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ORIGINAL RESEARCH

A study of an EOQ model where the demand depends on time and varying number of tourists using fuzzy triangular norms

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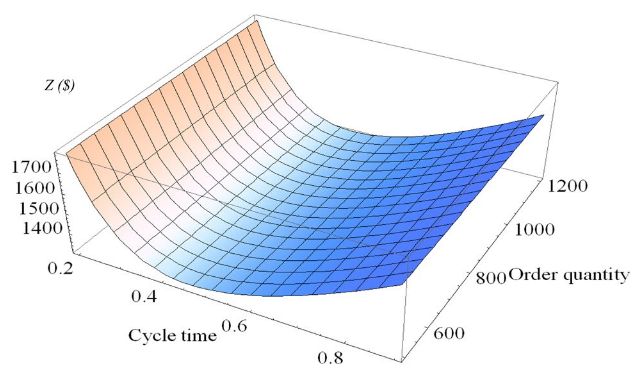
Abstract
The fundamental prerequisite of decision making is how to aggregate individual expert's preference information. For constructing various aggregation operators on intuitionistic fuzzy set, various kinds of t-norms and co-norms are the most significant tools. This work takes a closer look on various fuzzy triangular norms and uses them for the first time in an economic order quantity (EOQ) model. Traditional economic theory has a paradigm shift on objective factors affecting demand such as price and income. Recently, behavioural economics gives more weightage to psychological and social factors that affect our preferences and choices in markets differently. As per literature survey concern, this paper may indeed be the first to adopt the concept of the number of variations of a tourist's dependent demand in inventory management problems. Utilizing several triangular norms, we have characterized Attanassov (standard) and non-Attanassov's (non-standard) feasible region. A case study has been performed for numerical illustration and model validation. A solution algorithm, has also been developed which proves Attanassov's solution space is quite inferior to non-Attanassov's domain. Finally, sensitivity analysis and graphical illustrations are also provided to justify the model.

Graphical abstract



Keywords Inventory · Fuzzy sets · Intuitionistic fuzzy set · Triangular norms · Optimization

1 Introduction

In the first few decades, inventory models by considering various realistic assumptions have widely been studied. In general, demand is the most crucial factor in inventory management problems. According to the customer needs demand varies with various parameters like price, advertisement, stocks etc. and the demand function can be of different type such as ramp type, seasonal demand or it can follow some

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distribution, for example, Weibull distribution. A long decades ago, Goswami and Choudhuri (1991) examined the linear trend in demand. Kumar et al. (2012) analysed the ramp type demand in analysing inventory model. Recently, De et al. (2021) assumed the demand rate dependent on carbon emission, unit selling price and length of credit period offered by the retailer to the customers explicitly in a supplier-retailer-customer model.

However, the concept of seasonal demand in inventory management problem is not new. Demand in retailing is known to vary depending on the day of the week and time of year, around important holiday and seasons. Chen and Chang (2007) proposed two new methods for solving seasonal demand problem with variable lead time and resource constraints. Ehrenthal et al. (2014) investigated the demand seasonality in retail inventory management. Banerjee and Sharma (2010) considered an inventory model with seasonal demand where the distributor explores an alternative market in order to maximize the revenue. In supply chain management Chang and Chou (2013) studied seasonal demand inventory using a periodic review policy. Literature survey suggests no articles over the choice of tourists though tourism is one of the most flourishing sectors of many countries and it is considered to be the second largest industry in the world. Few of the various studies published on the tourism demand in stochastic or dynamical system. Nevertheless, there is a lack of adequate data and literature over the nature of tourism demand. Generally, the major demand in tourist spot is generated by the variation of tourists of those specific places. The demand in tourism can be sensitive, volatile and situation specific. The scale and the magnitude of demand differ with time and sometimes with seasons. Rosselló and Sansó (2017) used the case study of air arrivals and departures and studied tourism seasonality analysis.

Zadeh (1965) first developed the fuzzy set theory. Subsequently it was applied by Bellman and Zadeh (1970) in decision making problem. In real life, a person may assume that an object belongs to a set to a certain degree, but it is possible that the person is not so sure about it. In other words, there may be a hesitation or uncertainty about the membership degree of a decision variable. The problems were resolved with the help of intuitionistic fuzzy sets (IFS) theory, developed by Atanassov (1986). The concept of an intuitionistic fuzzy set (IFS) can be viewed as an alternative approach to define a fuzzy set in cases where available information is not sufficient for the definition of an imprecise concept by means of a conventional fuzzy set. In fuzzy set, the degree of acceptance is considered only but IFS is characterized by a membership function (acceptance) and a non-membership function (rejection) so that the sum of both values is less than one. Chen and Tan (1994) and Dymova and Sevastjanov (2011) proposed the score function of IFS as $S(x) = \mu(x) - \gamma(x)$. Researchers like De and Sana (2014), De et al. (2014) utilized this score function in a multi-period backlogging inventory management problem.

Moreover, there are some extensions on various kinds of fuzzy set theory like Singh et al. (2019) introduced knowledge measure using accuracy measure. Over the years, since the introduction of triangular norm (shortly t-norm) by Menger (1942), a lot of research has been done concerning both the theory and applications (disciplines like mathematics and computer science especially in artificial intelligence). Klement et al. (2004a, b, c) presented the basic analytical and algebraic properties of triangular norms in a series of three position papers. Bianchi (2015) discussed the strongest and the weakest t-norms. Moreover, for the cases of decision-making problems, Li and Liu (2014) discussed the linear optimization problem with max T composition and Lukasiewicz t-norm. Azadeh et al. (2015) considered an efficient model implementing the concept of trust in terms of performance measurement and utilized the t-norms and t-conorms as the final modelling tools. Based on Archimedean t-conorm and t-norm, Xia et al. (2012) extended some common aggregation techniques for discrete intuitionistic fuzzy numbers into more general forms. However, in some practical applications, we need to deal with the discrete intuitionistic fuzzy data having capabilities to solve plenty of problems related to the continuous intuitionistic fuzzy information. Kumar et al. (2014) evaluated the reliability of system in terms of membership function and non-membership function by using weakest t-norm. Lima et al. (2016) focused on interval-valued t-norms and t-conorms characterized by interval-valued homogenuous t-norms and t-conorms of interval-valued order. In addition, the recent literature survey shows that there are theoretical works available on t-norms, t-conorms and extended t-norms such as Bielawski and Tabor (2020) introduced T-convex hull of a fuzzy set in a notion of T-convexity and a metric which are based on a strict triangular norm T. Liu and Wang (2020) studied distributive laws on extended t-norms and t-conorms on fuzzy truth values. In multi-criteria decision making, Sarkar and Biswas (2019) analysed the more generalised forms of t-norms and t-conorms in terms of Archimedean t-norms and t-conorms in Pythagorean hesitant fuzzy number. Sun and Liu (2020) examined the additive generators of t-norms and t-conorms on bounded lattices which are important tools to study the representations of t-norms and t-conorms. Bejines et al. (2020) studied the t-norms on finite lattices. Petrik (2020) examined the dominance relation on the set of nilpotent t-norms and Archimedean t-norms.

In this paper, we study a simple EOQ model of a tourist spot where the demand is solely generated by the number of variations of tourists. This work considers on the demand for particular tourism related products though tourism is viewed as a major driving force of economic recovery and growth for some countries but this is not reflected in the attention of researchers yet. Also, we focus on the information aggregation of IFS theory. The existing research on utilizing the score function gives the membership value (α) that assumes greater value (α > β) than that of non-membership value (β) always. Moreover, as per literature survey concern, not a single article has been developed yet where the non-membership grade (β) exceeds the membership grade (α < β) of the proposed fuzzy variables. For an indefinite universe of discourse (solution space) it is a common phenomenon to get solution with minimum membership value instead of having greater membership value than their non-membership value exclusively. This article has been organised as follows: Sect. 2 includes various definitions on intuitionistic fuzzy set/fuzzy t-norms and the study of Attanassov and non-Attanassov's regions. Section 3 describes the model assumptions, notations and a case study, Sect. 4 develops crisp mathematical model, Sect. 5 expresses the fuzzy mathematical problem of the proposed model, Sect. 6 develops a solution algorithm, Sect. 7 includes numerical illustrations, Sect. 8 indicates the sensitivity analysis, Sect. 9 gives the graphical illustrations and finally a conclusion is made at Sect. 10 alone.

2 Preliminaries

Here we discuss some definitions associated with intuitionistic fuzzy set and triangular norms.

2.1 Definition -1: Intuitionistic fuzzy set (Atanassov 1986, 1999)

Let $X = (x_1, x_2, x_3 \dots \dots x_3)$ be a finite universal set. An Atanassov's IFS A in X is an object having the form $A = \{< x_i, \mu_A(x_i), \nu_A(x_i) >: x_i \epsilon X\}$ where the $\mu_A(x_i) : X \to [0,1]$ and $\nu_A(x_i) : X \to [0,1]$ define the degree of membership and degree of non-membership respectively. If the element $x_i \epsilon X$ to the set A, which is a subset of X, for every element of $x_i \epsilon X$ then $0 \le \mu_A(x_i) + \nu_A(x_i) \le 1$.

