



An alternative method for predicting internal friction angle of rock materials

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Abstract

The shear strength properties of rock materials, cohesion and internal friction angle, are determined by carrying out tri-axial strength test on cylindrical core specimens in laboratory. But determination of these parameters by triaxial tests in accordance with standards and suggested methods, particularly for weak, fractured and weathered rocks is extremely difficult and/or impossible due to difficulties related to preparation of test specimens suitable for this test. In addition, the tri-axial test requires high cost equipment and too much time for sample preparation and testing. In such cases, there is a need to precisely estimate the friction angle and estimation of rock shear strength properties using some indirect methods, as they are economical and easy to carry out. In this study, the traditional method, which is recommended to be used for the prediction of internal friction angle (ϕ) when triaxial test data is not available, was briefly assessed with its some limitations and an alternative method using theoretical tensile strength and uniaxial compressive strength to predict ϕ was proposed. Then the prediction performances of traditional and proposed methods were compared using a very large data set collected from published literature. The statistical reliability of the derived equations was assessed using F- and t-tests and according to the test results the prediction equations were found to be statistically reliable. The results indicated that the method proposed in this study using the theoretical tensile strength yields best predictions of ϕ when compared to those estimated from the traditional methods based on direct and Brazilian tensile strength values.

Keywords Internal friction angle · Rock material · Shear strength · Uniaxial compressive strength · Tensile strength

Introduction

The internal friction angle is one of the two main shear strength parameters of rock materials, determines the rate of increase in shear strength depending on normal stress and is used in many rock engineering applications. Shear strength of rock materials is often determined by Mohr–Coulomb (MC) failure criterion. The shear strength properties of rock materials, cohesion and internal friction angle, are determined by carrying out tri-axial strength test on cylindrical core specimens in laboratory and the test results are represented by Mohr Circles. The friction angles of rock specimens tested are obtained from the slopes of the

Mohr–Coulomb envelopes drawn as a tangent to the Mohr circles.

The theory of graphical representation of stresses in the form of Mohr circles is available in most standard textbooks on rock mechanics (i.e. Jaeger and Cook 1979) and the testing method for tri-axial test is given in standards (i.e. ASTM 2023) and suggested methods (ISRM 1981, 2007).

However, it is not always possible to determine this parameter by triaxial tests in accordance with standards and suggested methods, particularly for weak, fractured and weathered rocks from which preparation of cylindrical core specimens suitable for tri-axial test is extremely difficult and/or impossible. In addition, the tri-axial test requires high cost equipment and too much time for sample preparation and testing.

In such cases, there is a need to precisely estimate the friction angle and estimation of rock shear strength properties using some indirect methods, as they are economical and easy to carry out. For this purpose, there are two approaches. The first approach is the traditional method using Mohr

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circles, while the second approach is to obtain the internal friction angle from different parameters of rock materials such as strength, deformability or index properties.

In the first approach, the shear strength properties (c_{indr} and ϕ_{indr}) of rock materials are indirectly determined with the help of tangents drawn to the Mohr circles of the strength properties by considering the stress states effective at failure ($\sigma_1 = \sigma_c$ and $\sigma_3 = 0$ for uniaxial compressive strength -UCS- test, $\sigma_1 = 0$ and $\sigma_3 = -\sigma_t$ for direct tensile test and $\sigma_1 = 3\sigma_t$ and $\sigma_3 = -\sigma_t$ for Brazilian tensile test) for tensile and uniaxial compression tests as illustrated in Figs. 1a and b, respectively.

The magnitude of compressive stress is three times of the tensile stress at failure in Brazilian tensile test (Goodman 1989; Piratheepan et al. 2012; Serati et al. 2014). However, Serati et al. (2014) also stated that this ratio is valid for centre of the disc and stress tensor becomes almost negligible at relatively large distance from the load contact area. Therefore, the value of tensile strength determined from the Brazilian test is generally higher than the actual value (Li and Wong 2013). On the other hand, direct tensile test is not widely used due to difficulty of preparing test specimens and pure one-dimensional direct tensile loading (Gong et al. 2019).

The formulation of traditional method summarized above was derived by Piratheepan et al. (2012) and Sivakugan et al. (2014). Piratheepan et al. (2012) used BTS and UCS to determine the cohesion and internal friction angle, which are traditionally used and obtained by drawing tangents to Mohr circles. These researchers have formulated the theoretical equations (Eqs. 1 and 2), which allow the calculation of shear strength parameters of rock material using UCS and BTS test data, taking into account the stress state effective

at the time of failure for the Brazilian tensile (Indirect Diametrical Tensile-IDT) and uniaxial compression tests. The values of cohesion and internal friction angle obtained from these equations are the same as the values found from the traditional method, in which shear strength properties (c and ϕ) are determined by drawing tangents to Mohr circles. However, the proposed theoretical equations provide a more practical determination of the shear strength properties and the limitations of the traditional method are also valid for these equations, where σ_c is the UCS, σ_{IDT} is the tensile strength obtained from IDT test.

$$\sin\phi = a = \left(\frac{\sigma_c - 4\sigma_{\text{IDT}}}{\sigma_c - 2\sigma_{\text{IDT}}} \right) \tag{1}$$

$$c = \frac{\sigma_c(1 - a)}{2\cos\phi} \tag{2}$$

Sivakugan et al. (2014), conducted a theoretical study similar to that of Piratheepan et al. (2012) and derived Eqs. 3 and 4 for theoretical shear strength properties by considering the loading conditions applied in UCS (σ_c) and BTS (σ_t) tests.

$$\phi = \sin^{-1} \left(\frac{\sigma_c - 4\sigma_t}{\sigma_c - 2\sigma_t} \right) \tag{3}$$

$$c = \frac{0.5\sigma_c\sigma_t}{\sqrt{\sigma_t(\sigma_c - 3\sigma_t)}} \tag{4}$$

Sivakugan et al. (2014) compared the c and ϕ values determined from Eqs. 3 and 4 with the results of laboratory experiments using 35 rock specimens. These researchers

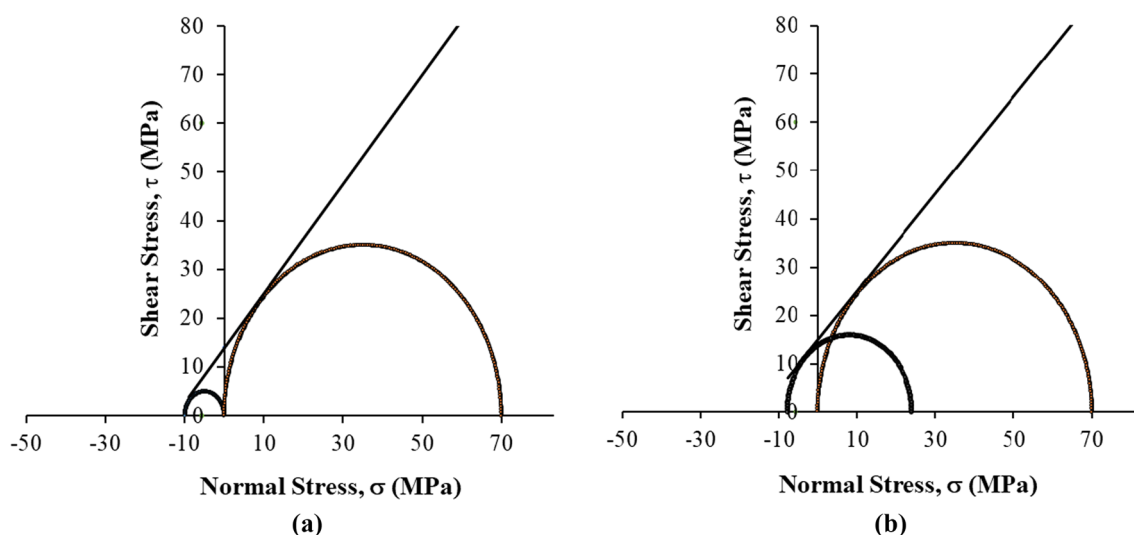


Fig. 1 Mohr envelope of a rock material obtained from the use of **a** UCS and direct tensile strength (DTS) test data and **b** UCS and Brazilian tensile strength (BTS) test data

stated that the cohesion values determined from these equations are more realistic than the internal friction angle values determined from the laboratory tests. Sivakugan et al. (2014) also obtained a few negative and very low internal friction angles using the traditional method. This situation revealed an important limitation of the traditional approach.

