



Gudermannian neural networks using the optimization procedures of genetic algorithm and active set approach for the three-species food chain nonlinear model

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Abstract

The present study is to investigate the Gudermannian neural networks (GNNs) using the optimization procedures of genetic algorithm and active-set approach (GA-ASA) to solve the three-species food chain nonlinear model. The three-species food chain nonlinear model is dependent upon the prey populations, top-predator, and specialist predator. The design of an error-based fitness function is presented using the sense of the three-species food chain nonlinear model and its initial conditions. The numerical results of the model have been obtained by exploiting the GNN-GA-ASA. The obtained results through the GNN-GA-ASA have been compared with the Runge–Kutta method to substantiate the correctness of the designed approach. The reliability, efficacy and authenticity of the proposed GNN-GA-ASA are examined through different statistical measures based on single and multiple neurons for solving the three-species food chain nonlinear model.

Keywords Gudermannian neural network · Three-dimensional food chain nonlinear model · Nonlinear differential system · Runge–Kutta scheme · Active-set algorithm · Statistical studies

1 Introduction

The study of two and three trophic-level based on food chain systems using the structure of logistic prey X , specialist Lotka–Volterra predator Y and top-predator Z (Freedman and Waltman 1977; Freedman and So 1985; Muratori and Rinaldi 1992; Kuznetsov and Rinaldi 1996; Rinaldi et al. 1996; El-Owaidy et al. 2001; Umar et al. 2019). The general

form of the state system based on the three species food chain nonlinear model is written as (Aziz-Alaoui 2002):

$$\begin{cases} X'(T) = a_0X(T) - b_0X^2(T) - \frac{v_0X(T)Y(T)}{X(T)+d_0}, X(0) = l_1, \\ Y'(T) = -a_1Y(T) + \frac{v_1X(T)Y(T)}{d_1+X(T)} - \frac{v_2Y(T)Z(T)}{Y(T)+d_2}, Y(0) = l_2, \\ Z'(T) = c_3Z^2(T) - \frac{v_3Z^2(T)}{Y(T)+d_3}, Z(0) = l_3. \end{cases} \quad (1)$$

The above system represents the three-dimensional food chain nonlinear model that has been investigated analytically/numerically with the prey population X , which implemented as a single food predator Y together with prey of a top-predator Z . The features of prey X along with the species Y present the modeling of Volterra scheme, which indicates the predator population reduces exponentially in the absence of prey. The association of species Z together with its prey Y is formed based on the Leslie–Gower scheme (Leslie and Gower 1960), which indicates the predator population reduces to the reciprocal of per capita availability of its most special food (Upadhyay et al. 1998). l_1, l_2 and l_3 are the positive initial conditions (Ics). The model parameter's detail of the three-dimensional food chain nonlinear model is expressed in the Table 1.

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Table 1 Illustrations of the three-dimensional food chain nonlinear model

Parameters	Specification
a_0	Prey growth rate X
b_0	Competition power among individuals-based species X
d_2	Elimination rate of Y per capita is $\frac{b_2}{2}$
d_0, d_1	Environment produce conservation to prey X
a_1	Rate at Y will decrease in the omission of X
d_3	Surplus loss in the species of Z due to severe insufficiency of its selected food Y
c_3	Development rate of Z
v_0, v_1, v_2, v_3	Obtained maximum values per capita by reducing the X
l_1, l_2, l_3	Positive ICs

The stochastic computing processes have been executed to solve a large variety of nonlinear systems, few of them are fractional singular systems (Sabir et al. 2021c, d, e), like higher order singular systems (Ayub et al. 2021; Sabir et al. 2021a), dengue fever system (Umar et al. 2020b, c, d, e), Sitr based COVID-19 models (Umar et al. 2020b, c, d, e, 2021a, b), delay singular function system (Khan et al. 2021; Sabir et al. 2021b), SIR system for spreading infection and treatment (Umar et al. 2021a, b), mosquito release system in the heterogeneous environment (Umar et al. 2020a), doubly singular nonlinear models (Raja et al. 2019; Sabir et al. 2020a, b, c), rank-constrained spectral clustering (Li et al. 2018a, b), zero-shot event detection system (Li et al. 2019), fuzzy K-means clustering associated discriminative embedding scheme (Li et al. 2018a, b), multiclass classification systems (Yan et al. 2020). dynamic affinity graph construction strategy for spectral clustering (Nie et al. 2020), enhanced multilayer piezoelectric transducer design (Naz et al. 2021), performance investigation of the heat sink of functionally graded material of the porous fin (Ahmad et al. 2021), impact of heat transfer in a Bodewadt flow model

in a set of Eq. (1). A brief summary of innovative insights and contributions of the presented study is listed in terms of salient features as follows:

- A novel application of artificial intelligent knacks via Gudermannian neural networks (GNNs) models optimized with genetic algorithm and active-set approach (GA-ASA), i.e., GNNs-GA-ASA is introduced to solve a mathematical model of the three-species food chain nonlinear systems (TS-FCNS).
- The design of an error-based fitness function is effectively portrayed for TS-FCNS for the dynamics of the prey populations, top-predator and specialist predator.
- The numerical results of the TS-FCNS have been obtained by exploiting computation heuristics of GNNs-GA-ASA and comparison with the outcomes of the Runge–Kutta method substantiated the correctness of the designed approach.
- The reliability, efficacy and authenticity of the proposed GNNs-GA-ASA are further scrutinized through different statistical measures based on single and multiple executions for solving the three-species food chain nonlinear model.