2.2 Definition-2: (α, β) level intervals or (α, β)-cuts

A set of (α, β)-cut, generated by IFS-A, where α and β $\in [0, 1]$ are fixed numbers such that $(\alpha + \beta) \in [0, 1]$ that defined as $A_{\alpha,\beta} = \left\{ \begin{matrix} (x, \mu_A(x), \nu_A(x)) : x \in X \\ \mu_A(x) \ge \alpha, \nu_A(x) \le \beta, \alpha, \beta \in [0,1] \end{matrix} \right\}$

(α, β) level intervals or (α, β)-cut denoted by $A_{\alpha,\beta}$ is defined as the crisp set of elements x which belongs to A at least to the degree α and which does belong to A at most to the degreeβ.

2.3 Definition-3

Let A and B be two Atanassov's IFS in the finite universal set X. The intersection of A and B is defined as follows: $A \cap B = \{x_i, min(\mu_A(x_i), \mu_B(x_i)), max(\nu_A(x_i), \nu_B(x_i))\}, \forall x_i \in X, i \in N$

2.4 Definition-4

Let $X = \{\dots x_{-2}, x_{-1}, x_0, x_1, x_2, \dots\}$ be the universe of discourse where $X \subseteq \mathbb{R}$. Let $\mu(x)$ and $\nu(x)$ be two fuzzy sets corresponding to the membership and non-membership function on X, then the following conditions may hold: (i) $\mu(x) \underset{\sim}{\subseteq} \nu(x)$ (ii) $\mu(x) \tilde{=} \nu(x)$ (iii) $\mu(x) \underset{\sim}{\supseteq} \nu(x)$

Thus, for decision making aspects, if $\Pi(x)$ is the degree of hesitancy then we write.

a) If (i) is true then score value is $S(x) \ge \nu(x) - \mu(x) + \Pi(x)$
b) If (ii) is true then score value is $S(x) = \Pi(x)$
c) If (iii) is true then score value is $S(x) \le \mu(x) - \nu(x) + \Pi(x)$

Such that (i) Attanassov inequality $0 \le \mu(x) + \nu(x) + \Pi(x) \le 1$, $\mu(x) \ge \nu(x)$ for $\mu(x) \in [0.5, 1]$; $\nu(x), \Pi(x) \in [0, 0.5]$ are satisfied otherwise (ii) Non-Attanassov's inequality $0 \le \mu(x) + \nu(x) + \Pi(x) \le 2$, $\mu(x), \nu(x), \Pi(x) \in (0, 1)$ with accuracy value $\mu(x) + \nu(x) - \mu(x)\nu(x)$ are satisfied.

2.5 Definition-5

Let X be the universe of discourse and A be the fuzzy set on X. Then there are two excluded middle axioms which are not valid for the fuzzy set unlike classical crisp set, we have Axiom of Excluded middle: $A \cup \overline{A} \ne X$ and Axiom of contradiction: $A \cap \overline{A} \ne \varphi$

2.6 Basic interval arithmetic

Let $A = [a_1, a_2]$ and $B = [b_1, b_2]$ then the usual operations $\{+, -, \times, \div\}$, namely addition, subtraction, multiplication are given below:

$A + B = [a_1 + b_1, a_2 + b_2], A - B = [a_1 - b_2, a_2 - b_1]$,

$A.B = [\min(a_1b_1, a_1b_2, a_2b_1, a_2b_2), \max(a_1b_1, a_1b_2, a_2b_1, a_2b_2)]$

$A/B = [\min(a_1/b_1, a_1/b_2, a_2/b_1, a_2/b_2), \max(a_1/b_1, a_1/b_2, a_2/b_1, a_2/b_2)]$,
$\delta A = [\delta a_1, \delta a_2]$ if $\delta \geq 0$ and $\delta A = [\delta a_2, \delta a_1]$ if $\delta < 0$.

2.7 Optimization for interval environment

We have the problem for interval valued coefficients of the variables of non-linear objective function as Minimize $Z(X) = \sum_{i=1}^{n} [a_{Li}, a_{Ri}] \prod_{j=1}^{k} x_j^{r_j}$ subject to $x_j > 0, j = 1, 2, \ldots n$ and $x \in S \in R^+$ where S is a feasible region of x. Now we can split $Z(X)$ in the form $Z(X) = [Z_L(X), Z_R(X)]$ where $Z_L(X) = \sum_{i=1}^{n} a_{Li} \prod_{j=1}^{k} x_j^{r_j}$ and $Z_R(X) = \sum_{i=1}^{n} a_{Ri} \prod_{j=1}^{k} x_j^{r_j}$ and the centre of the objective function is $Z_C(X) = \frac{1}{2}[Z_L(X) + Z_R(X)]$.

2.8 Conversion of a fuzzy number to its nearest interval number

Let $\tilde{A} = (a_1, a_2, a_3)$ be an arbitrary triangular fuzzy number with linear membership function $\mu_A(x) = \begin{Bmatrix} \frac{x-a_1}{a_2-a_1} \text{for } a_1 < x < a_2 \\ \frac{a_3-x}{a_3-a_2} \text{for } a_2 < x \leq a_3 \\ 0 \text{ for elsewhere} \end{Bmatrix}$

The α-cut of the membership function of A can be written as $[A_L(\alpha), A_R(\alpha)]$. Now as per Grzegorzewski (2002), the nearest interval can be obtained as $[C_L, C_R]$ where $C_L = \int_0^1 A_L(\alpha) d\alpha = \frac{a_1+a_2}{2}$ and $C_R = \int_0^1 A_R(\alpha) d\alpha = \frac{a_2+a_3}{2}$

2.9 Triangular norm [t-Norm]

The intersection of two fuzzy sets A and B is a binary operation on the unit interval; that is a function $\varphi : [0,1] \times [0,1] \to [0,1]$ for each $x \in X$. The function φ is called t-norm if the following axioms are satisfied $\forall a, b, d \in [0,1]$

a) Boundary condition: $\varphi(a, 1) = a$.
b) Monotonicity: $b \leq d \Rightarrow \varphi(a, b) \leq \varphi(a, d)$
c) Commutativity: $\varphi(a, b) = \varphi(b, a)$.
d) Associativity: $\varphi(a, \varphi(b, d)) = \varphi(\varphi(a, b), d)$
e) Continuity: φ is a continuous function.
f) Sub idempotency: $\varphi(a, a) < a$.
g) Strict Monotonicity: $a_1 < a_2$ and $b_1 < b_2 \Rightarrow \varphi(a_1, b_1) < \varphi(a_2, b_2)$.

i. Archimedean t-norm: A continuous t-norm that satisfies sub idempotency is called an Archimedean t-norm. If it also satisfies strict monotonicity, it is called a strict Archimedean t-norm. Here we write $\varphi(a, b) = Min(a, b), \forall a, b \in [0,1]$.
ii. Lukasiewicz t-norm [bounded difference/nilpotent archimedean] The formula is given by $\varphi(a, b) = Max(0, a + b - 1), \forall a, b \in [0,1]$ $= \begin{cases} 0 \text{ for } a + b = 1 \\ 2 - a - b \text{ for } 1 < a + b < 2 \\ 1 \text{ for } a + b = 2 \end{cases}$
iii. Hamacher t-norm [Hamacher product] Archimedean t-norm: $\varphi(a, b) = \begin{cases} 0, & \text{if } a = b = 0 \\ \frac{ab}{a+b-ab}, & \text{for } a \neq b \neq 0. \end{cases}$
iv. Nilpotent Minimum $\varphi(a, b) = \begin{cases} \text{Min}(a, b), \forall a, b \in [0,1] \text{ and } a + b > 1. \\ 0, \text{ Otherwise} \end{cases}$
v. Schweizer and Sklar Class of t-norms $\varphi(a, b) = (Max(0, a^p + b^p - 1))^{\frac{1}{p}}, p \neq 0$.
vi. Yager Class of t-norms $\varphi(a, b) = 1 - Min\left(1, ((1-a)^w + (1-b)^w)^{\frac{1}{w}}\right), w > 0$.
vii. Drastic t-norm $\varphi(a, b) = Min \begin{cases} a \text{ if } b = 1 \\ b \text{ if } a = 1 \\ 0 \text{ Otherwise} \end{cases}$
viii. Sugeno Class of involutive fuzzy complement $b = \frac{1-a}{1+\theta a} \forall \theta \in (-1, \infty)$
ix. Yager Class of involutive fuzzy Complement $b = (1 - a^w)^{1/w} \forall w \in (0, \infty)$
x. Equilibrium t-norms: If the values of a and b are same for any kinds of t-norms then such t-norms are called equilibrium t-norms, Here, $a = b \in [0,1]$.

