Indian J Phys (February 2024) 98(2):679–690
https://doi.org/10.1007/s12648-023-02800-1

ORIGINAL PAPER

Influence of a magnetic field on a nonlocal thermoelastic porous solid with memory-dependent derivative

S M Said*, M I A Othman and M G Eldemerdash

Department of Mathematics, Faculty of Science, Zagazig University, P.O. Box 44519, Zagazig, Egypt

Received: 06 February 2023 / Accepted: 02 June 2023 / Published online: 12 July 2023

Abstract: A novel model of a nonlocal magneto-thermoelastic porous solid in the context of the three-phase-lag model with a memory-dependent derivative is introduced. The effect of a magnetic field on a nonlocal thermoelastic porous medium in the context of a three-phase-lag model with memory-dependent derivatives was studied. The normal mode analysis is used to solve the problem of an isothermal boundary to obtain the exact expressions of physical fields. The numerical results are represented to estimate the effects of the magnetic field, time delay, and the nonlocal parameter on the behavior of all of the field variables such as temperature, displacement, and stresses. Comparisons are given for the results in the absence and presence of the magnetic field as well as the locality. Comparisons are also given for the results for different values of time delay. To the best of the author's knowledge, this model is reported for the first time. Some particular cases are also deduced from the present investigation.

Keywords: Porous thermoelastic solid; Magnetic field; Nonlocal parameter; The three-phase-lag model

Mathematics Subject Classification: 74Bxx; 35Qxx; 65Nxx; 65Txx

1. Introduction

The memory-dependent derivative (MDD) can be defined as the integral form over a sliding interval of a common derivative with a selected kernel function. This definition is better than the definition of fractional differentiation to reverse the influence of memory. There are several interesting phenomena, especially physical ones that have so-called memory-dependent influences, and this means that their current state depends not only on the position and the time but also on the previous cases. The definition of memory-dependent derivatives was introduced by Wang and Li [1]. An interesting application of MDD is given by Yu et al. [2]. They introduced the MDD instead of fractional calculus into the rate of heat flux in the Lord–Shulman model of generalized thermoelasticity. A model of two-temperature thermoelasticity theory with time delay and Taylor theorem with memory-dependent derivatives involving two temperatures was introduced by Ezzat et al. [3]. The mathematical model of thermoelectric viscoelastic materials with memory-dependent derivatives was proposed by Ezzat et al. [4]. Based on the generalized thermoelastic diffusion theory with memory-dependent derivative in both the generalized heat conduction law and the generalized diffusion law, the transient response is investigated by Li and He [5]. Very recently, several problems in generalized thermo-elasticity in the context of memory-dependent derivatives have been reported in the studies [6–13].

Magneto-thermo-elasticity, which deals with the interactions among strain, temperature, and electromagnetic fields, has drawn the attention of many researchers, because of the extensive uses in diverse fields, especially, geophysics for understanding the effect of the earth's magnetic field on seismic waves, damping of acoustic waves in a magnetic field, the emission of electromagnetic radiations from nuclear devices, development of a highly sensitive superconducting magnetometer, electrical power engineering, optics, etc. The problem of generalized electro-magneto-thermoelastic plane waves in a finite conductivity half-space with one relaxation time was discussed by Othman [14]. A novel model of the two-temperature generalized magneto-viscoelasticity with two relaxation times in a perfect conducting medium is established by Ezzat

*Corresponding author, E-mail: samia_said59@yahoo.com

et al. [15]. The reflection and transmission of the thermoelastic wave at a solid–liquid interface in the presence of initial stress and magnetic field in the context of Green-Lindsay model was discussed by Abo-Dahab and Abd-Alla [16]. The three-phase-lag model and Green–Naghdi theory without energy dissipation to study the effect of the gravity field and a magnetic field on wave propagations in a generalized thermoelastic problem for a medium with an internal heat source was applied as Said [17]. The effect of Thomson and initial stress in a thermo-porous elastic solid under Green-Naghdi electromagnetic theory was investigated by Abd-Elaziz et al. [18]. The effect of hydrostatic initial stress, gravity, and magnetic field in a fiber-reinforced thermoelastic solid with variable thermal conductivity was investigated by Said and Othman [19]. A dual-phase-lag model to discuss the effect of a magnetic field on thermoelastic micro-elongated solid with diffusion was applied by Alharbi et al. [20]. The literature Refs. [21–25] contains a wealth of studies on magneto-thermoelastic materials.

The theory of linear elastic materials with voids is one of the most significant generalizations of the classical theory of elasticity. This theory examines various types of geological and biological materials in order to fill the gaps left by the classical theory of elasticity and is concerned with materials that have a distribution of small (porous) voids. Iesan [26] discussed a hypothesis of thermoelastic materials with voids and without energy dissipation. The nonlinear theory of non-simple thermoelastic materials with voids was explored by Ciarletta and Scalia [27].The literature Refs. [28–32] contains a wealth of studies on porous thermoelastic materials.

Numerous authors have taken an interest in the theory of nonlocal elasticity as a result of its early success in resolving a long-standing issue in fracture mechanics. Based on the nonlocal thermoelasticity hypothesis, Inan and Eringen [33] looked into thermoelastic wave propagation in plates. By contrasting numerous results of both theories, Artan [34] demonstrated the superiority of the nonlocal theory. The literature Refs. [35–39] contains a wealth of studies on the nonlocal thermoelastic theory.

In the present study, the effect of a magnetic field on a nonlocal thermoelastic porous solid in the context of the three-phase-lag model with a memory-dependent derivative is discussed. The resulting non-dimensional equations are solved using normal mode analysis. A comparison is carried out between the considered variables in the absence and presence of the magnetic field as well as the locality. Comparisons are also given for the results for different values of time delay. Three-phase-lag model is very useful in the problems of nuclear boiling, exothermic catalytic reactions, phonon-electron interactions, phonon-scattering, etc. The numerical results are represented to estimate the effects of the magnetic field, time delay, and the nonlocal parameter on the behavior of all of the field variables such as temperature, displacement, and stresses. It is clear that the locality, time delay, and magnetic field have played a major role in the physical fields.

2. Formulation of the problem and basic equations

The problem of a nonlocal thermoelastic porous medium half-space ($x \geq 0$) was considered. A magnetic field with a constant intensity $\boldsymbol{H} = (0, 0, H_0)$, is acting parallel to the boundary plane. The displacement vector (Fig. 1)

$$u = u(x, y, t), \qquad v = v(x, y, t), \qquad w = 0. \tag{1}$$

The constitutive equations as Hetnarski and Eslami [40], Eringen et al. [41, 42], Inan and Eringen [33], and Wang and Dhaliwal [43]:

$$(1 - \varepsilon^2 \nabla^2)\, \sigma_{ij} = \lambda\, e_{kk} \delta_{ij} + 2\mu\, e_{ij} - \gamma\, \theta\, \delta_{ij} + b\, \varphi\, \delta_{ij}, \tag{2}$$

where $\varepsilon = a_0\, e_0$ is the elastic nonlocal parameter having a dimension of length, a_0, e_0 respectively, are an internal characteristic length and a material constant, σ_{ij} are the components of stress, e_{ij} are the components of strain, e_{kk} is the dilatation, λ, μ are elastic constants, α_t is the thermal expansion coefficient, $\theta = T - T_0$, where T is the temperature above the reference temperature T_0, φ is the change in volume fraction field of voids, δ_{ij} is the Kronecker's delta.

The equations of motion

$$\rho(1 - \varepsilon^2\nabla^2)\frac{\partial^2 u_i}{\partial t^2} = (1 - \varepsilon^2\nabla^2)\sigma_{ji,j} + (1 - \varepsilon^2\nabla^2)F_i, \quad F_i = \mu_0(\boldsymbol{J}\mathrm{x}\boldsymbol{H})_i \tag{3}$$

$$\beta\, \varphi_{,ii} - be - \alpha_1\, \varphi - \alpha_2\, \varphi_{,t} + \alpha_3\, \theta = \rho\alpha_4\, (1 - \varepsilon^2\nabla^2)\varphi_{,tt}, \tag{4}$$

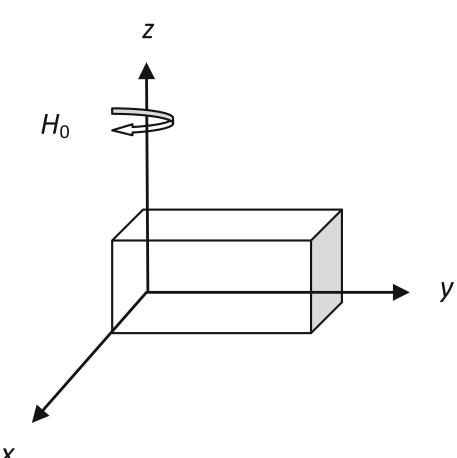

Fig. 1 Geometry of the problem

where $\beta,\ b,\ \alpha_1,\ \alpha_2,\ \alpha_3,\ \alpha_4$ are the material constants due to the presence of voids and $\mu_0\,(\boldsymbol{J}\times\boldsymbol{H})_i$ is Lorentz force due to the presence of a magnetic field.

