Multicriteria decision making approach using an efficient novel similarity measure for generalized trapezoidal fuzzy numbers

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Abstract

Multicriteria Decision Making (MCDM) has a huge role to play while ruling out one suitable alternative among a pool of alternatives governed by predefined multiple criteria. Some of the factors like imprecision, lack of information/data, etc., which are present in traditional MCDM processes have showcased their lack of efficiency and hence eventually it has paved the ways for the development of Fuzzy multicriteria decision making (FMCDM). In FMCDM processes, the decision makers can model most of the real-life phenomena by fuzzy information-based preferences. The availability of a wide literature on similarity measure (SM) emphasizes the vital role of SM of generalized fuzzy numbers (GFNs) to conduct accurate and precise decision making in FMCDM problems. Despite having few advantages, most of the existing approaches possessed a certain degree of counter intuitiveness and discrepancies. Thus, we have attempted to propose a novel SM for generalized trapezoidal fuzzy numbers (GTrFNs) which could deliberately overcome the impediments associated with the earlier existing approaches. Moreover, a meticulous comparative study with the existing approaches is also presented. This paper provides us with an improved method to obtain the similarity values between GTrFNs and the proposed SM consists of calculating the prominent features of fuzzy numbers such as expected value and variance. We use fourteen different sets of GTrFNs, to compare the fruition of the present approach with the existing SM approaches. Furthermore, to show the utility and applicability of our proposed measure, we illustrate few practical scenarios such as the launching of an electronic gadget by a company, a problem of medical diagnosis and finally, a proper anti-virus mask selection in light of the recent COVID-19 pandemic. The obtained results with our proposed SM, for the mentioned FMCDM problems, are analytically correct and they depict the efficiency and novelty of the present article.

Keywords Uncertainty/vagueness · Generalized trapezoidal fuzzy numbers · Similarity measure · Multicriteria decision making

1 Introduction

The presence of uncertainty usually makes things appear more complex and uncertainty-based processes usually take a much longer time than normal, but still, they cannot be completely ignored or eradicated from most of the real-life phenomena. Hence, the non-consideration of uncertainty in such processes may lead to glitches or irregularities, which existed in the age-old processes of decision-making.

Epistemic uncertainty and aleatoric uncertainty form the two major classifications of uncertainty. Factors representing paucity information about the environment, diminutive sample size, measurement-based uncertainty, etc., give rise to the epistemic variant of uncertainty (Dutta and Hazarika 2017). While aleatoric uncertainty may arise in decision making problems which are laid with factors like, non-extrinsic variability, stochasticity, time/space variation, etc. The classical probability theory (PT) is capable of handling aleatoric uncertainty, but however, it is not adequate to handle epistemic uncertainty with that ease. Thereafter, the introduction of the concept of fuzzy set theory (FST) by Zadeh (1965), came as a savior which could address the epistemic uncertainty. Since then, the multifarious applications of FST saw no bounds. Whenever we have a set of alternatives governed by certain definite criteria and we use

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linguistic variables to express the criteria weights, then we refer to this process of selecting the deserving alternative as MCDM. According to Zadeh (1975), a linguistic variable signifies either a word or a sentence, where the language used for expression is insignificant. To epitomize the uncertain (epistemic) behavior of linguistic variables, we generally prefer fuzzy numbers. Fuzzy numbers have entrusted the job of dealing with the uncertainty associated with the decision-making processes. Thereafter, we have the ranking or SMs which can compare between the fuzzy numbers. From time to time, SMs of GFNs have come out successful in accurate conduct of various decision-making processes. We encounter numerous examples of such kind in the literature like-in Chen (1996), successful attempts were made to devise a geometric distance-based SM so that the prime objectives of mental workload assessment and the associated risk factor analysis could be fulfilled. Thereafter it was Hsieh (1999), who developed an SM on the idea of “graded mean integration”. Chen and Chen (2001), devised an SM dependent on the center of gravity (COG) of the object/point. Likewise, Lee (2002) explored SM for group decision-making problems. Yong et al. (2004), introduced an SM based on the concept of the radius of gyration and they were successful in adding significant contributions in the field of pattern recognition. Consequently, another problem of risk assessment under fuzzy setting was tackled by Wei and Chen (2009), who developed an SM that produced quick and effective results. Later it was the idea of Xu et al. (2010), to merge the concepts of COG and geometric distance into developing an SM between two trapezoidal fuzzy numbers. Then it was Ye, who devised a cosine SM (Ye 2011) and an expected value-based SM (Ye 2012), for group decision-making problems which pushed the bounds a bit further. Moreover, Hejazi et al. (2011), utilized four significant concepts of height, geometric distance, perimeter, and area, to devise an SM for GFNs. Furthermore, Patra and Mondal (2015) reviewed the SM by Hejazi et al. (2011) and improved it for the more generalized case. FMCDM based on the fuzzy structured elements with incomplete weight information was developed by Wang et al. (2016). Later, Hesamian (2017); Hesamian and Chachi (2017), made astonishing contributions to the field of pattern recognition and approximate reasoning. Beg and Rashid (2017), enriched the literature in the field of cluster analysis by introducing a relatively new concept of equivalence into devising an SM. Khorshidi and Nikfalarzar (2017), improved the existing geometric distance-based SMs, by further incorporating the idea of ROG and heights. Xie et al. (2017) utilized the concept of span difference and center-width difference of fuzzy numbers to develop an efficient similarity measure. A year later, the novel initiative of defining the MADA (Modified Area Development Approach) methodology based on a D number was explained in Mo and Deng (2018). Thereafter, Garg (2018) gave a new

direction to decision-making processes, by developing an IFS-based cosine SM. Baccour (2018) pointed out some distance and SMs to aid the process of multicriteria decision-making; and likewise, Tourad and Abdali (2018) received recognition for devising an SM for GFNs and in a similar manner. Wei (2018) left no stone unturned, to develop a picture fuzzy set-based SM. It was in the same year that, Ahmed et al. (2018) utilized the idea of set theory to devise a novel SM. Phan et al. (2018) utilized their rich statistical knowledge to introduce fuzzy logic-based SM in uncorrelated multivariate time series. Later, Luo and Liang (2018), showed their keen interest in interval valued fuzzy sets and hence contributed, by devising an SM based on them. Consequently, Liu et al. (2018) devised a cosine SM and carried out the medical diagnosis by undertaking the study with hybrid intuitionistic fuzzy sets (IFS). While, Singhal et al. (2018) in their paper, proposed an SM between fuzzy soft sets. The application of IFSs in decision-making processes is so diverse that, in the year 2019, there were several works by researchers such as, an OWA operator based SM (Fei et al. 2019), IFSs based SM (Song et al. 2019), IFSs based SM having precise applications in the field of medical diagnosis and pattern recognition (Dhivya and Sridevi 2019). Thereafter, Peng and Garg (2019) extended the idea of SM to Pythagorean fuzzy sets. Later, Uluçay (2020), introduced a novel method for trapezoidal fuzzy multi-numbers for its application in the MCDM process. Few other works were carried out involving trapezoidal fuzzy multi-numbers which for instance, Uluçay et al. (2018) proposed arithmetic and geometric operators for them, and Şahin et al. (2020) formulated an improved hybrid vector similarity measure for such multi-numbers. Thereafter, Wu et al. (2020) tackled group decision making problems with the help of an improved SM for GFNs. A recent work by Dutta (2020) can be encountered, where MCDM problems are efficiently dealt with by GFNs and an advanced dice similarity measure is devised between them.

1.1 Setbacks of the existing approaches

Most of the existing approaches had very few advantages and they were with much more shortcomings. In our article, fourteen different sets of fuzzy numbers are considered as depicted in Table 1 (adapted from Dutta 2020), and our proposed approach very lucidly ascertains the drawbacks which had existed earlier. It is self-explanatory that the value of similarity between non-similar fuzzy numbers cannot be unity (1), unless and until the fuzzy numbers are the same. The Hsieh (1999) approach lacked efficiency as it did produce unit values of similarity for the profiles 2, 3, 5, 7, and 13, which are non-similar. The approach also did produce illogical results by giving identical values of similarity for some of the pairs like (2, 3), (4, 5), (6, 14), (7, 13), (8, 9),

and (10, 11). If we go by the Chen (1996) approach, we immediately face the counter-intuitive result as it produces unit SM value for completely different pairs of GFNs. It does so for profiles 2, 4, and 5. Likewise, if we carefully observe the SM values obtained for different pairs of profiles, we come up with yet another discrepancy, which is producing identical values of similarity for the pairs (2, 3), (4, 5), (6, 7), (8, 9), (10, 11) and (13, 14). The approach by Chen and Chen (2001) was also inefficient in a way, as it repeated the same discrepancy by Chen (1996) approach, producing similar results for the non-similar pairs (2, 4), (3, 5), (6, 14), (7, 13) and (8, 9). Several other approaches such as by Lee (2002) for the profiles (2, 3), (4, 5), (6, 7), (11, 12), (13, 14); by Wei and Chen (2009) for the pairs (6, 7), (10, 11), (13, 14); and by Hejazi et al. (2011) for the pairs (6, 7), (10, 11), (13, 14), were unable to distinguish between the non-similar pairs of FNs and as a result of their inability, they produced identical values of similarity for those profiles, which is illogical. The problem of producing identical SM values and the inability to distinguish between FNs were again repeated by approaches (Xu et al. 2010; Patra and Mondal 2015; Ye 2012). Precisely the similarity values for the pairs of profiles- (2, 3), (4, 5), (6, 14), (7, 13), (8, 9) in case of Xu et al. (2010); (2, 3), (4, 5), (6, 7), (10, 11), (13, 14) in case of Patra and Mondal (2015); and (1, 13), (2, 4), (3, 5) in case of Ye (2012), are identical and they reflected the non-consideration of some major key-points while devising these SMs. Likewise, Khorshidi and Nikfalazar (2017) approach obtained total similarity for non-similar FNs given in profile 11, which is counter-intuitive. For profiles 2 and 4; 3 and 5; 6 and 12; 10, 11 and 13, the Xie et al. (2017) approach obtained similar values of similarity which demonstrates a major flaw in the approach. Moreover, the approach by Wu et al. (2020) was unable in distinguishing between non-similar FNs in profiles 10 and 11, thereby producing identical similarity values.