2.10 Atannasov's domain of (non) membership grade

Let, $\alpha \in [0,1]$ and $\beta \in [0,1]$ be the membership grade and non-membership grade of a fuzzy parameter respectively. Then from arithmetic operation on intervals we write, $\alpha + \beta \in [0,2] \Rightarrow 0 \leq \alpha + \beta \leq 2$. However, as per Atannasov (1998, 1999) we write, $0 \leq \alpha + \beta \leq 1$. So, from the omitted part of the above set we get $1 < \alpha + \beta \leq 2$ which is a case of t-norms and our focus of attraction orients over here. The possible feasible regions are stated below:

$$\alpha + \beta \leq 1, \alpha \leq \beta, \alpha \leq 1, \beta \leq 1$$
$$\alpha + \beta \leq 2, \alpha \leq \beta, \alpha + \beta \geq 1, \beta \leq 1$$
$$\alpha + \beta \leq 1, \alpha \geq \beta, \beta \leq 1$$
$$\alpha + \beta \leq 2, \alpha \geq \beta, \alpha + \beta \geq 1, \alpha \leq 1$$

3 Model assumptions, notations and case study

In this section we consider some assumptions and notations followed by a case study for developing the proposed model.

Assumptions

(i) Replenishments are instantaneous.
(ii) The time horizon is infinite (weeks).
(iii) Shortages are not allowed. iv) The demand rate $D = de^{\rho t}Log(1 + n)$ (batches per week) where n is the variation of customers per time. $0 < \rho < 1$, d is a scale parameter. The notion is that, as $n \to 0, D \to 0$.

Notations

(i) Q: the order quantity per cycle (batches)
(ii) h: holding cost per batch per cycle ($)
(iii) b: setup cost per cycle ($)
(iv) T: cycle time in weeks
(v) Z: average cost of the inventory ($)

3.1 Case study

During COVID-19 pandemic situation, our research team visited one of the most magnificent hill resorts, Darjeeling, a city in West Bengal, India on November 2020. Darjeeling is well known for mainly their large selection of world's most expensive and exotically flavoured tea. It also helps Indian economy because of its international reputation and consumer recognition. For tourists and travellers, normally the best time to visit Darjeeling is from September to June every year. The variation of tourists is higher in autumn and spring than winter season. We talked with the manager of a famous tea shop "Nutmull" and came to know that generally the variation in number of tourists (n) was 1850 per week (approximately) for that shop before COVID-19 periods. But, due to COVID restrictions the manager experienced with some unfavourable situations where no demand was found in several weeks because of nonavailability of new customers (tourists) or found very poor demand due to very small number of variation of tourists in that place. However, our team noted the following data for research modelling. Normal threshold demand rate $d = 250$ batches per week, setup cost $b = \$300$ per cycle, holding cost per cycle per batch $h = \$\ 1.5$ and the time exponent coefficient $\rho = 0.05$. After careful investigation, we may adopt the following research questions.

(a) What will be the optimum replenishment and cycle time so that the total average inventory cost becomes minimum?
(b) In which region (Attanassov or non-Attanassov) the optimum decision will arise whenever some of the system parameters assume fuzzy flexibility and follow various triangular norms?

4 Crisp mathematical model

As per knowledge gained from real case study, we may formulate an EOQ model. Let the inventory starts with Q replenished quantity. The inventory depletes due to demand D where the demand is sensitive with the number of variations of tourist coming to the tourist spot. The inventory becomes zero at the end of the cycle time T. The governing differential equation is

$$\frac{dI}{dt} = -D = -de^{\rho t}Log(1 + n), I(T) = 0, \tag{1}$$

Solving (1), we get

$$I(t) = \frac{d}{\rho}\left\{e^{\rho T} - e^{\rho t}\right\}Log(1 + n), \tag{2}$$

The holding cost is given by

$$(\mathrm{HC}) = h\int_0^T I(t)dt = \frac{hd}{\rho}\left\{Te^{\rho T} - \frac{e^{\rho T} - 1}{\rho}\right\}Log(1 + n), \tag{3}$$

Maximum order quantity,

$$Q = I(0) = \frac{d}{\rho}\left\{e^{\rho T} - 1\right\}Log(1 + n), \tag{4}$$

Thus, the total average cost is given by

$$Z = \frac{1}{T}(SC + HC) = \frac{1}{T}\left[b + \frac{hd}{\rho}\left\{Te^{\rho T} - \frac{e^{\rho T} - 1}{\rho}\right\}Log(1 + n)\right] = \left[\frac{b}{T} + \frac{hd}{\rho T}\left\{Te^{\rho T} - \frac{e^{\rho T} - 1}{\rho}\right\}Log(1 + n)\right], \tag{5}$$

Therefore, the final problem becomes

$$\left\{ MinimizeZ = \left[\frac{b}{T} + \frac{hd}{\rho T}\left\{Te^{\rho T} - \frac{e^{\rho T}-1}{\rho}\right\}Log(1 + n)\right], SubjecttoQ = \frac{d}{\rho}\left\{e^{\rho T} - 1\right\}Log(1 + n)\ . \right. \tag{6}$$

5 Fuzzy mathematical model

In the classical EOQ model, the demand rate is assumed as constant which contradicts the practical situations in general. In practice, the demand rate might be flexible in nature. Here, we consider the demand rate as intuitionistic fuzzy number. Assuming $\tilde{\rho} = \langle \rho_1{'}, \rho_1, \rho_2, \rho_3, \rho_3{'} \rangle$ and $\tilde{d} = \langle d_1{'}, d_1, d_2, d_3, d_3{'} \rangle$ we get

$$\tilde{D} = \langle d_1{'} e^{\rho_1{'} t} Log(1+n), d_1 e^{\rho_1 t} Log(1+n), d_2 e^{\rho_2 t} Log(1+n), d_3 e^{\rho_3 t} Log(1+n), d_3{'} e^{\rho_3{'} t} Log(1+n) \rangle, \tag{7}$$

Thus, utilizing (6) the fuzzy total average cost becomes

$$\tilde{Z} = \left[\frac{b}{T} + \frac{h\tilde{d}}{\tilde{\rho}\,T} \left\{ T e^{\tilde{\rho} T} - \frac{e^{\tilde{\rho} T} - 1}{\tilde{\rho}} \right\} Log(1+n) \right], \tag{8}$$

and hence the problem under fuzzy environment is defined as

$$\left\{ Minimize \tilde{Z} = \left[\frac{b}{T} + \frac{h\tilde{d}}{\tilde{\rho} T} \left\{ T e^{\tilde{\rho} T} - \frac{e^{\tilde{\rho} T} - 1}{\tilde{\rho}} \right\} Log(1+n) \right], Subjectto \tilde{Q} = \frac{\tilde{d}}{\tilde{\rho}} \left\{ e^{\tilde{\rho} T} - 1 \right\} Log(1+n) \right. , \tag{9}$$

6 Solution algorithm

Step 1: We find the nearest interval for d as $d_l{'} = \frac{d_1{'} + d_2}{2}$, $d_l = \frac{d_1 + d_2}{2}$, $d_r = \frac{d_3 + d_3}{2}$ and $d_r{'} = \frac{d_2 + d_3{'}}{2}$. Similarly, for ρ we find $\rho_l{'} = \frac{\rho_1{'} + \rho_2}{2}$, $\rho_l = \frac{\rho_1 + \rho_2}{2}$, $\rho_r = \frac{\rho_2 + \rho_3}{2}$ and $\rho_r{'} = \frac{\rho_2 + \rho_3{'}}{2}$. Then we calculate $Z_l{'}, Z_l, Z_r$ and $Z_r{'}$ with $Q_l{'}, Q_l, Q_r$ and $Q_r{'}$ as

$$Z_l{'} = \left[\frac{b}{T} + \frac{hd_l{'}}{\rho_r{'} T} \left\{ T e^{\rho_l{'} T} - \frac{e^{\rho_l{'} T} - 1}{\rho_l{'}} \right\} Log(1+n) \right],$$

$$Q_l{'} = \frac{d_l{'}}{\rho_r{'}} \left\{ e^{\rho T} - 1 \right\} Log(1+n)$$

$$Z_l = \left[\frac{b}{T} + \frac{hd_l}{\rho_r T} \left\{ T e^{\rho_l T} - \frac{e^{\rho_l T} - 1}{\rho_l} \right\} Log(1+n) \right],$$

$$Q_l = \frac{d_l}{\rho_r} \left\{ e^{\rho_l T} - 1 \right\} Log(1+n),$$

$$Z_r = \left[\frac{b}{T} + \frac{hd_r}{\rho_l T} \left\{ T e^{\rho_r T} - \frac{e^{\rho_r T} - 1}{\rho_r} \right\} Log(1+n) \right],$$

$$Q_r = \frac{d_r}{\rho_l} \left\{ e^{\rho_r T} - 1 \right\} Log(1+n)$$

and $Z_r{'} = \left[\frac{b}{T} + \frac{hd_r{'}}{\rho_l{'} T} \left\{ T e^{\rho_r{'} T} - \frac{e^{\rho_r{'} T} - 1}{\rho_r{'}} \right\} Log(1+n) \right]$, $Q_r{'} = \frac{d_r{'}}{\rho_l{'}} \left\{ e^{\rho_r{'} T} - 1 \right\} Log(1+n)$ respectively.

