of grid operations and resources."

In this paper we form a dual problem of the multi-period N-1 reliable unit commitment and transmission switching problem, with the integer variables fixed to their optimal values, similar to the one in [17]. Next, we examine the economic concepts and relationships that appear in the dual. From a mathematical point of view, the dual MIP problem is not well-defined and difficult to economically interpret [18, 19]. The cuts that form the convex hull have defied general economic interpretation [20].

When multiple market participants benefit from a unit of traded good, as is true for generator startup or transmission switching, it can be called a club good [21]. A public good is a club good where all market participants benefit; therefore, we need and will only address club goods in this paper. A club good may require a two-part pricing system in order to meet non-confiscatory requirements while a private good requires only a single-part pricing system in markets without non-convexities. A private good can therefore be thought of as a degenerate club good. Binary variables represent club goods.

In this paper, we first present the multi-period unit commitment and transmission switching formulation with N-1 contingency constraints enforced; this formulation is a Mixed Integer Program (MIP). Section 3 then takes the MIP in Sect. 2, fixes the integer variables to their optimal solution values, and presents the LP formulation and its dual. Section 4 then presents the economic analysis of this problem. Section 5 identifies the terms representing the short term generation rent, the congestion rent, and the load payment. Section 6 provides computational results based on the IEEE 73-bus model (RTS96); within this section, we examine the impact on LMPs, generators, and transmission. Section 7 concludes this paper.

2 N-1 DCOPF optimal transmission switching unit commitment multi-period formulation

We modify the formulation in [17] to make it more general and to allow a more intuitive economic interpretation. Load has comparable bidding parameters to generation. In essence it is the mirror image of generation. For example, load can bid the value of consumption in a single period or can bid a single value for an entire eight hour shift using minimum run parameters. Our model is similar to that of ISOs. The motive is to survive any single contingency, but we do not include the contingency

[1] See Sect.1223.a.5 of the US Energy Policy Act of 2005.

redispatch costs in the objective. The model ensures that the system can survive any single contingency by explicitly enforcing N-1. The market model, a MIP, is:

N-1 MIP (NM1MIP):

$$\text{MS} = \text{Maximize} \quad \sum_{g,t}(-c_g P_{g0t} - SU_{gt} v_{gt} - cr_g^+ r_{g0t}^+ - cr_g^- r_{g0t}^-) \tag{1}$$

s.t.

$$\theta^- \leq \theta_{nct} \leq \theta^+ \quad \forall n, c, t \tag{2}$$

$$\sum_{k(.,n)} P_{kct} - \sum_{k(n,.)} P_{kct} + \sum_{g(n)} P_{g0t} = 0 \quad \forall n, c \in 0 \cup CT, t \tag{3a}$$

$$\sum_{k(.,n)} P_{kct} - \sum_{k(n,.)} P_{kct} + \sum_{g(n)} P_{gct} = 0 \quad \forall n, c \in CG, t \tag{3b}$$

$$P_{kct} - P_{kc}^+ N1_{kc} z_{kt} \leq 0 \quad \forall k, c, t \tag{4a}$$

$$-P_{kct} + P_{kc}^- N1_{kc} z_{kt} \leq 0 \quad \forall k, c, t \tag{4b}$$

$$B_k(\theta_{nct} - \theta_{mct}) + P_{kct} + M_k(2 - z_{kt} - N1_{kc}) \geq 0 \quad \forall k, c, t \tag{5a}$$

$$B_k(\theta_{nct} - \theta_{mct}) + P_{kct} - M_k(2 - z_{kt} - N1_{kc}) \leq 0 \quad \forall k, c, t \tag{5b}$$

$$z_{kt} \in \{0, 1\} \quad \forall k, t \tag{6}$$

$$P_{gct} - P_{gc}^+ N1_{gc} u_{gt} \leq 0 \quad \forall g, c \in 0 \cup CG, t \tag{7a}$$

$$-P_{gct} + P_{gc}^- N1_{gc} u_{gt} \leq 0 \quad \forall g, c \in 0 \cup CG, t \tag{7b}$$

$$P_{gct} - P_{gc,t-1} \leq r_{gct}^+ \quad \forall g, c \in 0 \cup CG, t \tag{8a}$$

$$r_{gct}^+ - R_{gct}^+ u_{g,t-1} - R_g^s v_{gt} \leq 0 \quad \forall g, c \in 0 \cup CG, t \tag{8b}$$

$$P_{gc,t-1} - P_{gct} \leq r_{gct}^- \quad \forall g, c \in 0 \cup CG, t \tag{9a}$$

$$r_{gct}^- - R_{gct}^- u_{g,t-1} \leq 0 \quad \forall g, c \in 0 \cup CG, t \tag{9b}$$

$$P_{gct} - P_{g0t} - u_{gt} R_{gt}^{c+} \leq 0 \quad \forall g, c \in CG, t \tag{10a}$$

$$N1_{gc} P_{g0t} - N1_{gc} P_{gct} - u_{gt} R_{gt}^{c-} \leq 0 \quad \forall g, c \in CG, t \tag{10b}$$

$$v_{gt} - w_{gt} = u_{gt} - u_{g,t-1} \quad \forall g, t \tag{11}$$

$$-u_{gt} + \sum_{q=t-UTg+1}^{t} v_{gq} \leq 0 \quad \forall g, t \in \{UT_g, \ldots, T\} \tag{12}$$

$$u_{gt} + \sum_{q=t-DTg+1}^{t} w_{gq} \leq 1 \quad \forall g, t \in \{DT_g, \ldots, T\} \tag{13}$$

$$0 \leq v_{gt} \leq 1 \quad \forall g, t \tag{14}$$

$$0 \leq w_{gt} \leq 1 \quad \forall g, t \tag{15}$$

$$u_{gt} \in \{0, 1\} \quad \forall g, t \tag{16}$$

$$r_{gct}^{+}, r_{gct}^{-} \geq 0 \quad \forall g, c, t. \tag{17}$$

Each decision variable is indexed for each state c, except for z_{kt} and the variables associated with the unit commitment formulation: u_{gt}, v_{gt}, and w_{gt}. State $c = 0$ represents the no-contingency, steady-state variables and constraints whereas all other states represent single generator or (non radial) transmission contingencies. We introduce a binary parameter for state c and element e: $N1_{ec}$. $N1_{kc} = 0$ represents the loss of transmission element k; $N1_{gc} = 0$ represents the loss of generator g. For $c = 0$, $N1_{e0} = 1$ for all e as this state reflects steady-state operations. There are N (transmission element or generator) contingencies. For $c > 0$, $N1_{ec} = 0$ if $c = e$; otherwise, $N1_{ec} = 1$. For each $c > 0$, $\sum_e N1_{ec} = N - 1$. For each e, $\sum_{c>0} N1_{ec} = N - 1$. For $c = 0$, $N1_{ec} = 1$.

Transmission switching is incorporated into the traditional Direct Current Optimal Power Flow (DCOPF) problem by modifying (4a)–(5b) to allow a line to be in service (closed), i.e. $z_{kt} = 1$, or out of service (open), i.e. $z_{kt} = 0$. Constraints (5a) and (5b) ensure that if a transmission element is opened, these constraints are satisfied no matter what the values are for the corresponding bus angles. The transmission element is considered opened if it is the contingency, i.e. $N1_{kc} = 0$, or it is chosen to be opened as a result of transmission switching, i.e. $z_{kt} = 0$. Further discussion on transmission switching can be found in [14–17].

Reserve constraints, such as spinning and non-spinning reserve constraints, are typically included in unit commitment models. The purpose of these reserve constraints within the unit commitment formulation is to ensure there is enough available capacity in order to survive any single contingency; once there is a contingency, spinning and non-spinning reserve are called upon so that the system can survive the contingency. These constraints are used as proxies to enforce N-1 since it is typically too computationally challenging to explicitly list every contingency. Reserve constraints are therefore a surrogate way to enforce N-1 reliability requirements whereas the ideal formulation would explicitly enforce the required contingency constraints within the program. Since this formulation enforces all N-1 contingency constraints explicitly, we do not include these proxy constraints, i.e. reserve constraints.

3 The chosen linear program and its dual

The set of feasible solutions to a MIP is disjoint; however, for a fixed set of binary variables, the resulting feasible solution set is either empty or convex. By setting the integer variables to their values in the best solution found, the resulting problem is a linear program and the resulting dual is well defined and has value for economic analysis. The linear program is optimal with respect to the fixed integer variables and the LP, its dual, and the MIP objective function have the same optimal value. If the MIP is solved to optimality, the integer variables can be fixed to their solution values thereby creating an LP and the resulting dual variables then form an economic equilibrium [22].

For the purpose of economic analysis, MIP duality can be considered in two parts. First, it is possible to determine the incremental value of a binary variable. With the MIP optimal solution determined, all binary variables, except one binary variable, are fixed to their optimal solution value; this single binary variable that was not fixed to its optimal value is instead fixed to its opposite, i.e. 0 to 1 or 1 to 0, value. The change in the objective function gives the incremental (switched) value of that binary variable. The change in market surplus for each asset can therefore be calculated. For complete analysis, all combinations of binary variables need to be considered, but this is practically impossible. For practical reasons we need to perform analysis that is cost beneficial. Second, for linear programs marginal information is almost free in the dual program. Consequently, duality analysis of the linear program can be helpful in marginal analysis and in choosing the binary variables for the incremental analysis.

Once the integer values are fixed, some constraints become redundant, thereby creating many choices for formulating the linear program and its dual. We will choose one that yields an intuitive economic interpretation. The following analysis holds even if the feasible MIP solution is not a global optimum.

If $N1_{kc} = 0$ or $z_{kt} = 0$, (5a) and (5b) are not binding constraints, their dual variables equal zero, and we can drop them from the formulation. If $N1_{kc} = 1$ and $z_{kt} = 1$, $2 - z_{kt} - N1_{kc} = 0$; (5a) and (5b) are then binding and together they form an equality constraint. We replace (5a) and (5b) with (5); also note that variable P_{kct} does not exist within the formulation for $z^*_{kt} = 0$ or $N1_{kc} = 0$.