1.2 Motivation and objectives of our work

It is worth mentioning that, most of the existing SMs are formulated in such a way that they are capable of distinguishing between two physical objects when the degree of similarity or dissimilarity between those objects is very high or large. However, in certain situations, when the objects are almost similar or they have minor differences, then either most similarity methods fail to be applied or the results obtained are not satisfactory. Thus, we aim to overcome this loophole that even when the objects have very small differences and they are not identical, our proposed SM must be capable of distinguishing between the two. As already in the previous subsection, we have presented the discrepancies exhibited by the existing methods, we are motivated to undertake this challenge and to devise a fruitful measure that overcomes

these lacunas. Furthermore, another drawback shown by some existing measures is their inability to evaluate the similarity value between a crisp valued fuzzy number and a non-crisp valued fuzzy number. Hence, with our proposed measure we not only plan to just overcome this setback but also achieve it effortlessly and efficiently.

In this present study, our main objectives can be listed down as under:

- (a) to devise a useful SM for GTrFNs which can overcome most of the existing discrepancies,
- (b) to compare our proposed measure with some other existent methods for check for its efficiency,
- (c) to illustrate an FMCDM method based on the proposed SM,
- (d) to validate the numerical applicability of our measure by demonstrating its usage in some real-life problems. Together with some other scenarios, one significant issue which we plan to tackle with our method is that of mask selection problem for ensuring maximum protection against the ongoing COVID-19 pandemic. We are all aware of the havoc and the outrage created by this pandemic and due to which the whole world is suffering. So, our motive is to show how our proposed measure can be applied to select a proper anti-virus mask to minimize the transmission rate of the virus.

1.3 Structure of the paper

The rest of the paper is structured as follows. In Sect. 2, few definitions related to GTrFNs are presented. Section 3, reviews the presented approach and the formula for calculating the expected value and variance of GTrFNs. Section 4 presents a comparative analysis of the previous SM approaches and brings some criticism while showing the superiority of the proposed SM approach. In Sect. 5, the proposed approach is illustrated for tackling FMCDM problems, and for validation, some numerical examples are discussed which include, launching of an electronic gadget; medical diagnosis; and anti-virus mask selection to prevent the transmission rate of COVID-19. Finally, the concluding remarks and future research scope are presented in Sect. 6.

2 Basic preliminaries

In this section, some basic concepts and essential backdrops related to GTrFNs which will be necessitated in the follow up are reviewed.

Definition 2.1 (Fuzzy Set; Zadeh 1965).

A fuzzy subset ζ on the universe of discourse Λ is well characterized by its membership function $\mu_{\zeta} : \Lambda \rightarrow [0, 1]$,

which assigns a real number in the interval $[0, 1]$, for every such element. Larger values of the membership function represent a higher degree of belongingness to the fuzzy set. Although fuzzy sets have great expressive power over the crisp valued sets, their optimum output depends on our capability to construct appropriate membership functions for different scenarios.

Definition 2.2 (α -level or α -cut or cut worthy set; Dutta 2011)

For any real number $\alpha \in [0, 1]$, the α -cut of a fuzzy set ζ in Λ is the crisp set denoted by ${}^\alpha\zeta$ and is defined as,

$${}^\alpha\zeta = \{x \in \Lambda : \mu_\zeta(x) \geq \alpha\}.$$

Thus, α -cut sets act as a connecting bridge between crisp sets and fuzzy sets.

For instance, let ζ be a fuzzy set whose membership function is given as,

$$\mu_\zeta(x) = \begin{cases} \frac{x-a}{b-a}, & a \leq x \leq b \\ \frac{c-x}{c-b}, & b \leq x \leq c. \end{cases}$$

To find the α -cut of ζ , we first set $\alpha \in [0, 1]$ to both left and right reference functions of ζ .

That is, $\alpha = \frac{x-a}{b-a}$ and $\alpha = \frac{c-x}{c-b}$.

Expressing x in terms of α we have,

$x = (b-a)\alpha + a$ and $x = c - (c-b)\alpha$, which gives the α -cut of ζ as,

$${}^\alpha\zeta = [(b-a)\alpha + a, c - (c-b)\alpha].$$

Definition 2.3 (Support; Zadeh 1965)

The support of a fuzzy subset is a crisp subset of Λ , where the elements of the set possess non-zero membership grades in ζ . It is denoted by $\text{supp}(\zeta)$ and defined as,

$$\text{supp}(\zeta) = \{x \in \Lambda : \mu_\zeta(x) > 0\}.$$

Support of a fuzzy set gives us an idea about the range which is suitable for mathematical calculations and for drawing useful inferences. It is to be noted that, when the supports of two fuzzy sets are disjoint then this implies that the two fuzzy sets are disjoint.

Definition 2.4 (Height; Zadeh 1965).

Any element of the set which acquires the maximum grade of membership among the entire set of elements of the set, is called the height of a fuzzy set.

For any universe of discourse Λ and any fuzzy set ζ , the height is denoted as $h(\zeta)$ and defined as,

$$h(\zeta) = \sup_{x \in \zeta} \mu_\zeta(x).$$

Whenever $h(\zeta) = 1$, the fuzzy set ζ is called normal fuzzy set or else called subnormal fuzzy set.

Definition 2.5 [Generalized fuzzy numbers (GFN); Chen 1985].

For any fuzzy set ζ on Λ to qualify as a generalized fuzzy number, it must satisfy atleast the following properties,

- ${}^\alpha\zeta$ must be a closed interval for every value of α in the interval $[0, 1]$,
- the support of ζ must be a bounded set,
- ζ should preferably be a sub-normal fuzzy set.

Mathematically, we can define the membership function of GFN, $\lambda = [\lambda_1, \lambda_2, \lambda_3, \lambda_4; w]$, where, $\lambda_1 \leq \lambda_2 \leq \lambda_3 \leq \lambda_4, 0 \leq w \leq 1$ as,

$$\mu_\lambda(x) = \begin{cases} 0, & x < \lambda_1 \\ w \frac{x-\lambda_1}{\lambda_2-\lambda_1}, & \lambda_1 \leq x < \lambda_2 \\ w, & \lambda_2 \leq x < \lambda_3 \\ w \frac{\lambda_4-x}{\lambda_4-\lambda_3}, & \lambda_3 \leq x < \lambda_4 \\ 0, & x \geq \lambda_4. \end{cases}$$

The above definition of λ is called generalized trapezoidal fuzzy number (GTrFN). For normal trapezoidal fuzzy number (TrFN), we have $w = 1$, and so λ takes the form $\lambda = [\lambda_1, \lambda_2, \lambda_3, \lambda_4]$. The requirement for a generalized triangular fuzzy number (GTFN) is $\lambda_2 = \lambda_3$ and $w < 1$, else if not satisfied it is called a normal triangular fuzzy number.

For a real number, all the four components must be the same, that is, $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4$ and $w = 1$.

Since GFNs have the unique ability to depict the confidence degree of the decision maker's opinions with the help of their heights, hence logically they are more preferred over their normal counterparts. Here, Fig. 1 depicts a GTrFN, $\lambda = [2, 4, 6, 8; 0.6]$ for better visualization.

Definition 2.6 (Similarity measure; Liu 1992).

SM is an important mathematical tool that is capable of measuring the degree of similarity or dissimilarity between two objects which may be sets, patterns, shapes, etc.

Formally speaking, whenever a real function $S_S : \mathbb{R} \times \mathbb{R} \rightarrow [0, 1]$ satisfies certain properties which are listed below as,

$$\mathbf{P-1:} \quad 0 \leq S_S(\zeta_1, \zeta_2) \leq 1.$$

$$\mathbf{P-2:} \quad S_S(\zeta_1, \zeta_2) = S_S(\zeta_2, \zeta_1).$$

$\mathbf{P-3:} \quad S_S(\zeta_1, \zeta_2) = 1 \Leftrightarrow \zeta_1 = \zeta_2$, where $\zeta_1, \zeta_2 \in \mathbb{R}$; only then the function can have the privilege to be coined the term, "similarity measure".

Fig. 1 GTrFN, $\lambda = [2, 4, 6, 8; 0.6]$

3 The proposed novel SM approach

Before moving straight into the proposed similarity measure for GTrFNs, we must beforehand learn about the importance of calculating the expected/mean value and variance for GTrFNs. Expected value and variance are treated as characteristic parameters of fuzzy numbers as, the expected value gives us an anticipated measure of how a fuzzy number behaves in a long term, while variance provides information about useful and unnecessary deviations of fuzzy numbers from their expected or mean values so that we can modify our definitions as per our requirements to produce optimum output. There have been numerous attempts in the literature to devise their mathematical form, most of which utilized credibility or possibility theory. However, instead of the possibilistic approach, and as an alternative method we utilize the concepts of FST to formulate the expressions for expected value and variance of GTrFNs via the α -cut technique. We choose GTrFNs in our study due to their enhanced ability to represent uncertain information in a better way with the help of their height parameters.

Interestingly, the idea which motivated us during our formulation process is that whenever we take the expected value or mean of any interval, we obtain the result as its midpoint. Also, the α -cut of any fuzzy number produces an interval. Therefore, after obtaining the α -cut we then equate each reference function of GTrFN equal to α so that we obtain the final expression for its expected value. Similarly, we obtain the expression for the variance of GTrFNs. It is noteworthy that our present work fills an existing research gap since the α -cut technique for evaluation is not frequently used. More precisely, it has not been previously explored for GTrFNs to calculate their expected values and variances. The detailed calculative procedure is illustrated in the subsequent section.

3.1 Expected value of a GTrFN

Expected value is also popularly known as the first moment or expectation or mean. In probabilistic and statistical terms, for any random variable, the expected value measures the center of distribution of the variable. That is, the long-term anticipated average value of the variable is none other than the expected value of the variable. Expected values can be determined for single and multiple discrete or continuous fuzzy numbers. Expected values help in choosing a scenario that is most likely to give the desired outcome.