Step 2: We define the membership and non-membership function for $\tilde{d}, \tilde{\rho}, \tilde{Q}$ and $\tilde{Z}$ respectively as

$$\mu_{\tilde{d}}(x) = \frac{x - d_l}{d_r - d_l}, d_l \le x \le d_r \text{and} \gamma_{\tilde{d}}(x) = \frac{d_r{'} - x}{d_r{'} - d_l{'}}, d_l{'} \le x \le d_r{'}, \tag{10}$$

$$\mu_{\tilde{\rho}}(x) = \frac{x - \rho_l}{\rho_r - \rho_l}, \rho_l \le x \le \rho_r \text{and} \gamma_{\tilde{\rho}}(x) = \frac{\rho_r{'} - x}{\rho_r{'} - \rho_l{'}}, \rho_l{'} \le x \le \rho_r{'}, \tag{11}$$

$$\mu_{\tilde{Q}}(x) = \begin{cases} \frac{x - Q_l}{Q_2 - Q_l}, Q_l \le x \le Q_2 \\ \frac{Q_r - x}{Q_r - Q_2}, Q_2 \le x \le Q_r \end{cases} \text{and} \gamma_{\tilde{Q}}(x) = \begin{cases} \frac{Q_2 - x}{Q_2 - Q_l{'}}, Q_l \le x \le Q_2 \\ \frac{x - Q_2}{Q_r{'} - Q_2}, Q_2 \le x \le Q_r \end{cases}, \tag{12}$$

$$\mu_{\tilde{Z}}(x) = \left(\frac{Z_l - x}{Z_r - Z_l} \right)^{\frac{1}{2}}, Z_l \le x \le Z_r \text{and} \gamma_{\tilde{Z}}(x) = \left(\frac{x - Z_l{'}}{Z_r{'} - Z_l{'}} \right)^{\frac{1}{2}}, Z_l{'} \le x \le Z_r{'}, \tag{13}$$

Step 3: We find α-cut and β-cut for $\tilde{Q}$ and $\tilde{Z}$ as $\{Q_l + \alpha(Q_2 - Q_l), Q_r - \alpha(Q_r - Q_2), Q_2 - \beta(Q_2 - Q_l{'}), Q_2 + \beta(Q_r{'} - Q_2)\}$ and $\{Z_l - \alpha^2(Z_r - Z_l), Z_l{'} + \beta^2(Z_r{'} - Z_l{'})\}$

We determine the compromise solution of $\tilde{d}$ and $\tilde{\rho}$ as $\frac{1}{2}\{d_l + \alpha(d_r - d_l) + d_r{'} - \beta(d_r{'} - d_l{'})\}$ and $\frac{1}{2}\{\rho_l + \alpha(\rho_r - \rho_l) + \rho_r{'} - \beta(\rho_r{'} - \rho_l{'})\}$ respectively.

Table 1 Solution of crisp model

Parameters	T Weeks	Q Batches	$Z(\$)$
$\mathbf{Z_{l'}}$	0.65	603.59	915.73
$\mathbf{Z_l}$	0.56	696.82	1058.51
$\mathbf{Z^*}$	0.45	864.15	1311.00
$\mathbf{Z_r}$	0.37	1058.92	1607.57
$\mathbf{Z_{r'}}$	0.32	1233.55	1869.86

Table 2 IFS solution for $Max = \alpha$

Solution space	α	β	T Weeks	Q Batches	$Z(\$)$
Region I	0.37	0.63	0.46	986.66	1296.36
Region II	0.76	1	0.46	910.46	1289.56
Region III	0.5	0.5	0.51	961.00	1153.5
Region IV	0.74	0.74	0.46	913.7	1303.25

Table 3 IFS solution for $Max = \frac{\alpha\beta}{\alpha+\beta-\alpha\beta}$

Solution space	α	β	T Weeks	Q Batches	$Z(\$)$
Region I	0.37	0.63	0.46	986.66	1296.36
Region II	0.76	1	0.46	910.48	1289.56
Region III	0.5	0.5	0.51	961.00	1153.5
Region IV	0.74	0.74	0.46	913.7	1303.25

Table 4 IFS solution for $Min = (1-\alpha)\beta$

Solution space	α	β	T Weeks	Q Batches	$Z(\$)$
Region I	0.37	0.63	0.46	986.66	1296.36
Region II	0.74	0.74	0.46	913.7	1303.25
Region III	0.5	0.5	0.51	961.00	1153.5
Region IV	0.74	0.64	0.45	914.98	1308.63

Step 4: Utilizing (10), (11), (12) and (13), the final intuitionistic fuzzy problem can be represented as

Table 5 IFS solution for $Max = \alpha - \beta$

Solution space	α	β	T Weeks	Q Batches	$Z(\$)$
Region I	0.37	0.63	0.46	986.66	1296.36
Region II	0.68	0.68	0.6	926.36	1354.23
Region III	0.5	0.5	0.51	961.00	1153.5
Region IV	0.74	0.64	0.45	914.98	1308.63

Table 6 IFS solution for $Min\beta = [(1-\alpha)^p]^{\frac{1}{p}}, p = 2$

Solution space	α	β	T Weeks	Q Batches	$Z(\$)$
Region I	0.37	0.63	0.46	986.66	1296.36
Region II	0.37	0.63	0.46	986.66	1296.36
Region III	0.5	0.5	0.51	961.00	1153.5
Region IV	0.64	0.64	0.46	933.9	1305.4

Table 7 IFS solution for $Max = \alpha\beta$

Solution space	α	β	T Weeks	Q Batches	$Z(\$)$
Region I	0.37	0.63	0.46	986.66	1296.36
Region II	0.76	1	0.46	910.48	1289.56
Region III	0.5	0.5	0.51	961.00	1153.5
Region IV	0.74	0.74	0.46	913.7	1303.25

Step 5: We solve the problem (14) for four different regions in each of the t-norms. The Archimedean t-norm

$$\begin{cases} Max/Mint-norm \\ Subjectto Z \le Z_l - \alpha^2(Z_r - Z_l), Z \le Z_l' + \beta^2(Z_r' - Z_l') \\ Q \ge Q_l + \alpha(Q_2 - Q_l), Q \le Q_r - \alpha(Q_r - Q_2), Q \ge Q_2 - \beta(Q_2 - Q_l'), Q \le Q_2 + \beta(Q_r' - Q_2) \\ Z = \left[\frac{b}{T} + \frac{hd}{\rho T}\left\{Te^{\rho T} - \frac{e^{\rho T}-1}{\rho}\right\}Log(1+n)\right], d = \frac{1}{2}\left\{d_l + \alpha(d_r - d_l) + d_r' - \beta(d_r' - d_l')\right\}, \\ \rho = \frac{1}{2}\left\{\rho_l + \alpha(\rho_r - \rho_l) + \rho_r' - \beta(\rho_r' - \rho_l')\right\}, Regioni, i = I, II, III, IV \end{cases}, \quad (14)$$

for the region I is shown in equation (15) as example:

$$\begin{cases} Max\frac{\alpha\beta}{\alpha+\beta-\alpha\beta} \\ Subjectto Z \le Z_l - \alpha^2(Z_r - Z_l), Z \le Z_l' + \beta^2(Z_r' - Z_l') \\ Q \ge Q_l + \alpha(Q_2 - Q_l), Q \le Q_r - \alpha(Q_r - Q_2), Q \ge Q_2 - \beta(Q_2 - Q_l'), Q \le Q_2 + \beta(Q_r' - Q_2) \\ Z = \left[\frac{b}{T} + \frac{hd}{\rho T}\left\{Te^{\rho T} - \frac{e^{\rho T}-1}{\rho}\right\}Log(1+n)\right], d = \frac{1}{2}\left\{d_l + \alpha(d_r - d_l) + d_r' - \beta(d_r' - d_l')\right\}, \\ \rho = \frac{1}{2}\left\{\rho_l + \alpha(\rho_r - \rho_l) + \rho_r' - \beta(\rho_r' - \rho_l')\right\}, \alpha + \beta \le 1, \alpha \le \beta, \alpha \le 1, \beta \le 1 \end{cases}, \quad (15)$$

Table 8 Sensitivity of IFS solution for $Max = \frac{\alpha\beta}{\alpha+\beta-\alpha\beta}$ over the region I