The solution of NM1MIP determines a set of optimal startup and shutdown variables, v^*_{gt} and w^*_{gt}; we enforce these values in the following linear program. Given an initial value for the unit commitment variable, u_{g0}, and recursively starting at $t = 1$, using (11): $u_{g,t} = v_{gt} - w_{gt} + u_{g,t-1}$ and with $u_{g,t-1} = u^*_{g,t-1}$, $v_{gt} = v^*_{gt}$, and $w_{gt} = w^*_{gt}$, $u^*_{g,t}$ is then uniquely determined. Therefore, (12)–(16) can be discarded and replaced by (14′) and (15′). The resulting linear program with its corresponding dual variables is:

$$\text{LP: MSLP} = \text{Maximize} \sum_{g,t} (-c_g P_{g0t} - SU_g v_{gt} - cr_g^+ r_{g0t}^+ - cr_g^- r_{g0t}^-) \tag{1}$$

s.t.

$$\theta_{nct} \le \theta^+ \quad \forall n, c, t \quad \alpha^+_{nct} \tag{2a}$$

$$-\theta_{nct} \le \theta^+ \quad \forall n, c, t \quad \alpha^-_{nct} \tag{2b}$$

$$\sum_{k(.,n)} P_{kct} - \sum_{k(n,.)} P_{kct} + \sum_{g(n)} P_{g0t} = 0 \quad \forall n, c \in 0 \cup CT, t \quad \lambda_{nct} \tag{3a}$$

$$\sum_{k(.,n)} P_{kct} - \sum_{k(n,.)} P_{kct} + \sum_{g(n)} P_{gct} = 0 \quad \forall n, c \in CG, t \quad \lambda_{nct} \tag{3b}$$

$$P_{kct} - P^+_{kc} z_{kt} N1_{kc} \le 0 \quad \forall k, c, t, z^*_{kt} = 1, N1_{kc} = 1 \quad \eta^+_{kct} \tag{4a′}$$

$$-P_{kct} + P_{kc}^{-} z_{kt} N1_{kc} \leq 0 \quad \forall k, c, t, z_{kt}^{*} = 1, N1_{kc} = 1 \quad \eta_{kct}^{-} \tag{4b$'$}$$

$$B_k(\theta_{nct} - \theta_{mct}) + P_{kct} = 0 \quad \forall k, c, t, z_{kt}^{*} = 1, N1_{kc} = 1 \quad \mu_{kct} \tag{5}$$

$$z_{kt} = z_{kt}^{*} \quad \forall k, t \quad \delta_{kt} \tag{6$'$}$$

$$P_{gct} - P_{gc}^{+} N1_{gc} u_{gt} \leq 0 \quad \forall g, c \in 0 \cup CG, t \quad \beta_{gct}^{+} \tag{7a}$$

$$-P_{gct} + P_{gc}^{-} N1_{gc} u_{gt} \leq 0 \quad \forall g, c \in 0 \cup CG, t \quad \beta_{gct}^{-} \tag{7b}$$

$$P_{gct} - P_{gc,t-1} - r_{gct}^{+} \leq 0 \quad \forall g, c \in 0 \cup CG, t \quad \omega_{gct}^{+} \tag{8a}$$

$$r_{gct}^{+} - R_{gt}^{+} u_{g,t-1} - R_{g}^{s} v_{gt} \leq 0 \quad \forall g, c \in 0 \cup CG, t \quad \chi_{gct}^{+} \tag{8b}$$

$$P_{gc,t-1} - P_{gct} - r_{gct}^{-} \leq 0 \quad \forall g, c \in 0 \cup CG, t \quad \omega_{gct}^{-} \tag{9a}$$

$$r_{gct}^{-} - R_{gt}^{-} u_{g,t-1} \leq 0 \quad \forall g, c \in 0 \cup CG, t \quad \chi_{gct}^{-} \tag{9b}$$

$$P_{gct} - P_{g0t} - R_{gt}^{c+} u_{gt} \leq 0 \quad \forall g, c \in CG, t \quad \chi_{gct}^{c+} \tag{10a}$$

$$N1_{gc} P_{g0t} - N1_{gc} P_{gct} - R_{gt}^{c-} u_{gt} \leq 0 \quad \forall g, c \in CG, t \quad \chi_{gct}^{c-} \tag{10b}$$

$$v_{gt} - w_{gt} - u_{gt} + u_{g,t-1} = 0 \quad \forall g, t \quad \tau_{gt} \tag{11}$$

$$v_{gt} = v_{gt}^{*} \quad \forall g, t \quad \sigma_{gt} \tag{14$'$}$$

$$w_{gt} = w_{gt}^{*} \quad \forall g, t \quad \psi_{gt} \tag{15$'$}$$

$$r_{gct}^{+}, r_{gct}^{-} \geq 0 \quad \forall g, c, t. \tag{17}$$

We now write the dual of the above linear program (note that $\mu_{kct} = 0$ for $z_{kt} = 0$ or $N1_{kc} = 0$):

$$\text{D: MSD} = \text{Minimize: } \theta^{+} \sum_{nct} (\alpha_{nct}^{+} + \alpha_{nct}^{-})$$

$$+ \sum_{gt} [v_{gt}^{*} \sigma_{gt} + w_{gt}^{*} \psi_{gt}] + \sum_{kt} z_{kt}^{*} \delta_{kt} \tag{18}$$

s.t.

$$\alpha_{nct}^{+} - \alpha_{nct}^{-} + \sum_{k \in k(n,.)} B_k \mu_{kct} - \sum_{k \in k(.,n)} B_k \mu_{kct} = 0 \quad \forall n, c, t \quad \theta_{nct} \tag{19}$$

$$\mu_{kct} + \lambda_{mct} - \lambda_{nct} + \eta_{kct}^{+} - \eta_{kct}^{-} = 0 \quad \forall k, c, t, z_{kt}^{*} = 1, N1_{kc} = 1 \quad P_{kct} \tag{20}$$

$$\sum_{c} (P_{kc}^{-} N1_{kc} \eta_{kct}^{-} - P_{kc}^{+} N1_{kc} \eta_{kct}^{+}) + \delta_{kt} = 0 \quad \forall k, t \quad z_{kt} \tag{21}$$

$$\sum_{c \in CT \cup 0} (\lambda_{n(g)ct}) - \beta^-_{g0t} + \beta^+_{g0t} + \omega^+_{g0t} - \omega^-_{g0t} - \omega^+_{g0,t+1} + \omega^-_{g0,t+1}$$
$$+ \sum_{c \in CG} (N1_{gc}\chi^{c-}_{gct} - \chi^{c+}_{gct}) = -c_g \quad \forall g, t \quad P_{g0t} \tag{22a}$$

$$\lambda_{n(g)ct} - \beta^-_{gct} + \beta^+_{gct} + \omega^+_{gct} - \omega^-_{gct} - \omega^+_{gc,t+1}$$
$$+ \omega^-_{gc,t+1} + \chi^{c+}_{gct} - N1_{gc}\chi^{c-}_{gct} = 0 \quad \forall g, c \in CG, t \quad P_{gct} \tag{22b}$$

$$\tau_{g,t} + \sigma_{gt} - \sum_{c \in 0 \cup CG} R^s_g \chi^+_{gct} = -SU_{gt} \quad \forall g, t \quad v_{gt} \tag{23}$$

$$-\tau_{g,t} + \psi_{gt} = 0 \quad \forall g, t \quad w_{gt} \tag{24}$$

$$\sum_{c \in 0 \cup CG} (-P^+_{gc} N1_{gc} \beta^+_{gct} + P^-_{gc} N1_{gc} \beta^-_{gct})$$
$$- \tau_{gt} + \tau_{g,t+1} - R_{gt} = 0 \quad \forall g, t \quad u_{gt} \tag{25}$$

$$-\omega^+_{gct} + \chi^+_{gct} \ge 0 \quad \forall g, c \in CG, t \quad r^+_{gct} \tag{26a}$$
$$-\omega^-_{gct} + \chi^-_{gct} \ge 0 \quad \forall g, c \in CG, t \quad r^-_{gct} \tag{26b}$$

$$-\omega^+_{g0t} + \chi^+_{g0t} \ge -cr^+_g \quad \forall g, t \quad r^+_{g0t} \tag{27a}$$
$$-\omega^-_{g0t} + \chi^-_{g0t} \ge -cr^-_g \quad \forall g, t \quad r^-_{g0t} \tag{27b}$$

$$\alpha^+_{nct}, \alpha^-_{nct}, \eta^+_{kct}, \eta^-_{kct}, \beta^+_{gct}, \beta^-_{gct},$$
$$\omega^+_{gct}, \omega^-_{gct}, \chi^+_{gct}, \chi^-_{gct}, \chi^{c+}_{gct}, \chi^{c-}_{gct} \ge 0 \quad \forall n, k, g, c, t, \tag{28}$$

where $R_{gt} = \sum_{c \in 0 \cup CG} (R^+_{g,t+1}\chi^+_{gc,t+1} + R^-_{g,t+1}\chi^-_{gc,t+1} + R^{c+}_{gt}\chi^{c+}_{gct} + R^{c-}_{gt}\chi^{c-}_{gct})$. Since $\theta^+ \neq \theta^-$, $\alpha^-_{nct}\alpha^+_{nct} = 0$. If $z^*_{kt} = 1$, $N1_{kc} = 1$, and $P^+_{kc} \neq P^-_{kc}$, then $\eta^+_{kct}\eta^-_{kct} = 0$. If $u^*_{gt} = 1$, $N1_{gc} = 1$, and $P^-_{gc} \neq P^+_{gc}$, then $\beta^+_{gct}\beta^-_{gct} = 0$. $\omega^+_{gct}\omega^-_{gct} = 0$. If $u^*_{g,t-1} = 1$, $\chi^+_{gct}\chi^-_{gct} = 0$ and $\chi^{c+}_{gc,t-1}\chi^{c-}_{gc,t-1} = 0$.

4 Economic analysis of the auction market

The primal MSLP problem maximizes the trade surplus for load and generation bids subject to the bid and system operating constraints. The dual objective function minimizes the resource cost of the goods subject to marginal cost relationships. The dual problem also yields an economic interpretation of the market and provides some settlement parameters. A market participant's surplus is the excess value over the settlement payment, or short-term profit, which is used in part to cover the investments costs.

In the dual formulation, the dual variables do not automatically 'clear' the market and do not allocate all costs unlike a naïve, simple energy-only market defined by a linear program. In a neoclassical market where there are no nonconvexities, the linear program produces dual variables that are market clearing prices. These market clearing prices cover the bid costs for sellers and have net value for buyers. This property is not present in non-neoclassical MIP markets. The market clearing settlement must be modified to satisfy this criterion. The market clearing price construct is valid if there are no binary variables or the binary variables can be relaxed to continuous variables without changing the optimal value and all generators and loads have convex bids and convex constraint sets. A positive minimum binding generation level violates this assumption.

Commodities in balancing equations that sum to zero (e.g. energy in (3a) and (3b)) are called private goods. The market clears this private good or commodity using the associated dual variable as the settlement price. There is a separate private good and a dual variable for each period, bus, and contingency and there is a one-to-one correspondence between injections and withdrawals. Notice that the dual variable of a private good does not appear in the dual objective function. When bids are constrained using convex constraints, e.g. upper and lower bounds, the private bid constraints are not traded but the associated dual variable produces scarcity rent or opportunity cost information. From complementary slackness conditions, if the constraint is not binding, the associated scarcity rent or opportunity cost is zero.

Unlike with private goods, there is no natural (endogenous) pricing, i.e. cost allocation, of club goods in this model. To a first approximation, the beneficiaries of a club good are those whose surplus decreases when the club good is removed from the market (MIP). The non-beneficiaries are those whose surplus either increases or is unchanged when the club good is removed from the market. This process becomes more complicated as the number of club goods and the combinations of club goods increase. For a club good, the binary variable representing the good appears in the objective function. The pricing of club goods must be decided by the exogenous settlement rules. We can think of the MIP optimization as finding the club goods that are optimal but not defining how to pay for them.

