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ORIGINAL PAPER

Novel analytical techniques for HIV-1 infection of CD4 + T cells on fractional order in mathematical biology

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Abstract: In this research, we present an analytical analysis of HIV-1 infection of CD4 + T cells with a conformable derivative model (CDM) in biology. An improved (Υ'/Υ)-expansion method is used to investigate this model analytically to construct a new exact traveling wave solution, namely, exponential function, trigonometric function, and the hyperbolic function, which can be further studied for more (FNEE) fractional nonlinear evolution equations in biology. Also, we provide some graphs in 2D plots that demonstrate how accurate the results will be produced using analytical approaches.

Keywords: HIV-1 infection of CD4 + T cells (CFM) biology model; Improved (Υ'/Υ)-expansion method; FNEE

1. Introduction

The human immunodeficiency virus (HIV) predominantly targets CD4 + T cells, the immune system's biggest white blood cells [1, 2]. HIV infection attacks all cells, but it is CD4 + T cells that are the most damaging and impair the immune system by eliminating them [3]. When the number of CD4 + T cells falls below a particular threshold, the cell-mediated immune system vanishes, the immune system weakens, and the body becomes vulnerable to infection [3]. Pearson [4] provided a straightforward mathematical model for HIV infection. This model has served as an inspiration for scientists working on HIV modeling [4–6]. The mathematical models for HIV described here are extremely helpful in understanding the dynamics of HIV infection [7–10]. A group of scientists led by Agosto et al. [11] constructed an HIV model that included HIV cell-to-cell transmission and studied the characteristics of CD4 + T cells in depth. HIV-1 latency and processes have been described by Ruelas et al. [12] in order to better understand this deadly retrovirus. Sun et al. [13] indicated the c-myc proto-oncogene in the context of HIV-1 infection. The literature has supplied several mathematical models that have applications in varied fields akin to engineering, physics, etc., to the last of those sciences [14–50]. In this regard, the 2019-nCoV pandemic that has afflicted the world has been thoroughly explored and studied [51–57]. Furthermore, COVID has impacted nearly every country on the globe in terms of economic, social, and psychological issues. Some key features of DNA have been observed by Cattani et al. [58] utilizing a mathematical model to describe them, and Owolabi et al. [59, 60] examined epidemiological models that included fractional order.

This study is arranged as follows: Sect. 2 offers an overview of the research model. The analytical solutions to this model and some diagrams for some of the analytical solutions we shall get are in Sect. 3. In the final part, we present the conclusion.

2. New algorithm scheme of the HIV-1 infection of CD4$^+$ T cell

Let us first consider the new biomathematical model as follows [61–63]

$$K_t^{\nu} f(t) = \beta_1 - \beta_2 f(t) - \beta_3 f(t) h(t)$$
$$K_t^{\nu} g(t) = \beta_3 f(t) h(t) - \beta_4 g(t) \quad (1)$$

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$$K_t^v h(t) = \beta_5 g(t) - \beta_6 h(t)$$

This model with *f(t), g(t), h(t),* $v \in (0,\ 1]$ and β_i, (i = 1, …, 6) is arbitrary constants that indicate the rate of production of CD4 + T cells, the rate of natural death rate, infected CD4 + cells from uninfected CD4 + cells, virus-producing cells' death, creation of vision viruses by infected cells, and virus particle death.

The conformable derivative (CD) of order v can be computed using the following formula:

$$M_t^v f(t) = \lim_{\varepsilon \to 0} \left(f\left(t + \varepsilon t^{1-v}\right) - f(t)/\varepsilon \right) \tag{2}$$

for all t > 0, $v \in (0,\ 1]$.

Using the transformation described below

$$\zeta = c\frac{t^v}{v} \tag{3}$$

Then Eq. (1), becomes

$$\begin{aligned} cf'(\zeta) &= \beta_1 - \beta_2 f(\zeta) - \beta_3 f(\zeta) h(\zeta) \\ cg'(\zeta) &= \beta_3 f(\zeta) h(\zeta) - \beta_4 g(\zeta) \\ ch'(\zeta) &= \beta_5 g(\zeta) - \beta_6 h(\zeta) \end{aligned} \tag{4}$$

In the following, the analytical scheme is used for generating new exact traveling wave solutions of the reduced Eq. (4).

3. New analytical solutions HIV-1 infection of CD4$^+$ T cell

Here, the extended analytical method has been used for getting new exact solutions for fractional Eq. (4).

3.1. Improved (Υ'/Υ) expansion method

In what's follows [64–70], let the polynomial form of solution for Eq. (4) can be presented as follows:

$$\begin{aligned} f(\zeta) &= \sum_{n=0}^{N} A_n F^n(\zeta) \\ g(\zeta) &= \sum_{n=0}^{U} B_n F^n(\zeta) \\ h(\zeta) &= \sum_{n=0}^{O} M_n F^n(\zeta) \end{aligned} \tag{5}$$

where $A_0,\ A_1,\ A_2 \ldots,\ A_N,\ B_0,\ B_1,\ B_2 \ldots,\ B_U$,and $M_0,\ M_1,\ M_2, \ldots\ M_O$ are constants, which can be determined by considering the derivative term of highest order with comparison of nonlinear terms of the governing equation, while $F(\varsigma)$ is introduced by

$$F(\zeta) = \frac{\Upsilon'(\zeta)}{\Upsilon(\zeta)} \tag{6}$$

where $\Upsilon = \Upsilon(\zeta)$ follows the ODE in the following form:

$$\Upsilon\Upsilon'' = \alpha\Upsilon^2 + \beta\Upsilon\Upsilon' + \delta(\Upsilon')^2 \tag{7}$$

where α, β, and δ are constants.