The variation of the magnetic and electric fields is perfectly conducting slowly moving medium and are given by Maxwell's equation: as Said [17],

$$\begin{gathered}\boldsymbol{J}=\nabla\times\boldsymbol{h}-\varepsilon_0\frac{\partial\boldsymbol{E}}{\partial t},\quad \nabla\times\boldsymbol{E}=-\mu_0\frac{\partial\boldsymbol{h}}{\partial t},\\ \boldsymbol{E}=-\mu_0\left(\frac{\partial u}{\partial t}\times\boldsymbol{H}\right),\nabla.\boldsymbol{h}=0.\nabla.\boldsymbol{E}=0.\end{gathered}\tag{5}$$

where μ_0 is the magnetic permeability, ε_0 is the electric permeability, $\boldsymbol{J}$ is the current density vector, $\dot{\boldsymbol{u}}$ is the particle velocity of the medium, and the small effect of the temperature gradient on $\boldsymbol{J}$ is also ignored. Expressing the components of the vector $\boldsymbol{J}$ in terms of displacement by eliminating the quantities $\boldsymbol{h}$ and $\boldsymbol{E}$ from Eq. (5), thus yield:

$$J_1=\frac{\partial h}{\partial y}+\mu_0\varepsilon_0 H_0\frac{\partial^2 v}{\partial t^2},J_2=-\frac{\partial h}{\partial x}-\mu_0\varepsilon_0 H_0\frac{\partial^2 u}{\partial t^2},J_3=0,\tag{6}$$

Substituting Eq. (6) into Eq. (3), we get

$$\begin{aligned}F_1&=-\,\mu_0 H_0\frac{\partial h}{\partial x}-\varepsilon_0\,\mu_0^2 H_0^2\frac{\partial^2 u}{\partial t^2},\\ F_2&=-\mu_0 H_0\frac{\partial h}{\partial y}-\varepsilon_0\,\mu_0^2 H_0^2\frac{\partial^2 v}{\partial t^2},\qquad F_3=0.\end{aligned}\tag{7}$$

The heat conduction equation as Purkait et al. [44] and Choudhuri [45]:

$$\begin{aligned}&K\ (1+\tau_\theta D_{w1})\nabla^2\theta_{,t}+K^*(1+\tau_v D_{w2})\nabla^2\theta\\ &\quad=\left(1+\tau_q D_{w3}+\frac{1}{2}\tau_q^2 D_{w_3}^2\right)(\rho\,C_E\,\theta_{,tt}+\gamma\,T_0\,e_{,tt}\\ &\qquad+\alpha_3\,T_0\,\varphi_{,tt}),\end{aligned}\tag{8}$$

where K^* is the coefficient of thermal conductivity, K is the additional material constant, C_E is the specific heat at constant strain, τ_v is the phase-lag of thermal displacement gradient, τ_θ is the phase-lag of temperature gradient and τ_q is the phase-lag of heat flux.

D_{w_i} is the memory-dependent derivative operator defined as

$$D_{w_i}f(t)=\frac{1}{w_i}\int_{t-w_i}^{t}L(t-\xi)f'(\xi)\,d\xi.\tag{9}$$

The parameter w_i is the time delay and $L(t-\xi)$ is the kernel function in which they can be chosen freely, see Caputo and Mainardi [46–48] for more explanations. In our present study, we choose $L(t-\xi)$ in the following form $L(t-\xi)=A+B\,(t-\xi)$.

Introducing Eqs. (2) and (7) in Eqs. (3), we get

$$\begin{aligned}\rho\,(1-\varepsilon^2\nabla^2)\frac{\partial^2 u}{\partial t^2}&=(\lambda+2\mu)\frac{\partial^2 u}{\partial x^2}+(\lambda+\mu)\frac{\partial^2 v}{\partial x\partial y}\\ &\quad+\mu\frac{\partial^2 u}{\partial y^2}-\gamma\frac{\partial\theta}{\partial x}+b\frac{\partial\varphi}{\partial x}-(1\\ &\quad-\varepsilon^2\nabla^2)\left(\mu_0 H_0\frac{\partial h}{\partial x}+\varepsilon_0\,\mu_0^2 H_0^2\frac{\partial^2 u}{\partial t^2}\right),\end{aligned}\tag{10}$$

$$\begin{aligned}\rho\,(1-\varepsilon^2\nabla^2)\frac{\partial^2 v}{\partial t^2}&=(\lambda+2\mu)\frac{\partial^2 v}{\partial y^2}+(\lambda+\mu)\frac{\partial^2 u}{\partial x\partial y}\\ &\quad+\mu\frac{\partial^2 v}{\partial x^2}-\gamma\frac{\partial\theta}{\partial y}+b\frac{\partial\varphi}{\partial y}-(1\\ &\quad-\varepsilon^2\nabla^2)\left(\mu_0 H_0\frac{\partial h}{\partial y}+\varepsilon_0\,\mu_0^2 H_0^2\frac{\partial^2 v}{\partial t^2}\right),\end{aligned}\tag{11}$$

For convenience, the following non-dimensional variables are used:

$$\begin{aligned}(x',y',\varepsilon',u',v')&=\frac{1}{l_0}(x,y,\varepsilon,u,v),(t',\tau_q',\tau_\theta',\tau_v')\\ &=\frac{c_0}{l_0}(t,\tau_q,\tau_\theta,\tau_v),\theta'=\frac{\gamma\theta}{(\lambda+2\mu)},\\ \sigma_{ij}'&=\frac{\sigma_{ij}}{\mu},\varphi'=\varphi,h'=\frac{h}{H_0},\\ l_0&=\sqrt{\frac{K^*}{\rho C_E T_0}},c_0=\sqrt{\frac{\lambda+2\mu}{\rho}}\end{aligned}\tag{12}$$

Using the above non-dimension variables, then employing $h=-\,H_0\,e$,

$$\begin{aligned}A_1(1-\varepsilon^2\nabla^2)\frac{\partial^2 u}{\partial t^2}&=(A_2-A_4\varepsilon^2\nabla^2)\frac{\partial^2 u}{\partial x^2}+(A_3\\ &\quad-A_4\varepsilon^2\nabla^2)\frac{\partial^2 v}{\partial x\partial y}+A_5\frac{\partial^2 u}{\partial y^2}-\frac{\partial\theta}{\partial x}\\ &\quad+A_6\frac{\partial\varphi}{\partial x},\end{aligned}\tag{13}$$

$$\begin{aligned}A_1\,(1-\varepsilon^2\nabla^2)\frac{\partial^2 v}{\partial t^2}&=(A_2-A_4\varepsilon^2\nabla^2)\frac{\partial^2 v}{\partial y^2}+(A_3\\ &\quad-A_4\varepsilon^2\nabla^2)\frac{\partial^2 u}{\partial x\partial y}+A_5\frac{\partial^2 v}{\partial x^2}-\frac{\partial\theta}{\partial y}\\ &\quad+A_6\frac{\partial\varphi}{\partial y},\end{aligned}\tag{14}$$

$$\begin{aligned}&A_7\,(1+\tau_\theta D_{w1})\nabla^2\theta_{,t}+\ (1+\tau_v D_{w2})\nabla^2\theta\\ &\quad=\left(1+\tau_q D_{w3}+\frac{1}{2}\tau_q^2 D_{w_3}^2\right)(A_8\theta_{,tt}+A_9 e_{,tt}+A_{10}\varphi_{,tt}),\end{aligned}\tag{15}$$

$$\varphi_{,ii}-A_{11}e-A_{12}\,\varphi-A_{13}\,\varphi_{,t}+A_{14}\theta=A_{15}(1-\varepsilon^2\nabla^2)\varphi_{,tt},\tag{16}$$

where A_i are given in the Appendix.

3. The analytical solution to the problem

The solution of the considered physical variable can be decomposed in terms of normal mode analysis as:

$$[u, v, \theta, \varphi, \sigma_{ij}](x, y, t) = [u^*, v^*, \theta^*, \varphi^*, \ \sigma^*_{ij}](x) \exp(mt + \mathrm{i}\, ay), \tag{17}$$

where $u^*(x)$, etc. is the amplitude of the function $u(x, y,\ t)$ etc., i is the imaginary unit, m is the complex frequency and a is the wave number in the $y-$ direction.

$$(N_1\mathrm{D}^4 - N_2\mathrm{D}^2 + N_3)\ u^* + (N_4\mathrm{D}^3 - N_5\mathrm{D})\ v^* + \mathrm{D}\,\theta^* - A_6\mathrm{D}\,\varphi^* = 0, \tag{18}$$

$$(N_8\mathrm{D}^3 - N_9\mathrm{D})\ u^* + (N_6 - N_7\mathrm{D}^2)\ v^* + \mathrm{i}a\,\theta^* - \mathrm{i}aA_6\,\varphi^* = 0, \tag{19}$$

$$A_{11}\mathrm{D}\,u^* + \mathrm{i}a\,A_{11}\,v^* - A_{14}\,\theta^* - (N_{10}\mathrm{D}^2 - N_{11})\,\varphi^* = 0, \tag{20}$$

$$N_{12}\mathrm{D}\,u^* + \mathrm{i}a\,N_{12}\,v^* - (N_{14}\mathrm{D}^2 - N_{15})\,\theta^* + N_{13}\,\varphi^* = 0, \tag{21}$$

Eliminating $v^*(x),\ \theta^*(x)$ and $\varphi^*(x)$ between Eqs. (18) – (21), the following ten-order ordinary differential equation satisfied by $u^*(x),\ v^*(x),\ \theta^*(x),\ \varphi^*(x)$ can be obtained:

$$(\mathrm{D}^{10} - H_1\mathrm{D}^8 + H_2\,\mathrm{D}^6 - H_3\,\mathrm{D}^4 + H_4\,\mathrm{D}^2 - H_5)\{u^*(x),\ v^*(x),\ \theta^*(x),\ \ \varphi^*(x)\} = 0, \tag{22}$$

where H_1, H_2, H_3, H_4, H_5 are given in the Appendix.