4.1 Power economics

As discussed above, power is a private good. Most ISOs clear the day-ahead market assuming each bid will perform as bid. However, the ISO starts up enough generators to assure the market can survive any single contingency (generator, load, or transmission). The dispatch must be able to operate within the emergency limits of transmission during a contingency. Generators are not immediately redispatched in a transmission contingency, that is, in (7) if $c \in CT$, $P_{gct} = P_{g0t}$ and transmission elements are allowed to temporarily exceed their steady state limits while procedures for regaining N-1 reliability are instituted outside the model. These procedures (called N-1-1 reliability) include startup of non-spinning reserve units, generator and/or load redispatch, and transmission switching to reestablish N-1 reliability. Reliability rules require that N-1 reliability be reestablished in thirty minutes.

We define the aggregate LMPs (ALMPs) at node n for time t as: $\lambda_{nt} = \sum_c \lambda_{nct}$. Also let $\beta^-_{gt} = \sum_c \beta^-_{gct}$, $\beta^+_{gt} = \sum_c \beta^+_{gct}$, $\omega^-_{gt} = \sum_c \omega^-_{gct}$, and $\omega^+_{gt} = \sum_c \omega^+_{gct}$. Summing (22b) over $c \in CG$ for each n and t and adding (22a) to it we have the identity in Table 1.

Table 1 Economic interpretation of the aggregate nodal price identity

$\lambda_{n(g)t} =$	Aggregate nodal price
$-c_g$	Marginal cost
$+\beta_{gt}^{-}$	Marginal value of the minimum operating level
$-\beta_{gt}^{+}$	Marginal value of the maximum operating level
$-\omega_{gt}^{+}$	Marginal value of ramping up in period t
$+\omega_{gt}^{-}$	Marginal value of ramping down in period t
$-\omega_{g,t+1}^{-}$	Marginal value of ramping down in period $t+1$
$+\omega_{g,t+1}^{+}$	Marginal value of ramping up in period t+1

For $c \in 0 \cup CT$, $P_{gct} = P_{g0t}$. For $c \in CG$, P_{gct} may be different from P_{g0t}. Here, we only require the feasibility of re-dispatch and no cost appears in the objective function for P_{gct} (where $c \neq 0$). Likewise, if (10a) and (10b) are inactive, then for $c \in CG$, $\lambda_{n(g)ct} = \beta_{gct}^{-} = \beta_{gct}^{+} = \omega_{gt}^{+} = \omega_{gt}^{-} = 0$. With (10a) and (10b) inactive, there is no link between P_{gct} for $c \in CG$ and P_{g0t}; thus, there is no cost consequence on the chosen value of P_{gct}. The only issue is obtaining a feasible solution. This does not mean that there is no impact from the $c \in CG$ variables. The impact is just not captured within the dual LP problem when the integer values are fixed; essentially, the formulation enforces that enough generation capacity is committed so that the system can survive any single contingency. This assumption understates expected costs of redispatch in a generator contingency in the day-ahead market. If the contingency is realized, the redispatch costs are incurred in the real-time market. This creates a difference between the day-ahead market and the optimal expected surplus commitment, but will not be addressed here. Instead, we are following the ISOs' procedure of only enforcing survivability of all single contingencies rather than incorporating the expected redispatch cost of a contingency into the formulation.

4.2 Economic analysis of generators as units in ISO markets

Since a generator must first be started up before it can generate, the startup variable can be thought of as a discrete real option sold by the generator to offer its power to the market. Loads, e.g. industrial processes, have similar startup characteristics to generators. For example, they can purchase options to buy. Since multiple market participants can jointly 'purchase' this discrete option, the startup variable can be thought of as the membership part of a club good. The costs or profits from club goods appear in the dual objective function and these costs or profits need to be allocated. Current allocation rules are established by the regulatory process.

Let $B_{gt}^{+} = \sum_{c \in CG \cup 0} P_{gc}^{+} N1_{gc} \beta_{gct}^{+}$. The product $u_{gt}^{*} B_{gt}^{+}$ is nonnegative and it gives the value or profit across all contingencies from the upper bound for generator g and period t; it is called the generator scarcity rent in period t. Let $B_{gt}^{-} = -\sum_{c \in CG \cup 0} P_{gc}^{-} N1_{gc} \beta_{gct}^{-}$. The product $u_{gt}^{*} B_{gt}^{-}$ is nonpositive and it gives the value or cost across all contingencies of enforcing the lower bound limit; it can be interpreted as the marginal opportunity cost of enforcing the binary variable, $u_{gt}^{*} = 1$. In the simplest case, this could represent the tradeoff between shutting down and restarting later versus 'riding through' this period at the minimum operating level. With minimum up and down time constraints, this relationship is more complex.

Now, let $\mathrm{TUP}(g)=\{t|\text{ for }u^*_{gt}=1\}$; $\mathrm{TUP}(g)$ is the set of periods the generator is up. Let $\mathrm{SUP}(g)=\{t|\text{ for }v^*_{gt}=1\}$; $\mathrm{SUP}(g)$ is the set of startup periods. Let $\mathrm{SD}(g)=\{t|\text{ for }w^*_{gt}=1\}$; $\mathrm{SD}(g)$ is the set of shutdown periods. Let $B_{gt}=B^+_{gt}+B^-_{gt}$. Assuming for simplicity that (8a)–(10b) are not binding and, therefore, $\omega^+_{gct}=\omega^-_{gct}=\chi^+_{gc,t+1}=\chi^-_{gc,t+1}=\chi^{c+}_{gct}=\chi^{c-}_{gct}=0$ for all g, c, and t, equation (25) becomes $B_{gt}+\tau_{gt}-\tau_{g,t+1}=0$. Define a startup and shutdown cycle as a set of consecutive periods where $u_{gt}=1\ \forall t\in[t',t''-1]$ where t' is the startup period and t'' is the shutdown period. For any start up and shutdown cycle,

$$\sum_{t'<=t<=t''-1}(B_{gt}+\tau_{gt}-\tau_{g,t+1})=\sum_{t'<=t<=t''-1}(B_{gt})+\tau_{g,t'}-\tau_{g,t''}=0.$$

As a result, $\sum_{t\in TUP(g)}B_{gt}+\sum_{t\in SUP(g)}\tau_{gt}-\sum_{t\in SD(g)}\tau_{g,t}=0$. Using $-\tau_{g,t'}=\sigma_{gt'}+SU_{gt'}$, $\tau_{g,t''}=\psi_{gt''}$ and rearranging, we have

$$\sum_{t\in TUP(g)}B_{gt}-\sum_{t\in SUP(g)}(SU_{gt}+\sigma_{gt})-\sum_{t\in SD(g)}\psi_{g,t}=0. \tag{29}$$

Substituting $\gamma_g=\sum_{t\in SUP(g)}\sigma_{gt}+\sum_{t\in SD(g)}\psi_{g,t}$, in (29) we have

$$\sum_{t\in TUP(g)}B_{gt}-\sum_{t\in SUP(g)}SU_{gt}-\gamma_g=0. \tag{30}$$

Multiplying (22a) by P_{g0t} and (22b) by P_{gct}, adding them together, and summing over t we obtain

$$\begin{aligned}-\sum_{c\in CT\cup 0,t}P_{g0t}\lambda_{n(g)ct}-\sum_{c\in CG,t}P_{gct}\lambda_{n(g)ct}\\ =\sum_{c\in CG\cup 0,t}[P_{gct}(\beta^+_{gct}-\beta^-_{gct})]+\sum_t c_gP_{g0t}.\end{aligned} \tag{31}$$

Note that (31) does not include ω^+_{g1} and ω^-_{g1} because it is assumed that the ramp rate constraints are inactive between the first period and the initial states. Observing that complementary slackness from (7a) and (7b) requires

$$P^+_{gc}N1_{gc}u^*_{gt}\beta^+_{gct}=P_{gct}\beta^+_{gct} \tag{32}$$

$$P^-_{gc}N1_{gc}u^*_{gt}\beta^-_{gct}=P_{gct}\beta^-_{gct}. \tag{33}$$

With $\sum_{c\in CG\cup 0}P^+_{gc}N1_{gc}u^*_{gt}\beta^+_{gct}=u^*_{gt}B^+_{gt}$ and $\sum_{c\in CG\cup 0}P^-_{gc}N1_{gc}u^*_{gt}\beta^-_{gct}=-u^*_{gt}B^-_{gt}$, we can sum (32) and (33) over c and t to get

$$-\sum_{c\in CG\cup 0,t}P_{gct}\beta^-_{gct}+\sum_{c\in CG\cup 0,t}P_{gct}\beta^+_{gct}=\sum_t u_{gt}B_{gt}=\sum_{t\in TUP(g)}B_{gt}. \tag{34}$$

Substituting (34) into (31) and using (29), we obtain

$$\sum_{c\in CT\cup 0,t}P_{g0t}\lambda_{n(g)ct}+\sum_{c\in CG,t}P_{gct}\lambda_{n(g)ct}+\sum_{t\in SUP(g)}\sigma_{gt}+\sum_{t\in SD(g)}\psi_{g,t}$$

$$= -\sum_{t\in SUP(g)} SU_{gt} - \sum_{t} c_g P_{g0t}. \tag{35}$$

With further substitutions, we have

$$\begin{aligned}
&\Big[\sum_{c\in CT\cup 0,t} P_{g0t}\lambda_{n(g)ct} + \sum_{c\in CG,t} P_{gct}\lambda_{n(g)ct}\Big] && \text{(LMP Revenues)}\\
&\quad + \gamma_g && \text{(Uplift/profits)}\\
&= -\sum_{t\in SUP(g)} SU_{gt} && \text{(Startup costs)}\\
&\quad -\sum_{c\in CG\cup 0,t} \rho_c c_g P_{gct}. && \text{(Operating costs)} \qquad (36)
\end{aligned}$$

If $\sigma_{gt} > 0$, $v_{gt} = v^*_{gt} = 1$ can be replaced by $v_{gt} \le v^*_{gt} = 1$ without changing the optimality properties of the linear program and γ_{gt} can be interpreted as the marginal value of an increase in capacity and is called the generator scarcity rent in period t. If $\sigma_{gt} < 0$, $v_{gt} = v^*_{gt} = 1$ can be replaced by $v_{gt} \ge v^*_{gt} = 1$ without changing the optimality properties of the linear program. If $\psi_{gt} > 0$, $w_{gt} = w^*_{gt} = 1$ can be replaced by $w_{gt} \le w^*_{gt} = 1$ without changing the optimality properties of the linear program. If $\psi_{gt} < 0$, $w_{gt} = w^*_{gt} = 1$ can be replaced by $w_{gt} \ge w^*_{gt} = 1$ without changing the optimality properties of the linear program.