Equation (7) can be rewritten as

$$\frac{d}{d\zeta}\left(\frac{\Upsilon'}{\Upsilon}\right) = \alpha + \beta\left(\frac{\Upsilon'}{\Upsilon}\right) + (\delta - 1)\left(\frac{\Upsilon'}{\Upsilon}\right)^2 \tag{8}$$

The generalized solutions of Eq. (7) are the four types of solutions as follows:

Case (1) if $\Lambda = \beta^2 + 4\alpha - 4\alpha\delta \geq 0$ and $\beta \neq 0$, then

$$F(\zeta) = \frac{\beta}{2(1-\delta)} + \frac{\beta\sqrt{\Lambda}}{2(1-\delta)}\left(\frac{\delta_1 \exp\left(\frac{\sqrt{\Delta}}{2}\zeta\right) + \delta_2 \exp\left(\frac{-\sqrt{\Delta}}{2}\zeta\right)}{\delta_1 \exp\left(\frac{\sqrt{\Delta}}{2}\zeta\right) - \delta_2 \exp\left(\frac{-\sqrt{\Delta}}{2}\zeta\right)}\right) \tag{9}$$

Case (2) If $\Lambda = \beta^2 + 4\alpha - 4\alpha\delta < 0$ and $\beta \neq 0$, then

$$F(\zeta) = \frac{\beta}{2(1-\delta)} + \frac{\beta\sqrt{-\Lambda}}{2(1-\delta)}\left(\frac{i\delta_1 \cos\left(\frac{\sqrt{-\Lambda}}{2}\zeta\right) - \delta_2 \sin\left(\frac{\sqrt{-\Lambda}}{2}\zeta\right)}{i\delta_1 \sin\left(\frac{\sqrt{-\Lambda}}{2}\zeta\right) + \delta_2 \cos\left(\frac{\sqrt{-\Lambda}}{2}\zeta\right)}\right) \tag{10}$$

Case (3) If $\Lambda = \alpha(1-\delta) \geq 0$ and $\beta = 0$, then

$$F(\zeta) = \frac{\sqrt{\Lambda}}{(1-\delta)}\left(\frac{\delta_1 \cos\left(\sqrt{\Lambda}\zeta\right) + \delta_2 \sin\left(\sqrt{\Lambda}\zeta\right)}{\delta_1 \sin\left(\sqrt{\Lambda}\zeta\right) - \delta_2 \cos\left(\sqrt{\Lambda}\zeta\right)}\right) \tag{11}$$

Case (4) If $\Lambda = \alpha(1-\delta) < 0$ and $\beta = 0$, then

$$F(\zeta) = \frac{\sqrt{-\Lambda}}{(1-\delta)}\left(\frac{i\delta_1 \cosh\left(\sqrt{-\Lambda}\zeta\right) - \delta_2 \sinh\left(\sqrt{-\Lambda}\zeta\right)}{i\delta_1 \sinh\left(\sqrt{-\Lambda}\zeta\right) - \delta_2 \cosh\left(\sqrt{-\Lambda}\zeta\right)}\right) \tag{12}$$

where, $\zeta = c\frac{t^v}{v}$ and $\alpha, \beta, \delta, \delta_1, \delta_2$ are real parameters.

3.2. Application of improved (Υ'/Υ) expansion method

In order to use this approach on our model, we must strike a balance between $f'(\zeta)$ with $f(\zeta)h(\zeta)$, $g'(\zeta)$ with $f(\zeta)h(\zeta)$ and $h'(\zeta)$ with $h(\zeta)$ in Eq. (4), and we obtain $N = U = 2$, $O = 1$. Then, the solutions of Eq. (4) yields

$$\begin{aligned} f(\zeta) &= A_0 + A_1F(\zeta) + A_2F^2(\zeta) \\ g(\zeta) &= B_0 + B_1F(\zeta) + B_2F^2(\zeta) \\ h(\zeta) &= M_0 + M_1F(\zeta) \end{aligned} \tag{13}$$

Inserting Eq. (13) into Eq. (4) with the aid of (8) yields a system of algebraic equations. By solving them, it gains.