Now, we evaluate the expected value of a GTrFN, $\eta = [\eta_1, \eta_2, \eta_3, \eta_4; w]$ having the membership function defined as,

$$\mu_{\eta}(x) = \begin{cases} 0, & x < \eta_1 \\ w \frac{x - \eta_1}{\eta_2 - \eta_1}, & \eta_1 \leq x < \eta_2 \\ w, & \eta_2 \leq x < \eta_3 \\ w \frac{\eta_4 - x}{\eta_4 - \eta_3}, & \eta_3 \leq x < \eta_4 \\ 0, & x \geq \eta_4 \end{cases}$$

by using the α -cut method as explained in the following theorem.

Theorem 1 Let $\eta = [\eta_1, \eta_2, \eta_3, \eta_4; w]$ be a GTrFN, where $\eta_1 \leq \eta_2 \leq \eta_3 \leq \eta_4$, $0 \leq w \leq 1$, its expected value is attained as.

$$E(\eta) = \frac{1}{4}w[\eta_1 + \eta_2 + \eta_3 + \eta_4]..$$

In other words, when we multiply the height of a GTrFN with the average of its components, then we obtain the required expected value for the fuzzy number.

Proof We first calculate the α -cut of the GTrFN η .

So, we have

$$\alpha = w \frac{x - \eta_1}{\eta_2 - \eta_1} \text{ and } \alpha = w \frac{\eta_4 - x}{\eta_4 - \eta_3} \\ \Rightarrow x = \eta_1 + \frac{1}{w} \{ \alpha(\eta_2 - \eta_1) \} \text{ and } \Rightarrow x = \eta_4 - \frac{1}{w} \{ \alpha(\eta_4 - \eta_3) \}.$$

$$\text{Therefore, } {}^{\alpha}\eta = \left[\eta_1 + \frac{1}{w} \{ \alpha(\eta_2 - \eta_1) \}, \eta_4 - \frac{1}{w} \{ \alpha(\eta_4 - \eta_3) \} \right].$$

We know that, α -cut of a fuzzy set/number always gives us an interval. Also, the mean value/expected value of an interval of the type $[p, q]$ is given by $\frac{p+q}{2}$.

Using similar intuition,

Expected value of η ,

$$\begin{aligned}
E(\eta) &= \frac{1}{2} \left[\int_0^w \left[\eta_1 + \frac{1}{w} \{ \alpha(\eta_2 - \eta_1) \} \right] d\alpha + \int_0^w \left[\eta_4 - \frac{1}{w} \{ \alpha(\eta_4 - \eta_3) \} \right] d\alpha \right] \\
&= \frac{1}{2} \left[\left[\eta_1 \alpha + \frac{1}{w} \left\{ \frac{\alpha^2}{2} (\eta_2 - \eta_1) \right\} \right]_0^w + \left[\eta_4 \alpha - \frac{1}{w} \left\{ \frac{\alpha^2}{2} (\eta_4 - \eta_3) \right\} \right]_0^w \right] \\
&= \frac{1}{2} \left[\eta_1 w + \frac{w^2}{2w} (\eta_2 - \eta_1) + dw - \frac{w^2}{2w} (\eta_4 - \eta_3) \right] \\
&= \frac{1}{4} [2\eta_1 w + \eta_2 w - \eta_1 w + 2\eta_4 w - \eta_4 w + \eta_3 w] \\
&= \frac{1}{4} [\eta_1 w + \eta_2 w + \eta_3 w + \eta_4 w] \\
&\Rightarrow E(\eta) = \frac{1}{4} w [\eta_1 + \eta_2 + \eta_3 + \eta_4], \text{ which concludes the proof.}
\end{aligned}$$

3.2 Variance of a GTrFN

In probabilistic terms, the term variance is used to denote how widely the numbers are distributed in a dataset. It is a measure of variability from the expected value or mean of fuzzy numbers. The variance of a fuzzy number is always a positive quantity; it might be zero at times, but cannot be negative. Variance does not differentiate between deviations based on their directions, rather it treats all deviations from the expected value to be the same, irrespective of their direction. Now, the variance of any set under consideration, say $A \in \mathbb{R}$, is given by the formula, $V(A) = E(A^2) - [E(A)]^2$.

Hence, similar intuition works for a fuzzy number η , which gives, $V(\eta) = E(\eta^2) - [E(\eta)]^2$.

The following theorem illustrates the mathematical expression for it, in case of GTrFNs.

Theorem 2 Let $\eta = [\eta_1, \eta_2, \eta_3, \eta_4; w]$ be a GTrFN, where $\eta_1 \leq \eta_2 \leq \eta_3 \leq \eta_4, 0 \leq w \leq 1$, and whose expected value is given by

$$E(\eta) = \frac{1}{4} w [\eta_1 + \eta_2 + \eta_3 + \eta_4],$$

then its variance has the form

$$\begin{aligned}
V(\eta) &= \frac{1}{6} w [\eta_1^2 + \eta_2^2 + \eta_3^2 + \eta_4^2 + \eta_1 \eta_2 + \eta_3 \eta_4] \\
&\quad - \frac{1}{16} w^2 [\eta_1 + \eta_2 + \eta_3 + \eta_4]^2.
\end{aligned}$$

In other words, when we take the difference between the expected value of the square of a fuzzy number and the square of the expected value of the fuzzy number under consideration, then we obtain the required value of variance.

Proof The proof is given in (Appendix 1).

3.3 Proposed approach

Having illustrated the procedure for calculating the expected value and variance of GTrFNs in the previous sections, we now devise our proposed definition of SM. For that, let us consider, $\lambda = [\lambda_1, \lambda_2, \lambda_3, \lambda_4; w_\lambda]$ and $\sigma = [\sigma_1, \sigma_2, \sigma_3, \sigma_4; w_\sigma]$ be two non-zero GTrFNs, where $\lambda_1 \leq \lambda_2 \leq \lambda_3 \leq \lambda_4, 0 \leq w_\lambda \leq 1$ and $\sigma_1 \leq \sigma_2 \leq \sigma_3 \leq \sigma_4, 0 \leq w_\sigma \leq 1$.

Then, the SM between λ and σ is proposed as,

$$\begin{aligned}
S_M(\lambda, \sigma) &= \frac{2[\sum \lambda_i \sigma_i + w_\lambda w_\sigma + E(\lambda)E(\sigma) + V(\lambda)V(\sigma)]}{\sum (\lambda_i^2 + \sigma_i^2) + w_\lambda^2 + w_\sigma^2 + E(\lambda)^2 + E(\sigma)^2 + V(\lambda)^2 + V(\sigma)^2}, \quad (1)
\end{aligned}$$

where, $\lambda_i, \sigma_i (i = 1, 2, 3, 4)$, w_λ -height of GTrFN λ , w_σ -height of GTrFN σ , $E(\lambda)$ -expected value of GTrFN λ , $E(\sigma)$ -expected value of GTrFN σ , $V(\lambda)$ -variance of GTrFN λ , $V(\sigma)$ -variance of GTrFN σ .

To prove it as a similarity measure, it must satisfy the following properties:

P-1: $0 \leq S_M(\lambda, \sigma) \leq 1$.

P-2: $S_M(\lambda, \sigma) = S_M(\sigma, \lambda)$.

P-3: $S_M(\lambda, \sigma) = 1 \Leftrightarrow \lambda = \sigma$.

Proof (P-1) Clearly by definition, $S_M(\lambda, \sigma) \geq 0$.

Now,

$$\begin{aligned}
&2 \left[\sum \lambda_i \sigma_i + w_\lambda w_\sigma + E(\lambda)E(\sigma) + V(\lambda)V(\sigma) \right] \\
&\leq \sum (\lambda_i^2 + \sigma_i^2) + w_\lambda^2 + w_\sigma^2 + E(\lambda)^2 + E(\sigma)^2 + V(\lambda)^2 + V(\sigma)^2 \\
&\Rightarrow 0 \leq S_M(\lambda, \sigma) \leq 1.
\end{aligned}$$

(P-2) Here,

$$S_M(\lambda, \sigma) = \frac{2[\sum \lambda_i \sigma_i + w_\lambda w_\sigma + E(\lambda)E(\sigma) + V(\lambda)V(\sigma)]}{\sum (\lambda_i^2 + \sigma_i^2) + w_\lambda^2 + w_\sigma^2 + E(\lambda)^2 + E(\sigma)^2 + V(\lambda)^2 + V(\sigma)^2}$$

$$= \frac{2[\sum \sigma_i \lambda_i + w_\sigma w_\lambda + E(\sigma)E(\lambda) + V(\sigma)V(\lambda)]}{\sum (\sigma_i^2 + \lambda_i^2) + w_\sigma^2 + w_\lambda^2 + E(\sigma)^2 + E(\lambda)^2 + V(\sigma)^2 + V(\lambda)^2}$$

$$= S_M(\sigma, \lambda).$$

(P-3) For $\lambda = \sigma$, we have $\lambda_i = \sigma_i, w_\lambda = w_\sigma, E(\lambda) = E(\sigma), V(\lambda) = V(\sigma)$, then clearly, $S_M(\lambda, \sigma) = 1$.

Conversely, let $S_M(\lambda, \sigma) = 1$, then,

$$2[\sum \lambda_i \sigma_i + w_\lambda w_\sigma + E(\lambda)E(\sigma) + V(\lambda)V(\sigma)]$$

$$= \sum (\lambda_i^2 + \sigma_i^2) + w_\lambda^2 + w_\sigma^2$$

$$+ E(\lambda)^2 + E(\sigma)^2 + V(\lambda)^2 + V(\sigma)^2$$

$$\Rightarrow (\lambda_i - \sigma_i)^2 + (w_\lambda - w_\sigma)^2 + (E(\lambda) - E(\sigma))^2 + (V(\lambda) - V(\sigma))^2 = 0$$

$$\Rightarrow \lambda_i = \sigma_i, w_\lambda = w_\sigma, E(\lambda) = E(\sigma), V(\lambda) = V(\sigma),$$

for $i = 1, 2, 3, 4$, if not then we arrive at a contradictory result of $S_M(\lambda, \sigma) \neq 1$

$$\Rightarrow \lambda = \sigma.$$

Hence, $S_M(\lambda, \sigma)$ is a similarity measure.