Note that $\sum_g \gamma_g = \sum_{gt}[v^*_{gt}\sigma_{gt} + w^*_{gt}\psi_{gt}]$, which appears in the objective function. If $\gamma_g > 0$, γ_g is called the total linear scarcity rent over the time horizon for generator g. If $\gamma_g < 0$, it can be interpreted as the marginal cost of enforcing the binary constraints. Once a unit is turned on, it must stay on in order to satisfy the minimum up time constraints. At times, a unit is forced to stay on and operate at its lower bound when its variable cost is higher than the unit's LMP. In essence, the market could be more efficient if the constraints were not binary. Nevertheless, dispatching other generators would be even more expensive. The full incremental cost of the binary constraint is calculated by fixing the binary variable to its opposite value and resolving the MIP while the rest of the binary variables remain at their previous fixed values. The difference in objective function value is the true incremental value of the asset or the cost of forcing it into the optimal. Incremental cost analysis can be a significant computational burden thereby making the marginal analysis from the dual variables more valuable.

4.3 Analysis of transmission assets

Transmission assets exist to move power from lower valued locations to higher valued locations; nevertheless, co-optimizing the network along with generation is still beneficial. For k, c, and t, Table 2 shows the economic relationship between the nodal price difference, $\lambda_{nct} - \lambda_{mct}$ (n and m are the terminal nodes of k), the susceptance value μ_{kct}, and the marginal values of flowgate capacity, η^+_{kct} and η^-_{kct}.

If the asset is not at its thermal capacity, $\eta^+_{kct} = \eta^-_{kct} = 0$ and the LMP difference is the marginal value of susceptance, μ_{kct}. For any loop within the network, summing

Table 2 Economic interpretation of (20)

$\lambda_{nct} - \lambda_{mct} =$	Difference in nodal prices
$+\eta^{+}_{kct} - \eta^{-}_{kct}$	Marginal value of another unit of capacity
$+\mu_{kct}$	Marginal value of another unit of susceptance

over the transmission elements (k) for loop L gives $\sum_{k\in L}(\lambda_{nct} - \lambda_{mct}) = 0$ and (20) becomes $0 = \sum_{k\in L}(\eta^{+}_{kct} - \eta^{-}_{kct}) + \sum_{k\in L}(\mu_{kct})$. If there are no capacity constrained elements in loop L, $\eta^{+}_{kct} = \eta^{-}_{kct} = \mu_{kct} = 0$ for all k in loop L.

To be included in the network, a transmission asset should increase the market surplus as expressed in the objective function. To find the optimal topology, each network must be a candidate for the optimal topology. Transmission switching, z_{kt}, is a club good. The dual objective function minimization includes $\sum_{k,t} z^{*}_{kt}\delta_{kt}$. Rewriting (21), we have

$$\underbrace{\sum_c P^{+}_{kc} N1_{kc}\eta^{+}_{kct} - \sum_c P^{-}_{kc} N1_{kc}\eta^{-}_{kct}}_{\text{Marginal value of increasing capacity}} = \underbrace{\delta_{kt}}_{\text{Marginal value of switching } k \text{ in } t}. \tag{37}$$

From (37), it is evident that even though δ_{kt} is free to be positive or negative, this dual variable will always be non-negative. Since the dual variables η^{-}_{kct} and η^{+}_{kct} must be non-negative along with the fact that P^{+}_{kc} is positive and P^{-}_{kc} is negative, this forces δ_{kt} to always be non-negative. This fact is easily explained. Equations (5a) and (5b) are rewritten as (5) for $z_{kt} = 1$ and $N1_{kc} = 1$ within MSLP. Equations (5a) and (5b) are never binding when $z_{kt} = 0$ or $N1_{kc} = 0$; thus, those constraints are not included in the LP. As a result, z_{kt} only affects transmission lines in (4a$'$) and (4b$'$). δ_{kt} therefore has the economic interpretation of reflecting the value of additional line capacity as z_{kt} increases the upper bound and decreases the lower bound, which is evident from (37).

Since for most flowgates, $-P^{-}_{kc} = P^{+}_{kc}$, the value of another unit of capacity is available for both the upper and lower bounds. The marginal value of another unit of thermal capacity is therefore $\eta_{kt} = \sum_c(\eta^{-}_{kct} + \eta^{+}_{kct})$. From (4a$'$) and (4b$'$), complementary slackness requires $P^{-}_{kc} z_{kt} N1_{kc}\eta^{-}_{kct} = \eta^{-}_{kct} P_{kct}$ and $P^{+}_{kc} z_{kt} N1_{kc}\eta^{+}_{kct} = \eta^{+}_{kct} P_{kct}$. Multiplying (37) by z^{*}_{kt}, we have (38), which leads to (39)

$$\sum_c P^{+}_{kc} z^{*}_{kt} N1_{kc}\eta^{+}_{kct} - \sum_c P^{-}_{kc} z^{*}_{kt} N1_{kc}\eta^{-}_{kct} = z^{*}_{kt}\delta_{kt} \tag{38}$$

$$\sum_c [(\eta^{+}_{kct} - \eta^{-}_{kct})P_{kct}] = \sum_c [(\lambda_{nct} - \lambda_{mct} - \mu_{kct})P_{kct}] = z^{*}_{kt}\delta_{kt}. \tag{39}$$

Note that $(\eta^{+}_{kct} - \eta^{-}_{kct})P_{kct}$ will never be negative. $\eta^{+}_{kct} - \eta^{-}_{kct}$ can only be negative when: (a) P_{kct} is at its lower bound, which is a negative number. Therefore, this product is still nonnegative; (b) $z^{*}_{kt} = 0$ or $N1_{kc} = 0$ and, therefore, $P_{kct} = 0$. Thus, (39) is always non-negative, which is obvious since $z^{*}_{kt}\delta_{kt}$ is always non-negative.

In an analogous way to generation and load, if $\delta_{kt} > 0$, $z_{kt} = z^{*}_{kt}$ can be replaced by $z_{kt} \le z^{*}_{kt}$ without changing the optimality properties of the linear program. For any line that is closed within the optimal solution, since (5) is rewritten into an equality constraint for $z_{kt} = 1$ and $N1_{kc} = 1$, this reduces the variable z_{kt} to only affect the capacity of the line and no longer reflects the ability to switch lines. Therefore, δ_{kt} is the marginal value of capacity and is called the total transmission scarcity rent.

If (5a) and (5b) are not reduced to (5), then δ_{kt} will have a different meaning. First, for this analysis we will assume that $N1_{kc} = 1$. With (5a) and (5b), δ_{kt} will be non-negative when $z^*_{kt} = 0$. Increasing the right hand side of (6′) will have two main affects on the LP: (a) this will increase the upper bound and decrease the lower bound on P_{kct}, thereby expanding the feasible set. (b) With (5a) and (5b) in the formulation, these constraints are inactive when $z^*_{kt} = 0$; therefore, Kirchhoff's laws are not enforced. An increase in z^*_{kt} from zero to a small number will keep (5a) and (5b) inactive; this translates into allowing a small flow on line k while being able to violate Kirchhoff's laws, which also expands the feasible set. These two results will only help the maximization formulation leading to δ_{kt} being non-negative whenever $z^*_{kt} = 0$ if (5a) and (5b) are used instead of (5).

Though increasing the right hand side of (6′) will improve the upper and lower bounds for line k, with (5a) and (5b), increasing z^*_{kt} above a value of one will create an infeasible solution since $B_k(\theta_{nct} - \theta_{mct}) + P_{kct}$ will then have to be greater than or equal to a positive number as well as less than or equal to a negative number. Thus, δ_{kt} may be positive or negative when $z^*_{kt} = 1$. With (5), whenever $z^*_{kt} = 1$, it was often the case that δ_{kt} was positive since by increasing z^*_{kt} the capacity of line k would increase, which expands the feasible set of the LP. When using (5a) and (5b) and when $z^*_{kt} = 1$, δ_{kt} is likely to have the opposite sign and a completely different interpretation.

4.4 Phase angle

ACOPF formulations include constraints on the angle difference between two connected buses; these constraints ensure angle stability. However, in a DCOPF model, an angle difference constraint can be subsumed by the line flow capacity constraints, (4a) and (4b). Restricting the angle difference between connected buses, i.e. $\theta^- \le \theta_{nct} - \theta_{mct} \le \theta^+$, indirectly places a bound on that line's flow, P_{kct} since $P_{kct} = B_k(\theta_{nct} - \theta_{mct})$. Instead of including angle difference constraints, the power flow constraint can contain the limit on angle difference. If the angle difference limit places a tighter bound on the line's flow than the thermal capacity constraints, then the capacity limits can be adjusted to enforce the angle constraints. In the formulation presented in Sect. 2, we employ limits on each bus angle (2) since it is not redundant and it conveniently provides a lower bound on M_k.

The phase angle constraint at a node is a reliability constraint, since it protects against a local (or system) outage. The objective function minimization includes the following: the product of the phase angle limit and marginal values of raising or lowering the phase angle limit across all contingencies, nodes, and periods $\theta^+ \sum_{nct}(\alpha^+_{nct} + \alpha^-_{nct})$.

Since the phase angle constraint is for system reliability, it can be classified as club good. For each transmission asset k, the marginal relationship between the value of the phase angle and susceptance, for each c, t, is described in Table 3.

Table 3 Economic interpretation of (19)

$\sum_{k\in k(n,.)} B_k\mu_{kct}$	Marginal value of increasing susceptance for $k\in k(n,.)$
$-\sum_{k\in k(.,n)} B_k\mu_{kct}$	Marginal value of increasing susceptance for $k\in k(.,n)$
$+\alpha^+_{nct}-\alpha^-_{nct}=0$	Marginal value of a radian of phase angle

5 Load payment, generation rent, and congestion rent

Once the integer variables are fixed to their optimal values, the resulting program is a linear program. Whether the integer variables are kept as variables and fixed to their solution values, as in (11), (14′), and (15′), or treated as parameters, the set of dual optimal solutions for the rest of the equations does not change. In this section, the integer variables are treated as parameters and set at their solution values; load is fixed as an input and represented by d_n. To identify the terms representing the load payment, the short term generation rent, and the congestion rent, we formulate the primal and the corresponding dual. Since v^*_{gt} is an input parameter, the startup cost term is not included in the primal objective. For these problems, (5a) and (5b) are not reduced to (5) as is the case in Sect. 3

$$\text{Primal: P} = \text{Maximize: } \sum_{gt}(-c_g P_{g0t} - cr^+_g r^+_{g0t} - cr^-_g r^-_{g0t}) \tag{1′}$$

s.t.