The paper is organized as: Sect. 2 defines the computational procedures based on GNNs-GA-ASA along with the statistical measures are provided in the next section. The result and discussion are provided in Sect. 3. The concluding remarks and future research directions are provided in Sect. 4.

2 Designed procedures: GNNs-GA-ASA

The mathematical formulations of the three-dimensional food chain nonlinear model together with derivatives are derived as:

$$\begin{aligned}
 [\hat{X}(T), \hat{Y}(T), \hat{Z}(T)] &= \left[\begin{array}{l} \sum_{q=1}^s r_{X,q} M(w_{X,q}T + n_{X,q}), \sum_{q=1}^s r_{Y,q} M(w_{Y,q}T + n_{Y,q}), \\ \sum_{q=1}^s r_{Z,q} M(w_{Z,q}T + n_{Z,q}), \end{array} \right], \\
 [\hat{X}'(T), \hat{Y}'(T), \hat{Z}'(T)] &= \left[\begin{array}{l} \sum_{q=1}^s r_{X,q} M'(w_{X,q}T + n_{X,q}), \sum_{q=1}^s r_{Y,q} M'(w_{Y,q}T + n_{Y,q}), \\ \sum_{q=1}^s r_{Z,q} M'(w_{Z,q}T + n_{Z,q}), \end{array} \right],
 \end{aligned}
 \tag{2}$$

(Awais et al. 2021), thin film flow model over a stretched surface (Uddin et al. 2021) and state estimation problems arising in underwater Markov chain maneuvering targets (Ali et al. 2021). All these utmost applications inspired the authors to explore/exploit/investigate artificial intelligence-based computational solver to solve the governing model of three-species food chain nonlinear model as presented

where, W indicates an unidentified weight vector, given as: $w = [w_x, w_y, w_z]$, for $W_X = [r_X, \omega_X, n_X]$, $W_Y = [r_Y, \omega_Y, n_Y]$, and $W_Z = [r_Z, \omega_Z, n_Z]$, where

$$\begin{aligned}
 r_X &= [r_{X,1}, r_{XT,2}, r_{X,3}, \dots, r_{X,s}], r_Y = [r_{Y,1}, r_{Y,2}, r_{Y,3}, \dots, r_{Y,s}], r_Z = [r_{Z,1}, r_{Z,2}, r_{Z,3}, \dots, r_{Z,s}] \\
 w_X &= [w_{X,1}, w_{XT,2}, w_{X,3}, \dots, w_{X,s}], w_Y = [w_{Y,1}, w_{Y,2}, w_{Y,3}, \dots, w_{Y,s}], w_Z = [w_{Z,1}, w_{Z,2}, w_{Z,3}, \dots, w_{Z,s}] \\
 n_X &= [n_{X,1}, n_{X,2}, n_{X,3}, \dots, n_{X,s}], n_Y = [n_{Y,1}, n_{Y,2}, n_{Y,3}, \dots, n_{Y,s}], n_Z = [n_{Z,1}, n_{Z,2}, n_{Z,3}, \dots, n_{Z,s}]
 \end{aligned}$$

A merit function, i.e., Gudermannian function $M(\Psi) = 2\tan^{-1}[\exp(\Psi)] - \frac{1}{2}\pi$ (Sabir et al. 2021c, d, e) is used as

Genetic algorithm is known as a famous, optimization global search scheme implemented to solve the linear/non-linear models. It is performed to tackle both constrained/unconstrained systems using the typical selection processes.

$$\begin{aligned}
 &= \left[\sum_{k=1}^r q_{X,k} \left(2\tan^{-1} e^{(w_{X,k}\Psi + n_{X,k})} - \frac{\pi}{2} \right), \sum_{k=1}^r q_{Y,k} \left(2\tan^{-1} e^{(w_{Y,k}\Psi + n_{Y,k})} - \frac{\pi}{2} \right), \right. \\
 &\quad \left. \sum_{k=1}^r q_{Z,k} \left(2\tan^{-1} e^{(w_{Z,k}\Psi + n_{Z,k})} - \frac{\pi}{2} \right) \right], \\
 [\hat{X}'(\Psi), \hat{Y}'(\Psi), \hat{Z}'(\Psi)] &= \left[\sum_{k=1}^r 2q_{X,k} w_{X,k} \left(\frac{e^{(w_{X,k}\Psi + n_{X,k})}}{1 + (e^{(w_{X,k}\Psi + n_{X,k})})^2} \right), \right. \\
 &\quad \left. \sum_{k=1}^r 2q_{Y,k} w_{Y,k} \left(\frac{e^{(w_{Y,k}\Psi + n_{Y,k})}}{1 + (e^{(w_{Y,k}\Psi + n_{Y,k})})^2} \right), \right. \\
 &\quad \left. \sum_{k=1}^r 2q_{Z,k} w_{Z,k} \left(\frac{e^{(w_{Z,k}\Psi + n_{Z,k})}}{1 + (e^{(w_{Z,k}\Psi + n_{Z,k})})^2} \right) \right], \tag{3}
 \end{aligned}$$