Parameters	% Change	α	β	T Weeks	Q Batches	Z (\$)	$\frac{Z-Z^*}{Z^*} \times 100\%$
b	− 30	0.5	0.5	0.46	780.00	1153.55	− 12.01
	− 10	0.42	0.58	0.43	976.18	1233.98	− 5.87
	+10	0.32	0.68	0.48	995.84	1355.62	+3.40
	+30	0.24	0.76	0.53	1011.47	1466.23	+11.84
h	− 30	0.5	0.5	0.46	961.00	1153.55	− 12.01
	− 10	0.42	0.58	0.48	976.34	1234.91	− 5.80
	+10	0.32	0.68	0.44	749.94	1354.73	+3.34
	+30	0.24	0.76	0.41	1011.13	1463.72	+11.65
n	− 30	0.39	0.61	0.47	761.86	1267.63	− 3.31
	− 10	0.37	0.62	0.46	985.3	1287.96	− 1.76
	+10	0.36	0.64	0.46	987.87	1303.91	− 0.54
	+30	0.35	0.65	0.45	989.94	1317.04	+0.46

Table 9 Sensitivity of IFS solution for $Max = \frac{\alpha\beta}{\alpha+\beta-\alpha\beta}$ over the region II

Parameter	% Change	α	β	T Weeks	Q Batches	Z (\$)	$\frac{Z-Z^*}{Z^*} \times 100\%$
b	− 30	0.98	1	0.38	868.63	1083.88	− 17.32
	− 10	0.83	1	0.44	896.25	1225.35	− 6.53
	+10	0.68	1	0.48	810.91	1350.16	+2.99
	+30	0.51	1	0.53	958.26	1461.88	+11.51
h	− 30	0.97	1	0.55	869.15	1086.75	− 17.10
	− 10	0.83	1	0.48	896.44	1226.27	− 6.46
	+10	0.68	1	0.44	811.11	1349.28	+2.92
	+30	0.52	1	0.41	783.12	1459.38	+11.32
n	− 30	0.79	1	0.47	903.78	1260.05	− 3.89
	− 10	0.77	1	0.46	908.49	1280.92	− 2.29
	+10	0.75	1	0.46	912.29	1297.31	− 1.04
	+30	0.73	1	0.45	915.49	1310.76	− 0.02

Table 10 Sensitivity of IFS solution for $Max = \frac{\alpha\beta}{\alpha+\beta-\alpha\beta}$ over the region III

Parameter	% Change	α	β	T Weeks	Q Batches	Z (\$)	$\frac{Z-Z^*}{Z^*} \times 100\%$
b	− 30	0.5	0.5	0.49	961.00	1153.55	− 12.01
	− 10	0.5	0.5	0.46	961.00	1153.55	− 12.01
	+10	0.5	0.5	0.56	961.00	1153.55	− 12.01
	+30	0.5	0.5	0.67	961.00	1153.55	− 12.01
h	− 30	0.5	0.5	0.7	961.00	1153.55	− 12.01
	− 10	0.5	0.5	0.51	961.00	1153.55	− 12.01
	+10	0.5	0.5	0.51	752.52	1153.55	− 12.01
	+30	0.5	0.5	0.51	961.00	1153.55	− 12.01
n	− 30	0.5	0.5	0.51	812.76	1153.55	− 12.01
	− 10	0.5	0.5	0.51	961.00	1153.55	− 12.01
	+10	0.5	0.5	0.51	961.00	1153.55	− 12.01
	+30	0.5	0.5	0.51	961.00	1153.55	− 12.01

Step 6: Optimize the problem (14) for each case defined in subsections as 2.7 we get the optimal solution of the decision variables α^*, β^*, T^*, Q^* and Z^* respectively.

7 Numerical example

We consider the data set obtained from the case study studied at Subsect. 3.1 for numerical illustrations. For numerical computations we take the help of solution algorithm developed at Sect. 6. Utilizing appropriate computer programming we have obtained all the optimum results that are recorded in Tables 1, 2, 3, 4, 5, 6, 7 respectively.

7.1 Discussion on numerical example

To illustrate the model, we have used six different t-norms. Applying the procedure of the solution algorithm, we summarize the computational results of the proposed crisp model in Table 1. The total average cost of the model is \$1311.00 with 0.45 week cycle time and an order quantity of 864.15 batches. Also, we have given a comparative study among all the models using six different t-norms each for four different regions given in Tables 2–7. The t-norm in Table 5 is widely used popular t-norm namely the Attanassov's t-norm. From the above results, we see that in region I, every t-norms give the same value. So, in region I the model is independent of t-norms but every t-norm gives better result than the crisp result. For the region II, the total average cost varies for different t-norms over the regions. The t-norms in Tables 2, 3 and 7 for region II, the total average cost value is \$1289.56 which is less than the average cost obtained in region I. The cost function in this region for Attanassov's t-norm (in Table 5) is maximum overall cost values. So, the Attanassov's t-norm is not suitable in region II. In Attanassov's region (region III), every t-norm gives

Table 11 Sensitivity of IFS solution for $Max = \frac{\alpha\beta}{\alpha+\beta-\alpha\beta}$ over the region IV

Parameter	% Change	α	β	T Weeks	Q Batches	Z(\$)	$\frac{Z-Z^*}{Z^*} \times 100\%$
b	− 30	0.97	0.97	0.38	868.83	1085.00	− 17.24
	− 10	0.82	0.82	0.43	898.16	1234.30	− 5.85
	+10	0.68	0.68	0.48	926.37	1354.23	+3.30
	+30	0.68	0.68	0.57	926.37	1354.23	+3.30
h	− 30	0.97	0.97	0.55	869.38	1088.02	− 17.01
	− 10	0.82	0.82	0.48	898.38	1235.33	− 5.77
	+10	0.68	0.68	0.44	926.36	1354.23	+3.30
	+30	0.68	0.68	0.44	926.36	1354.23	+3.30
n	− 30	0.78	0.78	0.47	906.34	1271.47	− 3.01
	− 10	0.75	0.75	0.46	911.50	1293.93	− 1.30
	+10	0.73	0.73	0.45	915.70	1311.62	+0.05
	+30	0.71	0.71	0.45	919.25	1326.19	+1.16

Table 12 Sensitivity of IFS solution for $Max = \alpha - \beta$ over the region I

Parameter	% change	α	β	T Weeks	Q Batches	Z(\$)	$\frac{Z-Z^*}{Z^*} \times 100\%$
b	− 30	0.5	0.5	0.53	780.00	1153.55	− 12.01
	− 10	0.42	0.58	0.43	976.18	1233.98	− 5.87
	+10	0.32	0.68	0.48	995.84	1355.62	+3.40
	+30	0.24	0.76	0.53	1011.47	1466.27	+11.84
h	− 30	0.5	0.5	0.74	961.00	1153.55	− 12.01
	− 10	0.42	0.58	0.48	976.34	1234.91	− 5.80
	+10	0.32	0.68	0.44	749.94	1354.73	+3.34
	+30	0.24	0.76	0.41	1011.13	1463.72	+11.65
n	− 30	0.39	0.61	0.47	761.86	1267.63	− 3.31
	− 10	0.37	0.62	0.46	985.3	1287.96	− 1.76
	+10	0.36	0.64	0.46	987.87	1303.91	− 0.54
	+30	0.35	0.65	0.45	989.94	1317.04	+0.46

Table 13 Sensitivity of IFS solution for $Max = \alpha - \beta$ over the region II

Parameter	% change	α	β	T Weeks	Q Batches	$Z(\$)$	$\frac{Z-Z^*}{Z^*} \times 100\%$
b	− 30	0.68	0.68	0.73	926.37	1354.23	+ 3.30
	− 10	0.68	0.68	0.66	926.37	1354.23	+ 3.30
	+ 10	0.66	0.69	0.48	806.95	1367.54	+ 4.31
	+ 30	0.49	0.77	0.52	963.19	1475.87	+ 12.57
h	− 30	0.58	0.58	0.89	945.57	1235.39	− 5.77
	− 10	0.68	0.68	0.73	926.37	1354.23	+ 3.30
	+ 10	0.66	0.69	0.44	807.17	1366.68	+ 4.25
	+ 30	0.49	0.76	0.40	778.91	1473.30	+ 12.38
n	− 30	0.61	0.61	0.47	938.84	1274.94	− 2.75
	− 10	0.68	0.68	0.62	926.37	1354.23	+ 3.30
	+ 10	0.68	0.68	0.55	926.37	1354.23	+ 3.30
	+ 30	0.68	0.68	0.55	926.37	1354.23	+ 3.30

Table 14 Sensitivity of IFS solution for $Max = \alpha - \beta$ over the region III