In the second approach, the shear strength properties are predicted by other parameters (such as UCS, point load strength index (I_{s50}), Poisson’s ratio etc.). In several studies (Turk and Dearman 1986; Esterhuizen et al. 2013; Armaghani et al. 2014), which were carried out to date, the issue of estimation of shear strength properties has been considered. Armaghani et al. (2014) carried out a research on predicting the shear strength properties of shales using some index properties, artificial neural networks and multivariate regression analyses. The index properties used by these researchers for predicting the shear strength properties (c , ϕ) were dry density (ρ_d), point load strength index ($I_{s(50)}$), Schmidt rebound value (SHn), Brazilian tensile strength ($\sigma_{t(Brazilian)}$) and P-wave velocity (V_p). The multivariate prediction equations derived by Armaghani et al. (2014) are given in Eqs. 5 and 6. Although these researchers obtained a high coefficient of determination in their study, the results are valid only for the shale rock unit used in their investigation. The necessity of determining a large number of index features to estimate the friction angle is another limitation of this study.

$$\phi = 0.0078 \rho_d - 0.106 \text{SHn} + 0.471 \sigma_{t(Brazilian)} + 2.181 I_{s(50)} + 0.004 V_p - 3.63 \tag{5}$$

$$c = -0.0004 \rho_d - 0.037 \text{SHn} + 0.448 \sigma_{t(Brazilian)} - 0.0496 I_{s(50)} - 0.003 V_p + 16.24 \tag{6}$$

Turk and Dearman (1986) stated that internal frictional angle of rock materials can be predicted from strain properties measured under uniaxial loading and recommended the following relationship between internal friction angle and Poisson’s ratio (ν). However, in order to use this relationship, strain measurements should be carried out.

$$\sin\phi = \frac{1 - \nu}{1 + \nu} \tag{7}$$

Esterhuizen et al. (2013) proposed the empirical equations given in Table 1 for predicting the internal friction angle using UCS depending on rock types and ranges of UCS.

Without any order implied by the principal stresses, the Mohr–Coulomb criterion can be expressed as given in Eq. 8 (Labuz and Zang 2012).

Table 1 The empirical equations recommended by Esterhuizen et al. (2013) to estimate internal friction angle of some rock types depending on ranges of uniaxial compressive strength (UCS)

Rock type	Prediction Equation	Applicable UCS (MPa) range
Shale and claystone	$\phi = 0.090 \text{ UCS} + 15$	10–80
Siltstone and sandstone	$\phi = 0.145 \text{ UCS} + 25$	40–200
Limestone	$\phi = 0.090 \text{ UCS} + 33$	40–200

$$\pm \frac{\sigma_1 - \sigma_2}{2} = a \frac{\sigma_1 + \sigma_2}{2} + b, \pm \frac{\sigma_2 - \sigma_3}{2} = a \frac{\sigma_2 + \sigma_3}{2} + b, \pm \frac{\sigma_3 - \sigma_1}{2} = a \frac{\sigma_3 + \sigma_1}{2} + b \tag{8}$$

where, $a = \frac{m-1}{m+1}$, $m = \frac{C_o}{T_o} = \frac{1+\sin\phi}{1-\sin\phi}$, $b = \frac{1}{m+1}$, $C_o = \frac{m}{m+1}$, $T_o = \frac{C_o}{2} (1 - \sin\phi)$ ve $0 \leq a < 1$.

T_o is the theoretical Mohr–Coulomb uniaxial tensile strength, and experimentally, a much lower tensile strength value is generally obtained when the failure plane is perpendicular to the direction of σ_3 ($\sigma_1 = 0$, $\sigma_3 = -T$). C_o is the theoretical MC UCS that is generally close to the experimental value, therefore, another symbol is not used (Labuz and Zang 2012).

The actual internal friction of rock materials can be determined by drawing the MC failure envelope obtained under compressive confining stresses ($\sigma_3 > 0$). However, if there is no data for confining stress of $\sigma_3 > 0$, the failure envelope can be drawn by the indirect method, which consider the Mohr circles of UCS and tensile strength. As given in Fig. 2, the failure envelope is tangent to the Mohr circle of the theoretical tensile strength test according to MC criterion. Therefore, in order to calculate the actual internal friction angle, the theoretical tensile strength (T_o) should be determined as well as the UCS. However, for this purpose, Mohr circles of direct or Brazilian tensile strength tests are generally used.

BTS is approximately 1.24 times greater than the direct tensile strength (AlAwad 2022), therefore, there is an order between these parameters as $\phi (\sigma_{t(direct)}) > \phi (\sigma_{t(Brazilian)}) > \phi (T_o) = \phi_{(actual)}$ (Fig. 3). In other words, since the experimentally determined values of tensile strength are lower than the theoretical value, the indirectly obtained internal friction angle values are generally determined to be greater than the actual internal friction angle value of the rock material. The level of this difference is controlled by both C_o/T_o and $T_o/\sigma_{t(exp)}$. For this reason, in order to estimate the internal friction angle with a high precision, it is necessary to determine the theoretical tensile strength rather than directly using the experimental tensile strength. If the theoretical tensile strength can be

Fig. 2 Mohr envelope obtained by using Mohr circles of theoretical uniaxial tensile and uniaxial compression strength test data (redrawn from Labuz and Zang 2012)

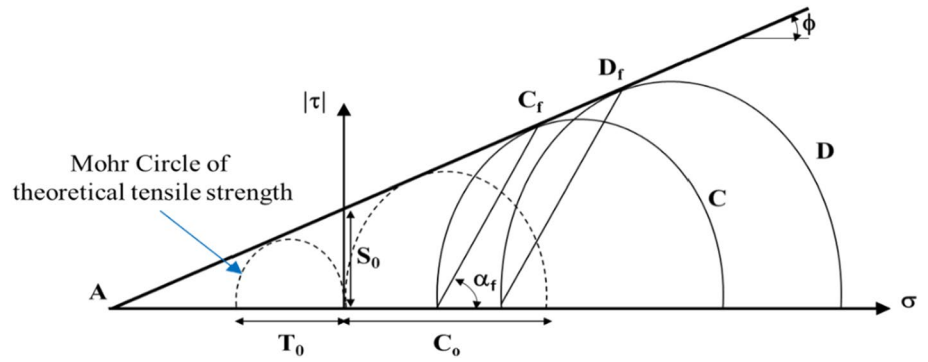
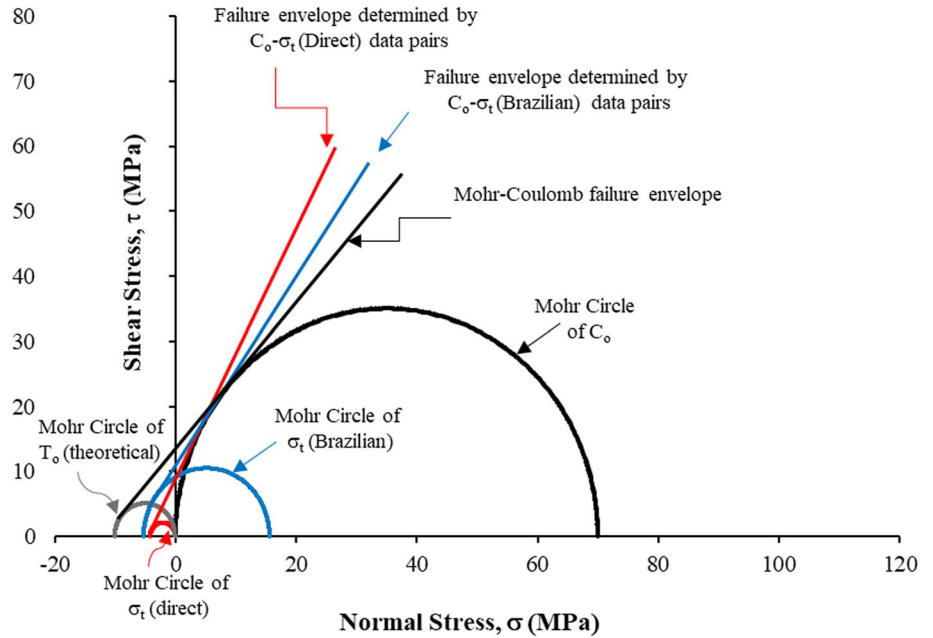


Fig. 3 The comparison of MC failure envelopes obtained using different types of tensile strength (theoretical, direct, Brazilian)



predicted with a high precision, it will also be possible to estimate the internal friction angle with a high precision. By considering this, this study aims to estimate the theoretical tensile strength and internal friction angle using a great number of data of different rock types available in literature.