The merit function is provided as:

$$\xi_f = \xi_{f-1} + \xi_{f-2} + \xi_{f-3} + \xi_{f-4} \tag{4}$$

$$\xi_{f-1} = \frac{1}{N} \sum_{j=1}^N \left[\hat{X}_j + a_0 \hat{X}_j + b X_j^2 + \frac{v_0 \hat{X}_j \hat{Y}_j}{d_0 + \hat{X}_j} \right]^2, \tag{5}$$

$$\xi_{f-2} = \frac{1}{N} \sum_{j=1}^N \left[\hat{Y}_j + a \hat{Y}_j - \frac{v_1 \hat{Y}_j \hat{Y}_j}{d_1 + \hat{X}_j} + \frac{v_2 \hat{Y}_j \hat{Z}_j}{d_2 + \hat{Y}_j} \right]^2, \tag{6}$$

$$\xi_{f-3} = \frac{1}{N} \sum_{j=1}^N \left[\hat{Z}_j - c_3 \hat{Z}_j + \frac{v_3 \hat{Z}_j}{d_3 + \hat{Y}_j} \right]^2, \tag{7}$$

$$\xi_{f-4} = \frac{1}{3} \left[(\hat{X}_0 - l_1)^2 + (\hat{Y}_0 - l_2)^2 + (\hat{Z}_0 - l_3)^2 \right], \tag{8}$$

where $\hat{X}_j = X(T_j), \hat{Y}_j = Y(T_j), \hat{Z}_j = Z(T_j), Nh = 1$, and $T_j = hJ$. \hat{X}_j, \hat{Y}_j and \hat{Z}_j indicate the proposed results of the system (1). Likewise, the Eqs. (5)–(7) represent an error function of the three-dimensional food chain nonlinear model and its ICs.

2.1 Optimization: GNN-GA-ASA

This section indicates the optimization procedures to solve the three-dimensional food chain nonlinear model using the stochastic procedures based on GNN-GA-ASA.

GA is usually applied to regulate the results of the accurate population to solve the numerous complex/steep models of ideal training. Recently, GA is implemented in the brain tumor images (Simi et al. 2020), hospitalization expenditure systems (Tao et al. 2019), Thomas–Fermi model (Sabir et al. 2018), feature diversity in cancer microarray (Sayed et al. 2019), radiation protective in the bismuth-borate glasses (Wilson 2019), nonlinear electric circuit models (Mehmood et al. 2020), heat conduction model (Raja et al. 2018), HIV infection model (Umar et al. 2020b, c, d, e), wire coating with Oldroyd 8-constant fluid model (Munir et al. 2019), prediction differential system (Sabir et al. 2020a, b, c), periodic differential model (Sabir et al. 2020a, b, c) and cloud service optimization procedures (Yang et al. 2019). ASA is applied in pricing American better-of option on two assets (Gao et al. 2020), pressure-dependent models of water distribution systems with flow controls (Piller et al. 2020), nonlinear optimization with polyhedral constraints (Hager and Tarzanagh 2020), numerical solution of the optimal control problem governed by partial differential equation (Azizi et al. 2020), electrodynamic frictional contact problems (Abide et al. 2021) and quadratic semidefinite program with general constraints (Shen et al. 2021). The optimization process-based GA-ASA is applied to control the slowness of GA.

2.2 Performance indices

The performance through statistics based on the semi-interquartile range (S.I.R), mean absolute deviation (MAD), variance account for (VAF) and Theil’s inequality coefficient

(TIC) along with the global representation are observed to solve the three-dimensional food chain nonlinear model, given as:

$$\begin{cases} \text{S.I.R} = -0.5(Q_1 - Q_3), \\ Q_1, Q_3 \text{ are the 1st 3rd quartiles,} \end{cases} \tag{9}$$

$$\begin{aligned} & [\text{MAD}_X, \text{MAD}_Y, \text{MAD}_Z] \\ & = \left[\sum_{j=1}^n |X_j - \hat{X}_j|, \sum_{j=1}^n |Y_j - \hat{Y}_j|, \sum_{j=1}^n |Z_j - \hat{Z}_j| \right] \end{aligned} \tag{10}$$