Set (1)

$$\left\{\begin{array}{l} A_0 = \mp\frac{A_1\alpha}{\ell}, A_2 = \mp\frac{1}{4}\frac{\ell A_1}{\alpha}, \ell = \left(\mp\beta + \sqrt{\beta^2 - 4\alpha\delta + 4\alpha}\right), \\ A_1 = A_1, c = c, B_0 = B_0, B_1 = -A_1, M_1 = -\frac{2c(\delta-1)}{\beta_3}, \\ \beta_1 = 0, \beta_2 = 0, \beta_4 = 0, \beta_6 = \beta_6, \\ B_2 = \frac{1}{4}\frac{\ell A_1}{\alpha}, M_0 = \mp\frac{4c\left(-\delta \mp \frac{\ell\beta}{2\alpha} + 1\right)\alpha}{\ell\beta_3} \end{array}\right\} \tag{14}$$

Set (2)

$$\left\{\begin{array}{l} c = c, A_0 = -B_0, A_1 = \frac{1}{2}\frac{\ell B_0}{\alpha}, A_2 = 0, B_0 = B_0, , \\ \beta_6 = \beta_6, \ell = \left(\mp\beta + \sqrt{\beta^2 - 4\alpha\delta + 4\alpha}\right), \\ B_2 = 0, M_0 = M_0, M_1 = -\frac{c(\delta-1)}{\beta_3}, \beta_1 = 0, B_1 = \frac{1}{2}\frac{\ell B_0}{\alpha}, \\ \beta_2 = \mp\frac{2\alpha\left(c\delta - c \pm \frac{1}{2}\frac{\beta_3\ell M_0}{\alpha} \pm \frac{c\beta\ell}{2\alpha}\right)}{\ell}, \beta_4 = \mp\frac{2\alpha\left(c\delta - c \pm \frac{1}{2}\frac{\beta_3\ell M_0}{\alpha} \pm \frac{c\beta\ell}{2\alpha}\right)}{\ell} \end{array}\right\} \tag{15}$$

Set (3)

$$\left\{\begin{array}{l} c = c, A_0 = A_0, A_1 = A_1, A_2 = 0, B_0 = B_0, B_1 = -A_1, \\ B_2 = 0, M_0 = \frac{c\left(B_0A_0\delta - B_0A_0 - B_0A_1\beta - A_1^2\alpha\right)}{\beta_3A_1(A_0 + B_0)}, \\ M_1 = -\frac{c(\delta-1)}{\beta_3}, \beta_1 = \frac{c\left(A_0^2\delta - A_0^2 - A_1^2\alpha - A_1\beta A_0\right)}{A_1}, \\ \beta_2 = \frac{c\left(A_0^2\delta - A_0^2 - A_1^2\alpha - A_1\beta A_0\right)}{A_1(A_0 + B_0)}, \\ \beta_4 = \frac{c\left(A_0^2\delta - A_0^2 - A_1^2\alpha - A_1\beta A_0\right)}{A_1(A_0 + B_0)}, \beta_6 = \beta_6 \end{array}\right\} \tag{16}$$

Set (4)

$$\left\{\begin{array}{l} c = c, A_0 = \frac{A_1\alpha}{\beta}, A_1 = A_1, A_2 = \frac{A_1(\delta-1)}{\beta}, B_0 = B_0, \\ B_1 = B_1, \beta_2 = -\frac{\beta c(A_1 + B_1)}{A_1}, \\ B_2 = -\frac{A_1(\delta-1)}{\beta}, M_0 = \frac{cB_1\beta}{\beta_3A_1}, M_1 = -\frac{2c(\delta-1)}{\beta_3}, \\ \beta_1 = 0, \beta_4 = 0, \beta_6 = \beta_6 \end{array}\right\} \tag{17}$$

Set (5)

$$\left\{\begin{array}{l} c = c, A_0 = \frac{A_1\alpha}{\beta}, A_1 = A_1, A_2 = \frac{A_1(\delta-1)}{\beta}, B_0 = B_0, \\ B_1 = B_1, \beta_4 = 0, \beta_6 = \beta_6, \\ B_2 = -\frac{A_1(\delta-1)}{\beta}, M_0 = \frac{cB_1\beta}{\beta_3A_1}, M_1 = -\frac{2c(\delta-1)}{\beta_3}, \\ \beta_1 = 0, \beta_2 = -\frac{\beta c(A_1 + B_1)}{A_1} \end{array}\right\} \tag{18}$$

Set (6)

$$\left\{\begin{array}{l} c = 0, A_0 = \frac{1}{2}\frac{A_1^2\delta - A_1\beta A_2 - A_1^2 + 2A_2^2\alpha}{A_2(\delta-1)}, A_1 = A_1, A_2 = A_2, \\ B_0 = \frac{1}{2}\left(\frac{A_1^2\delta - A_1^2 - A_1B_1 + A_1B_1\delta - A_1\beta A_2 + 2A_2^2\alpha}{A_2(\delta-1)}\right), \\ B_1 = B_1, B_2 = -A_2, M_0 = \frac{c(B_1\delta - A_2\beta + A_1\delta - A_1 - B_1)}{A_2\beta_3}, \\ M_1 = -\frac{2c(\delta-1)}{\beta_3}, \\ \beta_1 = \frac{1}{2}c \\ \left(\frac{-2A_1^3\delta + 2A_2^2\beta^2A_1 + 3A_2\beta A_1^2 - 4A_2^3\beta\alpha + A_1^3 + 4A_1\delta A_2^2\alpha - 4A_1A_2^2\alpha + A_1^3\delta^2 - 3A_2\beta A_1^2\delta}{A_2^2(\delta-1)}\right), \\ \beta_2 = -\frac{c(A_2\beta + B_1\delta - B_1)}{A_2}, \beta_4 = \frac{c(A_1\delta - A_1 - A_2\beta)}{A_2}, \beta_6 = \beta_6 \end{array}\right\} \tag{19}$$