Equation (22) can be factored as

$$(\mathrm{D}^2 - k_1^2)(\mathrm{D}^2 - k_2^2)(\mathrm{D}^2 - k_3^2)(\mathrm{D}^2 - k_4^2)(\mathrm{D}^2 - k_5^2)\,u^*(x) = 0, \tag{23}$$

where $k_n^2\,(n = 1,\ 2,\ 3,\ 4\,, 5)$ are the five roots of the following characteristic equation:

$$k^{10} - H_1\,k^8 + H_2\,k^6 - H_3\,k^4 + H_4\,k^2 - H_5 = 0 \tag{24}$$

The solution of Eq. (22), bounded as $x \to \infty$, can be expressed as:

$$u^*(x) = \sum_{n=1}^{5} G_n \exp(-k_n x), \tag{25}$$

$$v^*(x) = \sum_{n=1}^{5} R_{1n}\,G_n \exp(-k_n x), \tag{26}$$

$$\theta^*(x) = \sum_{n=1}^{5} R_{2n}\,G_n \exp(-k_n x), \tag{27}$$

$$\varphi^*(x) = \sum_{n=1}^{5} R_{3n}\,G_n \exp(-k_n \mathrm{x}), \tag{28}$$

Using the above equations, we get

$$\sigma^*_{xx}(x) = \sum_{n=1}^{5} R_{4n}\,G_n \exp(-k_n x), \tag{29}$$

$$\sigma^*_{xy}(x) = \sum_{n=1}^{5} R_{5n}\,G_n \exp(-k_n x), \tag{30}$$

where R_{in} are given in the Appendix.

4. Boundary conditions

In the physical problem, we should suppress the positive exponentials that are unbounded at infinity. The constants $G_n\ (n = 1, 2, 3, 4, 5)$ have been chosen such that the boundary conditions on the surface at $x = 0$ as Abbas et al. [49]:

$$v = 0, \theta = 0, \sigma_{xx} = -f_0\,G(y, t), \sigma_{xy} = 0, \varphi = \varphi_0\,. \tag{31}$$

where f_0 are constants and $G(y, t)$ are arbitraries functions.

Substituting the expressions of the variables considered into the above boundary conditions, we can obtain the following equations satisfied by the parameters:

$$\sum_{n=1}^{5} R_{1n}G_n = 0, \sum_{n=1}^{5} R_{2n}G_n = 0, \sum_{n=1}^{5} R_{4n}G_n = -f_0, \sum_{n=1}^{5} R_{5n}G_n = 0, \sum_{n=1}^{5} R_{3n}G_n = \varphi_0, \tag{32}$$

Solving the above system of Eqs. (32), we obtain a system of five equations. After applying the inverse of the matrix method, we have the values of the five constants $G_n,\ (\ n = 1, 2, 3, 4, 5)$, hence; we obtain the expressions of displacements, the thermal temperature, and the stress components.

$$\begin{pmatrix} G_1 \\ G_2 \\ G_3 \\ G_4 \\ G_5 \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} & R_{13} & R_{14} & R_{15} \\ R_{21} & R_{22} & R_{23} & R_{24} & R_{25} \\ R_{41} & R_{42} & R_{43} & R_{44} & R_{45} \\ R_{51} & R_{52} & R_{53} & R_{54} & R_{55} \\ R_{31} & R_{32} & R_{33} & R_{34} & R_{35} \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 0 \\ -f_0 \\ 0 \\ \varphi_0 \end{pmatrix}. \tag{33}$$

5. Special cases

(a) Equations of the 3PHL model when, $K, \tau_T,\ \tau_q,\ \tau_\nu > 0$ and the solutions are always (exponentially) stable if $\frac{2K\tau_T}{\tau_q} > \tau^*_\nu > K^*\tau_q$ as in Quintanilla and Racke [50].

(b) Equations of the GN-II theory without energy dissipation when, $K = \tau_T = \tau_q = \tau_v = 0$.
(c) Equations of the GN-III theory with energy dissipation when, $\tau_T = \tau_q = \tau_v = 0$.
(d) The corresponding equations for local thermoelastic porous solid without the influence of the magnetic field from the above-mentioned cases by taking $H_0 = 0$, then we have $A_4 = N_1 = N_4 = N_8 = 0$, thus we have

$$[N_3 - N_2 D^2]\, u^* - N_5 D\, v^* + D\,\theta^* - A_6 D\,\phi^* = 0, \tag{32}$$

$$N_9\, D\, u^* - [N_6 - N_7 D^2]\, v^* - i\, a\,\theta^* + i\, a A_6\,\varphi^* = 0, \tag{33}$$

$$A_{11}\, D\, u^* + i\, a A_{11}\, v^* - A_{14}\,\theta^* - [N_{10} D^2 - N_{11}]\,\varphi^* = 0, \tag{34}$$

$$N_{21}\, D\, u^* + i\, a N_{12} v^* - [N_{14} D^2 - N_{15}]\,\theta^* + N_{13}\,\varphi^* = 0, \tag{35}$$

where $N_i's$ are given in the Appendix and $D = \frac{d}{dx}$.
Eliminating $v^*(x)$, $\theta^*(x)$ and $\varphi^*(x)$ between Eqs. (32) – (35), the following ten-order ordinary differential equation satisfied by $u^*(x)$, $v^*(x)$, $\theta^*(x)$, $\varphi^*(x)$ can be obtained:

$$(D^8 - C D^6 + E D^4 - F D^2 + J)\{u^*(x),\ v^*(x),\ \theta^*(x),\ \varphi^*(x)\} = 0, \tag{36}$$

where C, E, F, J are given in the Appendix.

Equation (36) can be factored as

$$(D^2 - f_1^2)(D^2 - f_2^2)(D^2 - f_3^2)(D^2 - f_4^2)\, u^*(x) = 0, \tag{37}$$

where f_n^2 $(n = 1, 2, 3, 4)$ are the roots of the characteristic equation

The solution of Eq. (37), bounded as $x \to \infty$, can be expressed as:

$$u^*(x) = \sum_{n=1}^{4} Q_n \exp(-f_n x), \tag{38}$$

$$v^*(x) = \sum_{n=1}^{4} M_{1n}\, Q_n \exp(-f_n x), \tag{39}$$

$$\theta^*(x) = \sum_{n=1}^{4} M_{2n}\, Q_n \exp(-f_n x), \tag{40}$$

$$\varphi^*(x) = \sum_{n=1}^{4} M_{3n}\, Q_n \exp(-f_n x), \tag{41}$$

Using the above equations, we get

$$\sigma_{xx}^*(x) = \sum_{n=1}^{4} M_{4n}\, Q_n \exp(-f_n x), \tag{42}$$

$$\sigma_{xy}^*(x) = \sum_{n=1}^{4} M_{5n}\, Q_n \exp(-f_n x), \tag{43}$$

where M_{in} are given in the Appendix.

6. Boundary conditions

In the physical problem, we should suppress the positive exponentials that are unbounded at infinity. The constants Q_n $(n = 1, 2, 3, 4)$ have been chosen such that the boundary conditions on the surface at $x = 0$ as follows:

$$\theta = 0, \sigma_{xx} = -f_0\, Q(y,t), \sigma_{xy} = 0, \varphi = \varphi_0\,. \tag{44}$$

where f_0 are constants and $Q(y,t)$ are arbitraries functions.