Parameter	% change	α	β	T Weeks	Q Batches	$Z(\$)$	$\frac{Z-Z^*}{Z^*} \times 100\%$
b	− 30	0.56	0.44	0.40	948.92	1097.80	− 16.26
	− 10	0.5	0.5	0.46	961.00	1153.50	− 12.01
	+ 10	0.5	0.5	0.57	961.00	1153.50	− 12.01
	+ 30	0.5	0.5	0.67	961.00	1153.50	− 12.01
h	− 30	0.56	0.44	0.54	949.60	1100.74	− 16.04
	− 10	0.5	0.5	0.51	961.00	1153.50	− 12.01
	+ 10	0.5	0.5	0.51	792.52	1153.50	− 12.01
	+ 30	0.5	0.5	0.51	961.00	1153.50	− 12.01
n	− 30	0.5	0.5	0.51	812.76	1153.50	− 12.01
	− 10	0.5	0.5	0.51	961.00	1153.50	− 12.01
	+ 10	0.5	0.5	0.51	961.00	1153.50	− 12.01
	+ 30	0.5	0.5	0.51	961.00	1153.50	− 12.01

Table 15 Sensitivity of IFS solution for $Max = \alpha - \beta$ over the region IV

Parameter	% change	α	β	T Weeks	Q Batches	$Z(\$)$	$\frac{Z-Z^*}{Z^*} \times 100\%$
b	− 30	0.95	0.45	0.38	873.11	1108.34	− 15.46
	− 10	0.81	0.59	0.43	900.72	1246.14	− 4.95
	+ 10	0.68	0.68	0.48	926.37	1354.23	+ 3.30
	+ 30	0.68	0.68	0.57	926.37	1354.23	+ 3.30
h	− 30	0.95	0.45	0.53	873.70	1111.52	− 15.22
	− 10	0.81	0.59	0.48	900.93	1247.13	− 4.87
	+ 10	0.68	0.68	0.44	926.37	1354.23	+ 3.30
	+ 30	0.68	0.68	0.44	926.37	1354.23	+ 3.30
n	− 30	0.77	0.62	0.46	908.27	1279.96	− 2.37
	− 10	0.75	0.63	0.46	912.58	1300.24	− 0.82
	+ 10	0.73	0.65	0.45	916.80	1316.16	+ 0.39
	+ 30	0.71	0.66	0.45	920.00	1329.22	+ 1.39

the minimum objective value than that of crisp value as well as the average cost value obtained from some other regions. The cost function and decision variables assume exactly same value for any t-norm here. The region IV gives lesser cost value than crisp value but the cost value is maximum than any other regions. The cost value changes in this region due to different t-norms but the order quantity attains its minimum value for this region only for most of the t-norms. From the numerical example, we observe that the minimum cost value $1153.5 with optimum cycle length 0.51 week. Also, the nature of the t-norms in Tables 2,3 and 7 are quite similar as observed in numerical example. The behaviours of the t-norms in Tables 4,5 and 6 are different.

8 Sensitivity analysis

Here we compute sensitivity of two different t-norms for four different regions making a change of the parameters $\{b, h, n\}$ from − 30% to + 30% and this can be shown in the following Tables 8, 9, 10, 11, 12, 13, 14, 15. The rest of the t-norm values coincide one of these two for four different regions. Let $\text{Hamachert}-\text{norm}, \text{X} : Max = \frac{\alpha\beta}{\alpha+\beta-\alpha\beta}$ and $\text{Attanassovt}-\text{norm}, \text{Y} : Max = \alpha - \beta$

8.1 Discussion on sensitivity analysis

Table 8, 9, 10, 11 shows the sensitivity analysis of the parameters for the Hamacher t-norm for four regions. In region I, the − 30% change in the parameters, the cost value is much sensitive and it decreases up to − 12.01%. For a change of − 30% to + 30% in the parameters $\{b, h, n\}$, the nature of the cost function is increasing. The minimum value of the cost function reaches $1153.5 which coincides with the cost value obtained at Attanassov's region (region III). Also, for + 30% change in the parameter h, the cost function attains its maximum $1463.72. In region II, the cost function is more sensitive than in region I. For − 30% change in the parameter b, the cost function attains its minimum value $1083.88 with minimum cycle time 0.38 week. In Attanassov's region, the changes from − 30 to + 30% make no effect on the cost function. The cost function remains constant at $1153.55 for any change. For region IV, the cost function is sensitive for the change -30% to + 30% in the parameters. In this case, the maximum and minimum relative changes in the cost value are + 3.30% and -17.24% respectively.

Tables 12–15 shows in region I, the cost function is sensitive and increasing in nature for a change from − 30 to + 30%. The minimum value it attains is $1153.5 which is the cost value for Attanassov's region for all the t-norms. For region II, the cost function is less sensitive whenever a change from − 30 to + 30% is made. In region III, the cost function converges to $1153.5 for a change from − 30 to + 30% in the parameters. For region IV, the cost function is moderately sensitive and the minimum cost value is $1108.34.

Throughout the whole Tables 8, 9, 10, 11, 12, 13, 14, 15, we observe that the minimum cost value is $1083.88 for -30% change in the parameter b for the Hamacher t-norm for region II which is a non-Attanassov's region and the maximum cost value is $1475.87 for max t-norm for region II. From this we can conclude that the region II is much sensitive for the parameters and a minimum objective value can be obtained from a non-Attanassov's zone. The minimum order quantity is 749.94 batches obtained in region I for Hamacher t-norm with + 10% change in h and maximum order quantity (1011.47 batches) is obtained for + 30% change in b for Hamacher and max t-norm in region I.

Figure 1 corresponds the total average cost function which is convex in T and Q. Figure 2 shows the variations in the cost function for crisp environment and for both Hamacher t-norm and Attanassov's t-norm in different regions. In region I and III, the cost value remains exactly same but in region II and IV, the Hamacher t-norm gives better cost value than the Attanassov t-norm though in all cases the crisp cost is higher.

Figure 3 and Fig. 4 give a comparative study of the cost value for Hamacher t-norm and Attanassov t-norm whenever a % change is made in the parameter b. There is a similarity in the curve of the cost value for both the t-norms in regions I and IV but for region II the curve is completely different.

Here also for Fig. 5 and Fig. 6 the cost values for regions I and IV are same but for region II, the Hamacher t-norm gives the minimum cost value for -30% change in h.

Figure 7 and Fig. 8 reveal the graph of the sensitivity analysis of the parameter n for t-norm X & Y. From these two figures for regions, I and III, the graph of the cost function is equal but in region II for t-norm X the curve is strictly increasing whereas for t-norm Y the curve is increasing and then being fixed. The opposite behaviour has been observed for region IV.

Figure 9 explores the nature of the order quantity for three different environments. In crisp environment, the order quantity is fixed. In fuzzy environment, region I requires maximum order quantity for both t-norms X and Y but in region II, least replenishment is needed; though the crisp environment assumes the minimum replenishment quantity than that of the intuitionistic fuzzy environment.

9 Conclusion

This study expresses an EOQ model over varying number of tourist dependent demand rate. There is a vast literature on tourism demand modelling and forecasting in economics and other areas but in inventory management this is the first attempt in the global scenario. Through this model, we have

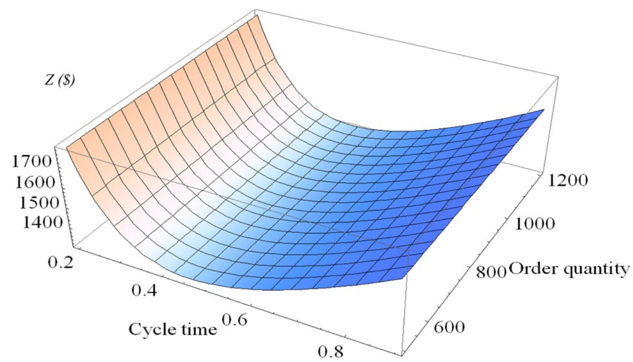


Fig. 1 Optimum total average cost vs. cycle time and order quantity (colour figure online)