Set (7)

$$\left\{\begin{array}{l} c = 0, A_0 = \frac{1}{2}\frac{A_1^2\delta - A_1\beta A_2 - A_1^2 + 2A_2^2\alpha}{A_2(\delta-1)}, A_1 = A_1, A_2 = A_2, \beta_2 = \beta_4, \\ B_0 = \frac{1}{2}\left(\frac{\begin{array}{r} cA_1^3\delta^2 - 3cA_2\beta A_1^2\delta - 2cA_1^3\delta + 2cA_2^2\beta^2A_1 + 3cA_2\beta A_1^2 - 4cA_2^3\beta\alpha \\ -\beta_4A_2A_1^2\delta + \beta_4A_2^2A_1\beta + \beta_4A_2A_1^2 - 2\beta_4A_2^3\alpha + cA_1^3 + 4cA_1\delta A_2^2\alpha - 4cA_1A_2^2\alpha \end{array}}{\beta_4A_2^2(\delta-1)}\right), \\ B_1 = -A_1, B_2 = -A_2, M_0 = \frac{cA_1\delta - 2cA_2\beta - \beta_4A_2 - cA_1}{A_2\beta_3}, M_1 = -\frac{2c(\delta-1)}{\beta_3}, \\ \beta_1 = \frac{1}{2}c\left(\frac{\begin{array}{r} -2A_1^3\delta + 2A_2^2\beta^2A_1 + 3A_2\beta A_1^2 - 4A_2^3\beta\alpha + A_1^3 \\ +4A_1\delta A_2^2\alpha - 4A_1A_2^2\alpha + A_1^3\delta^2 - 3A_2\beta A_1^2\delta \end{array}}{A_2^2(\delta-1)}\right), \beta_4 = \beta_4, \beta_6 = \beta_6 \end{array}\right\} \tag{20}$$

Set (8)

$$\left\{\begin{array}{l} c=0, A_0=A_0, A_1=A_1, A_2=A_2, B_0=\dfrac{A_0B_2}{A_2}, \\ B_1=\dfrac{A_1B_2}{A_2}, B_2=B_2, M_0=\dfrac{\beta_4B_2}{A_2\beta_3}, M_1=0, \beta_1=0, \\ \beta_2=-\dfrac{\beta_4B_2}{A_2}, \beta_4=\beta_4, \beta_6=\beta_6 \end{array}\right\} \tag{21}$$

Set (9)

$$\left\{\begin{array}{l} c=c, A_0=A_0, A_1=A_1, A_2=A_2, B_0=B_0, B_1=B_1, \\ B_2=B_2, M_0=0, M_1=0, \beta_1=0, \\ \beta_2=0, \beta_4=0, \beta_6=\beta_6 \end{array}\right\} \tag{22}$$

In view of set [1], Eq. (13) yields

$$\begin{aligned} f(\zeta) &= \mp\frac{A_1\alpha}{\ell}+A_1F(\zeta)\mp\frac{1}{4}\frac{\ell A_1}{\alpha}F^2(\zeta) \\ g(\zeta) &= B_0-A_1F(\zeta)+\frac{1}{4}\frac{\ell A_1}{\alpha}F^2(\zeta) \\ h(\zeta) &= \mp\frac{4c\left(-\delta\mp\frac{\ell\beta}{2\alpha}+1\right)\alpha}{\ell\beta_3}-\frac{2c(\delta-1)}{\beta_3}F(\zeta) \end{aligned} \tag{23}$$

Substituting Eqs. (23) in Eqs. (9–12), we have.

Case (1) If $\Lambda=\beta^2+4\alpha-4\alpha\delta\geq 0$ and $\beta\neq 0$ (Fig. 1), then

$$\begin{aligned} f_{1,1}(t) = &\mp\frac{A_1\alpha}{\ell}+A_1\left\{\frac{\beta}{2(1-\delta)}+\frac{\beta\sqrt{\Lambda}}{2(1-\delta)}\left(\frac{\delta_1\exp\left(\frac{\sqrt{\Delta}}{2}\zeta\right)+\delta_2\exp\left(\frac{-\sqrt{\Delta}}{2}\zeta\right)}{\delta_1\exp\left(\frac{\sqrt{\Delta}}{2}\zeta\right)-\delta_2\exp\left(\frac{-\sqrt{\Delta}}{2}\zeta\right)}\right)\right\} \\ &\mp\frac{1}{4}\frac{\ell A_1}{\alpha}\left\{\frac{\beta}{2(1-\delta)}+\frac{\beta\sqrt{\Lambda}}{2(1-\delta)}\left(\frac{\delta_1\exp\left(\frac{\sqrt{\Delta}}{2}\zeta\right)+\delta_2\exp\left(\frac{-\sqrt{\Delta}}{2}\zeta\right)}{\delta_1\exp\left(\frac{\sqrt{\Delta}}{2}\zeta\right)-\delta_2\exp\left(\frac{-\sqrt{\Delta}}{2}\zeta\right)}\right)\right\}^2 \end{aligned}$$