Substituting the expressions of the variables considered into the above boundary conditions, we can obtain the following equations satisfied by the parameters:

$$\sum_{n=1}^{4} M_{2n} Q_n = 0, \sum_{n=1}^{4} M_{4n} Q_n = -f_0, \sum_{n=1}^{4} M_{5n} Q_n = 0\,,$$
$$\sum_{n=1}^{4} M_{3n} Q_n = \varphi_0\,, \tag{45}$$

Solving the above system of Eqs. (45), we obtain a system of four equations. After applying the inverse of the matrix method, we have the values of the five constants Q_n, $(n = 1, 2, 3, 4)$. Hence, we obtain the expressions of displacements, the thermal temperature, and the stress components.

$$\begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{pmatrix} = \begin{pmatrix} M_{21} & M_{22} & M_{23} & M_{24} \\ M_{41} & M_{42} & M_{43} & M_{44} \\ M_{51} & M_{52} & M_{53} & M_{54} \\ M_{31} & M_{32} & M_{33} & M_{34} \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ -f_0 \\ 0 \\ \varphi_0 \end{pmatrix}. \tag{46}$$

7. Numerical results and discussion

In order to clarify the theoretical results obtained in the preceding section and compare these in the context of the three-phase-lag (3PHL) model, and study the effect of the magnetic field, nonlocal parameter, and memory-dependent derivative on a porous thermoelastic medium, we now present some numerical results for the physical constants

$\lambda = 2.9 \times 10^{10}\,\mathrm{N.m^{-2}}$, $\mu = 7.78 \times 10^{10}\,\mathrm{N.m^{-2}}$, $\rho = 8954\,\mathrm{kg.m^{-3}}$, $C_E = 383\,\mathrm{J.kg^{-1}.K^{-1}}$, $\alpha_t = 1.78 \times 10^{-3}\,\mathrm{K^{-1}}$, $f_0 = 0.5$,

$\varphi_0 = 0.01$, $\tau_q = 9 \times 10^{-7}\,\mathrm{s}$, $\tau_\theta = 7 \times 10^{-7}\,\mathrm{s}$, $\tau_\upsilon = 6 \times 10^{-7}\,\mathrm{s}$, $K^* = 386\,\mathrm{w.m^{-1}.K^{-1}.s^{-1}}$, $b = 1.6 \times 10^{10}\,\mathrm{N.m^{-2}}$, $\alpha_1 = 1.47 \times 10^{10}\mathrm{N.m^{-2}}$,

$\alpha_2 = 7.78 \times 10^{-10}\mathrm{N.m^{-2}}, \quad \alpha_3 = 2 \times 10^{11}\mathrm{N.m^{-2}},$
$\alpha_4 = 1.753 \times 10^{-10}\mathrm{N.m^{-2}}, \quad \beta = 2 \times 10^{10}\mathrm{N.m^{-2}},$
$m = m_0 + i\xi, \quad m_0 = -0.3,$

$\xi = -0.2, \quad K = 700\,\mathrm{w.m^{-1}.K^{-1}}, \quad a = 0.3, \quad A = 1,$
$B = -1, \quad T_0 = 293\,\mathrm{K}, \quad \varepsilon_0 = 0.2, \quad \mu_0 = 1.9, \quad y = -1.5.$

Figures 2, 3, 4, 5, 6 are graphed to describe the variation in the displacement component v, the thermodynamic temperature θ, the change in the volume fraction field φ and the stress components σ_{xx}, σ_{xy} with different values of $\varepsilon = 0.5, 0.3, 0.01$ (nonlocal parameter).

Figure 2 represents the change of displacement v with distance x and satisfies the boundary condition at $x = 0$, where v starts with decreasing to a minimum value in the range $0 \le x \le 4.7$ and converges to zero with increasing distance x. It is explained that when the value of ε is increasing, the value of v is decreasing. Figure 3 demonstrates the distribution of the thermodynamic temperature θ, it begins with decreasing to a minimum value in the range $0 \le x \le 0.12$,, then increases and approaches a zero

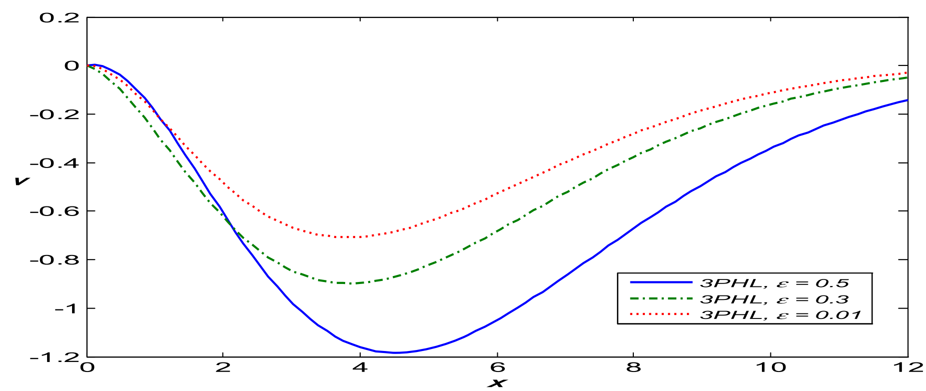

Fig. 2 Vertical displacement distribution v for different values of nonlocal parameter

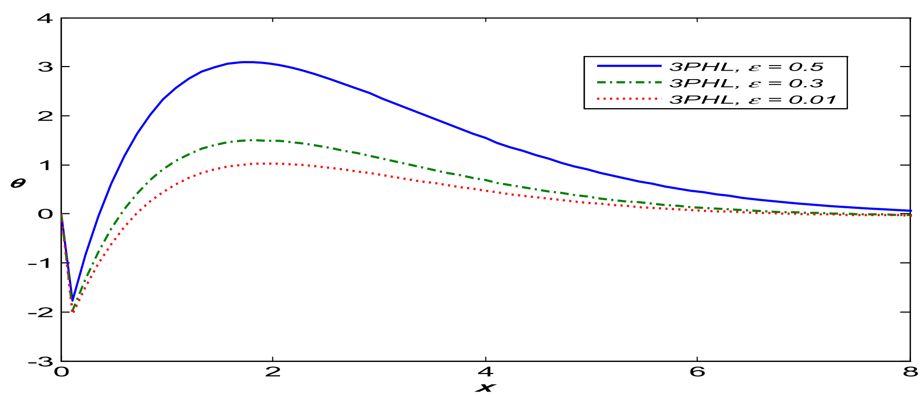

Fig. 3 Thermal temperature distribution θ for different values of nonlocal parameter

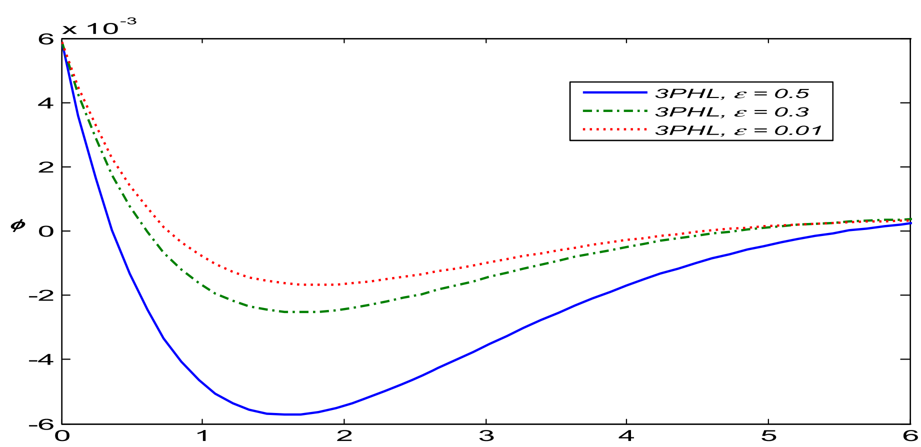

Fig. 4 Volume fraction field distribution φ for different values of nonlocal parameter

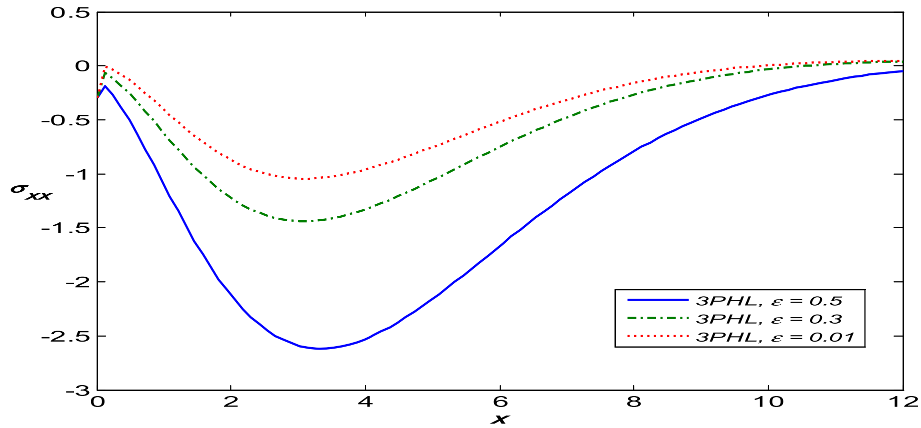

Fig. 5 Distribution of stress component σ_{xx} for different values of nonlocal parameter