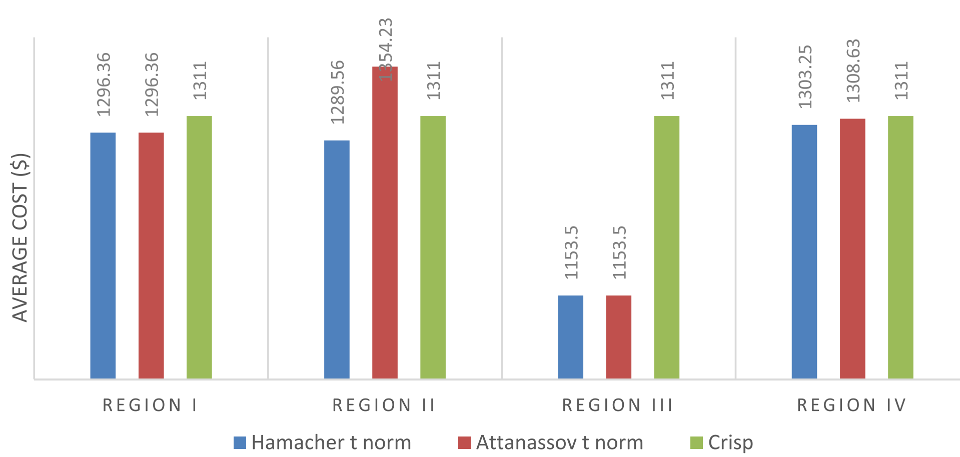
studied six different sets of triangular norms on IFS theory for four different regions/domains and obtained the optimal solution. Our main contribution lies in the practical application of fuzzy t-norms based on their flexibility and variability on IFS theory in inventory management problems. Our findings suggest that in Attanassov's zone where the sum of membership degree and non-membership degree is less than or equals one, all t-norms give the optimum value of the cost function (In region III, optimum cycle time is 0.51-week, optimum order quantity gets 961 batches and the corresponding system cost assumes \$1153.5 respectively with $\alpha^* = \beta^* = 0.5$). But some cases may arise whenever a percentage change is made for model parameter, in non-Attanassov zone over Hamacher t-norm works best and assumes minimum value (In region II, optimum average system cost assumes \$1083.88 with respect to the optimum cycle time 0.38 week and the optimum order quantity 868.63 batches with $\alpha^* = 0.98\beta^* = 1$; In region IV, the system cost becomes \$1085 with respect to the optimum order quantity 868.83 batches and cycle time 0.38 week with $\alpha^* = 0.97 = \beta^*$). However, we found some other optimal regions where the superiority of Attanassov's concepts have been ignored intelligently. Thus, from a decision makers' point of view the following observations should be taken care of:

(i) Fuzzy t-norm approach is profitable as it gives the minimum cost for all the cases.
(ii) Hamacher t-norm is more suitable whenever the sum of membership and non- membership degree exceeds one.
(iii) Every t-norm works best in Attanassov region where the membership degree is greater than the non-membership degree.
(iv) In any fuzzy optimization problem having greater non-membership degree relative to membership degree of the fuzzy variables, the techniques of selecting various t-norms might give new research.

10 Scope of future work

This model can be extended to n-layer supply chain model as well. Besides that, the concept of triangular norms can be applied to multiple-decision making problems and other fuzzy optimization models.

Fig. 2 The total average cost under different regions (colour figure online)



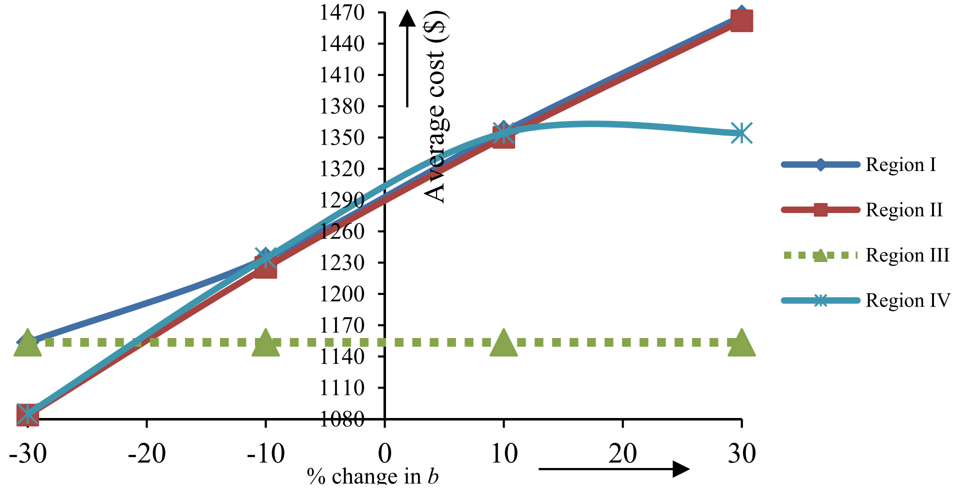


Fig. 3 Cost value for different regions for % change in b for Hamacher t-norm (colour figure online)

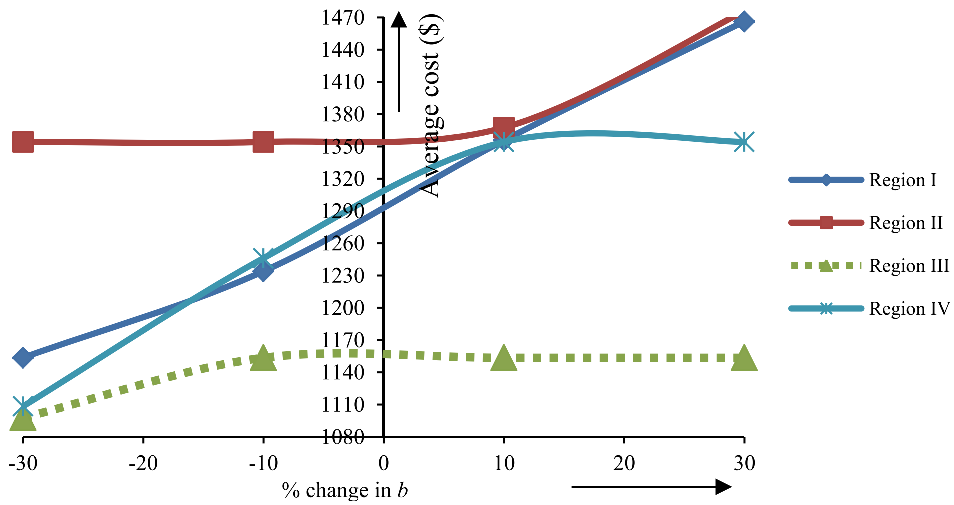


Fig. 4 Cost value for different regions for % change in b for Attanassov t-norm (colour figure online)

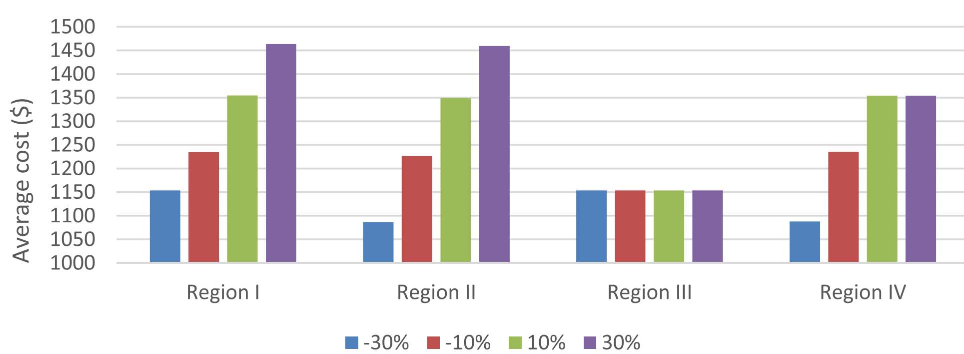


Fig. 5 Cost value for different regions for % change in h for Hamacher t-norm (colour figure online)

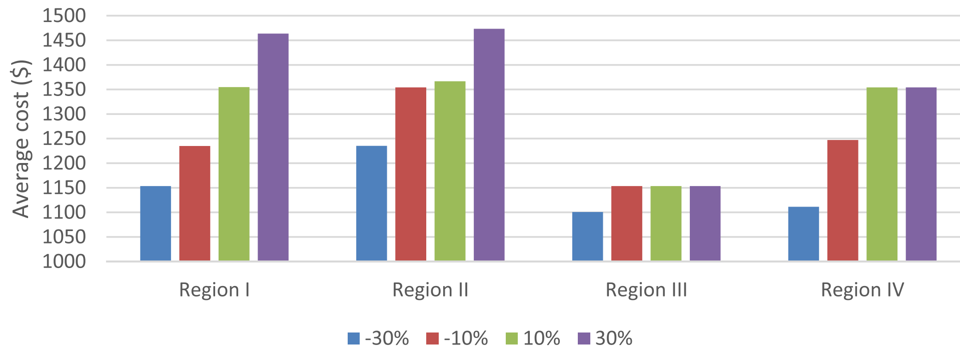


Fig. 6 Cost value for different regions for % change in *h* for Attanassov t-norm (colour figure online)

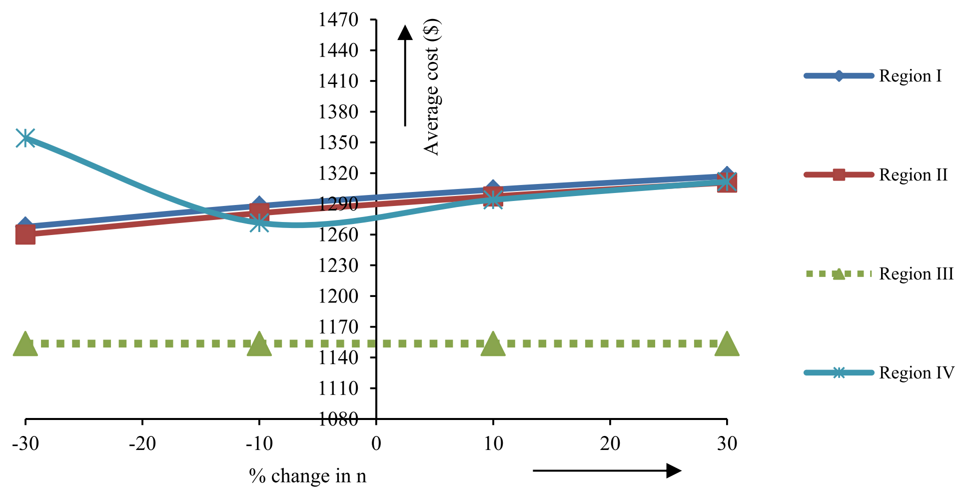


Fig. 7 Cost value for different regions for % change in *n* for Hamacher t-norm (colour figure online)