3 Results and discussions

The simplified form of the three-dimensional food chain nonlinear model using suitable parameter values is given as:

$$\begin{cases} X'(T) = 1.5X(T) - 0.06X^2(T) - \frac{X(T)Y(T)}{10+X(T)}, X_0 = 1.2, \\ Y'(T) = -Y(T) + \frac{2X(T)Y(T)}{10+X(T)} - \frac{0.405Y(T)Z(T)}{10+Y(T)}, Y_0 = 1.2, \\ Z'(T) = 1.5Z^2(T) - \frac{Z^2(T)}{20+Y(T)}, Z_0 = 1.2. \end{cases} \tag{13}$$

An objective function using the three-dimensional food chain nonlinear model is written as:

$$\begin{aligned} \xi_f = \frac{1}{N} \sum_{j=1}^N & \left(\left[\hat{X}'_j - 1.5\hat{X}_j + 0.06\hat{X}_j^2 + \frac{\hat{X}_j\hat{Y}_j}{10+\hat{X}_j} \right]^2 + \left[\hat{Y}'_j + \hat{Y}_j - \frac{2\hat{X}_j\hat{Y}_j}{10+\hat{X}_j} + \frac{0.405\hat{Y}_j\hat{Z}_j}{10+\hat{Y}_j} \right]^2 \right) \\ & + \frac{1}{3} \left[(\hat{X}_0 - 1.2)^2 + (\hat{Y}_0 - 1.2)^2 + (\hat{Z}_0 - 1.2)^2 \right] \end{aligned} \tag{14}$$

$$\begin{cases} [\text{VAF}_X, \text{VAF}_Y, \text{VAF}_Z] = \left[\left(1 - \frac{\text{var}(X_j - \hat{X}_j)}{\text{var}(X_j)} \right) * 100, \left(1 - \frac{\text{var}(Y_j - \hat{Y}_j)}{\text{var}(Y_{js})} \right) * 100, \right. \\ \left. \left(1 - \frac{\text{var}(Z_j - \hat{Z}_j)}{\text{var}(Z_j)} \right) * 100, \right] \\ [\text{EVAF}_X, \text{EVAF}_Y, \text{EVAF}_Z] = [100 - \text{VAF}_X, 100 - \text{VAF}_Y, 100 - \text{VAF}_Z]. \end{cases} \tag{11}$$

[TIC_X, TIC_Y, TIC_Z]

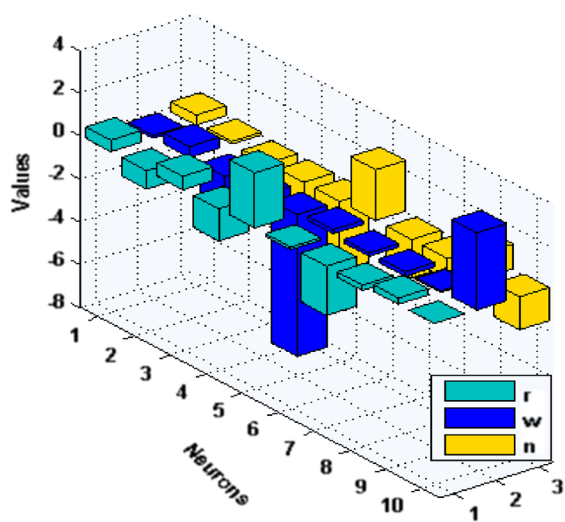
$$= \left[\frac{\sqrt{\frac{1}{n} \sum_{j=1}^n (X_j - \hat{X}_j)^2}}{\left(\sqrt{\frac{1}{n} \sum_{j=1}^n X_j^2} + \sqrt{\frac{1}{n} \sum_{j=1}^n \hat{X}_j^2} \right)}, \frac{\sqrt{\frac{1}{n} \sum_{j=1}^n (Y_j - \hat{Y}_j)^2}}{\left(\sqrt{\frac{1}{n} \sum_{j=1}^n Y_j^2} + \sqrt{\frac{1}{n} \sum_{j=1}^n \hat{Y}_j^2} \right)}, \right. \tag{12}$$

$$\left. \frac{\sqrt{\frac{1}{n} \sum_{j=1}^n (Z_j - \hat{Z}_j)^2}}{\left(\sqrt{\frac{1}{n} \sum_{j=1}^n Z_j^2} + \sqrt{\frac{1}{n} \sum_{j=1}^n \hat{Z}_j^2} \right)}, \right]$$

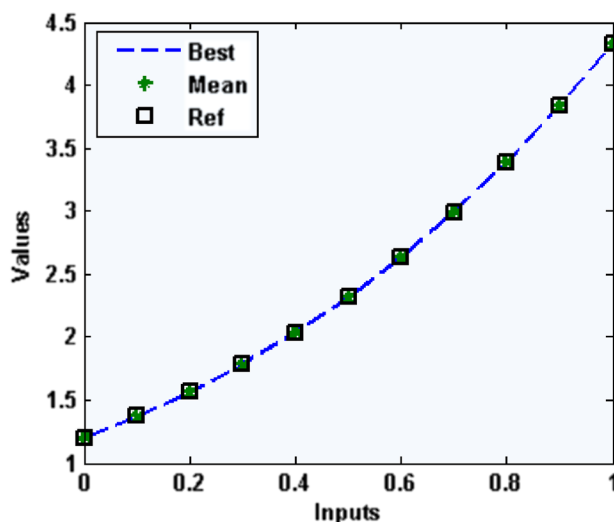
where \hat{X} , \hat{Y} and \hat{Z} are the proposed solutions.