In this study, first the proposed method was described and a database was compiled from previous studies. The database consists of internal friction angle (or major and minor principal stress data from triaxial tests), tensile strength (obtained from direct or Brazilian tensile tests) and UCS of rock materials. In order to predict the theoretical tensile strength (T_0), cohesion and internal friction angle values (c_{indr} and ϕ_{indr}) determined from the traditional method and UCS were used. In order to check the statistical reliability of the relationships found from the proposed method for

predicting the theoretical tensile strength, statistical analyses (F- and t-tests) were carried out. Then, the values of internal friction angle were calculated using method proposed in this study and they were compared with experimental ϕ values in the database used. Finally, the main conclusions drawn from this study and the recommendations for further studies were given.

Proposed method

In order to calculate the actual internal friction angle using the traditional method, as shown in Fig. 2, the theoretical tensile strength (T_0) should be determined as well as the C_0 . According to the MC failure criterion, the theoretical

tensile strength value is defined as given in Eq. 9 (Labuz and Zang 2012).

$$T_o = \frac{C_o}{2}(1 - \sin(\phi)) \tag{9}$$

where, C_o is the UCS (considering that it is equal to the experimental value, $C_o \approx \sigma_c$), ϕ is the internal friction angle, and T_o is the theoretical uniaxial tensile strength value.

As can be seen from Eq. 10, which is obtained by rearranging Eq. 9, the internal friction angle can be determined if the values of UCS (C_o) and theoretical tensile strength (T_o) are known.

$$\phi = \arcsin\left(1 - \frac{2T_o}{C_o}\right) \tag{10}$$

Alternatively, if the theoretical tensile strength is determined, the internal friction angle can be determined from the Mohr circles to be drawn (see Fig. 2). Therefore, accurate estimation of T_o is very important.

In this study, statistical analyses were carried out regarding the estimation of T_o using 153 UCS and tensile strength data pairs of different rock types which are given in the following section of this paper. The results of the statistical analyses indicated that the use of the values of cohesion (c_{indr}) and internal friction angle (ϕ_{indr}) as independent variables improved the prediction performance of T_o .

The relationships with the highest prediction performance obtained using the independent variable $[(C_o - c_{indr})/\phi_{indr}]$ are shown in Fig. 4 and given in Table 2. The statistical reliability and significance of these relationships were also examined using F- and t tests. As can be seen from Tables 3 and 4, all significance values for the 95% confidence level

Table 2 Exponential relationships determined between T_o and $(C_o - c_{indr})/\phi_{indr}$

	The traditional method used	
	$C_o - \sigma_t$ (Brazilian)	$C_o - \sigma_t$ (direct)
Prediction equation	$T_o_{\text{predict}} = 9.2935((C_o - c_{indr})/\phi_{indr})^{0.8592}$	$T_o = 10.232((C_o - c_{indr})/\phi_{indr})^{0.82}$
R^2	0.876	0.862
r	0.94	0.93

were determined less than 0.05. Therefore, the prediction equations derived for the estimation of T_o are considered as statistically reliable and significant.

The predicted values of T_o are obtained using the equations given in Table 2. The values of cohesion and internal friction angle used in these equations are determined by traditional method shown in Fig. 1, and the UCS is the experimentally determined value. In other words, the values of cohesion and internal friction angle are indirectly obtained using the traditional method given in Fig. 1 and are used as input variables for the prediction of T_o in the method proposed in this study.

The internal friction angle is calculated by substituting the predicted T_o values and experimental UCS values into Eq. 10. At the same time, the values of internal friction angle and cohesion can be determined by drawing Mohr circles of experimentally determined UCS and theoretical tensile strength test data. Since it allows the internal friction angle to be calculated more precisely and practically and for the purpose the use of Eq. 10 is recommended. The flow chart summarizing the calculation stages in the proposed method is depicted in Fig. 5.

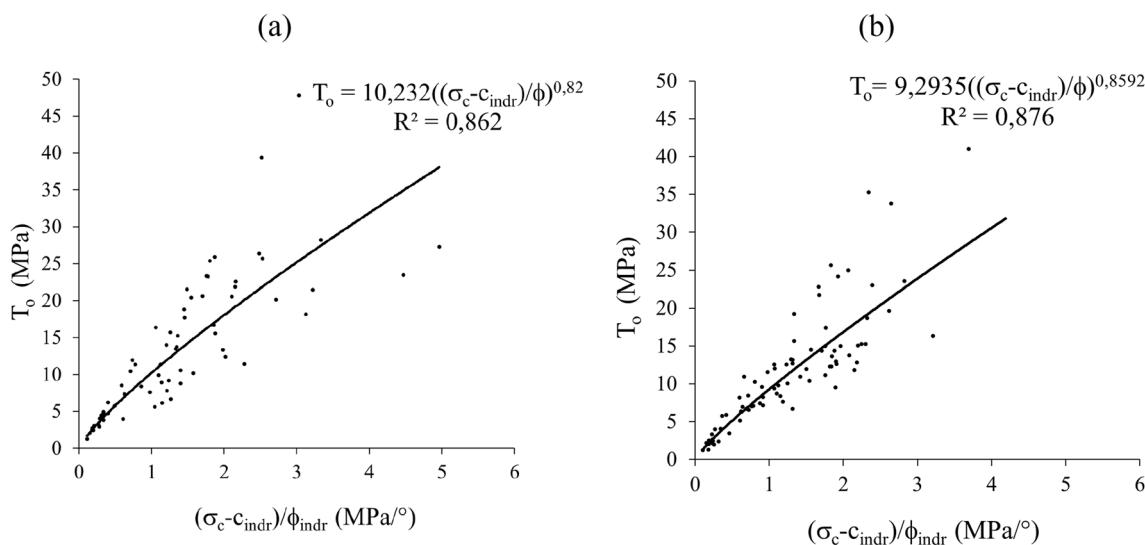


Fig. 4 Exponential relationships determined between T_o and $(C_o - c_{indr})/\phi_{indr}$ using **a** direct tensile and **b** Brazilian tensile test data

Table 3 F- and t test results regarding the equation obtained using direct tensile test data

F-test					
	Sum of squares	df	Mean square	F	Significance
Regression	39.327	1	39.327	430.929	0.000
Residual	6.297	69	0.091		
Total	45.624	70			
t test					
	Regression coefficients		Standardized coefficients	t	Significance
	B	Std. error			
$\ln(x_{\text{direct}})$	0.820	0.039	0.928	20.759	0.000
Constant	10.232	0.367		27.870	0.000

Table 4 F- and t test results regarding the equation obtained using Brazilian tensile test data

F-test					
	Sum of squares	df	Mean square	F	Significance
Regression	40.89	1	40.89	565.245	0.000
Residual	5.79	80	0.072		
Total	46.67	81			
t test					
	Regression coefficients		Standardized coefficients	t	Significance
	B	Std. error			
$\ln(x_{\text{indirect}})$	0.859	0.036	0.936	23.775	0.000
(Constant)	9.294	0.276		33.669	0.000

Data collection

In this study, a total of 153 strength data sets (internal friction angle, UCS and tensile strength) were compiled from the published literature. The rock materials of which data were used in this study belong to different rock groups (sedimentary, igneous and metamorphic and volcano-sedimentary).

As emphasized in the Introduction section, Brazilian tensile strength is approximately 1.24 times greater than the direct tensile strength for various rock types (AlAwad 2022). The ratio between the values of theoretical tensile strength, which were calculated by taking into account the experimental values used in this study, and the direct and Brazilian tensile strength are 2.33 and 1.79, respectively. Therefore, the Brazilian tensile strength is 1.30 (2.33/1.79) times greater than the direct tensile strength considering the data used in this study.