$$\begin{aligned} g_{1,1}(t) = &B_0-A_1\left\{\frac{\beta}{2(1-\delta)}+\frac{\beta\sqrt{\Lambda}}{2(1-\delta)}\left(\frac{\delta_1\exp\left(\frac{\sqrt{\Delta}}{2}\zeta\right)+\delta_2\exp\left(\frac{-\sqrt{\Delta}}{2}\zeta\right)}{\delta_1\exp\left(\frac{\sqrt{\Delta}}{2}\zeta\right)-\delta_2\exp\left(\frac{-\sqrt{\Delta}}{2}\zeta\right)}\right)\right\} \\ &+\frac{1}{4}\frac{\ell A_1}{\alpha}\left\{\frac{\beta}{2(1-\delta)}+\frac{\beta\sqrt{\Lambda}}{2(1-\delta)}\left(\frac{\delta_1\exp\left(\frac{\sqrt{\Delta}}{2}\zeta\right)+\delta_2\exp\left(\frac{-\sqrt{\Delta}}{2}\zeta\right)}{\delta_1\exp\left(\frac{\sqrt{\Delta}}{2}\zeta\right)-\delta_2\exp\left(\frac{-\sqrt{\Delta}}{2}\zeta\right)}\right)\right\}^2 \end{aligned} \tag{24}$$

$$\begin{aligned} h_{1,1}(t) = &\mp\frac{4c\left(-\delta\mp\frac{\ell\beta}{2\alpha}+1\right)\alpha}{\ell\beta_3} \\ &-\frac{2c(\delta-1)}{\beta_3}\left\{\frac{\beta}{2(1-\delta)}+\frac{\beta\sqrt{\Lambda}}{2(1-\delta)}\left(\frac{\delta_1\exp\left(\frac{\sqrt{\Delta}}{2}\zeta\right)+\delta_2\exp\left(\frac{-\sqrt{\Delta}}{2}\zeta\right)}{\delta_1\exp\left(\frac{\sqrt{\Delta}}{2}\zeta\right)-\delta_2\exp\left(\frac{-\sqrt{\Delta}}{2}\zeta\right)}\right)\right\} \end{aligned}$$

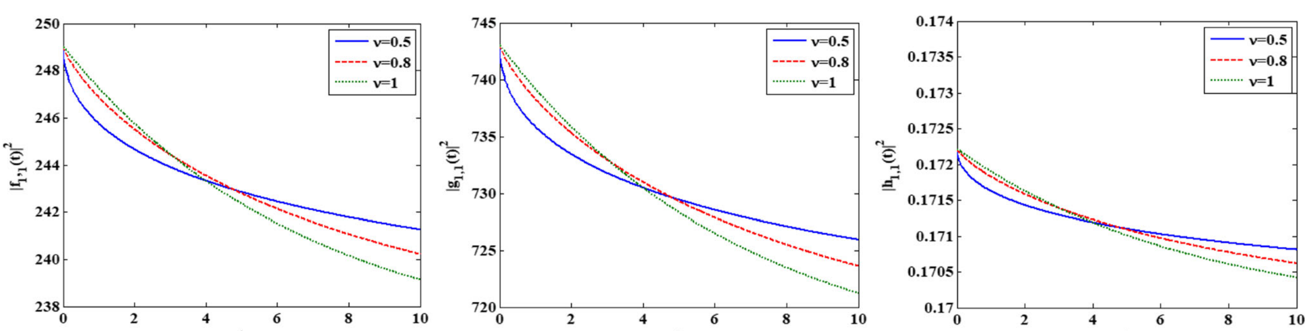
Case (2) If $\Lambda=\beta^2+4\alpha-4\alpha\delta<0$ and $\beta\neq 0$ (Fig. 2), then

$$\begin{aligned} f_{1,2}(t) = &\mp\frac{A_1\alpha}{\ell}+A_1\left\{\frac{\beta}{2(1-\delta)}+\frac{\beta\sqrt{-\Lambda}}{2(1-\delta)}\left(\frac{i\delta_1\cos\left(\frac{\sqrt{-\Lambda}}{2}\zeta\right)-\delta_2\sin\left(\frac{\sqrt{-\Lambda}}{2}\zeta\right)}{i\delta_1\sin\left(\frac{\sqrt{-\Lambda}}{2}\zeta\right)+\delta_2\cos\left(\frac{\sqrt{-\Lambda}}{2}\zeta\right)}\right)\right\} \\ &\mp\frac{1}{4}\frac{\ell A_1}{\alpha}\left\{\frac{\beta}{2(1-\delta)}+\frac{\beta\sqrt{-\Lambda}}{2(1-\delta)}\left(\frac{i\delta_1\cos\left(\frac{\sqrt{-\Lambda}}{2}\zeta\right)-\delta_2\sin\left(\frac{\sqrt{-\Lambda}}{2}\zeta\right)}{i\delta_1\sin\left(\frac{\sqrt{-\Lambda}}{2}\zeta\right)+\delta_2\cos\left(\frac{\sqrt{-\Lambda}}{2}\zeta\right)}\right)\right\}^2 \end{aligned}$$