The mathematical results of the stochastic procedures based on GNN-GA-ASA:

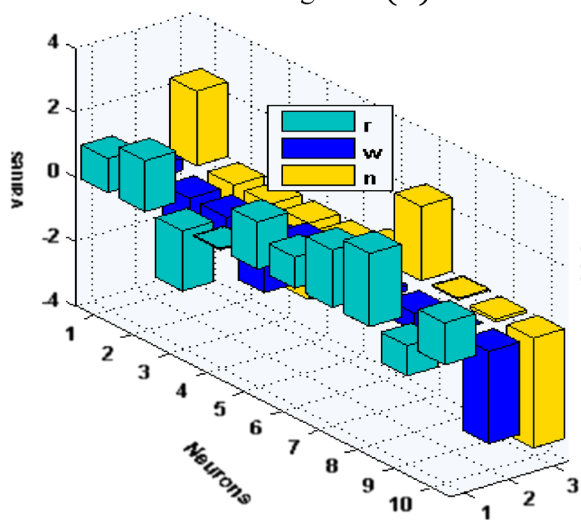
$$\begin{aligned} \hat{X}(\Psi) = & -4.1215(2\tan^{-1} e^{(-1.2073\Psi+2.0489)} - 0.5\pi) - 1.1601(2\tan^{-1} e^{-0.3909\Psi-0.3808} - 0.5\pi) \\ & - 0.0100(2\tan^{-1} e^{(1.7292\Psi+0.6374)} - 0.5\pi) - 0.6757(2\tan^{-1} e^{(1.0278\Psi-0.1355)} - 0.5\pi) \\ & - 3.7274(2\tan^{-1} e^{(-1.3829\Psi+3.4601)} - 0.5\pi) - 0.6447(2\tan^{-1} e^{(1.3228\Psi+1.3535)} - 0.5\pi) \\ & - 1.6067(2\tan^{-1} e^{(-0.3206\Psi-0.8162)} - 0.5\pi) + 0.3827(2\tan^{-1} e^{(0.4292\Psi+1.0041)} - 0.5\pi) \\ & 3.8605(2\tan^{-1} e^{(0.1557\Psi+2.6613)} - 0.5\pi) + 3.8418(2\tan^{-1} e^{(0.1831\Psi+2.4362)} - 0.5\pi), \end{aligned} \tag{15}$$