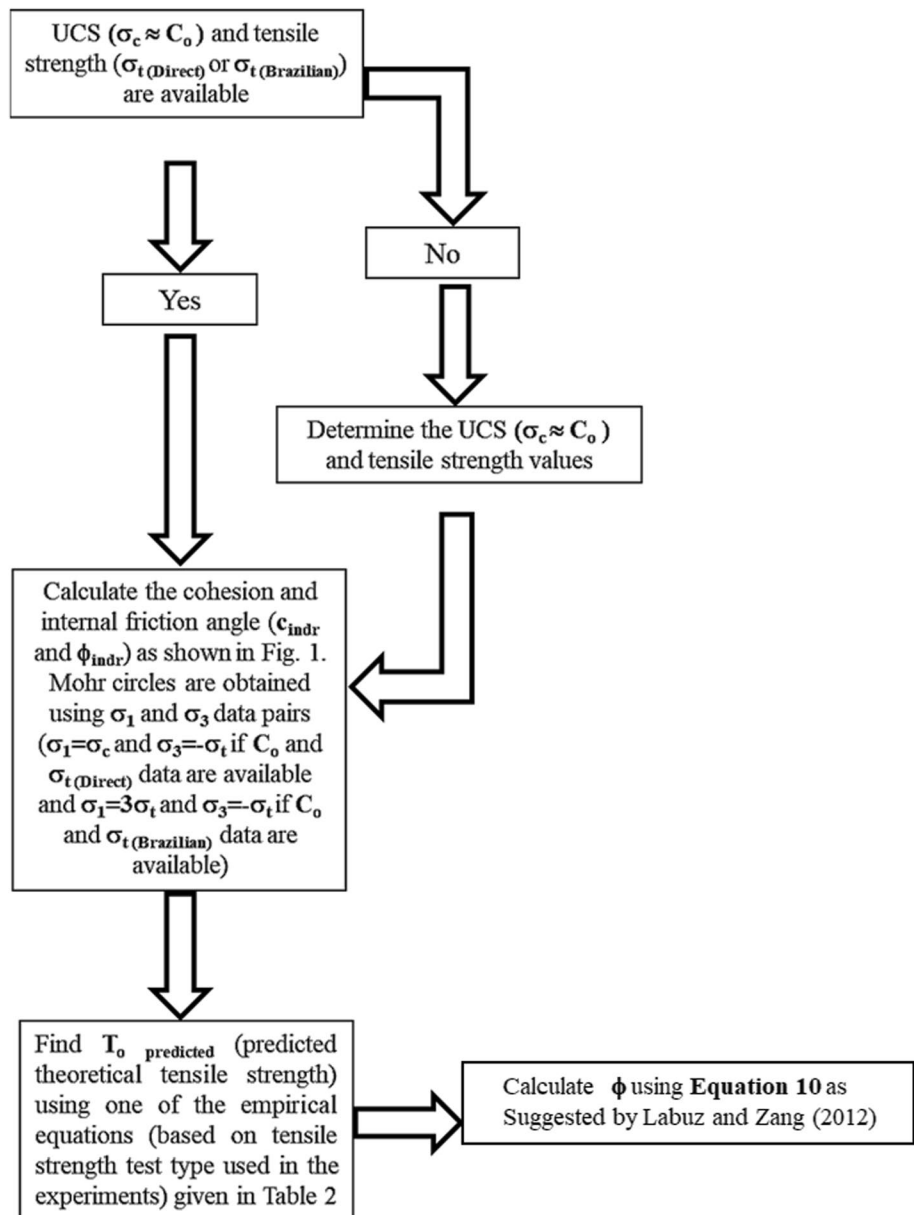
Since the tensile strength values obtained from the Brazilian and direct tensile tests are different from each other,

it is impossible to create a unique prediction equation. Therefore, two different groups were constructed considering the type of the tensile strength tests. In these data set, the number of the Brazilian and direct tensile strength tests are 82 and 71, respectively.

The availability of data for a large number of rock types belonging to sedimentary, igneous, metamorphic and volcano-sedimentary rock types in two different groups is an important advantage in terms of proving the validity of the proposed equations for different rock types.

Since the direct tensile test has not been widely used for a long time, except the data from Bell and Jermmy (2000), all direct tensile strength test data were collected from Shoerey (1997). The data set of Shoerey (1997) is available in his book entitled "Empirical Rock Failure Criteria" and the internal friction angle values (ϕ_{exp}) were calculated from the triaxial test data given in Appendix A of this book. Cohesion and internal friction angles (c_{indr} and ϕ_{indr}) were calculated from the traditional method using UCS and direct tensile strength data as shown in Fig. 1a

Fig. 5 Flow chart showing the steps followed to estimate internal friction angle using the method proposed in this study



(UCS and Brazilian tensile strength data are as given in Fig. 1b), experimentally determined internal friction angle (ϕ_{exp}) were calculated using triaxial test data (ϕ_{exp} values are obtained directly from sources given in Appendix 2), theoretical tensile strength (T_o) were calculated using ϕ_{exp} and UCS based on Eq. 9, predicted theoretical tensile strength ($T_{o(pre)}$) was calculated from Equation given in Table 2, predicted internal friction angle ϕ_{pre} was calculated from Eq. 10 and error values are given in Appendices 1 and 2. Since the Brazilian test is a widely used and popular test method, it could be possible to provide Brazilian tensile strength data in many published studies.

Estimation of internal friction angle from the proposed method

In this study, the values of internal friction angle estimated from the traditional method shown in Fig. 1 and the experimentally determined internal friction angles were compared for all rock types in the Appendix given at the end of this paper. The statistical data obtained from this comparison is given in Table 5.

As can be seen from Table 5, the error margins obtained from the traditional methods are quite high. In particular, the values of internal friction angle obtained from the

Table 5 Mean error and standard deviation values for internal friction angle (ϕ_{indr}) values obtained from indirect methods

	The name of traditional method	
	$C_0 - (\approx\sigma_c) - \sigma_t$ (Brazilian)	$C_0 - \sigma_t$ direct
Number of data	82	71
Mean absolute error	8.59°	21.32°
Standard deviation	6.03°	10.70°

Table 6 The values of mean error and standard deviation estimated from the method proposed in this study

	Proposed Method	
	$C_0 - \sigma_t$ (Brazilian)	$C_0 - \sigma_t$ direct
Number of specimen	82	71
Mean absolute error	5.43°	5.74°
Standard deviation	4.02°	3.99°

failure envelopes drawn using the data of direct tensile and UCS tests differ from the experimental values with an average of 21.32°. On the other hand, the mean absolute error obtained from the data of Brazilian tensile strength and UCS tests is 8.59° which is a significant deviation from the experimental values. If the standard deviations are taken into account, much higher prediction errors are

obtained when compared to the values of average error given above. For this reason, the values of friction angle obtained from traditional approaches (see Fig. 1) are far from the experimental values and do not represent experimental values. The mean errors and standard deviations obtained from the method proposed in this study are given in Table 6. The low values of mean error and standard deviation indicate that it is possible to estimate internal friction angles with higher precision by the proposed method when compared to the other traditional indirect estimation approaches.

The frequency histograms for the error and absolute error values are given in Fig. 6. On these histograms, normal distribution curves and vertical lines corresponding to 1 and 2-standard deviation error levels are also shown by different colors. By considering the 1-standard deviation from the mean (corresponding to approximately 70% of the data obtained), the absolute error ranges obtained in the estimation of internal friction angle are 9.7–31.1° and 2.56–14.62° for the traditional method based on direct tensile and Brazilian tests, respectively, suggesting that a very high prediction error such as 31.1° is possible for the estimation of internal friction angle. On the other hand, the prediction error of 14.62° is possible for the estimation of internal friction angle by traditional method based on Brazilian test data within the interval of 1-standard deviation away from the mean. If the outlier values (absolute error values within 2-standard