(2a), (2b), (8a), (9a), (17)

$$\sum_{k(.,n)} P_{kct} - \sum_{k(n,.)} P_{kct} + \sum_{g(n)} P_{g0t} = d_n \quad \forall n, c\in 0\cup CT, t \quad \lambda_{nct} \tag{3a′}$$

$$\sum_{k(.,n)} P_{kct} - \sum_{k(n,.)} P_{kct} + \sum_{g(n)} P_{gct} = d_n \quad \forall n, c\in CG, t \quad \lambda_{nct} \tag{3b′}$$

$$P_{kct} \le P^+_{kc} z^*_{kt} N1_{kc} \quad \forall k, c, t \quad \eta^+_{kct} \tag{4a″}$$

$$-P_{kct} \le P^+_{kc} z^*_{kt} N1_{kc} \quad \forall k, c, t \quad \eta^-_{kct} \tag{4b″}$$

$$-B_k(\theta_{nct}-\theta_{mct}) - P_{kct} \le M_k(2-z^*_{kt}-N1_{kc}) \quad \forall k, c, t \quad \mu^+_{kct} \tag{5a′}$$

$$B_k(\theta_{nct}-\theta_{mct}) + P_{kct} \le M_k(2-z^*_{kt}-N1_{kc}) \quad \forall k, c, t \quad \mu^-_{kct} \tag{5b′}$$

$$P_{gct} \le P^+_{gc} N1_{gc} u^*_{gt} \quad \forall g, c\in 0\cup CG, t \quad \beta^+_{gct} \tag{7a′}$$

$$-P_{gct} \le -P^-_{gc} N1_{gc} u^*_{gt} \quad \forall g, c\in 0\cup CG, t \quad \beta^-_{gct} \tag{7b′}$$

$$r^+_{gct} \le R^+_{gct} u^*_{g,t-1} + R^s_g v^*_{gt} \quad \forall g, c\in 0\cup CG, t \quad \chi^+_{gct} \tag{8b′}$$

$$r^-_{gct} \le R^-_{gt} u^*_{g,t-1} \quad \forall g, c\in 0\cup CG, t \quad \chi^-_{gct} \tag{9b′}$$

$$P_{gct} - P_{g0t} \leq R_{gt}^{c+} u_{gt}^* \quad \forall g, c \in CG, t \quad \chi_{gct}^{c+} \tag{10a$'$}$$

$$N1_{gc} P_{g0t} - N1_{gc} P_{gct} \leq R_{gt}^{c-} u_{gt}^* \quad \forall g, c \in CG, t \quad \chi_{gct}^{c-} \tag{10b$'$}$$

$$\text{Dual: } \pi = \text{Minimize: } \pi^D + \pi^G + \pi^K + \pi^0 \tag{40}$$

s.t.

$$\alpha_{nct}^+ - \alpha_{nct}^- + \sum_{k \in k(n,.)} B_k(\mu_{kct}^- - \mu_{kct}^+) - \sum_{k \in k(.,n)} B_k(\mu_{kct}^- - \mu_{kct}^+) = 0$$

$$\forall n, c, t \quad \theta_{nct} \tag{19$'$}$$

$$-\mu_{kct}^+ + \mu_{kct}^- + \lambda_{mct} - \lambda_{nct} + \eta_{kct}^+ - \eta_{kct}^- = 0 \quad \forall k, c, t \quad P_{kct} \tag{20$'$}$$

(22a), (22b), (26a)–(28)

$$\mu_{kct}^+, \mu_{kct}^- \geq 0 \quad \forall k, c, t. \tag{28b}$$

With the following identities

$$\pi^D = \sum_{nct} d_n \lambda_{nct} \tag{41}$$

$$\begin{aligned} \pi^G = & \sum_{g,c \in 0 \cup CG,t} (P_{gc}^+ N1_{gc} u_{gt}^* \beta_{gct}^+ - P_{gc}^- N1_{gc} u_{gt}^* \beta_{gct}^-) \\ & + \sum_{g,c \in 0 \cup CG,t} [(R_{gct}^+ u_{g,t-1}^* + R_g^s v_{gt}^*) \chi_{gct}^+ + R_{gt}^- u_{g,t-1}^* \chi_{gct}^-] \\ & + \sum_{g,c \in CG,t} (R_{gt}^{c+} u_{gt}^* \chi_{gct}^{c+} + R_{gt}^{c-} u_{gt}^* \chi_{gct}^{c-}) \end{aligned} \tag{42}$$

$$\pi^K = \theta^+ \sum_{nct} (\alpha_{nct}^+ + \alpha_{nct}^-) + \sum_{kct} P_{kc}^+ z_{kt}^* N1_{kc} (\eta_{kct}^+ + \eta_{kct}^-) \tag{43}$$

$$\pi^0 = \sum_{kct} [M_k (2 - z_{kt}^* - N1_{kc})(\mu_{kct}^+ + \mu_{kct}^-)]. \tag{44}$$

First, the load payment is easy to identify as (41). Next, let us assume that all ramp rates are inactive. The generation rent is: $\sum_{g,c \in 0 \cup CG,t} (P_{gc}^+ N1_{gc} u_{gt}^* \beta_{gct}^+ - P_{gc}^- N1_{gc} u_{gt}^* \beta_{gct}^-)$. Taking (22a) and multiplying by P_{g0t} and then using (32) and (33) as a substitution, we can get:

$$P_{g0t} \sum_{c \in CT \cup 0} (\lambda_{n(g)ct}) = -P_{gc}^+ N1_{g0} u_{gt}^* \beta_{g0t}^+ + P_{gc}^- N1_{g0} u_{gt}^* \beta_{g0t}^- - c_g P_{g0t}. \tag{45}$$

Doing the same for (22b), we get

$$P_{gct} \lambda_{n(g)ct} = -P_{gc}^+ N1_{gc} u_{gt}^* \beta_{gct}^+ + P_{gc}^- N1_{gc} u_{gt}^* \beta_{gct}^-. \tag{46}$$

Adding (45) and (46) and summing over g, c, and t, we get

$$-\sum_{gt}\left[\sum_{c\in CG}(P_{gct}\lambda_{n(g)ct})+P_{g0t}\left(\sum_{c\in CT\cup 0}(\lambda_{n(g)ct})+c_g\right)\right]$$
$$=\sum_{g,c\in 0\cup CG,t}(P^{+}_{gc}N1_{gc}u^{*}_{gt}\beta^{+}_{gct}-P^{-}_{gc}N1_{gc}u^{*}_{gt}\beta^{-}_{gct}). \qquad (47)$$

Equation (47) then shows that, when the ramp rate constraints are inactive, the generation rent is identified by the term in the dual's objective, which is identified by the right hand side of (47). Note that the left hand side of (47) actually has the negative generator payments plus cost inside the brackets. Based on the formulation of the primal, the $\lambda_{n(g)ct}$, i.e. the LMP, is generally negative as an increase in consumption will decrease the objective.

When the ramp rates are active, without loss of generality we will assume that the unit is not operating at its lower or upper bounds; otherwise, we have redundant constraints and then we will use (47) and (50) to identify the generation rent.

Suppose only the ramp up rate (8a) is active. We know $\omega^{+}_{gct}=\chi^{+}_{gct}$ and $(R^{+}_{gct}u^{*}_{g,t-1}+R^{s}_{g}v^{*}_{gt})\chi^{+}_{gct}=r^{+}_{gct}\chi^{+}_{gct}=\omega^{+}_{gct}(P_{gct}-P_{gc,t-1})$. This then allows us to have

$$\sum_{g,c\in 0\cup CG,t}[(R^{+}_{gct}u^{*}_{g,t-1}+R^{s}_{g}v^{*}_{gt})\chi^{+}_{gct}]$$
$$=\sum_{g,c\in 0\cup CG,t}(P_{gct}-P_{gc,t-1})\omega^{+}_{gct}$$
$$=\sum_{g,c\in 0\cup CG,t}[P_{gct}(\omega^{+}_{gct}-\omega^{+}_{gc,t+1})].$$

Multiplying (22a) by P_{g0t} and (22b) by P_{gct}, we get:

$$P_{g0t}\sum_{c\in CT\cup 0}(\lambda_{n(g)ct})+P_{g0t}(\omega^{+}_{g0t}-\omega^{+}_{g0,t+1})=-P_{g0t}c_g \qquad (48a)$$

$$\sum_{c\in CG}[P_{gct}\lambda_{n(g)ct}+P_{gct}(\omega^{+}_{gct}-\omega^{+}_{gc,t+1})]=0. \qquad (48b)$$

Summing over g and t we get

$$\left[\sum_{g,t}P_{g0t}\left(\sum_{c\in CT\cup 0}\lambda_{n(g)ct}\right)+P_{g0t}c_g+\sum_{c\in CG}P_{gct}\lambda_{n(g)ct}\right]$$
$$+\sum_{g,c\in 0\cup CG,t}P_{gct}(\omega^{+}_{gct}-\omega^{+}_{gc,t+1})=0. \qquad (49)$$

Equation (49) then becomes:

$$-\sum_{gt}\left[\sum_{c\in CG}(P_{gct}\lambda_{n(g)ct})+P_{g0t}\left(\sum_{c\in CT\cup 0}(\lambda_{n(g)ct})+c_g\right)\right]$$

$$= \sum_{g,c\in 0\cup CG,t} [(R^{+}_{gct}u^{*}_{g,t-1} + R^{s}_{g}v^{*}_{gt})\chi^{+}_{gct}]. \tag{50}$$

For the situation where (8a) is active with (7a′) and (7b′) inactive, the generation rent is defined as the right hand side of (50). If (8a) and (7a′) or (7b′) are active, we have redundant constraints. At that time, the right hand sides of (47) and (50) would identify the generation rent. This same process can be repeated for (9a) being active along with (10a′) or (10b′). With this process, the generation rent is defined by (42).

We now have identified the load payment and the generation rent. Since all integer variables are treated as parameters instead of fixed variables, we know that the primal objective contains only the total generation operational cost, i.e. startup and shutdown costs are not apart of the objective at this time. Thus, we know the dual objective is listed as the load payment, which is non-positive with this formulation, plus the generation rent plus the congestion rent, which are surpluses and therefore non-negative with this formulation. Thus, the congestion rent is equal to the remaining terms of the dual objective: $\theta^{+}\sum_{nct}(\alpha^{+}_{nct}+\alpha^{-}_{nct}) + \sum_{kct} P^{+}_{kc}z^{*}_{kt}N1_{kc}(\eta^{+}_{kct}+\eta^{-}_{kct}) + \sum_{kct}[M_k(2-z^{*}_{kt}-N1_{kc})(\mu^{+}_{kct}+\mu^{-}_{kct})]$, which we will show to be the case. First, note that (5a) and (5b) are always inactive constraints whenever $z^{*}_{kt}=0$ or $N1_{kc}=0$, which results in $\mu^{+}_{kct}=\mu^{-}_{kct}=0$. When that is not the case, $2-z^{*}_{kt}-N1_{kc}=0$. This tells us that $\sum_{kct}[M_k(2-z^{*}_{kt}-N1_{kc})(\mu^{+}_{kct}+\mu^{-}_{kct})]=0$ always at optimality, i.e. (44) is always zero at optimality. Multiplying (19′) by θ_{nct} to get: $\theta_{nct}(\alpha^{+}_{nct}-\alpha^{-}_{nct}) + \sum_{k\in k(n,.)}[\theta_{nct}B_k(\mu^{-}_{kct}-\mu^{+}_{kct})] - \sum_{k\in k(.,n)}[\theta_{nct}B_k(\mu^{-}_{kct}-\mu^{+}_{kct})]=0$.