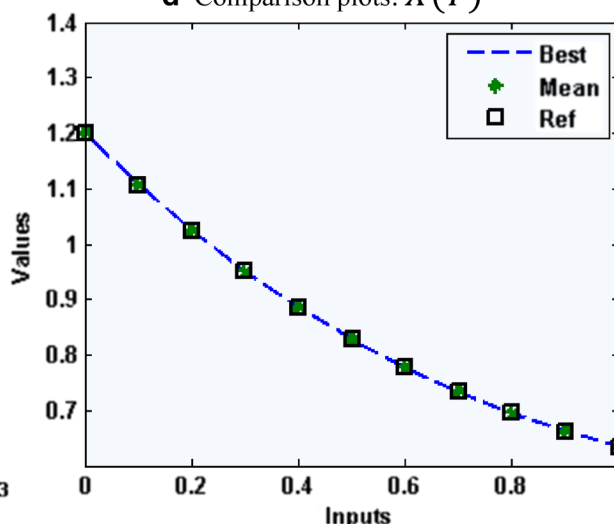
a Best weights: $X(T)$



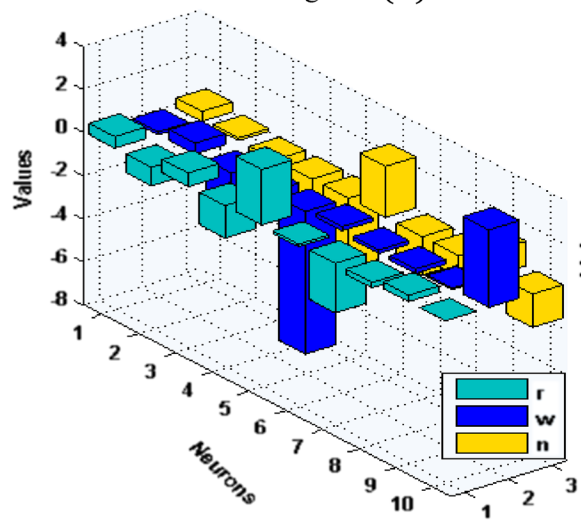
d Comparison plots: $X(T)$



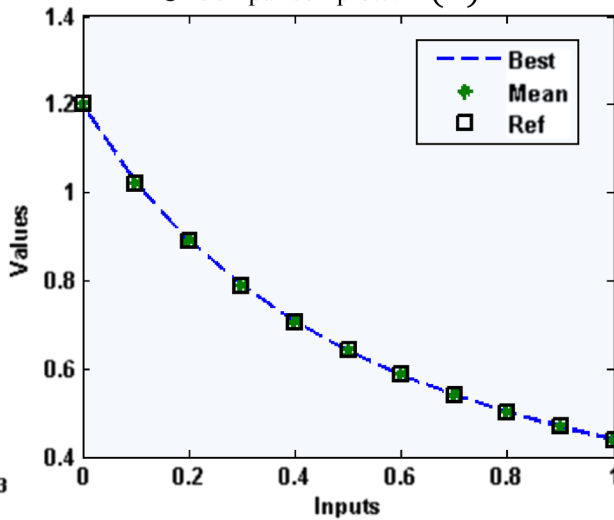
b Best weights: $Y(T)$



e Comparison plots: $Y(T)$



c Best weights: $Z(T)$



f Comparison plots: $Z(T)$

Fig. 1 Comparison of the results and best weight vectors for the three-dimensional food chain nonlinear model

Fig. 2 AE values and the performances based on MAD, TIC and EVAF for the three-dimensional food chain nonlinear model