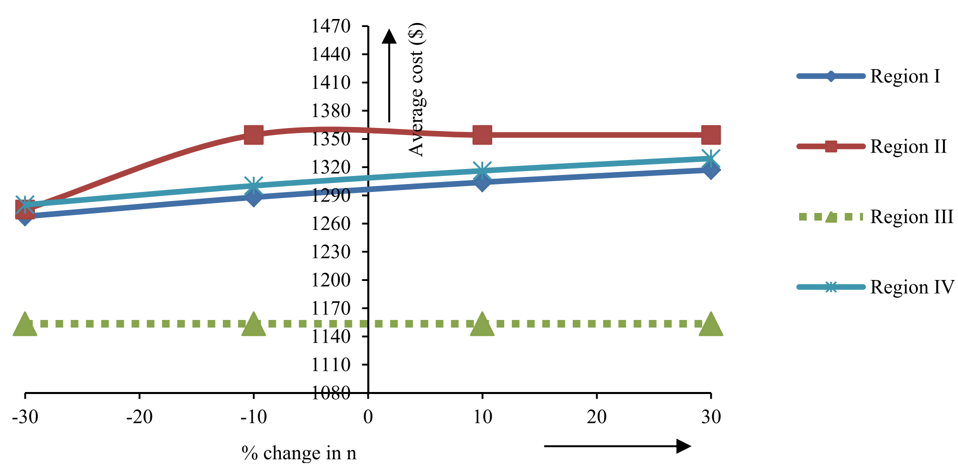


Fig. 8 Cost value for different regions for % change in *n* for Attanassov t-norm (colour figure online)

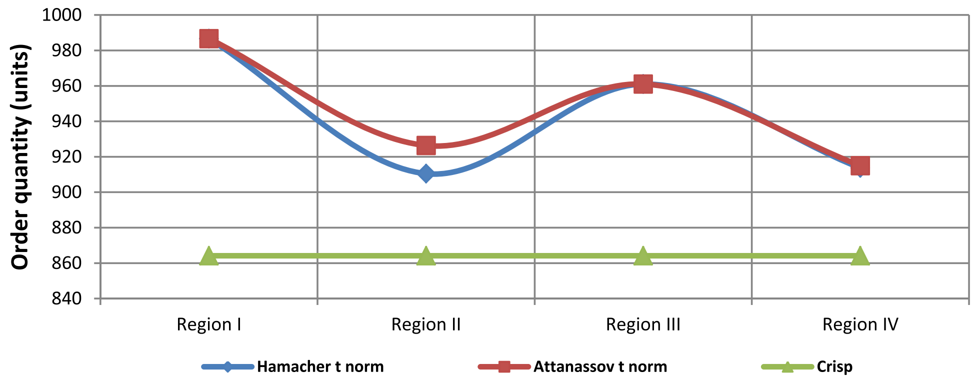


Fig. 9 Order quantity for four different regions for crisp and t-norms (colour figure online)

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Declarations

Conflict of interest The authors declare that they have no conflict of interest regarding the publication of this article.

References

Atanassov K (1986) Intuitionistic fuzzy sets and system. Fuzzy Sets Syst 20:87–96

Atanassov K (1999) Intuitionistic fuzzy Sets: theory and applications. Physica Verlag, Heidelberg

Azadeh A, Zia NP, Saberi M, Hussain FK, Yoon JH, Hussain OK, Sadri S (2015) A trust-based performance measurement modeling using t-norm and t-conorm operator. Appl Soft Comput 30:491–500

Banerjee S, Sharma A (2010) Inventory model for seasonal demand with option to change the market. Comput Ind Eng 59:807–818

Bejines C, Brutenicova M, Chasco MJ, Elorza J, Janis V (2020) The number of t-norms on some special lattices. Fuzzy sets systems 408:26

Bellman RE, Zadeh LA (1970) Decision making in a fuzzy environment. Manage Sci 17:B141–B164

Bianchi M (2015) The logic of the strongest and the weakest t-norms. Fuzzy Sets System 276:31–42

Bielawski J, Tabor J (2020) Convex hull of a fuzzy sets and triangular norms. Fuzzy Sets System 417:93

Chen KK, Chang C-T (2007) A seasonal demand inventory model with variable lead time and resource constraints. Appl Math Model 31:2433–2445

Chen S-M, Tan J-M (1994) Handling multicriteria fuzzy decision-making problems based on vague set theory. Fuzzy Sets Syst 67(2):163–172

Chang C-T, Chou H-C (2013) A coordination system for seasonal demand problems in the supply chain. Appl Math Model 37:3674–3686

De SK, Sana SS (2014) A multi-period production-inventory model with capacity constraints for multi-manufacturers-a global optimality in intuitionistic fuzzy environment. Appl Math Comput 242:825–841

De SK, Goswami A, Sana SS (2014) An interpolating by pass to pareto optimality in intuitionistic fuzzy technique for an eoq model with time sensitive backlogging. Appl Math Comput 230:664–674

De SK, Mahata GC, Maity S (2021) Carbon emission sensitive deteriorating inventory model with trade credit under volumetric fuzzy system. Int J Inteligent Syst 36(10):5530–5572

Dymova L, Sevastjanov P (2011) Operations on intuitionistic fuzzy values in multiple criteria decision making. Sci Res Institut Math Comput Sci 1:41–48

Ehrenthal JCF, Honhon D, Woensel TV (2014) Demand seasonality in retail inventory management. Eur J Oper Res 238:527–539

Goswami AC, KS, (1991) An EOQ model for deteriorating items with shortages and a linear trend in demand. J of Oper Research Society 42(12):1105–1110

Grzegorzewski P (2002) Nearest interval approximation of a fuzzy number. Fuzzy Setsems 130:321–330

Klement EP, Mesiar R, Pap E (2004a) Triangular norms. Position paper I: basic analytical and algebraic properties. Fuzzy Sets and System 143:5–26

Klement EP, Mesiar R, Pap E (2004b) Triangular norms. Position paper II: general constructions and parametized families. Fuzzy Sets and System 145:411–438

Klement EP, Mesiar R, Pap E (2004c) Triangular norms. Position paper III: continuous t-norms. Fuzzy Sets and System 145:439–454

Kumar M (2014) Applying weakest t-norm based approximate intuitionistic fuzzy arithmetic operations on different types of intuitionistic fuzzy numbers to evaluate reliability of PCBA fault. Appl Soft Comput 23:387–406

Kumar RS, De SK, Goswami A (2012) Fuzzy EOQ models with ramp type demand rate, partial backlogging and time dependent deterioration rate. Int J Math Operat Res 4:473–502

Li P, Lin Y (2014) Linear optimization with bipolar fuzzy relational equation constraints using the Lukasiewicz triangular norm. Soft Comput. https://doi.org/10.1007/s00500-013-1152-1

Lima L, Bedregal B, Bustince H, Barrenechea E, Rocha M (2016) An interval extension of homogeneous and pseudo-homogeneous t-norms and t-conorms. Inform Sci 355:328–347

Liu Z-Q, Wang X-P (2020) Distributivity between extended t-norms and t-conorms on fuzzy truth values. Fuzzy Sets Systems 408:44

Menger K (1942) Statistical metrices. Proc Natl Acad Sci USA 28(12):535–537

Petrik M (2020) Dominance on continuous Archimedean triangular norms and generalised Mulholland inequality. Fuzzy Sets Systems 403:88

Rossello J, Sanso A (2017) Yearly, monthly and weekly seasonality of tourism demand: a decomposition analysis. Tour Manage 60:379–389

Sarkar A, Biswas A (2019) Multi criteria decision-making using Archimedean aggregation operators in Pythagorean hesitant fuzzy environment. Int J Intell Syst 1–26

Singh S, Lalotra S, Sharma S (2019) Dual concepts in fuzzy theory: Entropy and knowledge measure. Int J Intell Syst 1–26

Sun X-R, Liu H-W (2020) The additive generators of t-norms and t-conorms on bounded lattices. Fuzzy Sets Syst 408:13

Xia MM, Xu ZS, Zhu B (2012) Some issues on intuitionistic fuzzy aggregation operators based on Archimedean t-conorm and t-norm. Knowledge Based Syst 31:78–88

Zadeh LA (1965) Fuzzy sets. Inf Control 8:338–356