Remark 1 By the first property, we have the bounds for the SM which implies $0 \leq S_M(\lambda, \sigma) \leq 1$, and therefore to mean a total dissimilarity between λ and σ , it would imply $S_M(\lambda, \sigma) = 0$. However, for the total similarity between them we must have $S_M(\lambda, \sigma) = 1$. Hence, it is evident that, for maximum similarity between λ and σ , we should have maximum value of $S_M(\lambda, \sigma)$ accordingly.

3.4 Propositions

Proposition 3.4.1 For two normal triangular numbers $\lambda = [\lambda_1, \lambda_2, \lambda_3; 1]$ and $\sigma = [\sigma_1, \sigma_2, \sigma_3; 1]$, then by our proposed approach,

$$S_M(\lambda, \sigma)$$

$$= \frac{2[\lambda_1 \sigma_1 + 2\lambda_2 \sigma_2 + \lambda_3 \sigma_3 + 1 + E(\lambda)E(\sigma) + V(\lambda)V(\sigma)]}{(\lambda_1^2 + 2\lambda_2^2 + \lambda_3^2 + \sigma_1^2 + 2\sigma_2^2 + \sigma_3^2) + 2 + E(\lambda)^2 + E(\sigma)^2 + V(\lambda)^2 + V(\sigma)^2}$$

(2)

Proposition 3.4.2 Let suppose $\lambda = [\lambda_1, \lambda_2, \lambda_3, \lambda_4; 1]$ and $\sigma = [\sigma_1, \sigma_2, \sigma_3, \sigma_4; 1]$ be two normal trapezoidal fuzzy numbers, then our proposed approach would imply,

$$S_M(\lambda, \sigma) = \frac{2[\sum \lambda_i \sigma_i + 1 + E(\lambda)E(\sigma) + V(\lambda)V(\sigma)]}{\sum (\lambda_i^2 + \sigma_i^2) + 2 + E(\lambda)^2 + E(\sigma)^2 + V(\lambda)^2 + V(\sigma)^2}.$$

(3)

Proposition 3.4.3 For two normal TrFNs $\lambda = [0, 0, 0, 0; 1]$ and $\sigma = [\sigma_1, \sigma_2, \sigma_3, \sigma_4; 1]$, our approach would yield,

$$S_M(\lambda, \sigma) = \frac{2}{(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_4^2) + 2 + E(\sigma)^2 + V(\sigma)^2}.$$

(4)

Proposition 3.4.4 For GTrFNs with identical heights w_1 , $\lambda = [0, 0, 0, 0; w_1]$ and $\sigma = [0, 0, 0, 0; w_1]$, we have,

$$S_M(\lambda, \sigma) = \frac{2w_1 w_1}{w_1^2 + w_1^2} = \frac{2w_1^2}{2w_1^2} = 1.$$

(5)

For the particular case, when $\lambda = [0, 0, 0, 0; 1]$ and $\sigma = [0, 0, 0, 0; 1]$, clearly we have, $S_M(\lambda, \sigma) = 1$.

Table 1 Profiles of fuzzy numbers

Profiles	λ	σ
Profile 1	[0.1, 0.2, 0.3, 0.4;1.0]	[0.1, 0.2, 0.3, 0.4;1.0]
Profile 2	[0.1, 0.25, 0.25, 0.4;0.6]	[0.1, 0.25, 0.25, 0.4;0.8]
Profile 3	[0.1, 0.25, 0.25, 0.4;1.0]	[0.1, 0.25, 0.25, 0.4;0.8]
Profile 4	[0.1, 0.2, 0.3, 0.4;0.6]	[0.1, 0.2, 0.3, 0.4;0.8]
Profile 5	[0.1, 0.2, 0.3, 0.4;1.0]	[0.1, 0.2, 0.3, 0.4;0.8]
Profile 6	[0.2, 0.3, 0.5, 0.6;1.0]	[0.2, 0.3, 0.3, 0.4;1.0]
Profile 7	[0.2, 0.3, 0.5, 0.6;1.0]	[0.3, 0.4, 0.4, 0.5;1.0]
Profile 8	[0.4, 0.4, 0.5, 0.5;1.0]	[0.7, 0.7, 0.8, 0.8;1.0]
Profile 9	[0.4, 0.4, 0.5, 0.5;1.0]	[0.6, 0.6, 0.9, 0.9;1.0]
Profile 10	[0.1, 0.2, 0.2, 0.2;1.0]	[0.15, 0.25, 0.25, 0.25;1.0]
Profile 11	[0.1, 0.2, 0.2, 0.2;1.0]	[0.1, 0.1, 0.1, 0.2;1.0]
Profile 12	[0.0, 0.225, 0.225, 0.45;0.225]	[0.45, 0.675, 0.675, 0.9;0.225]
Profile 13	[0.2, 0.3, 0.5, 0.6;1.0]	[0.1, 0.4, 0.4, 0.7;1.0]
Profile 14	[0.2, 0.3, 0.5, 0.6;1.0]	[0.2, 0.5, 0.5, 0.8;1.0]

Table 2 Comparative analysis of similarity measure obtained from the proposed approach and existing approaches

Profiles	Hsieh (1999)	Chen (1996)	Chen and Chen (2001)	Lee (2002)	Wei and Chen (2009)	Hejazi et al. (2011)	Proposed SM approach	
Profile 1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	
Profile 2	1.0000	1.0000	0.7500	1.0000	0.7833	0.5979	0.9743	
Profile 3	1.0000	1.0000	0.8000	1.0000	0.8211	0.6641	0.9817	
Profile 4	1.0000	1.0000	0.7500	1.0000	0.7881	0.7500	0.9744	
Profile 5	1.0000	1.0000	0.8000	1.0000	0.8247	0.6680	0.9818	
Profile 6	0.9090	0.9000	0.6480	0.7500	0.8002	0.6448	0.9732	
Profile 7	1.0000	0.9000	0.7200	0.7500	0.8002	0.6448	0.9891	
Profile 8	0.7692	0.7000	0.4900	0.2500	0.7000	0.7000	0.9230	
Profile 9	0.7692	0.7000	0.4900	0.4000	0.6222	0.5011	0.9172	
Profile 10	0.9375	0.9500	0.9025	0.6666	0.9500	0.9500	0.9948	
Profile 11	0.9375	0.9500	0.9183	0.5000	0.9500	0.9500	0.9900	
Profile 12	0.6896	0.5500	0.3025	0.5000	0.5500	0.5500	0.6505	
Profile 13	1.0000	0.9000	0.7200	0.8333	0.8809	0.8738	0.9897	
Profile 14	0.9090	0.9000	0.6480	0.8333	0.8809	0.8738	0.9792	
Profiles	Xu et al. (2010)	Patra andYe Mondal (2012)	Khorshidi and Nikfalazar (2017)				Xieet al. (2017)	Wu et al. (2020)
Profile 1	1.0000	1.0000	1.0000	1.0000			1.0000	1.0000
Profile 2	0.9701	0.8850	0.9600	0.8557			0.7500	0.7863
Profile 3	0.9701	0.8850	0.9756	0.8667			0.8000	0.8233
Profile 4	0.9652	0.8800	0.9600	0.8544			0.7500	0.7912
Profile 5	0.9652	0.8800	0.9756	0.8650			0.8000	0.8277
Profile 6	0.8917	0.8100	0.9273	0.8809			0.1353	0.7141
Profile 7	0.9127	0.8100	0.9710	0.8817			0.2231	0.7100
Profile 8	0.7158	0.7000	0.8831	0.9759			0.0025	0.6782
Profile 9	0.7158	0.6300	0.8734	0.8608			0.0183	0.5379
Profile 10	0.9526	0.9500	0.9676	0.9999			0.3679	0.9467
Profile 11	0.9600	0.9500	0.9474	1.0000			0.3679	0.9467
Profile 12	0.5737	0.5500	0.6190	0.9185			0.1353	0.5378
Profile 13	0.9127	0.9000	1.0000	0.9897			0.3679	0.8711
Profile 14	0.8917	0.9000	0.9780	0.9888			0.2636	0.8558

Here, numbers in 'italics' represent unacceptable or absurd values.

Proposition 3.4.5 For two GTrFNs $\lambda = [0, 0, 0, 0; w_1]$ and $\sigma = [0, 0, 0, 0; w_2]$ with different heights w_1 and w_2 respectively, we have,

$$S_M(\lambda, \sigma) = \frac{2w_1w_2}{w_1^2 + w_2^2}. \quad (6)$$

4 Comparative analysis

To showcase the efficiency of the proposed SM approach over the earlier approaches, we hereby present a comparative analysis of the outcomes, where the discrepancies of the existing methods are fruitfully addressed by the presented approach.

In Table 1 which we have adapted from Dutta (2020), there we present fourteen different profiles of fuzzy numbers. In Table 2, we have compared the values of similarity obtained under various methods, and to visualize the profiles

of fuzzy numbers, we have enclosed the corresponding graphical illustrations in Figs. 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14 and 15. (pl. refer to Appendix 2 section).

4.1 Inferences from the analysis

Even though ample light has been thrown in Sect. 1.1, mentioning a few of the drawbacks that existed, we hereby illustrate in detail, in the following sequel. Now, it is evident from Table 2 that, all of the discussed methods including the present approach attained unit similarity measure value for the first profile, which is as expected. Hence, we enumerate the key points, under each method as follows,

Hsieh (1999) approach vs proposed approach:

- For profiles 2, 3, 5, 7, and 12, Hsieh's approach was inefficient to distinguish between non-similar pairs of FNs, whereas by the proposed method the respective values are 0.9743, 0.9817, 0.9818, 0.9891, and 0.6505.
- Contradicting results were obtained via Hsieh's approach as well, for the profiles (2, 3), (4, 5), (6, 14), (7, 13), (8, 9) and (10, 11), whereas by proposed method the respective values for the profiles are (0.9743, 0.9817), (0.9744, 0.9818), (0.9732, 0.9792), (0.9891, 0.9897), (0.9230, 0.9172) and (0.9948, 0.9900).