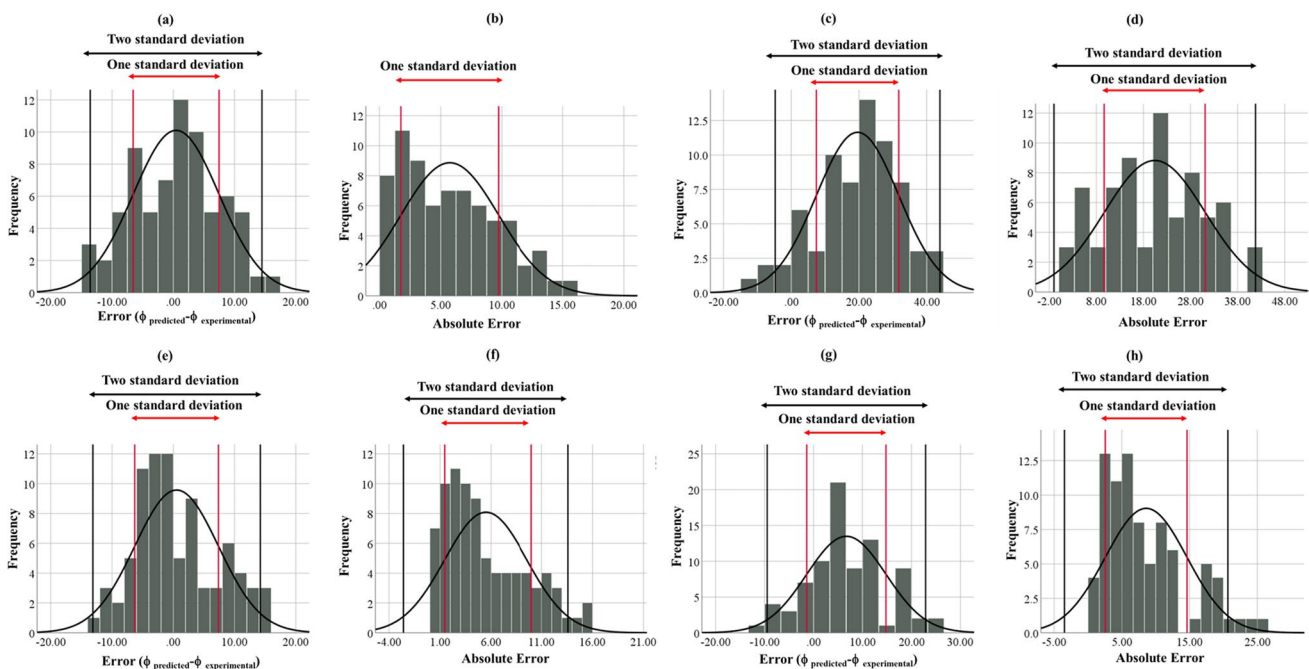


Fig. 6 Histograms of error and absolute error values obtained from **a, b** the proposed method using direct tensile test data, **c, d** the traditional method using direct tensile test data, **e, f** the proposed method

using the Brazilian test data, and **g, h** the traditional method using Brazilian test data

deviations from the mean) are taken into account, absolute prediction error values are 41.8° and 20.65° for the traditional method based on direct tensile and Brazilian tests, respectively. However, the values of mean absolute error are about 5° for the method proposed in this study. Moreover, the values of prediction errors can only reach to maximum values of 9.73° and 9.56° within the interval of 1-standard deviation for the proposed method based on direct tensile and Brazilian tests, respectively. For this reason, it is considered that the method proposed in this study has a higher performance for predicting the internal friction angle of rock materials when compared to the traditional methods.

On the other hand, if the error values given in Fig. 6c are carefully examined, it can be considered that the error values in traditional predictions using direct tensile tests ($\phi_{\text{prediction}} - \phi_{\text{experimental}}$) are completely positive, with a few exceptions. It means that the values of overestimated internal friction angle are calculated by traditional method based on direct tensile tests. As can be seen from the error values obtained from the proposed method (Fig. 6a), there is an error distribution fitting to the normal distribution with a mean close to zero. The comparison of error values obtained for each rock type is also given in Fig. 7. As can be clearly seen from this figure, the errors obtained from the proposed

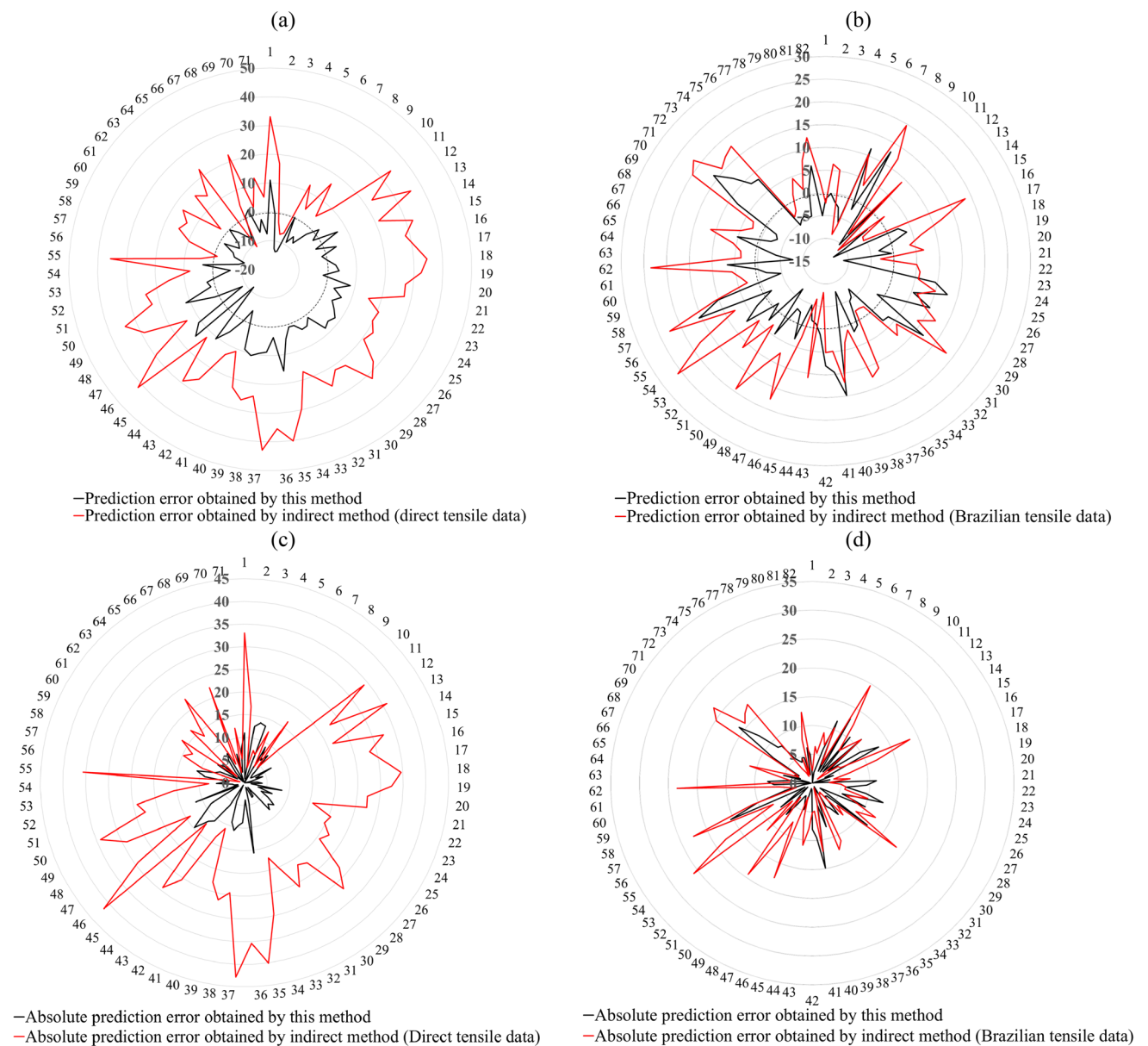


Fig. 7 Comparison of error and absolute error values obtained from different methods for each rock type: errors and absolute errors obtained using **a, c** direct tensile strength data and **b, d** Brazilian tensile strength data

method are generally close to zero, while the errors obtained from the traditional methods change in a much wider range. This range includes errors above 40° and 25° when direct tensile strength and Brazilian tensile strength data is used, respectively.

Conclusions

In this study, the traditional method, which is recommended to be used for the prediction of internal friction angle (ϕ) when triaxial test data is not available, was briefly assessed with its some limitations and an alternative method using theoretical tensile strength (T_0) and UCS to predict ϕ was proposed. The prediction performances of traditional and proposed methods were compared using a very large data collected from published literature. The main conclusions drawn from the study are given below.

The estimates for ϕ from the traditional method based on UCS and tensile strength yield very high absolute errors and standard deviations. Particularly, the values of internal

friction angle predicted from the traditional method based on direct tensile strength data can reach approximately 1.9 times the experimental values for a few rock types. So it can be inferred that traditional method based on direct tensile strength data for the prediction of ϕ is generally inappropriate.

The prediction equations derived to estimate theoretical tensile strength needed for proposed method have very high correlation coefficient. The statistical reliability of the derived equations was assessed using F- and t-tests and according to the test results the prediction equations were found to be statistically reliable.

The average ratios between ϕ values predicted from the proposed method and the values of experimentally determined ϕ values were found to be 1.03 and 1.04 for direct tensile strength data and Brazilian tensile strength data, respectively. Therefore, the method proposed in this study using the theoretical tensile strength yields best predictions of ϕ when compared to those estimated from the traditional methods based on direct and Brazilian tensile strength values.