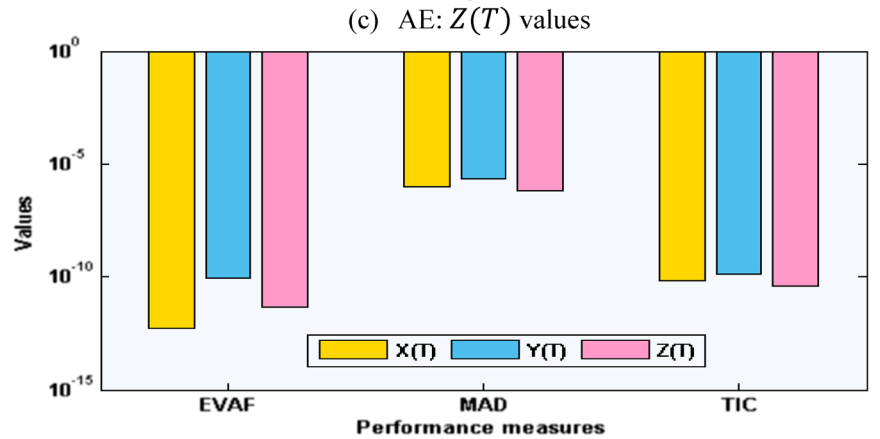
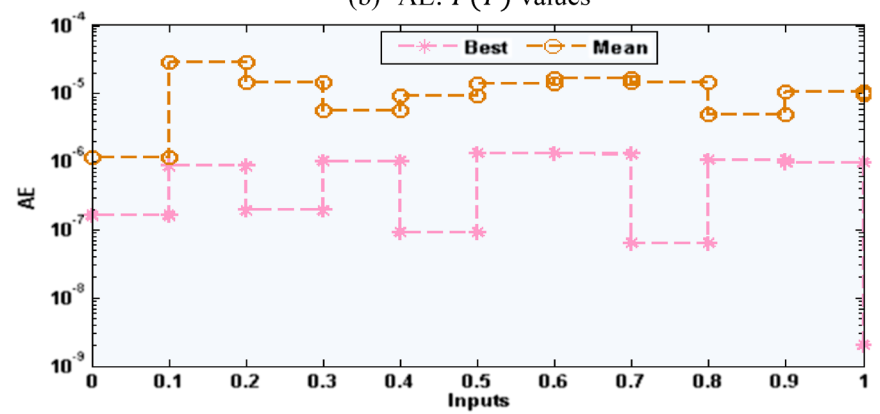
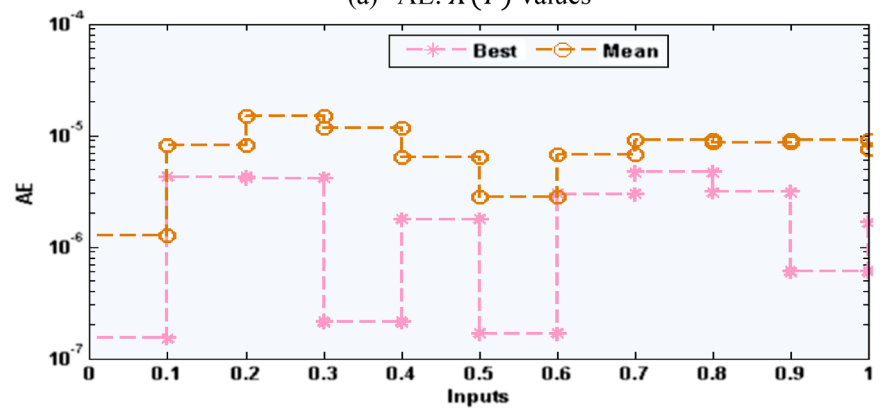
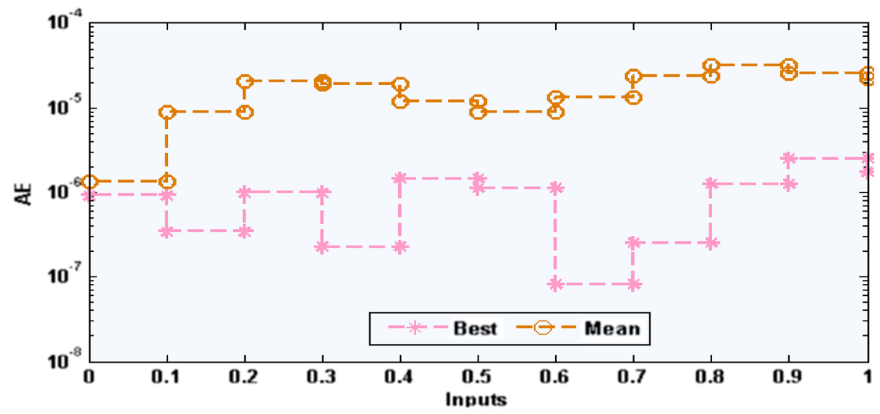
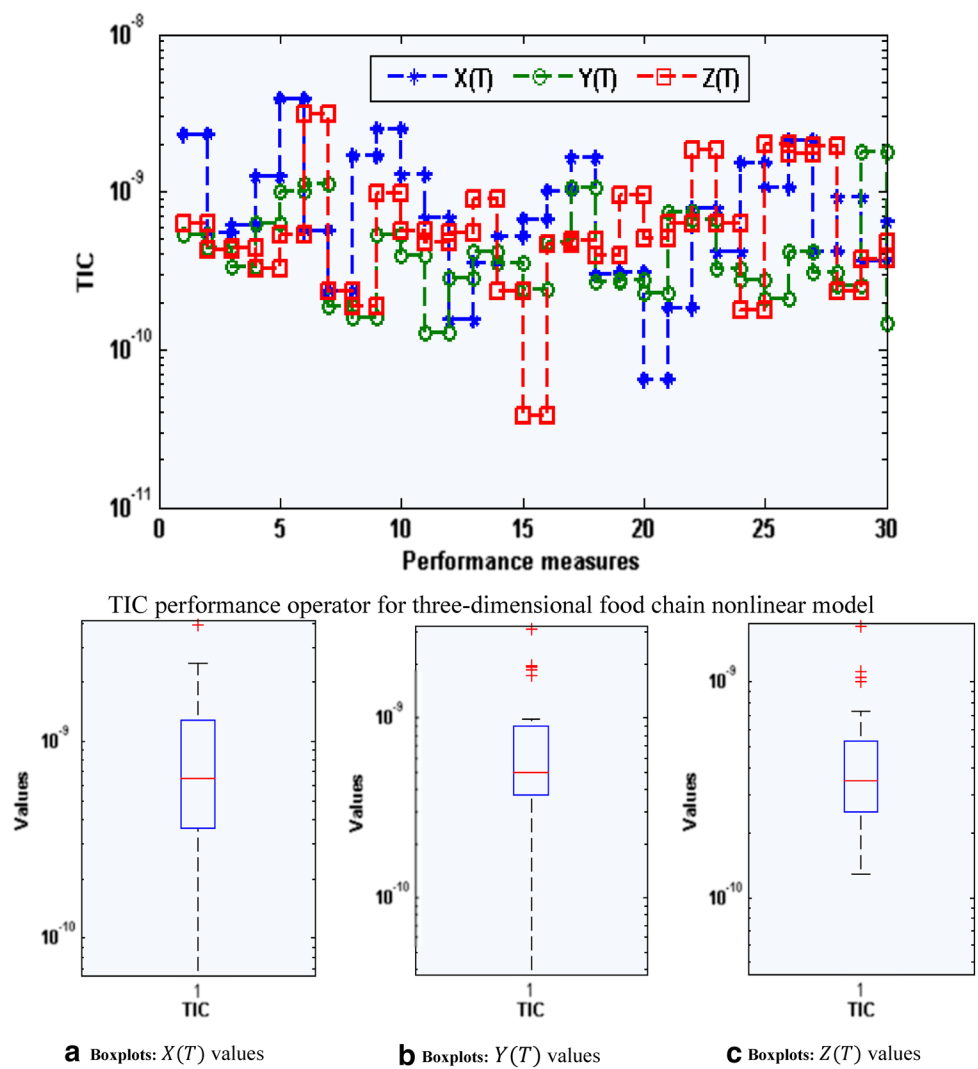