Using complementary slackness from (2a) and (2b):

$$\theta^{+}(\alpha^{+}_{nct}+\alpha^{-}_{nct}) + \sum_{k\in k(n,.)} \theta_{nct}B_k(\mu^{-}_{kct}-\mu^{+}_{kct}) - \sum_{k\in k(.,n)} \theta_{nct}B_k(\mu^{-}_{kct}-\mu^{+}_{kct}) = 0.$$

If we sum over n, c, and t, we get:

$$\sum_{nct}\left[\theta^{+}(\alpha^{+}_{nct}+\alpha^{-}_{nct}) + \sum_{k\in k(n,.)} \theta_{nct}B_k(\mu^{-}_{kct}-\mu^{+}_{kct}) - \sum_{k\in k(.,n)} \theta_{nct}B_k(\mu^{-}_{kct}-\mu^{+}_{kct})\right]$$
$$= \sum_{nct}[\theta^{+}(\alpha^{+}_{nct}+\alpha^{-}_{nct})] + \sum_{kct}(\theta_{nct}-\theta_{mct})B_k(\mu^{-}_{kct}-\mu^{+}_{kct}) = 0$$
$$= \sum_{nct}[\theta^{+}(\alpha^{+}_{nct}+\alpha^{-}_{nct})] - \sum_{kct} P_{kct}(\mu^{-}_{kct}-\mu^{+}_{kct}) = 0.$$

Thus:

$$\sum_{nct}[\theta^{+}(\alpha^{+}_{nct}+\alpha^{-}_{nct})] = \sum_{kct} P_{kct}(\mu^{-}_{kct}-\mu^{+}_{kct}). \tag{51}$$

Next, we take (20′), multiply by P_{kct}, rearrange the equation, and sum over k, c, and t to get:

$$-\sum_{kct} P_{kct}(\lambda_{mct}-\lambda_{nct}) = \sum_{kct} P_{kct}(-\mu^{+}_{kct}+\mu^{-}_{kct}+\eta^{+}_{kct}-\eta^{-}_{kct}). \tag{52a}$$

It can easily be seen that the left hand side of (52a) is the congestion rent identity. With (51), (52a) becomes:

$$-\sum_{kct} P_{kct}(\lambda_{mct}-\lambda_{nct}) = \sum_{nct}[\theta^{+}(\alpha^{+}_{nct}+\alpha^{-}_{nct})] + \sum_{kct} P_{kct}(\eta^{+}_{kct}-\eta^{-}_{kct}). \quad (52b)$$

Using the complementary slackness conditions from (4a″) and (4b″), we have (52c) below, which states that the congestion rent is defined by (43)

$$\begin{aligned} &-\sum_{kct} P_{kct}(\lambda_{mct}-\lambda_{nct}) \\ &\quad = \sum_{nct}[\theta^{+}(\alpha^{+}_{nct}+\alpha^{-}_{nct})] + \sum_{kct} P^{+}_{kc} z^{*}_{kt} N1_{kc}(\eta^{+}_{kct}+\eta^{-}_{kct}). \end{aligned} \quad (52c)$$

To summarize, the following identities hold
Load Payment:

$$\sum_{nct} d_n \lambda_{nct}$$

Generation Rent:

$$\begin{aligned} &\sum_{g,c\in 0\cup CG,t} (P^{+}_{gc} N1_{gc} u^{*}_{gt} \beta^{+}_{gct} - P^{-}_{gc} N1_{gc} u^{*}_{gt} \beta^{-}_{gct}) \\ &\quad + \sum_{g,c\in 0\cup CG,t} [(R^{+}_{gct} u^{*}_{g,t-1} + R^{s}_{g} v^{*}_{gt})\chi^{+}_{gct} + R^{-}_{gt} u^{*}_{g,t-1} \chi^{-}_{gct}] \\ &\quad + \sum_{g,c\in CG,t} (R^{c+}_{gt} u^{*}_{gt} \chi^{c+}_{gct} + R^{c-}_{gt} u^{*}_{gt} \chi^{c-}_{gct}). \end{aligned}$$

Congestion Rent:

$$\theta^{+}\sum_{nct}(\alpha^{+}_{nct}+\alpha^{-}_{nct}) + \sum_{kct} P^{+}_{kc} z^{*}_{kt} N1_{kc}(\eta^{+}_{kct}+\eta^{-}_{kct}).$$

6 Computational results

We also examine the economics of the transmission switching problem in a wholesale electricity market, through analysis of the computational results. We solved the IEEE RTS96 unit commitment transmission switching problem over a 24 hour period using the formulation presented in [17]. In RTS96, load is fixed, i.e., $P^{-}_{gc} = P^{+}_{gc}$ and $c_g = 0$, $\forall g \in D$. The primal problem maximizes the surplus for load and generation bids subject to system operating constraints. Since load is perfectly inelastic, the computation reduces to a minimization of the dispatch cost. The lowest dispatch cost found is \$3,129,778, [17].

We analyze the Locational Marginal Prices (LMPs), as well as marginal values associated with generator limits and transmission constraints. Additionally, we discuss the generator short term profits and the need for uplift, as well as the marginal values on the switching decision variables. Since we present a full N-1 formulation, a number of the marginal values associated with locations, generators, or elements in a given time period must be aggregated over all contingency constraints to show the full marginal impact on surplus, as discussed throughout this paper. Prices are examined by presenting their locational and temporal variation, as well as the variation across contingency constraints. We analyze the frequency of the various marginal relaxation values in this problem, and describe the frequency by building empirical distribution functions (EDF) of the appropriate combinations of dual variables. The EDF describes the proportion of the data that falls below a particular value. Dual variables for this problem were computed from the solution to the linear program (LP) with the MIP binary variables fixed at their values in the best solution found.

6.1 Aggregate LMPs

We define the aggregate LMP (ALMP) as $\lambda_{nt} = \sum_c \lambda_{nct}$. In other words, it is the sum of the marginal cost of enforcing the power balance constraints across all N-1 contingency states plus the marginal cost for the no contingency state. The formulation presented in Sect. 2 starts enough generators as well as chooses appropriate generator dispatch values in order to survive any one generator or transmission contingency. The individual LMP for a particular bus-state-hour combination is λ_{nct}, which is the dual variable on the power balance constraint, (3a) and (3b). For this specific test case, once the integer variables are fixed to their solution values, the generator contingency constraints and variables are only linked to the objective function by ramp rate constraints. Since these ramp rate constraints are inactive and since the cost of ramping in a contingency is zero for this test case, $\lambda_{nct} = 0\ \forall c \in CG$.

Figure 1 shows the maximum and minimum aggregate LMPs by hour. Periods 3–5 have the same ALMP across all buses. These periods all have low load levels.

Figure 2 plots the aggregate LMPs across bus and hour. ALMPs tend to fall when demand is falling and after units are committed even though demand may be increasing. For example, bus 73 experiences an ALMP drop of 88% (from \$13.02 to \$1.55) from hour 7 to 8, while hourly demand increases from 74% to 86% of daily peak load.

For $c \in 0 \cup CT$, the individual LMPs, λ_{nct}, vary considerably. Table 4 gives a summary of the top 10 ALMPs, along with the range of the individual λ_{nct} values across all $c \in 0 \cup CT$. The individual λ_{nct} values provide useful information about enforcing constraints in individual contingencies, but they do not represent system marginal costs. For example, at bus 13 in hour 11, the marginal cost of power balance under the no contingency state is \$0.00/MWh. However, note that this does not reflect the true cost to the system to deliver another MW unit to bus 13 in hour 11. If bus 13's load is to increase by another MW for hour 11, then all contingency states must also adjust their load level since the load level will be the same over all contingency states. The true marginal cost to the system to deliver another MW to a bus is therefore based on the ALMP, not any of the individual LMPs. For $n = 13$ and

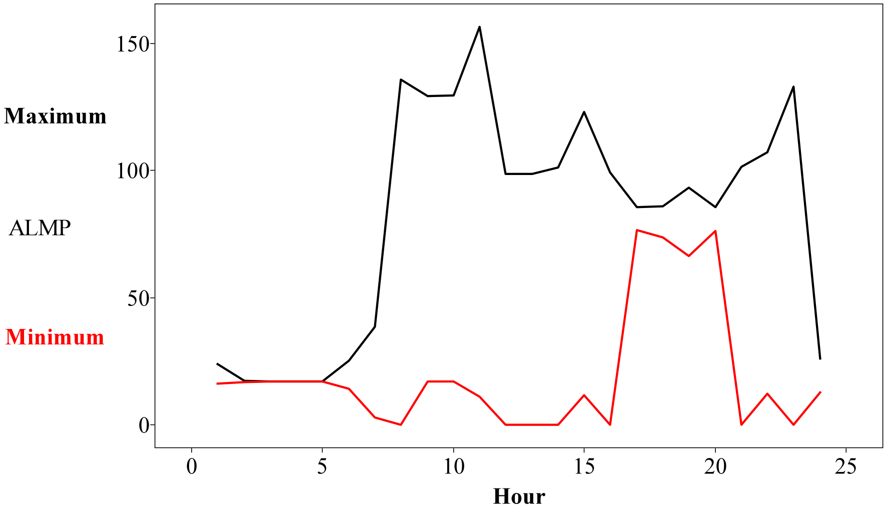
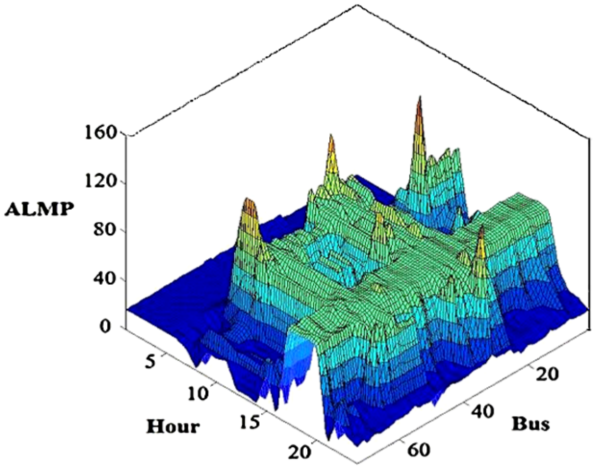


Fig. 1 Maximum and minimum ALMP by hour

Fig. 2 ALMP by hour and bus



$t = 11$, the individual LMP λ_{nct} for transmission contingency $c = 21$ is \$115.45; the individual LMP for transmission contingency $c = 117$ is $-$\$0.76. The ALMP at bus 13 in hour 11 is \$156.49.