$$\begin{aligned} g_{1,2}(t) = &B_0-A_1\left\{\frac{\beta}{2(1-\delta)}+\frac{\beta\sqrt{-\Lambda}}{2(1-\delta)}\left(\frac{i\delta_1\cos\left(\frac{\sqrt{-\Lambda}}{2}\zeta\right)-\delta_2\sin\left(\frac{\sqrt{-\Lambda}}{2}\zeta\right)}{i\delta_1\sin\left(\frac{\sqrt{-\Lambda}}{2}\zeta\right)+\delta_2\cos\left(\frac{\sqrt{-\Lambda}}{2}\zeta\right)}\right)\right\} \\ &+\frac{1}{4}\frac{\ell A_1}{\alpha}\left\{\frac{\beta}{2(1-\delta)}+\frac{\beta\sqrt{-\Lambda}}{2(1-\delta)}\left(\frac{i\delta_1\cos\left(\frac{\sqrt{-\Lambda}}{2}\zeta\right)-\delta_2\sin\left(\frac{\sqrt{-\Lambda}}{2}\zeta\right)}{i\delta_1\sin\left(\frac{\sqrt{-\Lambda}}{2}\zeta\right)+\delta_2\cos\left(\frac{\sqrt{-\Lambda}}{2}\zeta\right)}\right)\right\}^2 \end{aligned} \tag{25}$$

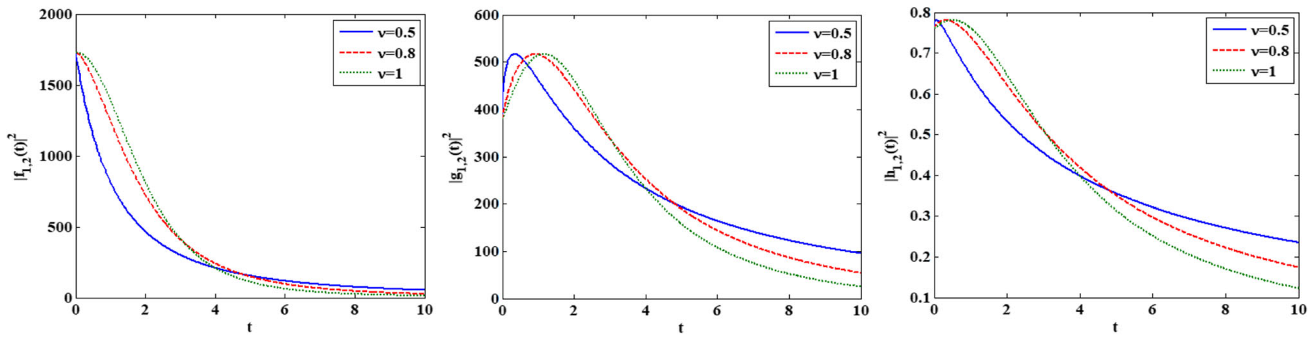
$$\begin{aligned} h_{1,2}(t) = &\mp\frac{4c\left(-\delta\mp\frac{\ell\beta}{2\alpha}+1\right)\alpha}{\ell\beta_3} \\ &-\frac{2c(\delta-1)}{\beta_3}\left\{\begin{array}{r}\dfrac{\beta}{2(1-\delta)}+ \\ \dfrac{\beta\sqrt{-\Lambda}}{2(1-\delta)}\left(\dfrac{i\delta_1\cos\left(\frac{\sqrt{-\Lambda}}{2}\zeta\right)-\delta_2\sin\left(\frac{\sqrt{-\Lambda}}{2}\zeta\right)}{i\delta_1\sin\left(\frac{\sqrt{-\Lambda}}{2}\zeta\right)+\delta_2\cos\left(\frac{\sqrt{-\Lambda}}{2}\zeta\right)}\right)\end{array}\right\} \end{aligned}$$

Case (3) If $\Lambda=\alpha(1-\delta)\geq 0$ and $\beta=0$ (Fig. 3), then



(a) $\left|f_{1,1}(t)\right|^2$ **(b)** $\left|g_{1,2}1(t)\right|^2$ **(c)** $\left|h_{1,1}(t)\right|^2$

Fig. 1 Analytical solution of Eqs. (24) with different v at $\alpha=0.5$, $\delta_2=0.02$, $c=0.1, \delta=\delta_1=\beta=A_1=\beta_3=B_0=2$



(a) $\left|f_{1,2}(t)\right|^2$ **(b)** $\left|g_{1,2}(t)\right|^2$ **(c)** $\left|h_{1,2}(t)\right|^2$