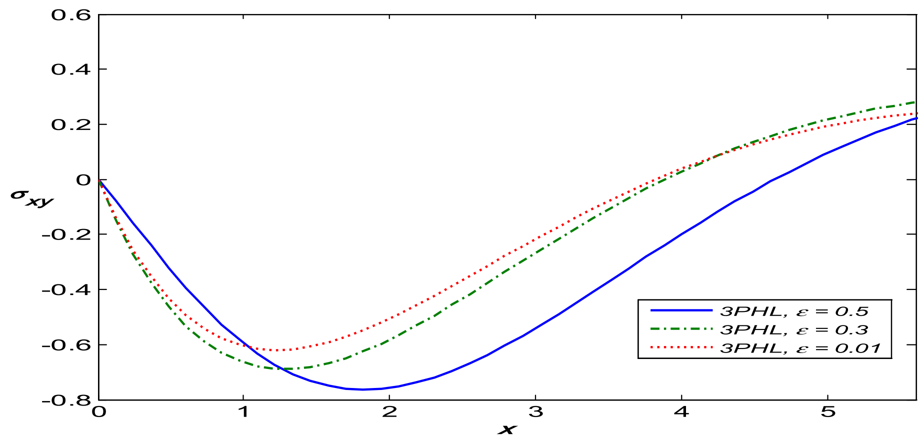

Fig. 6 Distribution of stress component σ_{xy} for different values of nonlocal parameter

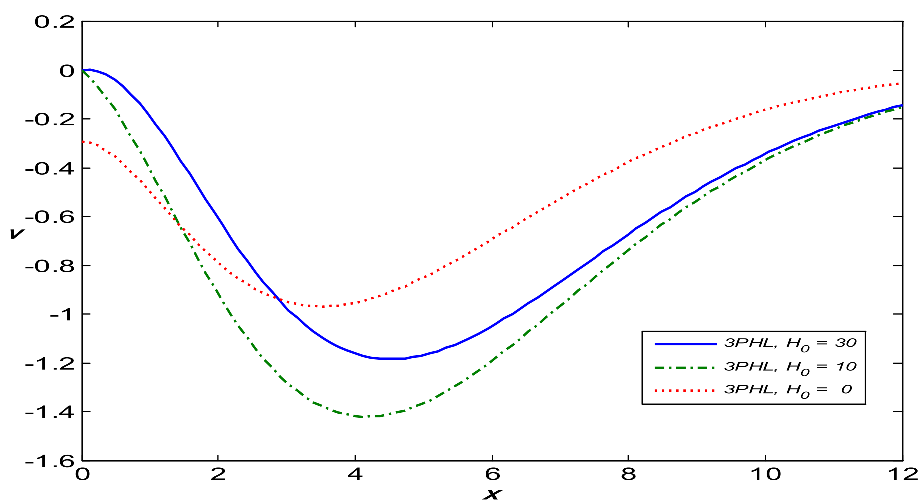

Fig. 7 Effect of different values of magnetic field on vertical displacement v

value. The nonlocal parameter increases the magnitude of θ. Figure 4 shows that when the magnitude of ε is increasing magnitude of the volume fraction field φ is decreasing, where it starts from positive values for all value of ε, then decreases in the range $0 \leq x \leq 1.5$, then increases and converge to zero at $x \geq 1.5$. Figure 5 exhibits that the distribution of the stress component σ_{xx} always begins from negative values. For all values of ε, the values of σ_{xx} start with increasing in the range $0 \leq x \leq 0.12$, then decreasing to a minimum value in the range $0.12 \leq x \leq 3.4$, and finally increasing to a maximum value and becoming constant. Figure 6 depicts the distribution of the stress component

σ_{xy}, based on the three-phase-lag theory and different value of ε, the magnitudes of the stress component σ_{xy} decrease to a minimum value in the range $0 \leq x \leq 1.5$, but increase in the range $1.5 \leq z \leq 5.5$, also the magnitude of σ_{xy} decrease while the value of ε increase.

The influence of different values of the magnetic field, according to the three-phase-lag model on the displacement component v, the thermodynamic temperature θ, the volume fraction field φ, and the stress component σ_{xy}, in Figs. 7, 8, 9, 10. Figure 7 displays the effect of the magnetic field on the vertical displacement v, where the values

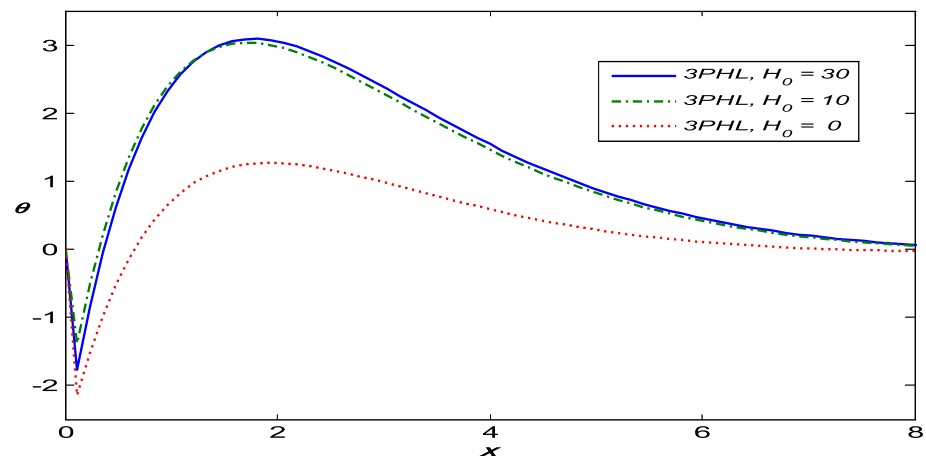

Fig. 8 Effect of different values of magnetic field on thermal temperature distribution θ

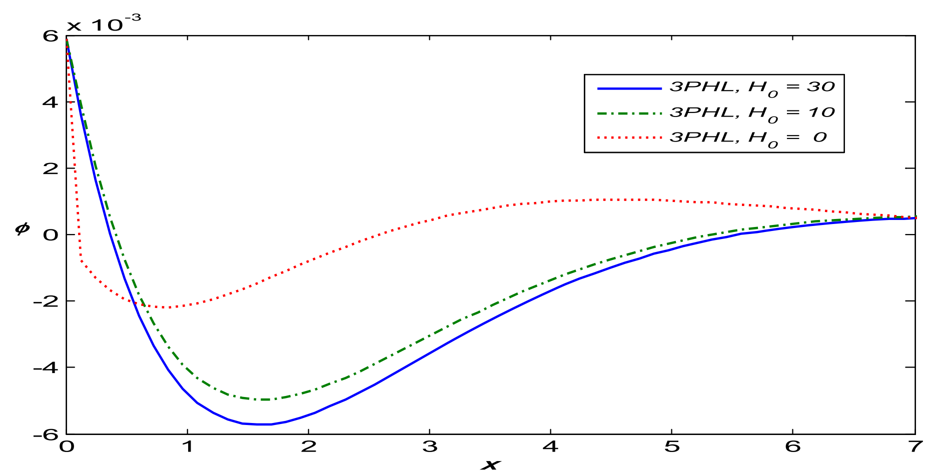

Fig. 9 Effect of different values of magnetic field on volume fraction field distribution φ

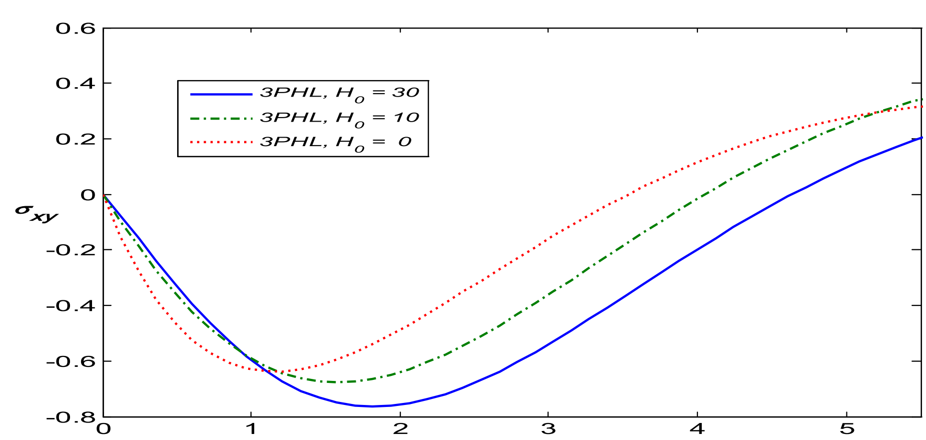

Fig. 10 Effect of different values of magnetic field on stress component σ_{xy}

of v begin from zero except at the $H_0 = 0$ starts from negative, then the values of v decrease in the range $0 \leq x \leq 4.2$, then increase at $x \geq 4.2$ and go to zero. Figure 8 shows that the variance of the thermodynamic temperature θ, begins with decreasing to a minimum value in the range $0 \leq x \leq 0.12$ for $(H_0 = 30, 10, 0)$ then increases in the range $0.12 \leq x \leq 2$ and converges to zero for $x \geq 2$. Figure 9 exhibits that the distribution of the volume fraction field φ, it is observed that due to the presence of a magnetic field, the volume fraction field φ appreciably decreased for $H_0 = 30, 10$ in comparison with $H_0 = 0$, it begins from positive values, then decreases to a minimum