Chen (1996) approach vs proposed approach:

- For profiles 2, 4, and 5, Chen's approach was unable to distinguish, whereas by the proposed method the respective values are 0.9743, 0.9744, and 0.9818.
- Identical results were obtained via Chen's approach for the profiles (2, 3), (4, 5), (6, 7), (8, 9), (10, 11) and (12, 13), whereas by proposed method the respective values for the profiles are (0.9743, 0.9817), (0.9744, 0.9818), (0.9732, 0.9891), (0.9230, 0.9172), (0.9948, 0.9900) and (0.6505, 0.9897).

Chen and Chen (2001) approach vs proposed approach:

- Chen and Chen's in a similar way produced counter intuitive results for the profiles (2, 4), (3, 5), (6, 14), (7, 13), and (8, 9), whereas by proposed method the respective values for the profiles are (0.9743, 0.9744), (0.9817, 0.9818), (0.9732, 0.9792), (0.9891, 0.9897), and (0.9230, 0.9172).

Lee (2002) approach vs proposed approach:

- The inefficiency of Lee's approach also reflected, when it could not distinguish between the profiles 2, 3, 4, and 5, whereas by the proposed method the respective values are 0.9743, 0.9817, 0.9744, and 0.9818.
- Lee's approach also continued the legacy of producing illogical results as most of the earlier methods of that time, for the profiles (2, 3), (4, 5), (6, 7), (11, 12), and (13, 14), whereas by proposed method the respective values for the profiles are (0.9743, 0.9817), (0.9744, 0.9818), (0.9732, 0.9891), (0.9900, 0.6505), and (0.9897, 0.9792).

Wei and Chen (2009) approach vs proposed approach:

- Just as most of the methods failed, Wei & Chen's approach was no exception and it too failed to distinguish between different profiles of FNs. They produced non-acceptable values of similarity for the profiles (6, 7), (10, 11), and (13, 14), whereas by proposed method the respective values for the profiles are (0.9732, 0.9891), (0.9948, 0.9900), and (0.9897, 0.9792).

Hejazi et al. (2011) approach vs proposed approach:

- Hejazi et al. similarity method produced identical values of similarity for the pairs of profiles (6, 7), (10, 11), and (13, 14). On the other hand, with our proposed measure distinct similarity values are obtained for the same pairs which are respectively, (0.9732, 0.9891), (0.9948, 0.9900), and (0.9897, 0.9792).

Xu et al. (2010) approach vs proposed approach:

- Similar to most other methods, Xu et al. similarity approach produced identical values for the pairs (2, 3), (4, 5), (6, 14), (7, 13), and (8, 9), which indicated the structural flaw of their version of SM. However, it is evident from Table 2 that, the similarity values with our proposed method are non-identical.

Patra and Mondal (2015) approach vs proposed approach:

- Patra and Mondal's similarity measure was inefficient to distinguish between pairs (2, 3), (4, 5), (6, 7), (10, 11), and (13, 14). Contrary to that, our proposed measure obtained (0.9743, 0.9817), (0.9744, 0.9818), (0.9732,

0.9891), (0.9948, 0.9900), and (0.9897, 0.9792) as similarity values.

Ye (2012) approach vs proposed approach:

- Even with Ye's similarity measure as well, identical values of similarity were obtained for the profiles (2, 4) and (3, 5).
- Moreover, the method obtained a unit similarity value or total similarity between two non-similar fuzzy numbers given in Profile 13, which is illogical and it corresponds to a major flaw in its definition. However, such irregularities are not observed with our proposed measure.

Khorshidi and Nikfalazar (2017) approach vs proposed approach:

- A major flaw observed in Khorshidi and Nikfalazar's similarity measure is its inability to distinguish between two different fuzzy numbers considered in profile 11, where it evaluated the similarity value as unity. Also, if we observe carefully, then we notice that it obtained the similarity value as 0.9999 which is almost equal to 1 and thereby implying total similarity for the fuzzy numbers considered in profile 10 as well.
- Whereas, our proposed measure attains 0.9948 and 0.9900 as similarity values for profiles 10 and 11 respectively.

Xie et al. (2017) approach vs proposed approach:

- Xie et al. similarity were inefficient on several occasions since it also obtained identical similarity measures values. For instance, it obtained similarity value as -0.7500 for profile pairs (2, 4); 0.8000 for pairs (3, 5); 0.1353 for pairs (6, 12); and 0.3679 for profiles 10, 11, and 13. However, our measure obtained distinct values of similarity for those profiles as evident from Table 2.
- In addition, for profiles 8 and 9, Xie et al. method obtained significantly small values of similarity unlike most of the methods under study. For profile 8 it evaluated 0.0025 and for profile 9 it obtained 0.0183 as similarity values, whereas with our approach we calculated them as 0.9230 and 0.9172 , respectively.

Wu et al. (2020) approach vs proposed approach:

- With Wu et al. method, most of the similarity measures values evaluated for the profiles of fuzzy numbers are

acceptable. However, one such irregularity is still found for the profile pair (9, 10) as it obtained the identical similarity value of 0.9467 for both of them. On the other hand, our approach attains distinct values of 0.9172 and 0.9948 for profiles 9 and 10, respectively and hence our method is more desirable.

Hence, all arguments indicate the fact that the proposed similarity measure approach is highly adequate and well equipped to overcome the scantiness and impediments of the existing SM approaches.

Remark 2 It is to be noted that, although the similarity values obtained with our method are distinct, yet they are close to each other to a certain extent. The reason is that the fuzzy numbers considered in the respective profiles are chosen in such a way that they are almost similar with very minute differences so that most of the existing similarity measure methods fail to distinguish between them and where our proposed method comes out successful in distinguishing between the two.

Remark 3 Consequently, if the fuzzy numbers considered have larger dissimilarities as in Profile 12, our method is certain to obtain smaller values of similarity. For instance, with our proposed method the similarity value between GTrFNs, $\sigma_1 = [0.0, 0.1, 0.1, 0.2; 0.2]$, $\eta_1 = [0.70, 0.75, 0.80, 0.85; 0.9]$ and, $\sigma_2 = [0.02, 0.04, 0.06, 0.08; 0.10]$, $\eta_2 = [0.91, 0.92, 0.93, 0.94; 0.95]$ are obtained as $S_M(\sigma_1, \eta_1) = 0.2721$ and $S_M(\sigma_2, \eta_2) = 0.1115$ which are smaller in value. Likewise, we may verify for other fuzzy numbers as well.

Remark 4: Hence, we observe that for larger differences SM values obtained are distinct for almost all SM methods. But for smaller differences, most of the existing methods fail. We have thus devised our SM to overcome this loophole/discrepancy.

5 Applying the proposed SM approach to an FMCDM problem

Here in this section, by tackling an MCDM problem, the proposed SM is validated against two very important features, which are its novelty and applicability.

As already mentioned, briefly, the process which brings out the best alternative amongst a pool of alternatives considered or even determines the preference ordering of alternatives is termed as MCDM. The given set of alternatives are governed by some predefined multicriteria. It is worth noticing that, the type of attribute(s) or criteria(s), has two broad classifications viz., the profit/benefit criteria need to be maximized for obtaining the feasible solution for any

problem, whereas the other is the cost criteria which needs to be minimized similarly. There are also provisions when a cost criterion can take the form of benefit criteria, as beautifully explained by Zizovic et al. (2017). When a solution can achieve both the objectives of maximizing the profit as well as minimizing the cost, then the solution is termed as an ideal one (Xu and Yang 2001).

Suppose we consider, $A = \{A_1, A_2, \dots, A_p\}$ to be the collection of alternative(s) and $C = \{C_1, C_2, \dots, C_q\}$ be the collection of criteria(s), respectively. Also, let us assume GTrFNs of the type, $\eta = [\eta_{ij1}, \eta_{ij2}, \eta_{ij3}, \eta_{ij4}; w_{ij}]$ to represent the criteria(s). The reason for such consideration is to address the vague or uncertain nature of the criteria(s). Here, w_{ij} as usual denotes the height of the GTrFN η , and $\eta_{ij1} \leq \eta_{ij2} \leq \eta_{ij3} \leq \eta_{ij4}$, where $i = 1, 2, \dots, p$, $j = 1, 2, \dots, q$; $\eta_{ijk} \in \mathbb{R}$; $k = 1, 2, 3, 4$.

Depending upon the set of alternative(s) and criteria(s), the next thing we do is to construct a fuzzy judgement matrix, say $D_{ij} = [\delta_{ij}]$. Having known about the classifications of criteria-types, let us denote the profit/benefit criteria by χ^B and the cost criteria by χ^C , respectively. Once we have done that, there is a step-by-step which we need to follow while selecting the deserving alternative, which is illustrated as follows.

Step 1 Having formed the judgement matrix, $D_{ij} = [\delta_{ij}]_{p \times q} = [\delta_{ij1}, \delta_{ij2}, \delta_{ij3}, \delta_{ij4}; w_{ij}]$, the next prime concern is to evaluate the normalized judgement matrix $\bar{R}_{ij} = [\sigma_{ij}]_{p \times q} = [\sigma_{ij1}, \sigma_{ij2}, \sigma_{ij3}, \sigma_{ij4}; w'_{ij}]$, where obtaining the normalized components may vary depending on the type of criteria(s) (Xu et al. 2010). The normalized components may be calculated as under,

(a) For profit criteria(i.e., $C_j \in \chi^B$) (b) For cost criteria(i.e., $C_j \in \chi^C$)

$$\begin{cases} \sigma_{ij1} = \delta_{ij1} / \max_i \{\delta_{ij4}\} \\ \sigma_{ij2} = \delta_{ij2} / \max_i \{\delta_{ij3}\} \\ \sigma_{ij3} = \delta_{ij3} / \max_i \{\delta_{ij2}\} \wedge 1 \\ \sigma_{ij4} = \delta_{ij4} / \max_i \{\delta_{ij1}\} \wedge 1 \end{cases} \quad \begin{cases} \sigma_{ij1} = \min_i \{\delta_{ij1}\} / \delta_{ij4} \\ \sigma_{ij2} = \min_i \{\delta_{ij2}\} / \delta_{ij3} \\ \sigma_{ij3} = \min_i \{\delta_{ij3}\} / \delta_{ij2} \wedge 1 \\ \sigma_{ij4} = \min_i \{\delta_{ij4}\} / \delta_{ij1} \wedge 1 \end{cases}$$

$$\& w'_{ij} = w_{ij} / \max_i \{w_{ij}\} \quad \& w'_{ij} = \min_i \{w_{ij}\} / w_{ij}.$$

Step-2 Although it is complicated to realize a perfectly ideal alternative in the realworld, yet for the sake of evaluation and a useful theoretical construct, we need to evaluate the SM of the available set of alternatives with an ideal

alternative. For instance, let us suppose that the ideal alternative A^* is $[1, 1, 1, 1; 1]$.