Appendix 1

The parameters calculated using C_0 and $\sigma_{t(Direct)}$ data pairs [All ϕ_{exp} values in this table, Except the data from Bell and Jermy (2000), all data was derived from the triaxial test data in Sheorey (1997)].

References	Rock type	ϕ_{indr}	c_{indr}	ϕ_{exp}	T_0	$T_{0(pre)}$	ϕ_{pre}	Error (this method)	Absolute error (this method)	Error (indirect method)	Absolute error (indirect method)
Barat (1995)	Schist	56.45	14	54.15	8.81	13.46	45.27	8.88	8.88	- 4.49	4.49
Bell and Jermy (2000)	Dolerite	60.76	18.4	55.5	12.43	18.21	47.92	7.58	7.58	- 5.26	5.26
Betourney et al. (1991)	Quartzite	58.13	23.99	43.34	26.39	21.54	48.07	- 4.73	4.73	- 29.61	29.61
Betourney et al. (1991)	Andesite	66.55	23.37	35.14	47.82	25.41	50.77	- 15.63	15.63	- 40.12	40.12
Betourney et al. (1991)	Diorite	56.98	21.43	43.45	22.60	19.25	47.22	- 3.77	3.77	- 35.42	35.42
Betourney et al. (1991)	Basalt	53.44	16.35	36	20.42	14.63	44.81	- 8.81	8.81	- 42.83	42.83
Betourney et al. (1991)	Rhyolite	53.05	18.71	35.76	23.29	16.25	45.24	- 9.48	9.48	- 24.47	24.47
Betourney et al. (1991)	Quartzdiorite	58.54	21.83	58.56	11.41	20.09	47.84	10.72	10.72	1.03	1.03
Betourney et al. (1991)	Granite	62.96	26.91	56.93	18.17	26.07	50.12	6.81	6.81	- 9.74	9.74
Betourney et al. (1991)	Sandstone	65.63	15.74	54.84	13.32	17.93	48.97	5.87	5.87	- 14.74	14.74

References	Rock type	ϕ_{indr}	c_{indr}	ϕ_{exp}	T_0	$T_{0(\text{pre})}$	ϕ_{pre}	Error (this method)	Absolute error (this method)	Error (indirect method)	Absolute error (indirect method)
Betourney et al. (1991)	Limestone	55.61	8.73	44.65	8.39	9.02	42.88	1.77	1.77	– 13.12	13.12
Betourney et al. (1991)	Gneiss	65.12	36.04	58.94	23.47	34.90	51.88	7.06	7.06	– 16.61	16.61
Betourney et al. (1991)	Granodiorite	48.27	15.87	48.33	10.55	13.46	42.62	5.71	5.71	– 2.14	2.14
Betourney et al. (1991)	Limestone	59.69	17.35	47.68	16.71	16.99	47.31	0.37	0.37	– 16.54	16.54
Betourney et al. (1991)	Quartzdiorite	58.27	26.22	51.4	20.18	23.22	48.47	2.93	2.93	– 13.59	13.59
Betourney et al. (1991)	Granite breccia	65.03	40.19	58.19	27.31	38.07	52.24	5.95	5.95	– 22.61	22.61
Betourney et al. (1991)	Gneiss	27.28	13.62	36.86	8.95	11.39	29.39	7.47	7.47	10.89	10.89
Betourney et al. (1991)	Diorite	39.98	13.72	37.8	11.39	11.29	38.04	– 0.24	0.24	– 4.54	4.54
Betourney et al. (1991)	Lamprophyre	56.77	20.95	45.04	20.55	18.83	47.06	– 2.02	2.02	– 22.35	22.35
Betourney et al. (1991)	Quartzdiorite	50.31	13.72	49.38	9.17	12.19	42.81	6.57	6.57	– 2.34	2.34
Betourney et al. (1991)	Agglomeratauff	45.39	14.85	43.51	15.61	17.16	41.11	2.40	2.40	– 12.19	12.19
Borecki et al. (1982)	Sandstone	57.15	18.49	35.96	25.90	17.10	46.68	– 10.72	10.72	– 26.43	26.43
Borecki et al. (1982)	Sandstone	54.73	18.34	36.64	23.28	16.37	45.78	– 9.14	9.14	– 22.96	22.96
Chan et al (1972)	Quartzite	60.35	29.66	53.99	21.44	26.71	49.64	4.35	4.35	– 10.93	10.93
Dayre and Giraud (1986)	Granodiorite	59.58	31.27	49.03	28.20	27.48	49.58	– 0.55	0.55	– 13.45	13.45
Dlugosz et al. (1981)	Sandstone	62.97	12.5	41.28	17.71	13.90	47.14	– 5.86	5.86	– 25.47	25.47
Everling (1960)	Sandstone	61.81	16.01	37.04	25.37	16.60	47.73	– 10.69	10.69	– 29.25	29.25
Glushko and Kirnichanskiy (1974)	Sandstone	60.07	5.61	54.23	3.97	6.78	42.63	11.60	11.60	– 12.36	12.36
Gnirk and Cheatham (1963)	Limestone	66.38	19.47	35.26	39.38	21.77	50.03	– 14.77	14.77	– 41.57	41.57
Gnirk and Cheatham (1965)	Marble	59.87	13.84	35.52	21.55	14.16	46.46	– 10.94	10.94	– 29.24	29.24
Hobbs (1964)	Coal	67.66	1.57	38.01	3.06	2.87	39.78	– 1.77	1.77	– 34.03	34.03
Hobbs (1964)	Coal	57.39	1.52	37.08	2.07	2.22	35.05	2.03	2.03	– 22.80	22.80
Hobbs (1964)	Coal	68.42	2.89	36.23	6.20	4.83	42.94	– 6.71	6.71	– 35.87	35.87
Hobbs (1964)	Coal	58.14	1.93	39.31	2.48	2.72	36.65	2.66	2.66	– 21.36	21.36
Hobbs (1964)	Coal	66.49	2.64	37.51	4.97	4.25	41.74	– 4.23	4.23	– 32.38	32.38
Hobbs (1964)	Coal	64.7	1.54	39.89	2.46	2.60	38.39	1.50	1.50	– 28.43	28.43
Hobbs (1964)	Coal	66.84	1.34	38.45	2.48	2.46	38.65	– 0.20	0.20	– 31.12	31.12
Hobbs (1964)	Coal	68.7	2.02	38.61	4.04	3.64	41.43	– 2.82	2.82	– 34.57	34.57
Hobbs (1964)	Coal	66.31	2.62	37.72	4.86	4.20	41.63	– 3.91	3.91	– 32.40	32.40