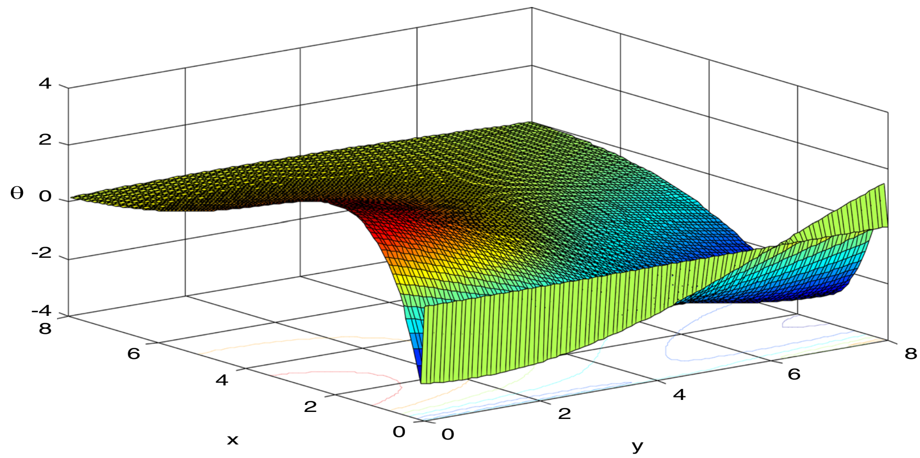

Fig. 11 3D distribution of thermal temperature θ versus components of distance

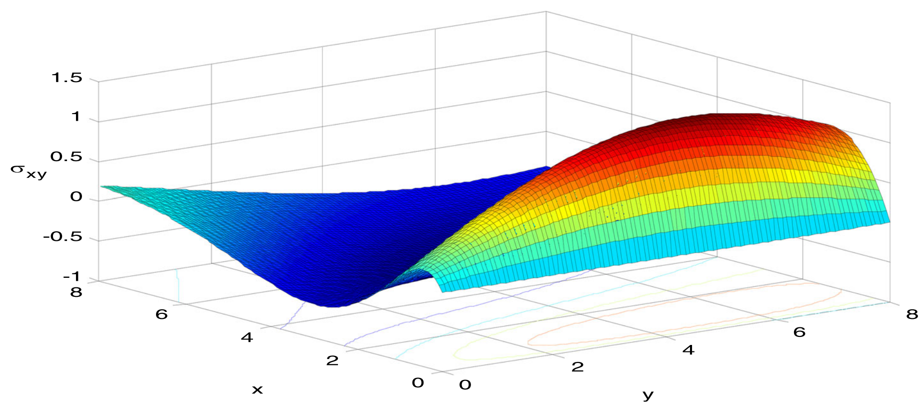

Fig. 12 3D distribution of stress component σ_{xy} versus components of distance

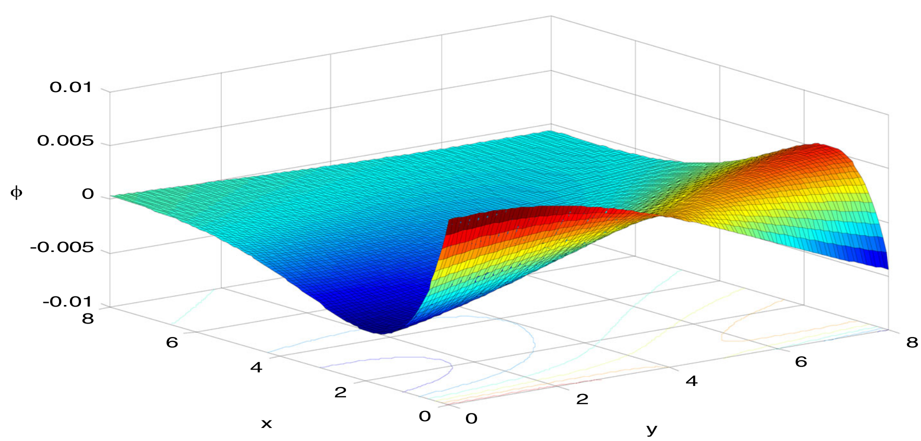

Fig. 13 3D distribution of volume fraction field φ versus components of distance

value in the range $0 \le x \le 1.8$ and converge to zero with increasing distance x at $x \ge 1.8$ for all values of $(H_0 = 30, 10, 0)$. Figure 10 depicts that the distribution of the stress component σ_{xy}, where the values of the stress component σ_{xy} agree with the boundary condition and decrease in the range $0 \le x \le 1.3$, but increase in the range $1 \le x \le 5.5$.

Figures 11, 12, and 13 are giving 3D surface curves for the thermodynamic temperature θ, the stress component σ_{xy} and the change in the volume fraction field φ to study the nonlocal porous thermoelastic solid under the effect of the magnetic field in the context of the three-phase lag (3PHL) model. These figures are very important to study the dependence of these physical quantities on the vertical component of distance.

8. Conclusions

In this problem, we studied the effect of the magnetic field and the memory-dependent derivative on a nonlocal thermoelastic porous solid in the context of the three-phase-lag (PHL) model. The resulting non-dimensional equations were solved by using the normal mode analysis. We can get the following conclusions based on the above discussions:

a) The locality has played a major role in the physical fields which are fairly clear from Figs. 1, 2, 3, 4, 5.
b) The magnetic field has played a major role in the physical fields which are pretty clear from Figs. 6, 7, 8, 9.
c) The vertical distance has played a major role in the physical fields which are pretty clear from Figs. 10, 11, 12.
d) All physical quantities distributions have converged to zero with increasing distance x, and all functions are continuous.
e) The method that was used in the present article is applicable to a wide range of problems in hydrodynamics and thermoelasticity.
f) Three-phase-lag model is very useful in the problems of nuclear boiling, exothermic catalytic reactions, phonon-electron interactions, phonon-scattering, etc.

Appendix

$$A_1 = 1 + \frac{\varepsilon_0 \mu_0^2 H_0^2}{\rho},$$
$$A_2 = 1 + \frac{\mu_0 H_0^2}{\rho\, c_0^2},$$
$$A_3 = \frac{\lambda + \mu + \mu_0 H_0^2}{\rho\, c_0^2},$$
$$A_4 = \frac{\mu_0 H_0^2}{\rho\, c_0^2},$$
$$A_5 = \frac{\mu}{\rho\, c_0^2},$$
$$A_6 = \frac{b}{\rho\, c_0^2},$$
$$A_7 = \frac{k\, c_0}{k^* l_0},$$
$$A_8 = \frac{\rho c_E c_0^2}{k^*},$$
$$A_9 = \frac{\gamma^2 T_0 c_0^2}{k^*(\lambda + 2\mu)},$$
$$A_{10} = \frac{\alpha_3 A_9}{\gamma},$$
$$A_{11} = \frac{b\, l_0^2}{\beta},$$
$$A_{12} = \frac{\alpha_1\, l_0^2}{\beta},$$
$$A_{13} = \frac{\alpha_2\, c_0\, l_0}{\beta},$$
$$A_{14} = \frac{\alpha_3\, l_0^2\,(\lambda + 2\mu)}{\beta\gamma},$$
$$A_{15} = \frac{\rho\, \alpha_4\, c_0^2}{\beta},$$

$$N_1 = \varepsilon^2 A_4, \quad N_2 = m^2 A_1 \varepsilon^2 + A_2 + \varepsilon^2 A_4 a^2,$$
$$N_3 = m^2 A_1 + \varepsilon^2 A_1 m^2 a^2 + A_5 a^2, \quad N_4 = \mathrm{i}\, a A_4 \varepsilon^2,$$
$$N_5 = \mathrm{i}\, a\,(A_3 + A_4 \varepsilon^2 a^2),$$
$$N_6 = m^2 A_1 + \varepsilon^2 A_1 m^2 a^2 + A_2 a^2 + \varepsilon^2 A_4 a^4,$$
$$N_7 = m^2 A_1 \varepsilon^2 + a^2 \varepsilon^2 A_4 + A_5, \quad N_8 = \mathrm{i}\, a A_4 \varepsilon^2,$$
$$N_9 = \mathrm{i} a A_3 + \mathrm{i} a^3 A_4 \varepsilon^2, \quad N_{10} = 1 + m^2 A_{15} \varepsilon^2,$$
$$N_{11} = a^2 + A_{12} + m A_{13} + m^2 A_{15} + m^2 A_{15} \varepsilon^2 a^2,$$
$$N_{12} = m^2 A_9 (1 + G_3 + \frac{1}{2} G_4), \quad N_{13} = \frac{A_{10} N_{12}}{A_9},$$
$$N_{14} = m A_7 (1 + G_1) + (1 + G_2),$$
$$N_{15} = N_{14} a^2 + m^2 A_8 (1 + G_3 + \frac{1}{2} G_4),$$