Step-3 Eventually our SM expression between the alternatives set A_i and the ideal alternative A^* takes the following form,

$$S_M(A_i, A^*) = \frac{1}{q} \sum_{j=1}^q \frac{2[\sigma_{ij1} + \sigma_{ij2} + \sigma_{ij3} + \sigma_{ij4} + w'_{ij} + E(A_i)]}{\sigma_{ij1}^2 + \sigma_{ij2}^2 + \sigma_{ij3}^2 + \sigma_{ij4}^2 + 6 + w_{ij}'^2 + E(A_i)^2 + V(A_i)^2}. \quad (7)$$

From the evaluated SM values, the title of “best alternative” is conferred upon the alternative having the utmost similarity value with the ideal alternative. Equivalently, when we arrange the alternatives in the decreasing order of their similarity values, we obtain the required preference ordering of alternatives.

5.1 Numerical illustration

In this section, we present some empirical applications of our proposed method in a few practical real-life scenarios which indicate that our SM is well-structured, reasonable, and applicable.

5.1.1 Launching of electronic gadgets

Suppose we have the information that a company is going to launch a new electronic, and for that purpose, they are counting on a set of four alternatives A_1, A_2, A_3 and A_4 to choose their desired one. However, they have set up certain

grounds based on which the alternatives may be evaluated for their quality, endurance, and durability. Precisely, certain criteria(s) are set up by the company, which are C_1 is the investment return, C_2 is the net earning, C_3 is the funding total, and C_4 is the loss of investment.

It is evident that, out of the four above-mentioned criteria(s), C_1 and C_2 denote the profit/benefit criteria, while the remaining criteria(s), C_3 and C_4 are the cost criteria respectively. We present the judgement matrix in Table 3.

Step 1 Following the procedure for obtaining the normalized components as previously discussed, we normalize the judgement matrix and depict it in Table 4, while being very careful on the criteria-types (Table 5).

Step 2 For the particular scenario, we adopt the ideal alternative as, $A^* = [1, 1, 1, 1; 1]$.

Step 3 Finally, we evaluate the similarity measure values, $S_M(A_i, A^*)$, and we obtain,

$$S_M(A_1, A^*) = 0.9166, S_M(A_2, A^*) = 0.8577,$$

$$S_M(A_3, A^*) = 0.8807, S_M(A_4, A^*) = 0.8460.$$

Since the SM of the alternative A_1 with A^* is obtained as maximum, so A_1 results as the best suitable alternative. Consequently, the desired preference ordering is obtained to be $A_1 > A_3 > A_2 > A_4$.

Moreover, the ranking order comparisons obtained under various similarity methods are presented in the following table below.

Here, N/A indicates “not applicable”.

5.1.2 Medical diagnosis

The process of medical diagnosis is often a challenging one as it requires physical examination of the patient and also, carefully going through the background history of diseases.

The vague or unclear representation of information provided by patients requiring medical attention makes the exact identification of disease much more difficult. In this regard, the application of fuzzy sets and their ability to represent uncertain information helps us attain convincing results, and hence, we impose GTrFNs with the motive to effectively conduct the medical decision-making process. Generally, the process of medical decision-making comprises a set of diagnoses and a set of symptoms. Based on the maximum similarity or least dissimilarity between the patient and the set of diagnoses, we determine the disease which is causing the patient's medical condition, provided we represent the information about a patient with respect to the set of all possible symptoms by an ordered set. Hereby, we present a numerical illustration for better understanding.

For convenience, let us consider a set that represents the set of diagnoses,

$$P = \{P_1(\text{Jaundice}), P_2(\text{Asthma}), P_3(\text{Tuberculosis}), P_4(\text{Typhoid})\},$$

and equivalently we consider a set of symptoms,

$$S = \{s_1(\text{Fatigue}), s_2(\text{Breathing problem}), s_3(\text{Chest Pain}), s_4(\text{Loss of appetite})\}.$$

Suppose a patient Q with respect to the given set of symptoms has representation in the form of GTrFNs as shown below,

Table 3 Judgement matrix
 $D_{ij} = [\delta_{ij}]_{p \times q}$

Alt	Crit			
	C_1	C_2	C_3	C_4
A_1	[5, 6, 7, 8;0.8]	[3, 4, 5, 6;0.5]	[4, 5, 6, 7;0.6]	[0.4, 0.5, 0.6, 0.7;0.2]
A_2	[9, 10, 11, 12;0.9]	[5, 5.5, 6, 6.5;0.7]	[5, 5.5, 6, 6.5;0.5]	[1.4, 1.7, 2, 2.3;0.4]
A_3	[5, 5.5, 6, 6.5;0.4]	[4, 4.5, 5, 5.5;0.5]	[3, 3.5, 4, 4.5;0.3]	[0.8, 0.9, 1, 1.1;0.4]
A_4	[8, 9, 10, 11;0.3]	[3, 3.5, 4, 4.5;0.4]	[3, 3.5, 4, 4.5;0.4]	[0.5, 0.6, 0.7, 0.8;0.3]

Table 4 Normalized judgement matrix $\bar{R}_{ij} = [\sigma_{ij}]_{p \times q}$

Alt	Crit			
	C_1	C_2	C_3	C_4
A_1	[0.42, 0.54, 0.7, 0.89;0.89]	[0.46, 0.67, 0.91, 1;0.72]	[0.43, 0.58, 0.8, 1;0.5]	[0.57, 0.84, 1, 1;1]
A_2	[0.75, 0.91, 1, 1;1]	[0.77, 0.92, 1, 1;1]	[0.46, 0.58, 0.73, 0.9;0.6]	[0.17, 0.25, 0.35, 0.5;0.5]
A_3	[0.42, 0.5, 0.6, 0.73;0.45]	[0.62, 0.75, 0.91, 1;0.72]	[0.67, 0.88, 1, 1;1]	[0.36, 0.5, 0.67, 0.88;0.5]
A_4	[0.67, 0.82, 1, 1;0.34]	[0.46, 0.58, 0.73, 0.9;0.57]	[0.67, 0.88, 1, 1;0.75]	[0.5, 0.71, 1, 1;0.67]

Table 5 Best alternative as per different similarity methods

Similarity measure methods	Similarity values between				Best alternative	Ranking order
	(P_1, Q)	(P_2, Q)	(P_3, Q)	(P_4, Q)		
Hsieh (1999)	0.8031	0.8002	0.7923	0.8514	A_4	$A_4 > A_1 > A_2 > A_3$
Chen (1996)	0.8075	0.7056	0.7181	0.7381	A_1	$A_1 > A_4 > A_3 > A_2$
Chen and Chen (2001)	N/A	N/A	N/A	N/A	–	–
Lee (2002)	0.4083	0.3541	0.3466	0.3835	A_1	$A_1 > A_4 > A_2 > A_3$
Wei and Chen (2009)	0.6148	0.5997	0.5741	0.6067	A_1	$A_1 > A_4 > A_2 > A_3$
Hejazi et al. (2011)	0.4420	0.3787	0.3649	0.3528	A_1	$A_1 > A_2 > A_3 > A_4$
Xu et al. (2010)	N/A	N/A	N/A	N/A	–	–
Patra and Mondal (2015)	0.5701	0.5960	0.5487	0.5611	A_2	$A_2 > A_1 > A_4 > A_3$
Ye (2012)	0.8176	0.7450	0.7197	0.7365	A_1	$A_1 > A_2 > A_4 > A_3$
Khorshidi and Nikfalazar (2017)	N/A	N/A	N/A	N/A	–	–
Xie et al. (2017)	0.0364	0.0380	0.0296	0.0274	A_2	$A_2 > A_1 > A_3 > A_4$
Wu et al. (2020)	N/A	N/A	N/A	N/A	–	–
Proposed approach	0.9166	0.8577	0.8807	0.8460	A_1	$A_1 > A_3 > A_2 > A_4$

$$Q(\text{Patient}) = \left\{ \begin{array}{l} \langle s_1, [0.6, 0.7, 0.8, 0.9; 0.5] \rangle, \langle s_2, [0.3, 0.4, 0.5, 0.6; 0.36] \rangle, \langle s_3, [0.45, 0.50, 0.55, 0.60; 0.42] \rangle, \\ \langle s_4, [0.42, 0.52, 0.62, 0.72; 0.5] \rangle \end{array} \right\}.$$

Consequently, the set of diagnoses P_i ($i = 1, 2, 3, 4, 5$) with respect to all possible symptoms have the following representations in the form of GTrFNs,

From the similarity values obtained, the maximum similarity is found for the pair (P_1, Q) . Hence, the patient Q is more likely to suffer from illness caused by P_2 (Asthma).