Fig. 2 Analytical solution of Eqs. (25) with different v at $\alpha = 3$, $\delta_2 = 0.8$, $c = 0.1, \delta = \delta_1 = \beta = A_1 = \beta_3 = B_0 = 2$

$$f_{1,3}(t) = \mp\frac{A_1\alpha}{\hbar} + A_1\left\{\frac{\sqrt{\Lambda}}{(1-\delta)}\left(\frac{\delta_1\cos\left(\sqrt{\Lambda}\zeta\right) + \delta_2\sin\left(\sqrt{\Lambda}\zeta\right)}{\delta_1\sin\left(\sqrt{\Lambda}\zeta\right) - \delta_2\cos\left(\sqrt{\Lambda}\zeta\right)}\right)\right\} \mp\frac{1}{4}\frac{\hbar A_1}{\alpha}\left\{\frac{\sqrt{\Lambda}}{(1-\delta)}\left(\frac{\delta_1\cos\left(\sqrt{\Lambda}\zeta\right) + \delta_2\sin\left(\sqrt{\Lambda}\zeta\right)}{\delta_1\sin\left(\sqrt{\Lambda}\zeta\right) - \delta_2\cos\left(\sqrt{\Lambda}\zeta\right)}\right)\right\}^2$$

$$g_{1,3}(t) = B_0 - A_1\left\{\frac{\sqrt{\Lambda}}{(1-\delta)}\left(\frac{\delta_1\cos\left(\sqrt{\Lambda}\zeta\right) + \delta_2\sin\left(\sqrt{\Lambda}\zeta\right)}{\delta_1\sin\left(\sqrt{\Lambda}\zeta\right) - \delta_2\cos\left(\sqrt{\Lambda}\zeta\right)}\right)\right\} + \frac{1}{4}\frac{\hbar A_1}{\alpha}\left\{\frac{\sqrt{\Lambda}}{(1-\delta)}\left(\frac{\delta_1\cos\left(\sqrt{\Lambda}\zeta\right) + \delta_2\sin\left(\sqrt{\Lambda}\zeta\right)}{\delta_1\sin\left(\sqrt{\Lambda}\zeta\right) - \delta_2\cos\left(\sqrt{\Lambda}\zeta\right)}\right)\right\}^2 \tag{26}$$

$$h_{1,3}(t) = \mp\frac{4c(-\delta+1)\alpha}{\hbar\beta_3} - \frac{2c(\delta-1)}{\beta_3}\left\{\frac{\sqrt{\Lambda}}{(1-\delta)}\left(\frac{\delta_1\cos\left(\sqrt{\Lambda}\zeta\right) + \delta_2\sin\left(\sqrt{\Lambda}\zeta\right)}{\delta_1\sin\left(\sqrt{\Lambda}\zeta\right) - \delta_2\cos\left(\sqrt{\Lambda}\zeta\right)}\right)\right\}$$

where, $\hbar = \sqrt{-4\alpha\delta + 4\alpha}$.

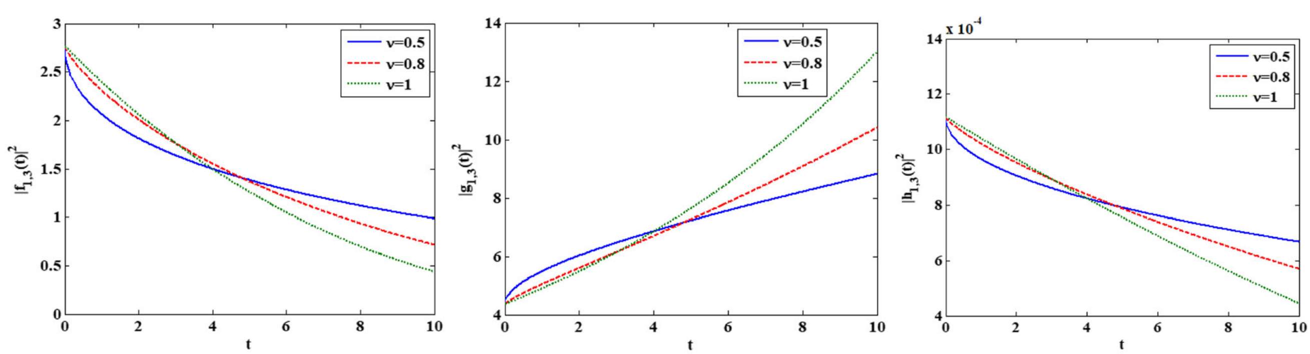
Case (4) If $\Lambda = \alpha(1-\delta) < 0$ and $\beta = 0$, then

$$f_{1,4}(t) = \mp\frac{A_1\alpha}{\hbar} + A_1\left\{\frac{\sqrt{-\Lambda}}{(1-\delta)}\left(\frac{i\delta_1\cosh\left(\sqrt{-\Lambda}\zeta\right) - \delta_2\sinh\left(\sqrt{-\Lambda}\zeta\right)}{i\delta_1\sinh\left(\sqrt{-\Lambda}\zeta\right) - \delta_2\cosh\left(\sqrt{-\Lambda}\zeta\right)}\right)\right\} \mp\frac{1}{4}\frac{\hbar A_1}{\alpha}\left\{\frac{\sqrt{-\Lambda}}{(1-\delta)}\left(\frac{i\delta_1\cosh\left(\sqrt{-\Lambda}\zeta\right) - \delta_2\sinh\left(\sqrt{-\Lambda}\zeta\right)}{i\delta_1\sinh\left(\sqrt{-\Lambda}\zeta\right) - \delta_2\cosh\left(\sqrt{-\Lambda}\zeta\right)}\right)\right\}^2$$