References	Rock type	ϕ_{indr}	c_{indr}	ϕ_{exp}	T_0	$T_{0(\text{pre})}$	ϕ_{pre}	Error (this method)	Absolute error (this method)	Error (indirect method)	Absolute error (indirect method)
Hobbs (1964)	Coal	66	2.09	41.93	3.27	3.46	40.44	1.49	1.49	- 27.24	27.24
Hobbs (1964)	Coal	51.74	1.2	39.58	1.25	1.68	30.95	8.63	8.63	- 13.99	13.99
Hobbs (1964)	Coal	63.01	1.58	36.3	2.69	2.56	37.78	- 1.48	1.48	- 28.96	28.96
Hobbs (1964)	Coal	61.11	6.62	32.27	11.98	7.93	43.77	- 11.50	11.50	- 34.13	34.13
Hobbs (1964)	Coal	57.2	3.52	37.76	4.64	4.39	39.27	- 1.51	1.51	- 22.76	22.76
Hobbs (1964)	Coal	58.02	3.12	36.62	4.40	4.04	39.02	- 2.40	2.40	- 24.13	24.13
Hobbs (1964)	Coal	52.85	3.65	40.17	3.85	4.24	37.55	2.62	2.62	- 15.64	15.64
Hobbs (1964)	Coal	45.03	3.24	39.22	2.89	3.57	33.07	6.15	6.15	- 7.90	7.90
Hobbs (1964)	Coal	69.65	2.12	38.63	4.44	3.90	42.05	- 3.42	3.42	- 35.60	35.60
Hossaini and Vutukuri (1993)	Sandstone	58.38	6.8	34.48	10.42	7.69	42.84	- 8.36	8.36	- 27.45	27.45
Initskaya (1969)	Sandstone	54.9	22.17	43.51	21.86	19.17	46.62	- 3.11	3.11	- 15.66	15.66
Kuntys (1964)	Sandstone	57.92	16.56	39.94	20.64	15.83	46.50	- 6.56	6.56	- 20.45	20.45
Kwasniewski (1983)	Sandstone	57.2	11.89	40.76	14.03	11.92	44.85	- 4.09	4.09	- 19.72	19.72
Kwasniewski (1983)	Sandstone	56.5	12.63	38.77	15.69	12.38	44.85	- 6.08	6.08	- 21.91	21.91
Kwasniewski (1983)	Sandstone	52.63	15.52	36.21	18.80	13.87	44.27	- 8.06	8.06	- 20.62	20.62
Misra (1972)	Sandstone	61.66	5.23	35.96	8.55	6.61	42.93	- 6.97	6.97	- 28.07	28.07
Misra (1972)	Sandstone	65.62	6.18	37.13	11.34	8.32	45.17	- 8.04	8.04	- 31.93	31.93
Misra (1972)	Sandstone	61.49	5.63	41.91	7.36	6.99	43.19	- 1.28	1.28	- 22.38	22.38
Misra (1972)	Sandstone	61.17	12.27	42.93	15.24	13.17	46.42	- 3.49	3.49	- 21.55	21.55
Misra (1972)	Sandstone	62.09	22.25	45.5	25.70	21.88	49.11	- 3.61	3.61	- 25.89	25.89
Misra (1972)	Sandstone	63.47	11.43	45.76	13.77	13.07	46.95	- 1.19	1.19	- 20.85	20.85
Misra (1972)	Sandstone	59.86	12.39	44.97	13.50	12.93	45.99	- 1.02	1.02	- 17.38	17.38
Murrel (1965)	Sandstone	67.63	7.84	35.9	16.41	10.70	46.91	- 11.01	11.01	- 33.02	33.02
Ramamurthy (1989)	Sandstone	49.6	4.47	38	4.67	4.82	37.10	0.90	0.90	- 16.89	16.89
Rao et al. (1983)	Sandstone	53.73	13.31	56.66	6.70	12.41	44.03	12.63	12.63	3.40	3.40
Rao et al. (1983)	Sandstone	48.91	11.77	55.21	5.62	10.59	41.52	13.69	13.69	7.40	7.40
Rao et al. (1983)	Sandstone	49.19	12.89	55.39	6.14	11.45	42.06	13.33	13.33	6.60	6.60
Schwartz (1964)	Marble	47.02	5.69	37.11	5.74	5.74	37.11	0.00	0.00	- 12.39	12.39
Schwartz (1964)	Granite	57.48	11.9	54.02	7.80	11.98	44.96	9.06	9.06	- 4.24	4.24
Singh et al. (1992)	Quartzite	55.5	16.05	53.4	10.21	14.84	45.50	7.90	7.90	- 4.34	4.34
Singh et al. (1992)	Sandstone	60.2	9	50.85	7.61	10.02	44.76	6.09	6.09	- 16.47	16.47
Stowe (1969)	Limestone	51.06	12.02	45.06	9.94	11.03	42.51	2.55	2.55	- 10.45	10.45

ϕ_{indr} : Internal friction angle calculated using indirect method; c_{indr} : Cohesion calculated using indirect method; ϕ_{exp} : Internal friction angle determined triaxial test data; T_0 : Theoretical MC uniaxial tensile strength; $T_{0(\text{pre})}$: Predicted theoretical MC uniaxial tensile strength; ϕ_{pre} : Internal friction angle predicted from the method proposed in this study

Appendix 2

The results obtained using C_0 and σ_t (Brazilian) data pairs compiled from literature.

References	Rock type	ϕ_{indr}	c_{indr}	ϕ_{exp}	T_0	$T_{0(\text{pre})}$	ϕ_{pre}	Error (this method)	Absolute error (this method)	Error (indirect method)	Absolute error (indirect method)
Arzua and Alejano (2003)	Granite	54.65	13.61	57.59	6.67	11.79	46.41	11.18	11.18	- 2.94	2.94
Arzua and Alejano (2003)	Granite	64.51	15.6	59.52	9.55	16.16	50.00	9.52	9.52	4.99	4.99
Arzua and Alejano (2003)	Granite	60.76	16	54.91	11.16	15.12	48.92	5.99	5.99	5.85	5.85
Choi et al 2022	Igneous	52.56	23.41	54.56	12.82	18.24	47.41	7.15	7.15	- 2	2
Choi et al 2022	Metamorphic	48.99	21.52	51.29	12.66	16.26	45.87	5.42	5.42	- 2.3	2.3
Choi et al 2022	Sedimentary	49.06	25.93	51.28	15.29	19.10	46.50	4.78	4.78	- 2.22	2.22
Dintwe et al (2019)	Tuff	41.24	2.40	30	2.15	1.83	35.14	- 5.14	5.14	11.24	11.24
Ergüler (2007)	Siltstone	37.79	11.27	40	8.22	8.67	38.57	1.43	1.43	- 2.21	2.21
Ergüler (2007)	Mudstone	48.85	7.77	42.5	6.72	6.76	42.36	0.14	0.14	6.35	6.35
Ergüler (2007)	Mudstone	50.27	8.05	45	6.53	7.08	43.05	1.95	1.95	5.27	5.27
Ergüler (2007)	Siltstone	28.54	18.16	37.5	11.96	13.24	34.53	2.97	2.97	- 8.96	8.96
Ergüler (2007)	Siltstone	41.57	18.63	48.5	10.41	13.55	42.32	6.18	6.18	- 6.93	6.93
Ergüler (2007)	Siltstone	41.34	20.23	31	21.71	14.52	42.51	- 11.51	11.51	10.34	10.34
Ergüler (2007)	Mudstone	53.09	19.99	49.5	14.37	16.03	47.10	2.40	2.40	3.59	3.59
Ergüler (2007)	Marl	52.63	14.28	33	19.23	11.93	45.86	- 12.86	12.86	19.63	19.63
Ergüler (2007)	Ignimbrite	50.98	1.17	38.5	1.24	1.35	36.08	2.42	2.42	12.48	12.48
Ergüler (2007)	Siltstone	36.74	3.9	44	2.38	3.47	33.64	10.36	10.36	- 7.26	7.26
Ergüler (2007)	Marl	51.99	14.25	43	13.17	11.81	45.62	- 2.62	2.62	8.99	8.99
Ergüler (2007)	Mudstone	21.67	5.12	33	3.44	4.78	21.51	11.49	11.49	- 11.3	11.33
Ergüler (2007)	Marl	37.64	2.95	36.5	2.43	2.73	32.99	3.51	3.51	1.14	1.14
Ergüler (2007)	Ignimbrite	38.17	3.13	43.5	2.01	2.88	33.57	9.93	9.93	- 5.33	5.33
Ergüler (2007)	Ignimbrite	41.42	2.18	47	1.30	2.14	33.89	13.11	13.11	- 5.58	5.58
Ergüler (2007)	Marl	59.55	3.23	41	4.09	3.73	43.35	- 2.35	2.35	18.55	18.55
Ergüler (2007)	Mudstone	42.96	2.25	31	2.51	2.22	34.85	- 3.85	3.85	11.96	11.96
Eum (2002)	Basaltic intact rock	54.42	6.46	43.24	6.35	6.19	43.86	- 0.62	0.62	11.18	11.18
Eum (2002)	Basaltic intact rock	58.17	12.72	45.71	12.69	11.80	47.38	- 1.67	1.67	12.46	12.46
Heidarzadeh et al. (2021)	Carbonatite	54.04	18.35	47.32	14.99	15.10	47.16	0.16	0.16	6.72	6.72
Heidarzadeh et al. (2021)	Syenite	53.23	15.08	49.41	10.93	12.61	46.25	3.16	3.16	3.82	3.82
Heidarzadeh et al. (2021)	Carbonatite and Syenite	54.25	17.8	47.65	14.42	14.75	47.13	0.52	0.52	6.6	6.6
Heng et al. (2020)	Shale	47.23	27	36.22	24.15	16.38	46.27	- 10.05	10.05	11.01	11.01
Hosseini and Khodaryari (2019)	Sandstone	58.84	10.15	47.94	9.39	9.84	46.89	1.05	1.05	10.9	10.9
Kahraman et al. (2004)	Dol. Limestone	55.55	21.19	53	13.78	17.49	48.11	4.89	4.89	2.55	2.55
Kahraman et al. (2004)	Limestone	65.18	19.26	47.5	23.01	19.71	50.81	- 3.31	3.31	17.68	17.68