The 10 largest ALMPs vary from \$156/MWh to \$107/MWh and all are higher than the most expensive generator dispatched at \$101/MWh. They occur at various buses and time periods without a simple pattern except the obvious result that ALMPs are generally higher at buses with less generation than load. The individual LMPs range from $-$\$4.19 at bus 61 in hour 10 under transmission contingency 116 to \$115 at bus 13 in hour 11 under transmission contingency 21. Although generators and loads would have no concern other than the ALMP, extreme values of the individual LMPs have economic and reliability interpretations. An individual LMP with a high

Table 4 10 Largest ALMPs λ_{nt}

n	t	λ_{nt}	λ_{n0t}	Min LMP, (state c): $\min_{c\in CT}\{\lambda_{nct}\}$, $(\mathrm{argmin}_c\lambda_{nct})$	Max LMP, (state c): $\max_{c\in CT}\{\lambda_{nct}\}$, $(\mathrm{argmax}_c\lambda_{nct})$
13	11	156.49	0.00	−0.76 (117)	115.45 (21)
31	8	135.87	0.00	−0.67 (3)	73.78 (116)
31	23	133.03	0.03	−0.01 (40)	100.24 (53)
61	10	129.45	0.22	−4.19 (116)	102.86 (97)
61	9	129.31	0.04	−1.67 (116)	104.58 (97)
12	11	124.68	0.00	−0.77 (117)	83.48 (21)
38	15	123.04	0.00	−0.36 (70)	43.44 (66)
32	8	117.81	0.00	−0.67 (3)	55.72 (116)
32	23	112.52	0.03	−0.01 (40)	80.25 (53)
38	22	107.28	0.01	−0.64 (117)	40.08 (66)

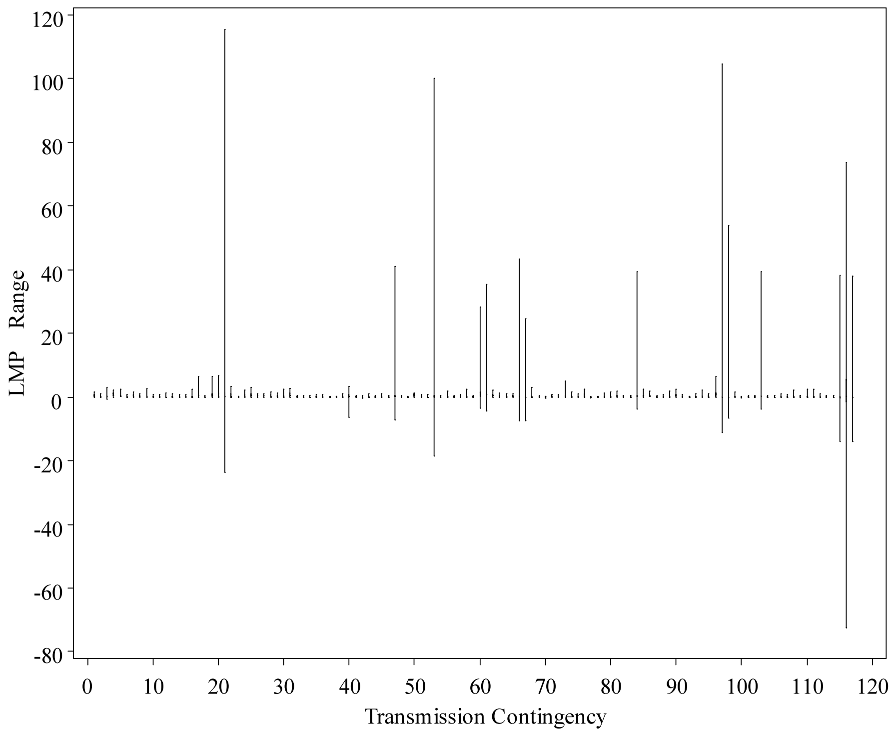


Fig. 3 Range of individual LMPs, λ_{nct} vs. $c \in CT$

value indicates a contingency constraint that the system pays a high dispatch cost to enforce. Figure 3 illustrates the range of individual LMPs against $c \in CT$, taken across all hours and buses. It is seen that the majority of transmission contingencies have very little impact on LMPs, having values at or near 0 across all hours and buses.

Table 5 10 Smallest ALMPs λ_{nt}

n	t	λ_{nt}	λ_{n0t}	$\text{Min}_{c\in CT}\{\lambda_{nct}\}$, $(\text{argmin}_c \lambda_{nct})$	$\text{Max}_{c\in CT}\{\lambda_{nct}\}$, $(\text{argmax}_c \lambda_{nct})$
70	21	0.00	0.11	−67.75 (116)	13.26 (115)
70	16	0.00	0.26	−71.92 (116)	12.63 (115)
70	13	0.00	0.16	−68.16 (116)	13.28 (115)
70	12	0.00	0.16	−68.16 (16)	13.28 (115)
70	14	0.00	0.16	−71.43 (116)	14.54 (117)
70	8	0.00	0.00	−72.63 (116)	19.53 (115)
46	23	0.00	0.03	−18.46 (53)	3.14 (60)
73	16	0.06	0.26	−37.32 (116)	14.98 (115)
73	21	1.49	0.11	−32.72 (116)	16.61 (115)
73	8	1.56	0.00	−40.56 (116)	22.52 (115)

However, the enforcement of certain transmission contingencies creates a wide range of individual LMPs. For instance, the enforcement of transmission contingency 21 results in LMPs ranging from −\$23.69 at bus 23 in hour 11, to \$115.45 at bus 22 in hour 24. The smallest ALMPs occur at various buses and time periods (see Table 5) without a simple pattern except the ALMPs at bus 70, a hydro location, are zero for several periods. Most buses with low ALMPs are without load.

6.2 Generators

Of the 99 generators in the RTS96 system, 51 make short run profits ranging from \$358,546 to \$26,121, and 19 receive uplift payments totaling \$157,874 or about 4% of total costs; 29 generators are not dispatched. The optimization is indifferent to choosing between generating units at a single node having the same characteristics. In hours 8 and 21, there are four hydro units with zero cost at bus 70 that are collectively operating at less than capacity. Bus 13 has three Oil/Steam generators each with variables costs of \$80.6/MWh, a high startup cost (\$6510), and minimum run times of 12 hours. The ALMP rises to \$156.5/MWh in hour 11 but the units are uneconomic due to the startup costs and minimum run time. When operating at its upper bound, a generator may have a scarcity value. Positive scarcity values are as high as \$125/MWh. When operating at its lower bound, a generator may have a negative scarcity value or opportunity cost. Negative scarcity values are as low as −\$85/MWh. Most generators operate at their upper or lower bounds and only a few operate in between; this is a result due to using linear generator costs. Figure 4 displays the empirical distribution of the marginal values of reducing the minimum run level or increasing the maximum output in all generator-hour combinations, i.e. $\sum_c(\beta^+_{gct} - \beta^-_{gct})$; β^+_{gct} and β^-_{gct} are the dual variables of (7a) and (7b) respectively in Sect. 3.

The scarcity rent, or short term profit, is calculated as revenue from ALMPs, less production costs, less startup costs. The calculation includes the generator's or load's value as reserves in a contingency. Figure 5 presents the empirical distribution of generator scarcity rents. The generators that receive uplift payments are listed in Table 6 along with their uplift payments. Uplift payments can be caused by integer based

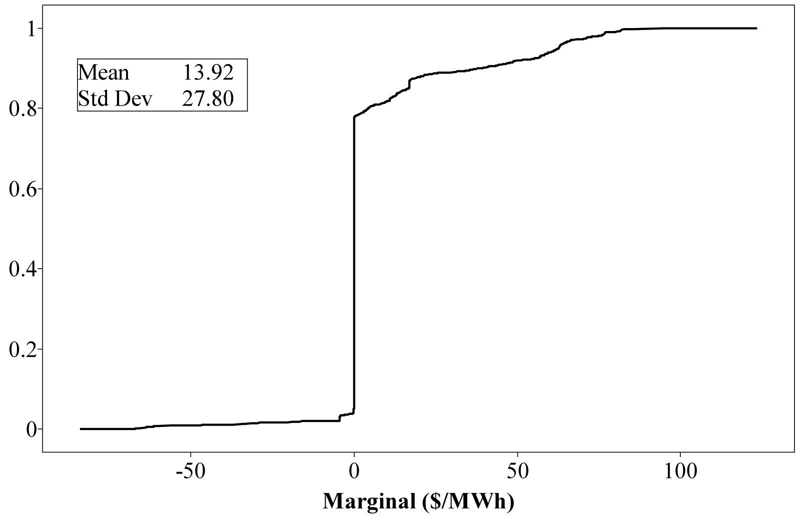


Fig. 4 Empirical distribution—value of an additional unit of capacity of lower minimum run level for generators

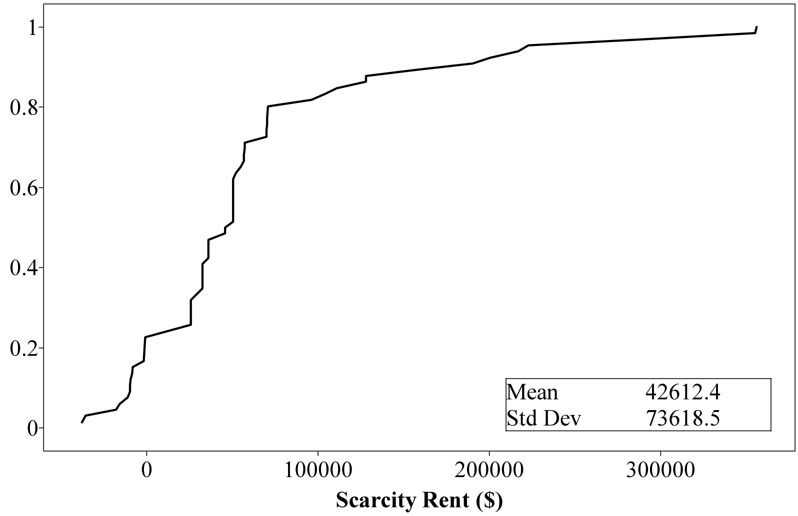


Fig. 5 Empirical distribution of scarcity rents (short term profits from LMPs)

constraints, such as minimum run levels and minimum up/down time constraints, as well as startup costs.

Table 6 Generators paid uplift

g	LMP revenue	Variable bid cost	Startup bid cost	LMP profits
45	231168	244332	0	−31164
46	169584	198644	0	−29061
78	226078	237376	0	−11298
75	96887	107951	0	−11064
9	20607	25495	4754	−9642
11	18287	23100	4754	−9567
10	16603	21000	4754	−9151
80	88328	90247	6510	−8429
12	105389	106795	6510	−7916
42	130514	136592	0	−6078
43	91587	91587	4754	−4754
44	44183	44183	4754	−4754
76	37457	37457	4754	−4754
77	29400	29400	4754	−4754
52	1618	2661	571	−1614
49	571	968	571	−968
50	571	968	571	−968
51	571	968	571	−968
53	571	968	571	−968

6.3 Transmission

We examine the transmission lines in a similar manner. Transmission lines have three constraints within the DCOPF problem: capacity in both directions and a line flow constraint, which contains the susceptance of the line and the phase angle at each end of the line. The flow capacity in one direction is the negative of the flow capacity in the other direction. Consequently, a relaxation in either direction will expand the feasible region and, therefore, may increase the surplus. Since $\eta^-_{kct}, \eta^+_{kct} \geq 0$, the marginal relaxation value in the negative direction is $-\eta^-_{kct}$. In this problem very few transmission branches are utilized at capacity; this is a result of enforcing all N-1 contingencies. Figure 6 shows the marginal value of transmission constraint relaxation, either increasing the upper (positive) bound or reducing the lower (negative) bound, $\eta_{kt} = \sum_c \eta_{kct} = \sum_c(-\eta^-_{kct} + \eta^+_{kct})$. For over 95% of the transmission constraint-hours, the marginal transmission capacity values are zero.