$$g_{1,4}(t) = B_0 - A_1\left\{\frac{\sqrt{-\Lambda}}{(1-\delta)}\left(\frac{i\delta_1\cosh\left(\sqrt{-\Lambda}\zeta\right) - \delta_2\sinh\left(\sqrt{-\Lambda}\zeta\right)}{i\delta_1\sinh\left(\sqrt{-\Lambda}\zeta\right) - \delta_2\cosh\left(\sqrt{-\Lambda}\zeta\right)}\right)\right\} + \frac{1}{4}\frac{\hbar A_1}{\alpha}\left\{\frac{\sqrt{-\Lambda}}{(1-\delta)}\left(\frac{i\delta_1\cosh\left(\sqrt{-\Lambda}\zeta\right) - \delta_2\sinh\left(\sqrt{-\Lambda}\zeta\right)}{i\delta_1\sinh\left(\sqrt{-\Lambda}\zeta\right) - \delta_2\cosh\left(\sqrt{-\Lambda}\zeta\right)}\right)\right\}^2 \tag{27}$$

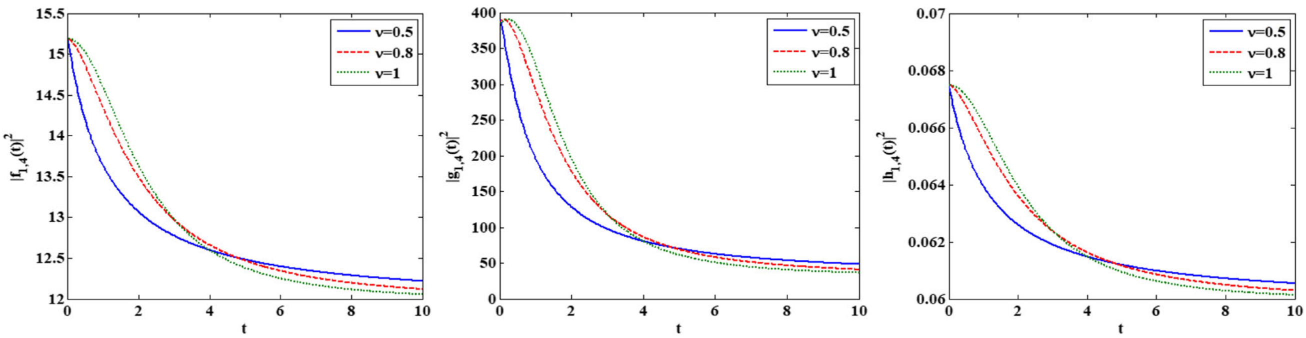
$$h_{1,4}(t) = \mp\frac{4c(-\delta+1)\alpha}{\hbar\beta_3} - \frac{2c(\delta-1)}{\beta_3}\left\{\frac{\sqrt{-\Lambda}}{(1-\delta)}\left(\frac{i\delta_1\cosh\left(\sqrt{-\Lambda}\zeta\right) - \delta_2\sinh\left(\sqrt{-\Lambda}\zeta\right)}{i\delta_1\sinh\left(\sqrt{-\Lambda}\zeta\right) - \delta_2\cosh\left(\sqrt{-\Lambda}\zeta\right)}\right)\right\}$$

where $\zeta = c\frac{t^v}{v}$ and $\alpha, \beta, \delta, \delta_1, \delta_2$ are real parameters. For simplicity, the sets [2–9] should be omitted here (Fig. 4).



(a) $\left|f_{1,3}(t)\right|^2$ **(b)** $\left|g_{1,3}(t)\right|^2$ **(c)** $\left|h_{1,3}(t)\right|^2$

Fig. 3 Analytical solution of Eqs. (26) with different v at $\alpha = 3$, $\delta_2 = 0.8$, $c = \delta = \delta_1 = 0.02, A_1 = \beta_3 = B_0 = 2$



(a) $\left|f_{1,4}(t)\right|^2$ **(b)** $\left|g_{1,4}(t)\right|^2$ **(c)** $\left|h_{1,4}(t)\right|^2$

Fig. 4 Analytical solution of Eqs. (26) with different v at $\alpha = 3$, $\delta_2 = 0.8$,$c = 0.1$, $A_1 = \beta_3 = B_0 = \delta = \delta_1 = 2$

4. Conclusions

In conclusion, we introduced a novel fractional model in biology, namely HIV-1 infection of CD4 + T cells. Here, an extended (Υ'/Υ) method has been studied for constructing new exact traveling wave solutions such as exponential function, trigonometric function and hyperbolic function which are shown graphically in 2D plots to show the dynamical behavior of the proposed model for a different fractal order to see how unique our solutions are, as they are all fresh and different. Therefore, we came to the conclusion that the analytical findings presented here are both useful and fascinating. We want to propose a simple and trustworthy way to the research that will be conducted for the future of human beings.

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