References	Rock type	ϕ_{indr}	c_{indr}	ϕ_{exp}	T_0	$T_{0(\text{pre})}$	ϕ_{pre}	Error (this method)	Absolute error (this method)	Error (indirect method)	Absolute error (indirect method)
Kahraman et al. (2004)	Travertine	56.95	12.36	49.4	10.04	11.26	46.89	2.51	2.51	7.55	7.55
Kahraman et al. (2004)	Limestone	64.4	15.25	52.8	13.67	15.80	49.90	2.90	2.90	11.6	11.6
Kahraman et al. (2004)	Travertine (Limra)	61.01	6.5	41.7	8.43	7.01	46.19	- 4.49	4.49	19.31	19.31
Kahraman et al. (2004)	Limestone	57.08	11.74	53.9	7.64	10.80	46.77	7.13	7.13	3.18	3.18
Kahraman et al. (2004)	Travertine	53.13	9.60	41.8	9.61	8.54	44.73	- 2.93	2.93	11.33	11.33
Kahraman et al. (2004)	Travertine	48.23	8.66	43.6	7.05	7.37	42.49	1.11	1.11	4.63	4.63
Kahraman et al. (2004)	Limestone	64.79	14.4	46.9	17.40	15.20	49.85	- 2.95	2.95	17.89	17.89
Kahraman et al. (2004)	Travertine	53.63	8.26	45.9	7.10	7.56	44.41	1.49	1.49	7.73	7.73
Kahraman et al. (2004)	Travertine	67.05	6.09	41.2	10.25	7.72	47.97	- 6.77	6.77	25.85	25.85
Kainthola et al (2015)	Quartzite	41.71	44.5	35.95	41.01	28.66	45.36	- 9.41	9.41	5.76	5.76
Kainthola et al (2015)	Slate	41.5	31.87	31.53	33.78	21.49	44.15	- 12.62	12.62	9.97	9.97
Kainthola et al (2015)	Quartz mica schist	42.22	12.88	35.88	12.04	9.89	41.30	- 5.42	5.42	6.34	6.34
Kainthola et al (2015)	Limestone	41.88	24.84	33.47	24.96	17.38	43.47	- 10.00	10.00	8.41	8.41
Kazerani and Zhao (2008)	Granite	55.89	20.5	53	12.31	15.58	48.17	4.83	4.83	2.89	2.89
Kazerani and Zhao (2008)	Granite	56.29	18.51	53	12.31	15.75	47.93	5.07	5.07	3.29	3.29
Min et al (2019)	Granite gneiss	62.88	30	59.2	16.37	25.39	51.36	7.84	7.84	3.68	3.68
Min et al (2019)	Gneiss A	56.93	24.6	53.1	15.27	18.66	49.04	4.06	4.06	3.83	3.83
Min et al (2019)	Gneiss B	54.89	25.9	49.12	18.73	19.23	48.54	0.58	0.58	5.77	5.77
Moon and Yang (2020)	Basalt	53.88	6.71	41.43	6.97	6.35	43.79	- 2.36	2.36	12.45	12.45
Moon and Yang (2020)	Basalt	60.79	23.86	51.76	19.67	21.33	50.11	1.65	1.65	9.03	9.03
Moon and Yang (2020)	Scoria	44.8	4.33	26.49	5.76	3.94	38.42	- 11.93	11.93	18.31	18.31
Paşamehmetoğlu et al. (1981)	Andesite	54.81	20.31	50	14.99	16.66	47.73	2.27	2.27	4.81	4.81
Paşamehmetoğlu et al. (1981)	Andesite	50.61	12.18	48	8.74	10.14	44.60	3.40	3.40	2.61	2.61
Paşamehmetoğlu et al. (1981)	Andesite	49.13	9.88	46	7.44	8.33	43.31	2.69	2.69	3.13	3.13
Paşamehmetoğlu et al. (1981)	Andesite	51.81	2.94	32	4.00	3.03	40.04	- 8.04	8.04	19.81	19.81
Paşamehmetoğlu et al. (1981)	Andesite	53.52	24.73	32	35.27	19.36	47.91	- 15.91	15.91	21.52	21.52
Paşamehmetoğlu et al. (1981)	Andesite	48.62	18.90	33	22.78	14.47	45.30	- 12.30	12.30	15.62	15.62
Paşamehmetoğlu et al. (1981)	Andesite	50.82	6.59	34	8.16	5.99	42.56	- 8.56	8.56	16.82	16.82
Paşamehmetoğlu et al. (1981)	Andesite	50.70	4.64	33	5.92	4.43	41.27	- 8.27	8.27	17.70	17.70

References	Rock type	ϕ_{indr}	c_{indr}	ϕ_{exp}	T_0	$T_{0(\text{pre})}$	ϕ_{pre}	Error (this method)	Absolute error (this method)	Error (indirect method)	Absolute error (indirect method)
Paşamehmetoğlu et al. (1981)	Andesite	44.87	13.51	48	8.36	10.49	42.65	5.35	5.35	- 3.13	3.13
Paşamehmetoğlu et al. (1981)	Andesite	43.84	10.87	46	7.16	8.64	41.42	4.58	4.58	- 2.16	2.16
Paşamehmetoğlu et al. (1981)	Andesite	46.30	7.02	45	5.13	6.04	40.92	4.08	4.08	1.30	1.30
Paşamehmetoğlu et al. (1981)	Andesite	46.53	2.59	42	2.15	2.57	37.22	4.78	4.78	4.53	4.53
Paşamehmetoğlu et al. (1981)	Andesite	53.53	20.11	52	12.95	16.21	47.27	4.73	4.73	1.53	1.53
Paşamehmetoğlu et al. (1981)	Andesite	52.74	16.70	45	14.51	13.66	46.41	- 1.41	1.41	7.74	7.74
Paşamehmetoğlu et al. (1981)	Andesite	50.42	10.80	38	11.54	9.12	44.12	- 6.12	6.12	12.42	12.42
Sriapai et al. (2012)	Salt	22.42	12.68	28.85	9.81	10.31	27.14	1.71	1.71	- 6.43	6.43
Stoxreiter et al. (2020)	Granite	57.73	20.97	56.8	11.85	17.99	48.78	8.02	8.02	0.93	0.93
Stoxreiter et al. (2020)	Sandstone	55.78	22.32	52.4	15.08	18.35	48.34	4.06	4.06	3.38	3.38
Stoxreiter et al. (2020)	Marble	38.87	22.39	26.88	25.65	15.68	41.69	- 14.81	14.81	11.99	11.99
Wang and He (2023)	Sandstone	22.67	12.06	17.79	12.57	9.84	27.18	- 9.39	9.39	4.88	4.88
Wang and He (2023)	Shale	36.39	16.44	31.31	15.64	11.96	39.25	- 7.94	7.94	5.08	5.08
Wang and He (2023)	Diorite	24.79	32.91	32.87	23.54	22.75	33.92	- 1.05	1.05	- 8.08	8.08
Wang et al (2015)	Carbonate Rock	39.54	2.5	35.51	2.06	2.18	33.75	1.76	1.76	4.03	4.03
Wang et al (2015)	Carbonate Rock	41.63	3.2	35.84	2.67	2.65	35.99	- 0.15	0.15	5.79	5.79
Wang et al (2015)	Carbonate Rock	59.19	2.5	35.73	3.30	2.59	42.35	- 6.62	6.62	23.46	23.46
Wei et al (2020)	Gypsum	49.58	7.36	27	10.91	6.50	42.46	- 15.46	15.46	22.58	22.58
Yasar (2021)	Tuff	46	15.08	40.2	13.25	11.64	43.51	- 3.31	3.31	5.80	5.80
Yasar (2021)	Tuff	49.04	13.84	41.36	12.58	11.13	44.43	- 3.07	3.07	7.68	7.68

ϕ_{indr} : Internal friction angle calculated using indirect method; c_{indr} : Cohesion calculated using indirect method; ϕ_{exp} : Internal friction angle determined by triaxial tests; T_0 : Theoretical MC uniaxial tensile strength; $T_{0(\text{pre})}$: Predicted theoretical MC uniaxial tensile strength; ϕ_{pre} : Internal friction angle estimated from the method proposed in this study

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Declarations

Conflict of interest The author declares that they have no conflict of interest.

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