$$\begin{aligned} P_1(\text{Jaundice}) &= \left\{ \begin{array}{l} \langle s_1, [0.15, 0.16, 0.17, 0.18; 0.2] \rangle, \langle s_2, [0.10, 0.20, 0.30, 0.40; 0.4] \rangle, \langle s_3, [0.6, 0.7, 0.8, 0.9; 0.8] \rangle, \\ \langle s_4, [0.45, 0.55, 0.65, 0.75; 0.75] \rangle \end{array} \right\}, \\ P_2(\text{Asthma}) &= \left\{ \begin{array}{l} \langle s_1, [0.50, 0.53, 0.56, 0.59; 0.37] \rangle, \langle s_2, [0.10, 0.15, 0.20, 0.25; 0.30] \rangle, \langle s_3, [0.2, 0.3, 0.4, 0.5; 0.6] \rangle, \\ \langle s_4, [0.40, 0.50, 0.60, 0.70; 0.5] \rangle \end{array} \right\}, \\ P_3(\text{Tuberculosis}) &= \left\{ \begin{array}{l} \langle s_1, [0.80, 0.83, 0.86, 0.89; 0.8] \rangle, \langle s_2, [0.60, 0.65, 0.70, 0.75; 0.5] \rangle, \langle s_3, [0.3, 0.4, 0.5, 0.6; 0.4] \rangle, \\ \langle s_4, [0.10, 0.20, 0.30, 0.40; 0.7] \rangle \end{array} \right\}, \\ P_4(\text{Typhoid}) &= \left\{ \begin{array}{l} \langle s_1, [0.20, 0.25, 0.30, 0.35; 0.4] \rangle, \langle s_2, [0.16, 0.22, 0.28, 0.34; 0.28] \rangle, \langle s_3, [0.78, 0.82, 0.86, 0.90; 0.8] \rangle, \\ \langle s_4, [0.91, 0.92, 0.93, 0.94; 0.85] \rangle \end{array} \right\}. \end{aligned}$$

Here, our objective is to classify the patient Q , into one of the diagnoses P_1, P_2, P_3 or P_4 . Depending on the highest value of similarity obtained between the patient Q and any of P_1, P_2, P_3 or P_4 , we can identify the exact disease of the patient.

The evaluated similarity values with respect to our proposed measure is given below,

$$\begin{aligned} S_M(P_1, Q) &= 0.7961, S_M(P_2, Q) = 0.9198, \\ S_M(P_3, Q) &= 0.8997, S_M(P_4, Q) = 0.8247. \end{aligned}$$

Further, the comparison results under different similarity measures are depicted in Table 6.

5.1.3 Antivirus mask selection to prevent COVID-19 transmission

Since the onset of the havoc and outrage caused by the COVID-19 pandemic, the demand for face masks has seen an unprecedented spike. A wide range of masks are normally available in the market are namely, surgical masks (M_1), N95 masks or particulate respirators (M_2), disposable

Table 6 Determination of the patient's suffering illness under various methods

Similarity measures	Similarity values between				Patient Q suffering from disease
	(P_1, Q)	(P_2, Q)	(P_3, Q)	(P_4, Q)	
Hsieh (1999)	0.8128	0.8614	0.8543	0.7524	P_2 (Asthma)
Chen (1996)	0.7400	0.8312	0.8200	0.6637	P_2 (Asthma)
Chen and Chen (2001)	0.3969	0.5856	0.5217	0.2964	P_2 (Asthma)
Lee (2002)	0.5506	0.5921	0.5616	0.3602	P_2 (Asthma)
Wei and Chen (2009)	0.4959	0.6757	0.6423	0.4718	P_2 (Asthma)
Hejazi et al. (2011)	0.3480	0.5149	0.5071	0.3116	P_2 (Asthma)
Xu et al. (2010)	0.9235	0.9331	0.9312	0.8920	P_2 (Asthma)
Patra and Mondal (2015)	0.6300	0.7747	0.7376	0.5675	P_2 (Asthma)
Ye (2012)	0.6560	0.8753	0.8509	0.6946	P_2 (Asthma)
Khorshidi and Nikfalazar (2017)	0.7383	0.8739	0.8591	0.7720	P_2 (Asthma)
Xie et al. (2017)	0.2068	0.2627	0.1095	0.0297	P_2 (Asthma)
Wu et al. (2020)	0.4846	0.6709	0.6041	0.4530	P_2 (Asthma)
Proposed approach	0.7961	0.9198	0.8997	0.8247	P_2 (Asthma)

masks (M_3), thick-coated medical protective masks (M_4), and gas masks (M_5). The concerned and responsible citizens before buying a mask for themselves keep in mind, the following four attributes, namely, its ability to re-utilize or reuse (A_1), rate of leakage (A_2), high filtration capability (A_3), and material texture or quality (A_4). The attribute values for each type of face mask are determined based on the evaluation index provided by people for each type of mask and are presented via GTrFNs as shown in Table 7.

As it is evident from the above decision matrix, the components of GTrFNs are already represented in their normalized form, so there is no need for further normalization. The step-by-step is illustrated below.

Step 1 Identification of the attribute-type.

For our considered scenario, the attributes A_1, A_3 and A_4 are of benefit-type, while A_2 is a cost-type attribute.

Step 2 Determination of the ideal solution (IS)/ ideal mask.

The ideal solution $M^* = (M_1^*, M_2^*, M_3^*, M_4^*)$ is constructed using the formulae given below,

$$M_j^* = \langle T_j^*, I_j^*, F_j^* \rangle = \langle \max_i \{T_{ij}\}, \min_i \{I_{ij}\}, \min_i \{F_{ij}\} \rangle$$

for benefit-type attribute,

$$M_j^* = \langle T_j^*, I_j^*, F_j^* \rangle = \langle \min_i \{T_{ij}\}, \max_i \{I_{ij}\}, \max_i \{F_{ij}\} \rangle$$

for cost-type attribute, where $i = 1, 2, \dots, 6$; $j = 1, 2, 3, 4$.

Therefore, the ideal solution for the given decision matrix $R = (r_{ij})_{6 \times 4}$ with the help of above two equations, is evaluated as,

$$M^* = \left\langle [0.80, 0.83, 0.86, 0.90; 0.8], [0.15, 0.19, 0.23, 0.27; 0.01], [0.70, 0.75, 0.80, 0.85; 0.9], [0.80, 0.86, 0.92, 0.98; 0.8] \right\rangle$$

Step 3 Determining the similarity measure values.

The evaluated similarity values for each mask -type with our proposed measure is obtained as,

$$S_M(M^*, M_1) = 0.9334, S_M(M^*, M_2) = 0.9932, S_M(M^*, M_3) = 0.7577, S_M(M^*, M_4) = 0.7995, S_M(M^*, M_5) = 0.8072.$$

Step 4 Ranking of the masks.

After evaluation of the SM values between the set of masks and the ideal mask, the highest similarity measure value is obtained for the M_2 (N95-mask), and therefore it is the most appropriate mask or the best buying option to minimize the chances of COVID-19 virus transmission.

It is evident from Table 8 that, our result for the best suitable mask coincides with the outcomes obtained with other existing methods as well. This demonstrates the validity and credibility of our newly constructed SM.

Also, we present the similarity measure results obtained under various existing measures in Table 8 below.

5.2 Advantages/superiority and limitations of our proposed SM approach

The advantages of our proposed SM approach are stated below as:

1. Our presented approach is highly efficient and well equipped to overcome the scantiness and impediments of the existing SM approaches as is evident from the comparative study conducted in Sect. 4. Moreover, the

Table 7 Decision Matrix $R = (r_{ij})_{6 \times 4}$, for different mask types and their attribute values in terms of SVNNS

Attributes	Mask types			
	A_1	A_2	A_3	A_4
M_1	$\begin{bmatrix} 0.60, 0.70, 0.80, 0.90; \\ 0.7 \end{bmatrix}$	$\begin{bmatrix} 0.20, 0.30, 0.40, 0.60; \\ 0.01 \end{bmatrix}$	$\begin{bmatrix} 0.50, 0.60, 0.70, 0.80; \\ 0.6 \end{bmatrix}$	$\begin{bmatrix} 0.70, 0.72, 0.74, 0.76; \\ 0.65 \end{bmatrix}$
M_2	$\begin{bmatrix} 0.80, 0.83, 0.86, 0.89; \\ 0.8 \end{bmatrix}$	$\begin{bmatrix} 0.15, 0.19, 0.23, 0.27; \\ 0.1 \end{bmatrix}$	$\begin{bmatrix} 0.70, 0.75, 0.80, 0.85; \\ 0.8 \end{bmatrix}$	$\begin{bmatrix} 0.80, 0.86, 0.92, 0.98; \\ 0.7 \end{bmatrix}$
M_3	$\begin{bmatrix} 0.50, 0.55, 0.60, 0.65; \\ 0.4 \end{bmatrix}$	$\begin{bmatrix} 0.30, 0.40, 0.50, 0.60; \\ 0.9 \end{bmatrix}$	$\begin{bmatrix} 0.60, 0.62, 0.64, 0.66; \\ 0.5 \end{bmatrix}$	$\begin{bmatrix} 0.20, 0.40, 0.60, 0.80; \\ 0.45 \end{bmatrix}$
M_4	$\begin{bmatrix} 0.20, 0.60, 0.60, 0.80; \\ 0.6 \end{bmatrix}$	$\begin{bmatrix} 0.60, 0.70, 0.80, 0.90; \\ 0.8 \end{bmatrix}$	$\begin{bmatrix} 0.30, 0.45, 0.60, 0.75; \\ 0.9 \end{bmatrix}$	$\begin{bmatrix} 0.69, 0.74, 0.79, 0.84; \\ 0.8 \end{bmatrix}$
M_5	$\begin{bmatrix} 0.75, 0.76, 0.77, 0.78; \\ 0.75 \end{bmatrix}$	$\begin{bmatrix} 0.80, 0.83, 0.86, 0.89; \\ 0.9 \end{bmatrix}$	$\begin{bmatrix} 0.70, 0.71, 0.72, 0.73; \\ 0.68 \end{bmatrix}$	$\begin{bmatrix} 0.50, 0.60, 0.70, 0.80; \\ 0.5 \end{bmatrix}$

Table 8 Similarity values obtained for different mask types under different methods

SM methods	Similarity values between pairs of masks					Best mask	Worst mask
	(M^*, M_1)	(M^*, M_2)	(M^*, M_3)	(M^*, M_4)	(M^*, M_5)		
Hsieh (1999)	0.8819	0.9995	0.7964	0.7798	0.8215	M_2	M_4
Chen (1996)	0.8631	0.9994	0.7381	0.6969	0.7456	M_2	M_4
Chen and Chen (2001)	0.6184	0.7110	0.2225	0.4111	0.4575	M_2	M_3
Lee (2002)	0.5429	0.9875	0.4073	0.4608	0.3895	M_2	M_5
Wei and Chen (2009)	0.6031	0.8270	0.3617	0.5380	0.5220	M_2	M_3
Hejazi et al. (2011)	0.4649	0.6580	0.1976	0.4448	0.3952	M_2	M_3
Xu et al. (2010)	0.9426	0.9997	0.9239	0.9127	0.9412	M_2	M_4
Patra and Mondal (2015)	0.7825	0.9589	0.5157	0.5948	0.6289	M_2	M_3
Ye (2012)	0.8924	0.9756	0.4869	0.6746	0.6713	M_2	M_3
Khorshidi and Nikfalazar (2017)	0.8368	0.9257	0.6299	0.7501	0.7052	M_2	M_3
Xie et al. (2017)	0.0959	0.6706	0.0173	0.0849	0.0429	M_2	M_3
Wu et al. (2020)	0.5967	0.8512	0.3750	0.5144	0.5167	M_2	M_3
Proposed approach	0.9334	0.9932	0.7577	0.7995	0.8072	M_2	M_3

values of similarity obtained with our method are more acceptable, more commonsensical, and are in tone with human intuition.