$$
\begin{aligned}
s_1 &= \mathrm{i}aN_1 - N_8, s_2 = N_9 - \mathrm{i}aN_2, s_3 = \mathrm{i}aN_3,\\
s_4 &= \mathrm{i}aN_4 + N_7, s_5 = \mathrm{i}aN_5 + N_6, s_6 = N_8N_{10},\\
s_7 &= N_9N_{10} + N_8N_{11}, s_8 = N_9N_{11} - \mathrm{i}aN_6N_{11},\\
s_9 &= N_7N_{10}, s_{10} = N_7N_{11} + N_6N_{10}, s_{11}\\
&= N_6N_{11} - a^2A_6A_{11}, s_{12} = \mathrm{i}aN_{10}, s_{13}\\
&= \mathrm{i}a(N_{11} - A_6A_{14}), s_{14}\\
&= N_1N_6N_{10}N_{14} + N_1N_7N_{10}N_{15} + N_1N_7N_{11}N_{14}\\
&\quad + N_2N_7N_{10}N_{14} + N_4N_8N_{10}N_{15}, s_{15}\\
&= N_4N_8N_{11}N_{14} + N_4N_9N_{10}N_{14} + N_5N_8N_{10}N_{14},\\
s_{16} &= N_7N_{10}N_{12} - A_6A_{11}N_7N_{14} + A_{14}N_1N_7N_{13}\\
&\quad + A_{14}N_4N_8N_{13} + N_1N_6N_{10}N_{15} + N_1N_6N_{11}N_{14},\\
s_{17} &= N_2N_6N_{10}N_{14} + N_1N_7N_{11}N_{15} + N_2N_7N_{10}N_{15}\\
&\quad + N_2N_7N_{11}N_{14} + N_3N_7N_{10}N_{14} + N_4N_8N_{11}N_{15},\\
s_{18} &= N_4N_9N_{10}N_{15} + N_4N_9N_{11}N_{14} + N_5N_8N_{10}N_{15}\\
&\quad + N_5N_8N_{11}N_{14} + N_5N_9N_{10}N_{14} + \mathrm{i}aN_4N_{10}N_{12},\\
s_{19} &= \mathrm{i}aN_8N_{10}N_{12} + a^2N_1N_{10}N_{12} - \mathrm{i}aA_6A_{11}N_4N_{14}\\
&\quad - \mathrm{i}aA_6A_{11}N_8N_{14} - A_6A_{11}N_1N_{14}a^2,\\
s_{20} &= -A_{11}N_7N_{13} + N_6N_{10}N_{12} + N_7N_{11}N_{12}\\
&\quad - A_6A_{11}N_6N_{14} - A_6A_{11}N_7N_{15} - A_6A_{14}N_7N_{12}\\
&\quad + A_{14}N_1N_6N_{13}, s_{21}\\
&= A_{14}N_2N_7N_{13} + A_{14}N_4N_9N_{13} + A_{14}N_5N_8N_{13}\\
&\quad + N_1N_6N_{11}N_{15} + N_2N_6N_{10}N_{15} + N_2N_6N_{11}N_{14},\\
s_{22} &= N_3N_6N_{10}N_{14} + N_2N_7N_{11}N_{15} + N_3N_7N_{10}N_{15}\\
&\quad + N_3N_7N_{11}N_{14} + N_4N_9N_{11}N_{15} + N_5N_8N_{11}N_{15},\\
s_{23} &= N_5N_9N_{10}N_{15} + N_5N_9N_{11}N_{14} - \mathrm{i}aA_{11}N_4N_{13}\\
&\quad - \mathrm{i}aA_{11}N_8N_{13} + \mathrm{i}aN_4N_{11}N_{12} + \mathrm{i}aN_5N_{10}N_{12},\\
s_{24} &= \mathrm{i}aN_8N_{11}N_{12} + \mathrm{i}aN_9N_{10}N_{12} - A_{11}N_1N_{13}a^2\\
&\quad + N_1N_{11}N_{12}a^2 + N_2N_{10}N_{12}a^2 - \mathrm{i}aA_6A_{11}N_4N_{15},\\
s_{25} &= -\mathrm{i}aA_6A_{11}N_5N_{14} - \mathrm{i}aA_6A_{14}N_4N_{12} - \mathrm{i}aA_6A_{11}N_8N_{15}\\
&\quad - \mathrm{i}aA_6A_{11}N_9N_{14} - \mathrm{i}aA_6A_{14}N_8N_{12}\\
&\quad - A_6A_{11}N_1N_{15}a^2, s_{26}\\
&= -A_6A_{11}N_2N_{14}a^2 - A_6A_{14}N_1N_{12}a^2,\\
s_{27} &= -A_{11}N_6N_{13} + N_6N_{11}N_{12} - A_6A_{11}N_6N_{15}\\
&\quad - A_6A_{14}N_6N_{12} + A_{14}N_2N_6N_{13} + A_{14}N_3N_7N_{13},\\
s_{28} &= A_{14}N_5N_9N_{13} + N_2N_6N_{11}N_{15} + N_3N_6N_{10}N_{15}\\
&\quad + N_3N_6N_{11}N_{14} + N_3N_7N_{11}N_{15} + N_5N_9N_{11}N_{15},\\
s_{29} &= -\mathrm{i}aA_{11}N_5N_{13} - \mathrm{i}aA_{11}N_9N_{13} + \mathrm{i}aN_5N_{11}N_{12}\\
&\quad + \mathrm{i}aN_9N_{11}N_{12} - A_{11}N_2N_{13}a^2 + N_2N_{11}N_{12}a^2,\\
s_{30} &= N_3N_{10}N_{12}a^2 - \mathrm{i}aA_6A_{11}N_5N_{15} - \mathrm{i}aA_6A_{14}N_5N_{12}\\
&\quad - \mathrm{i}aA_6A_{11}N_9N_{15} - \mathrm{i}aA_6A_{14}N_9N_{12}\\
&\quad - A_6A_{11}N_2N_{15}a^2,\\
s_{31} &= -A_6A_{11}N_3N_{14}a^2 - A_6A_{14}N_2N_{12}a^2,\\
s_{32} &= A_{14}N_3N_6N_{13} + N_3N_6N_{11}N_{15} + N_3N_{11}N_{12}a^2\\
&\quad - A_{11}N_3N_{13}a^2 - A_6A_{11}N_3N_{15}a^2 - A_6A_{14}N_3N_{12}a^2,
\end{aligned}
$$

$$
H_1 = \frac{L_1}{L}, H_2 = \frac{L_2}{L}, H_3 = \frac{L_3}{L}, H_4 = \frac{L_4}{L}, H_5 = \frac{L_5}{L},
$$

$$
\begin{aligned}
L_1 &= s_{14} + s_{15}, L_2 = s_{16} + s_{17} + s_{18} + s_{19},\\
L_3 &= s_{20} + s_{21} + s_{22} + s_{23} + s_{24} + s_{25} + s_{26},\\
L_4 &= s_{27} + s_{28} + s_{29} + s_{30} + s_{31}, L_5 = s_{32},\\
L &= N_1N_7N_{10}N_{14} + N_4N_8N_{10}N_{14},
\end{aligned}
$$

$$
\begin{aligned}
R_{1n} &= \frac{s_2k_n^2 + s_3}{k_n(s_4k_n^2 - s_5)}, R_{2n}\\
&= \frac{k_n(s_6k_n^4 - s_7k_n^2 + s_8) + R_{1n}(s_9k_n^4 - s_{10}k_n^2 + s_{11})}{s_{12}k_n^2 - s_{13}},\\
R_{3n} &= \frac{k_nN_{12} - \mathrm{i}aN_{12}R_{1n} + (N_{14}k_n^2 - N_{15})R_{2n}}{N_{13}},\\
R_{4n} &= \frac{-(\lambda + 2\mu)k_n + \mathrm{i}a\lambda R_{1n} - (\lambda + 2\mu)R_{2n} + bR_{3n}}{\mu(1 - \varepsilon^2k_n^2 + \varepsilon^2a^2)},\\
R_{5n} &= \frac{\mathrm{i}a - k_nR_{1n}}{1 - \varepsilon^2k_n^2 + \varepsilon^2a^2},
\end{aligned}
$$