Fig. 3 TIC performances along with the boxplots based GNN-GA-ASA for three-dimensional food chain nonlinear model



$$\begin{aligned}
 \hat{X}(\Psi) = & 1.0561(2\tan^{-1}e^{(0.3584\Psi+2.3141)} - 0.5\pi) + 1.5786(2\tan^{-1}e^{(-1.0463\Psi-0.6359)} - 0.5\pi) \\
 & - 1.8630(2\tan^{-1}e^{(-0.6936\Psi-0.3845)} - 0.5\pi) - 0.0488(2\tan^{-1}e^{(-1.7367\Psi-2.3627)} - 0.5\pi) \\
 & 1.4624(2\tan^{-1}e^{(0.3281\Psi-0.6821)} - 0.5\pi) + 0.9619(2\tan^{-1}e^{(0.2794\Psi+0.3296)} - 0.5\pi) \\
 & 1.7775(2\tan^{-1}e^{(-0.1763\Psi+2.3006)} - 0.5\pi) + 2.2604(2\tan^{-1}e^{(-0.5332\Psi+0.0478)} - 0.5\pi) \\
 & - 0.9345(2\tan^{-1}e^{(0.0191\Psi-0.0942)} - 0.5\pi) + 1.3007(2\tan^{-1}e^{(-2.8534\Psi-3.4033)} - 0.5\pi),
 \end{aligned} \tag{16}$$

$$\begin{aligned}
 \hat{X}(\Psi) = & 0.5342(2\tan^{-1}e^{(0.1335\Psi+0.4637)} - 0.5\pi) - 0.8565(2\tan^{-1}e^{(0.5032\Psi+0.1273)} - 0.5\pi) \\
 & 0.6277(2\tan^{-1}e^{(-2.2469\Psi-0.7075)} - 0.5\pi) - 1.4959(2\tan^{-1}e^{(-1.1810\Psi-3.0498)} - 0.5\pi) \\
 & 2.5696(2\tan^{-1}e^{(-6.6073\Psi-3.6405)} - 0.5\pi) - 0.1184(2\tan^{-1}e^{(0.1933\Psi+2.3627)} - 0.5\pi) \\
 & - 2.2969(2\tan^{-1}e^{(-0.1593\Psi-1.7907)} - 0.5\pi) - 0.2461(2\tan^{-1}e^{(-0.2011\Psi-0.9760)} - 0.5\pi) \\
 & 0.2974(2\tan^{-1}e^{(-0.0698\Psi-1.1331)} - 0.5\pi) + 0.0016(2\tan^{-1}e^{(3.5979\Psi-1.5463)} - 0.5\pi),
 \end{aligned} \tag{17}$$

Figures 1, 2 and 3 illustrates the best weight vectors, result comparisons and the values of AE to solve the three-dimensional food chain nonlinear model using the stochastic procedures based on GNNs-GA-ASA. The best weight values are illustrated in the three-dimensional food chain nonlinear model in Fig. 1a–c for 30 variables or 10 neurons. These weight vectors are established in Eqs. (15–17). The comparative performance of the results for the three-dimensional food chain nonlinear model is illustrated in Fig. 1d–f. The plots of the AE have been established in Fig. 2a–c for the three-dimensional food chain nonlinear model. The statistical operator plots along with the performances of the boxplots are illustrated in Fig. 3 to solve the three-dimensional food chain nonlinear model. The convergence measures are plotted using the TIC, MAD and EVAF to solve the three-dimensional food chain nonlinear model. The complexity of GNNs-GA-ASA in terms of execution time consumed for learning of the weights of the networks is calculated and it is found in the close vicinity of 30 ± 10 for the single runs of the algorithm.

4 Conclusions

This study aims to investigate the Gudermannian neural networks (GNNs) using the optimization procedures of genetic algorithm and active-set approach (GA-ASA) to solve the three-species food chain nonlinear model. An error function is constructed using the three classes of the three-species food chain nonlinear model names as prey populations, top-predator and specialist predator and its initial conditions. The exactness of the scheme GNN-GA-ASA is observed by comparing the proposed results and the reference Runge–Kutta results to solve the three-dimensional food chain nonlinear model. The AE values are found in good measures to solve the three-dimensional food chain nonlinear model, i.e. around 10^{-05} – 10^{-07} . The performances of the operators TIC, EVAF and MAD proved the good illustrations to solve the three-dimensional food chain nonlinear model. The statistical Mean, S.I.R, Min, Max, MED and STD performances for 30 independent runs validate the correctness of the proposed stochastic procedures based on GNN-GA-ASA. Furthermore, the global performances through statistical trials of MED and S.I.R have been competently applied to solve the three-dimensional food chain nonlinear model.

In the future, the proposed stochastic procedures based on GNN-GA-ASA are accomplished to solve the environmental economic systems (Kiani et al. 2021; Nisar et al. 2021), information security models (Masood et al. 2019, 2020, 2021) and fluid dynamic models (Awan et al. 2020, 2021; Raja et al. 2020; Umar et al. 2020b, c, d, e).

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Data availability No data is used to support this study.

Declarations

Conflict of interest There is no conflict of interest. All authors contributed equally.

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