- Our present approach does not fail to evaluate the similarity measure between a crisp valued number and a non-crisp valued number, while some of the existing methods were deprived of this ability.
- Moreover, “expected value” and “variance” are deterministic parameters and they express the nature or the behavior exhibited by data sets including fuzzy numbers. Hence, unlike other existing methods in the literature, our proposed method has the unique ability to incorporate both parameters into a single SM. This direction has not been previously undertaken and hence the novelty of our work.

- Our proposed method has the capability and flexibility of applying to a wide variety of MCDM problems (some of which have been discussed above in Sect. 5.1), and obtaining logical and feasible outcomes as well.

However, some of the limitations of the work might include:

- we have utilized the traditional α -cut method for the determination of expected value as well as the variance of GTrFNs. However, several other methods are widely in practice such as the expected interval method for calculation of expected value, which motivates us to think about the expression for variance via the expected interval method in the future.

2. we may think of further refining our similarity measure expression so that even for almost similar but non-identical fuzzy numbers, we obtain an enhanced degree of distinction between them, than that which exists at present.
3. there is even scope to use possibility theory in determining the forms for expected value and variance via possibility and necessity functions. Further, we may devise new ranking and scoring functions for GTrFNs.

6 Conclusion

The similarity measure is a significant concept in human cognition which has multifarious applications in the field of taxonomy, psychology, ecology, information retrieval, machine learning, etc. Several factors such as the position, shape, and area may affect the similarity measure result between fuzzy numbers, and as such their real-world applications become much more varied and complicated. Although SM is a significant numerical index in fuzzy sets which accounts for the similar or dissimilar degree amongst them, however, most of the practical applications of SM were designed for the standard or normalized fuzzy numbers rather than the non-standardized fuzzy numbers, which are much prevalent in the real-world. Interestingly, SMs are preferred over their distance counterparts as they preserve the fuzzy form of fuzzy numbers and they substantially reduce the loss of information which generally occurs in the defuzzification process of evaluating the distance between fuzzy sets using distance measure methods. In this present work, a novel SM for GTrFNs is proposed which takes into consideration the expected value and variance of fuzzy numbers. GTrFNs have more flexibility and are convenient in defining the accuracy of data. Most importantly, the regularity condition of fuzzy numbers is relaxed in the case of GTrFNs. To validate the stability and superiority of our proposed method, we have carried out a comparative study of our method with other existing methods considering fourteen different profiles of GTrFNs. The experimental results indicate that our proposed SM can overcome the drawbacks of the existing SMs and that our presented measure has better efficiency and desirable qualities of its own. Then, we have described the process of handling an FMCDM problem with our proposed SM approach. Moreover, to establish the applicability of our measure we provide numerical illustrations of few practical scenarios, the most crucial of which is the selection of a proper anti-virus masks as a preventive measure against the COVID-19 outbreak. Even though the selection of masks is a daunting task to accomplish, we have shown that our proposed measure is capable of handling the problem in the context of an FMCDM scenario very effortlessly and diligently. Furthermore, we present

a comparative study of the outcomes obtained under various similarity methods for each of the numerical scenarios, which are found to be in logical agreement, and thus veracity of our proposed method is established.

The contributions and originality of our work can be pointed out as follows:

- Evaluated the mathematical form for “expected value” and “variance” of GTrFNs. Expected value and variance calculation of FNs have always been an area of interest for the researchers and our novel approach explores the α -cut method for their evaluation.
- Formulated an SM for GTrFNs which takes into account the components of GTrFNs, their heights, expected values, and variances. The idea is to have a simple description yet yield efficient outcomes.
- Illustrated an FMCDM method based on the proposed measure.
- Few practical examples are contemplated to demonstrate the applicability of our presented measure, which for instance are- launching of electronic gadgets; a problem of medical decision-making, and selection of proper anti-virus masks for ensuring maximum protection against the COVID-19 pandemic.

In the future research direction, we shall seek probable applications of our proposed SM in some other allied decision-making fields which for instance may be, data mining, supply chain management, pattern recognition, clustering analysis, information fusion, risk analysis, game theory, expert systems, optimization problems, etc. We plan to incorporate the standard deviation of GTrFNs into devising fruitful SMs and to rigorously test our SMs in complex decision-making problems and observe their utility therein. In addition, we shall focus on reaching a consensus among the decision-makers involved in a group decision-making process. Different preference relations such as interval-valued fuzzy preference relation, intuitionistic fuzzy preference relations, trapezoidal fuzzy preference relations, etc. shall be considered in group decision-making problems with our proposed measure. For instance, some relevant works by Yu et al. (2019) and Zhang et al. (2020), shall be referred and consequently, novel methodologies shall be proposed.

Appendix 1

Proof We have already obtained the expression for expected value of GTrFN, which is

$$E(\eta) = \frac{1}{4}w[\eta_1 + \eta_2 + \eta_3 + \eta_4] \quad \text{and also, } V(\eta) = E(\eta^2) - [E(\eta)]^2.$$

So, we proceed by evaluating first, $E(\eta^2)$.

$$\begin{aligned}
 E(\eta^2) &= \frac{1}{2} \left[\int_0^w \left[\eta_1 + \frac{1}{w} \{ \alpha(\eta_2 - \eta_1) \} \right]^2 d\alpha + \int_0^w \left[\eta_4 - \frac{1}{w} \{ \alpha(\eta_4 - \eta_3) \} \right]^2 d\alpha \right] \\
 &= \frac{1}{2} \left[\int_0^w \left\{ \eta_1^2 + \frac{1}{w^2} \alpha^2 (\eta_2 - \eta_1)^2 + \frac{2\eta_1(\eta_2 - \eta_1)\alpha}{w} \right\} d\alpha + \int_0^w \left\{ \eta_4^2 + \frac{1}{w^2} \alpha^2 (\eta_4 - \eta_3)^2 - \frac{2\eta_4(\eta_4 - \eta_3)\alpha}{w} \right\} d\alpha \right] \\
 &= \frac{1}{2} \left[\left[\eta_1^2 \alpha + \frac{\alpha^3}{3} \frac{1}{w^2} (\eta_2 - \eta_1)^2 + \frac{2\eta_1(\eta_2 - \eta_1)\alpha^2}{w} \right]_0^w + \left[\eta_4^2 \alpha + \frac{\alpha^3}{3} \frac{1}{w^2} (\eta_4 - \eta_3)^2 - \frac{2\eta_4(\eta_4 - \eta_3)\alpha^2}{w} \right]_0^w \right] \\
 &= \frac{1}{2} \left[\eta_1^2 w + \frac{w^3}{3w^2} (\eta_2 - \eta_1)^2 + \eta_1 (\eta_2 - \eta_1) \frac{w^2}{w} + \eta_4^2 w + \frac{w^3}{3w^2} (\eta_4 - \eta_3)^2 - \eta_4 (\eta_4 - \eta_3) \frac{w^2}{w} \right] \\
 &= \frac{1}{2} \left[\frac{(\eta_2^2 + \eta_1^2 - 2\eta_1\eta_2 + 3\eta_1\eta_2)w}{3} + \frac{(\eta_4^2 + \eta_3^2 - 2\eta_3\eta_4 + 3\eta_3\eta_4)w}{3} \right] \\
 &\Rightarrow E(\eta^2) = \frac{1}{6} w [\eta_1^2 + \eta_2^2 + \eta_3^2 + \eta_4^2 + \eta_1\eta_2 + \eta_3\eta_4].
 \end{aligned}$$

Therefore, $V(\eta) = \frac{1}{6} w [\eta_1^2 + \eta_2^2 + \eta_3^2 + \eta_4^2 + \eta_1\eta_2 + \eta_3\eta_4]$
 $- \left[\frac{1}{4} w [\eta_1 + \eta_2 + \eta_3 + \eta_4] \right]^2$.
 $\Rightarrow V(\eta) = \frac{1}{6} w [\eta_1^2 + \eta_2^2 + \eta_3^2 + \eta_4^2 + \eta_1\eta_2 + \eta_3\eta_4] - \frac{1}{16} w^2$
 $[\eta_1 + \eta_2 + \eta_3 + \eta_4]^2$..

Hence, the proof.

Appendix 2

See Figs. 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14 and 15.

Fig. 2 Profile-1

Fig. 3 Profile-2 **Fig. 4** Profile-3 **Fig. 5** Profile-4

Fig. 6 Profile-5

Fig. 7 Profile-6

Fig. 8 Profile-7

Fig. 9 Profile-8

Fig. 10 Profile-9

Fig. 11 Profile-10

Fig. 12 Profile-11

Fig. 13 Profile-12

Fig. 14 Profile-13

Fig. 15 Profile-14

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