$$
\begin{aligned}
h_1 &= N_9 - \mathrm{i}aN_2, h_2 = \mathrm{i}aN_3, h_3 = \mathrm{i}aN_5 + N_6, h_4\\
&= -N_9N_{10}, h_5 = N_9N_{11} - \mathrm{i}aA_6A_{11}, h_6 = N_7N_{10},\\
h_7 &= N_7N_{11} + N_6N_{10}, h_8 = N_6N_{11} - a^2A_6A_{11},\\
h_9 &= \mathrm{i}aN_{10}, h_{10} = \mathrm{i}a(N_{11} - A_6A_{14}),\\
h_{11} &= N_7N_{10}N_{12} - A_6A_{11}N_7N_{14} + N_2N_6N_{10}N_{14}\\
&\quad + N_2N_7N_{10}N_{15} + N_2N_7N_{11}N_{14},\\
h_{12} &= N_3N_7N_{10}N_{14} + N_5N_9N_{10}N_{14},\\
h_{13} &= -A_{11}N_7N_{13} + N_6N_{10}N_{12} + N_7N_{11}N_{12}\\
&\quad - A_6A_{11}N_6N_{14} - A_6A_{11}N_7N_{15} - A_6A_{14}N_7N_{12}\\
&\quad + A_{14}N_2N_7N_{13}, h_{14}\\
&= N_2N_6N_{10}N_{15} + N_2N_6N_{11}N_{14} + N_3N_6N_{10}N_{14}\\
&\quad + N_2N_7N_{11}N_{15} + N_3N_7N_{10}N_{15} + N_3N_7N_{11}N_{14}, h_{15}\\
&= N_5N_9N_{10}N_{15} + N_5N_9N_{11}N_{14} + \mathrm{i}aN_5N_{10}N_{12}\\
&\quad + \mathrm{i}aN_9N_{10}N_{12} + N_2N_{10}N_{12}a^2 - \mathrm{i}aA_6A_{11}N_5N_{14}, h_{16}\\
&= -\mathrm{i}aA_6A_{11}N_9N_{14} - A_6A_{11}N_2N_{14}a^2,\\
h_{17} &= -A_{11}N_6N_{13} + N_6N_{11}N_{12} - A_6A_{11}N_6N_{15}\\
&\quad - A_6A_{14}N_6N_{12} + A_{14}N_2N_6N_{13} + A_{14}N_3N_7N_{13},\\
h_{18} &= A_{14}N_5N_9N_{13} + N_2N_6N_{11}N_{15} + N_3N_6N_{10}N_{15}\\
&\quad + N_3N_6N_{11}N_{14} + N_3N_7N_{11}N_{15} + N_5N_9N_{11}N_{15},\\
h_{19} &= -\mathrm{i}aA_{11}N_5N_{13} - \mathrm{i}aA_{11}N_9N_{13} + \mathrm{i}aN_5N_{11}N_{12}\\
&\quad + \mathrm{i}aN_9N_{11}N_{12} - A_{11}N_2N_{13}a^2 + N_2N_{11}N_{12}a^2,\\
h_{20} &= +N_3N_{10}N_{12}a^2 - \mathrm{i}aA_6A_{11}N_5N_{15} - \mathrm{i}aA_6A_{14}N_5N_{12}\\
&\quad - \mathrm{i}aA_6A_{11}N_9N_{15} - \mathrm{i}aA_6A_{14}N_9N_{12}\\
&\quad - A_6A_{11}N_2N_{15}a^2,\\
h_{21} &= -A_6A_{11}N_3N_{14}a^2 - A_6A_{14}N_2N_{12}a^2,\\
h_{22} &= A_{14}N_3N_6N_{13} + N_3N_6N_{11}N_{15} + N_3N_{11}N_{12}a^2\\
&\quad - A_{11}N_3N_{13}a^2 - A_6A_{11}N_3N_{15}a^2 - A_6A_{14}N_3N_{12}a^2,
\end{aligned}
$$

$$
\begin{aligned}
C &= \frac{L_7}{L_6}, E = \frac{L_8}{L_6}, F = \frac{L_9}{L_6}, J = \frac{L_{10}}{L_6}, L_7 = h_{11} + h_{12},\\
L_8 &= h_{13} + h_{14} + h_{15} + h_{16},\\
L_9 &= h_{17} + h_{18} + h_{19} + h_{20} + h_{21}, L_{10} = h_{22},\\
L_6 &= N_2N_7N_{10}N_{14},
\end{aligned}
$$

$$M_{1n} = \frac{h_1 f_n^2 + h_2}{f_n(N_7 f_n^2 - h_3)}, M_{2n} = \frac{f_n(h_4 f_n^2 + h_5) + M_{1n}(h_6 f_n^4 - h_7 f_n^2 + h_8)}{h_9 f_n^2 - h_{10}},$$

$$M_{3n} = \frac{f_n N_{12} - \mathrm{i}aN_{12}M_{1n} + (N_{14} f_n^2 - N_{15})M_{2n}}{N_{13}},$$

$$M_{4n} = \frac{-(\lambda + 2\mu)f_n + \mathrm{i}a\lambda M_{1n} - (\lambda + 2\mu)M_{2n} + bM_{3n}}{\mu},$$

$$M_{5n} = \mathrm{i}a - f_n M_{1n},$$

Funding Open access funding provided by The Science, Technology & Innovation Funding Authority (STDF) in cooperation with The Egyptian Knowledge Bank (EKB). The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Conflict of interest The authors declare that they have no conflict of interest.

References

[1] J L Wang, H F Li and Com. *Math. Appl.* **62** 1562 (2011)
[2] Y-J Yu *J. Eng. Sci.* **811** 23 (2014)
[3] M A Ezzat *AS EL-Karamany and AA EL-Bary Mech Adv Mater Struct* **23** 545 (2016)
[4] M A Ezzat *Sys. An Int.* **19** 539 (2017)
[5] Y Li and T He *Math. Mech. Solids* **24** 1438 (2019)
[6] S Kant and S Mukhopadhyay *Math. Mech. Solids* **24** 2392 (2019)
[7] S Mondal *P Pal and M Kanoria Acta Mech.* **230** 179 (2019)
[8] S M Said, E M Abd-ELaziz and M I A Othman *Steel Comp. Struct.* **36** 617 (2020)
[9] N Sarkar and S De *Ind. J. Phys.* **95** 1203 (2021)
[10] R Tiwari and R Kumar *Appl. Phys. A* **128** Art. 19 (2022)
[11] A M Abd-Alla *Math. Comp.* **155** 235 (2004)
[12] M I A Othman *J. Num. Meth. Heat Fluid Flow* **29** 4788 (2019)
[13] S Gupta, R Dutta, S Das and AK Verma *J. Therm. Stress*. (2023) Doi. https://doi.org/10.1080/01495739.2023.2202718
[14] MIA Othman Multi *Mater. Struct.* **1** 231 (2005)
[15] M A Ezzat *J. Phys.* **87** 329 (2009)
[16] SM Abo-Dahab and AM Abd-Alla *J. Vib. Control* **22** 2885 (2016)
[17] SM Said *J. Com. Appl. Math.* **291** 142 (2016)
[18] E M Abd-ELaziz *M Marin and MIA Othman Symmetry* **11** 413 (2019)
[19] SM Said and MIA Othman Struct *Eng. Mech. An Int. J.* **74** 425 (2020)
[20] AM ALharbi, MIA Othman and EM Abd-ELaziz *Int. J. Struct. Stab. Dynam.* **21** Art. 2150189 (2021)
[21] AM ALharbi, SM Said, EM Abd-ELaziz and MIA Othman *Int. J. Struct. Stab. Dynam.* **22** Art. 2250007 (2022)
[22] AK Yadav Mech *Mach. An Int. J.* **50** 4117 (2022)
[23] AK Yadav *J. Therm. Stress.* **44** 86 (2021)
[24] S Gupta *Bas. Des. Struct. Mach.* **51** 764 (2023)
[25] AK Yadav *J. Eng. Phys. Thermophys.* **94** 1663 (2021)
[26] D Iesan *Acta Mechanica* **60** 67 (1986)
[27] M Ciarletta, A Scalia and Z Angew *Maths. Mech.* **73** 67 (1993)
[28] AK Yadav Waves Rand *Comp. Media* (2021). https://doi.org/10.1080/17455030.2021.1956014
[29] AK Yadav *J. Ocean Eng. Sci*. **6** 376 (2021)
[30] S Gupta, R Dutta and S Das *Int. J. Numer. Meth. Heat & Fluid Flow* **32** 3697 (2022)
[31] AM ALharbi, SM Said and MIA Othman *Steel Compos. Struct*. **40** 881 (2021)
[32] S Gupta and R Dutta *Rand. Comp. Media* (2021). https://doi.org/10.1080/17455030.2021.2021315
[33] E Inan and AC Eringen *Int. J. Eng. Sci*. **29** 831 (1991)
[34] R Artan *Int. J. Eng. Sci*. **34** 943 (1996)
[35] S Gupta, R Dutta and S Das *J. Ocean Eng. Sci*. In Press
[36] S Gupta, S Das, R Dutta and AK Verm *J. Ocean Eng. Sci*. In Press
[37] S Gupta *Adv. Mater. Struct.* **30** 449 (2023)
[38] A K Yadav and E Carrera *Adv. Mater. Struct.* (2022). https://doi.org/10.1080/15376494.2022.2130484
[39] SM Said Geomech *Eng.* **32** 137 (2023)
[40] RB Hetnarski and MR Eslami, *Springer Science Business Media*, B.V., New York, **41** 227 (2009)
[41] AC Eringen and DGB Edelen Int *J. Eng. Sci.* **10** 233 (1972)
[42] A C Eringen *J. Appli. Phys.* **54** 4703 (1983)
[43] J Wang and R S Dhaliwal *J. Therm. Stress.* **16** 71 (1993)
[44] P Purkaita, A Sur and M Kanoria *Adv. Appl. Math. Mech.* **5** 28 (2017)
[45] R S Choudhuri *J. Therm. Stress.* **30** 231 (2007)
[46] M Caputo *J. Aco. Soc. Amer.* **56** 897 (1971)
[47] M Caputo and F Mainardi *Pure Appl. Geophys.* **91** 134 (1971)
[48] M Caputo and F Mainardi *La Rivista del Nuovo Cimento* **1** 161 (1971)
[49] I Abbas and A Hobiny *S Vlase and M Marin Mathematics* **10** 2168 (2022)
[50] R Quintanilla and R Racke *Int. J. Heat Mass Transfer* **51** 24 (2008)