This optimal transmission switching with unit commitment formulation problem enforces all N-1 contingencies and transmission constraints have emergency thermal ratings to allow for additional utilization of transmission capacity during a contingency event. Very few transmission-contingency-hour combinations require a utilization of transmission capacity at or above the normal transmission line thermal rating.

The marginal susceptance value, μ_{kct} is the value of the marginal change in B_k in contingency c and period t. This marginal susceptance value is relevant when the susceptance is variable, e.g. for phase shifters or transformers. Let the aggregate

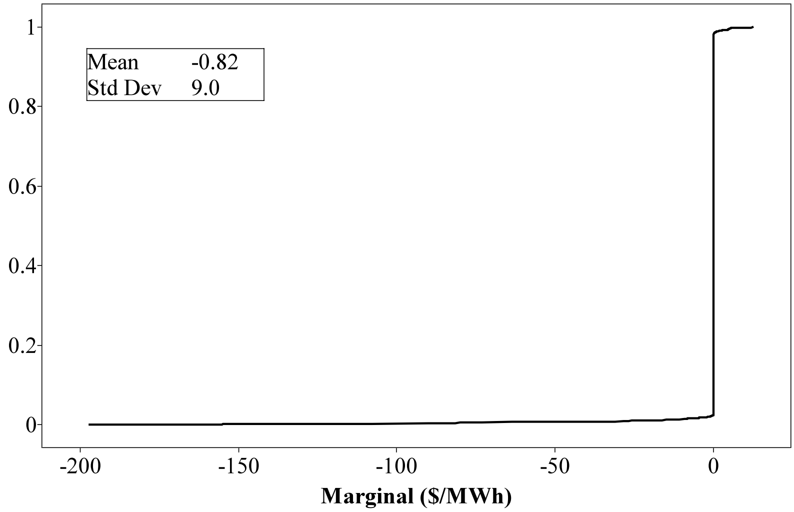


Fig. 6 Empirical distribution of transmission line relaxation values in single periods

Table 7 Largest and smallest marginal susceptance values

10 most negative marginal susceptance values			10 most positive marginal susceptance values		
Marginal value	t	k	Marginal value	t	k
−57.66	11	22	54.25	23	52
−42.83	10	98	25.89	8	52
−42.70	9	98	25.42	11	17
−35.12	21	115	23.51	21	116
−34.97	14	115	23.16	10	93
−34.28	16	115	23.10	9	93
−32.97	12	115	22.96	8	53
−32.97	13	115	22.27	11	7
−31.81	11	20	22.08	22	97
−30.55	16	97	21.71	14	116

marginal susceptance value for a given transmission line in a given hour be $\mu_{kt} = \sum_c \mu_{kct}$.

The 10 largest transmission line-hour marginal susceptance values (in either direction) are presented in Table 7. The empirical distribution of these marginal susceptance values by transmission line and hour is presented in Fig. 7.

After the MIP is solved, the integer variables are set to their solution values. This creates an LP with the constraint $z_{kt} = z^*_{kt}$. The dual variable of this constraint is, δ_{kt}, which is the marginal value of enforcing the binary transmission switching constraint. Section 4.3 discusses how the interpretation of this value varies based on the chosen LP formulation. The following results are based on an LP formulation with all line

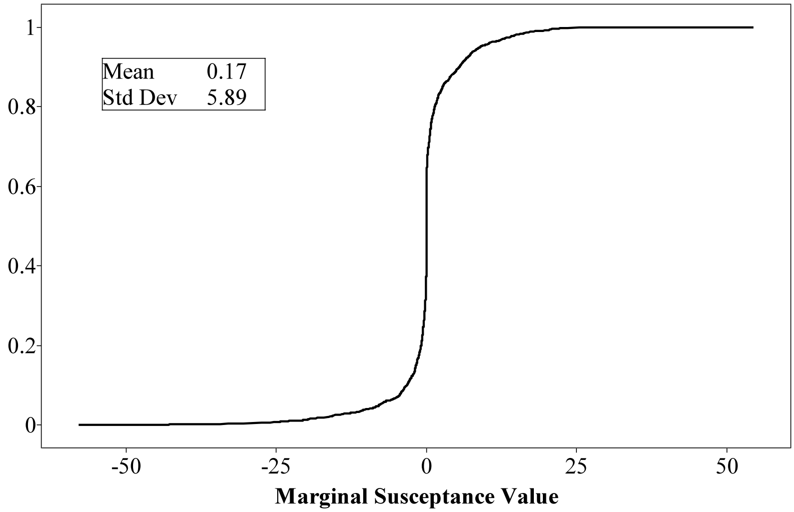


Fig. 7 Empirical distribution of marginal susceptance values in single periods

variables and constraints included no matter if the line is opened or closed, i.e. we do not reduce (5a) and (5b) to (5).

In NM1MIP, variable z_{kt} only affects (4a)–(6). When examining only its affect on (4a) and (4b), if you increase z^*_{kt} you are relaxing the original problem, i.e. by increasing z^*_{kt} in (6′) you are increasing the upper bounds in (4a) and (4b). If this was the only impact that z^*_{kt} has on the problem, then δ_{kt} must then be non-negative even though it is a dual variable for an equality constraint. However, z^*_{kt} also affects (5a) and (5b). When $z^*_{kt} = 0$, we know that (5a) and (5b) are not binding as the value of M_k is chosen to ensure this is the case when $z^*_{kt} = 0$. Therefore, increasing the value of z^*_{kt} when $z^*_{kt} = 0$ does not affect (5a) and (5b) since the constraints are inactive and would remain inactive for minor increases in z^*_{kt}. Therefore, we know that δ_{kt} will be non-negative when $z^*_{kt} = 0$. When $z^*_{kt} = 1$, (5a) and (5b) are active constraints so the same conclusion does not hold. In fact, for this RTS96 test case, when $z^*_{kt} = 1$ the δ_{kt} variables are all negative. When z^*_{kt} is increased beyond one, this causes infeasibility through (5a) and (5b). This, however, does not guarantee that δ_{kt} must be non-positive when $z^*_{kt} = 1$; rather, the results show that it is possible for δ_{kt} to be negative. For further discussion on the economic interpretation of δ_{kt}, refer back to Sect. 4.3. The distribution of δ_{kt} is presented in Fig. 8. Of course, the full incremental value of the asset is obtained by forcing z^*_{kt} to its opposite value and resolving the MIP. The difference in objective function value is then the true incremental value of the asset or the cost of forcing it into the optimal solution.

7 Conclusion

In this paper, we analyzed the multi-period N-1-reliable unit commitment and transmission switching MIP problem by fixing all integer variables to their optimal values

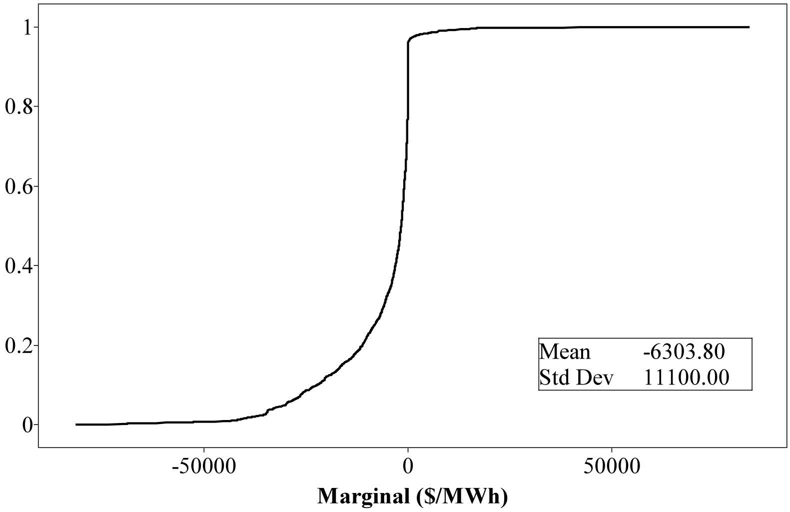


Fig. 8 Empirical distribution marginal transmission switching values

and, thus, forming an LP and its dual; we then presented economic interpretations and examined the sensitivity of this problem. The combination of duality theory and computational results of the marginal system costs of the RTS96 system unit commitment problem with N-1 reliability and transmission switching has been presented. This paper presents empirical distributions of the duality concepts of the transmission switching problem. These distributions describe the frequency by which marginal values occur in the solution. This information is a complement to the theory, and is intended to provide further insight into the economics of the formulation.

Sensitivity analysis can help to find relaxations of soft constraints and their interaction with reliability constraints that are also soft. Such control and flexibility can be categorized as a "smart grid application" where there is a co-optimization of both generator or loads and transmission topology. Empirical distributions of binding transmission constraint dual variables (presumably over a much longer period than 24 hours) can be used to predict relaxation costs that will lead to infrequent relaxations that can improve market efficiency. These predictions could be used in conjunction with analytical computation of contingency probabilities, and analysis of conductor properties to create relaxation costs that allow thermal constraint relaxations for limited periods of time within reliability rules, while maintaining an acceptable expected loss of life on conductors. Such a methodology could lead to thermal overload pricing. RTOs today do relax transmission line thermal ratings for a predefined price; for instance, $4,000/MWh is used in NYISO, $5,000/MWh in CAISO, and $2,000/MW in SPP [23–25]. An improved methodology for determining these relaxation prices, which analytically attempts to balance the tradeoffs between solution feasibility, economics, and system reliability, could be an important subject for further research.

The probability of an individual contingency is very low and the requirement of N-1 is to guarantee load can be served when there is any single contingency. The emphasis in this paper is on minimizing the current forward dispatch cost and surviving the contingencies. Essentially, the objective does not include the expected re-dispatch cost that is incurred once or if there is a contingency. Extending this research to investigate the costs associated with probabilistic contingency events due to random generator failures may provide insight into possible settlement mechanisms where stochastic events are considered. The operational costs in each contingency state could be considered in a formulation that maximizes expected surplus if the probability distribution of contingencies is correctly defined. Distributions that account for uncertainty in load and the variable output of renewable resources could also be included. This is especially relevant for Day Ahead Markets.

Better modeling for both reliability and market efficiency increases societal benefits. Optimal transmission switching and intelligent constraint relaxations that appropriately factor in risks associated with expected line loss of life due to low probability overloads, while allowing for an increase in total surplus, could eventually be part of a smarter and more flexible electric grid. Hardware upgrades to enable these concepts are relatively cheap, and more efficient software has very low marginal costs, most of which are maintenance costs. In this paper we have demonstrated, through computational results and sensitivity analysis, an economic investigation of these modeling enhancements on the IEEE 73-bus test system (RTS96) over 24 hours. A model of this size provides useful insights, but larger models would prove to be even more informative. Further savings may be obtained if thermal constraint relaxations with an associated cost were incorporated into the overall MIP formulation, thereby allowing relaxations to affect the unit commitment and transmission switching decisions. Further analysis could be performed by extending the formulation and analysis methods presented in these two papers onto larger models representative of actual RTO/ISO markets. Such analysis could potentially lead to policy recommendations relating to transmission switching, relaxation of constraints, and relaxation costs in RTO/ISO markets.

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