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RESEARCH ARTICLE

Highly dispersive optical solitons with differential group delay for Sasa-Satsuma equation having multiplicative white noise

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Abstract This paper is about the retrieval of highly dispersive optical solitons for Sasa-Satsuma equation with differential group delay in presence of white noise. There are four integration schemes that make this retrieval possible. A full spectrum of optical solitons have been revealed from these schemes. The parametric restrictions for the existence of such solitons are also presented. The displayed surface plots support the analytical findings.

Keywords Stochaticity · Weiner process · Birefringence

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Introduction

Sasa-Satsuma equation (SSE) was formulated as the perturbed version of the well-known nonlinear Schrödinger's equation about three decades ago. This proposed model provided an accurate description of the soliton propagation through optical fibers. The three Hamiltonian perturbative effects are from soliton self-frequency shifts and the self-steepening effects. This model gained popularity and extensive research was conducted with it for decades. While it is only the scalar version of the model that was mostly studied thus far, it is about time to consider SSE further along with a freshly new perspective.

While a plethora of studies have been conducted with regards to stochastic nonlinear evolution equation, the current paper turns the page to give the model an effect of unprecedented novelty [1–30]. The model is first considered with higher order dispersions, namely the dispersive effects

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are from first order to sixth order with the effect of chromatic dispersion (CD) being included. Thus, the dispersive effects stem from inter-model dispersion, CD, third-order dispersion (3OD), fourth-order dispersion (4OD) and fifth-order dispersion (5OD) and finally the sixth-order dispersion (6OD). These dispersive effects together constitute the highly dispersive optical solitons. The self-phase modulation is from Kerr law. Next, this SSE is considered with differential group delay and thus the two-component model is considered. Finally, from a practical perspective, The effect of white noise is included. The resulting coupled stochastic differential equation is studied in fiber optics and its soliton solutions are retrieved.

The coupled model is addressed using a few integration algorithms this led to the retrieval of a full spectrum of optical solitons. It will be observed that the effect of white noise is only present in the phase component of the solitons and not in the amplitude part. The details are all enumerated in the rest of the paper after the model is presented with the technicalities as stated are illustrated.

Governing model

Highly dispersive stochastic in dimensionless form for the first time, the SSE in birefringent fibers with Kerr law nonlinearity, and multiplicative white noise in the Itô sense is expressed as:

$$\begin{aligned} &iu_t + ia_{11}u_x + a_{12}u_{xx} + ia_{13}u_{xxx} + a_{14}u_{xxxx} \\ &\quad + ia_{15}u_{xxxxx} + a_{16}u_{xxxxxx} + \left(c_1|u|^2 + d_1|v|^2\right)u \\ &\quad + i\left(e_1|u|^2 + f_1|v|^2\right)u_x \\ &\quad + i\left(g_1\left(|u|^2\right)_x + h_1\left(|v|^2\right)_x\right) \\ &\quad u + \sigma_1 u\frac{dW_1(t)}{dt} = 0, \end{aligned} \tag{1}$$

and

$$\begin{aligned} &iv_t + ia_{21}v_x + a_{22}v_{xx} + ia_{23}v_{xxx} \\ &\quad + a_{24}v_{xxxx} + ia_{25}v_{xxxxx} + a_{26}v_{xxxxxx} \\ &\quad + \left(c_2|v|^2 + d_2|u|^2\right)v \\ &\quad + i\left(e_2|v|^2 + f_2|u|^2\right)v_x \\ &\quad + i\left(g_2\left(|v|^2\right)_x + h_2\left(|u|^2\right)_x\right)v + \sigma_2 v\frac{dW_2(t)}{dt} = 0. \end{aligned} \tag{2}$$

In the prior system, u(x,t) and v(x,t) are complex-valued functions that reflect the wave profiles & $i^2 = -1$. The first terms in the above system represent the linear temporal evolution. The constants $\left(a_{lk}, l = 1, 2, k = 1, 2, \dots, 6\right)$ correspond to the coefficients of IMD, CD, 3OD, 4OD, 5OD and 6OD respectively. The parameters $c_j, d_j, (j = 1, 2)$ are the coefficients of SPM and cross-phase modulation respectively. The coefficients of nonlinear dispersion terms are denoted by the constants $e_j, f_j, g_j, h_j, (j = 1, 2)$ Finally, $\sigma_j, (j = 1, 2)$ represent the noises strength coefficients & $W_j(t), (j = 1, 2)$ give the standard Wiener processes, such that $dW_j(t)/dt, (j = 1, 2)$ represent white noise. Also, the terms $dW_j(t)/dt, (j = 1, 2)$ are the temporal derivative of the standard Wiener processes.

Mathematical preliminaries

To analyze the stochastic systems (1) and (2), we make the following assumption (3):

$$u(x,t) = P_1(z)e^{i\left[\eta_1(x,t)+\sigma_1 W_1(t)-\sigma_1^2 t\right]},$$
$$v(x,t) = P_2(z)e^{i\left[\eta_2(x,t)+\sigma_2 W_2(t)-\sigma_2^2 t\right]},$$

and

$$z = x - Vt, \eta_l(x,t) = -\kappa_l x + \Omega_l t, \tag{3}$$

where $\kappa_l, \Omega_l, (l = 1, 2)$ & V are nonzero real-valued constants. The frequencies of the solitons may be calculated from the phase component $\kappa_l, (l = 1, 2)$, while $\Omega_l, (l = 1, 2)$ arise the wave numbers and the velocity soliton is denoted by V. The functions $P_1(z), P_2(z)$ & $\eta_l(x,t)$ are real functions that reflect the amplitude and phase components of solitons, respectively. Inputting (3) into Eqs. (1) and (2) yields the following:

$$\begin{aligned} \Re_1 :\ & a_{16}P_1^{(6)} + \left(a_{14} + 5\kappa_1 a_{15} - 15a_{16}\kappa_1^2\right)P_1^{(4)} \\ & + \left(a_{12} + 3\kappa_1 a_{13} - 6\kappa_1^2 a_{14} - 10\kappa_1^3 a_{15} + 15\kappa_1^4 a_{16}\right)P_1'' \\ & + \left[a_{11}\kappa_1 - a_{12}\kappa_1^2 - a_{13}\kappa_1^3 + a_{14}\kappa_1^4\right. \\ & \left.+a_{15}\kappa_1^5 - a_{16}\kappa_1^6 - \left(\Omega_1 - \sigma_1^2\right)\right]P_1 \\ & + \left(c_1 + \kappa_1 e_1\right)P_1^3 + \left(d_1 + \kappa_1 f_1\right)P_1 P_2^2 = 0, \end{aligned} \tag{4}$$

$$\begin{aligned} \Re_2 :\ & a_{26}P_2^{(6)} + \left(a_{24} + 5\kappa_2 a_{25} - 15a_{26}\kappa_2^2\right)P_2^{(4)} \\ & + \left(a_{22} + 3\kappa_2 a_{23} - 6\kappa_2^2 a_{24} - 10\kappa_2^3 a_{25} + 15\kappa_2^4 a_{26}\right)P_2'' \\ & + \left[a_{21}\kappa_2 - a_{22}\kappa_2^2 - a_{23}\kappa_2^3 + a_{24}\kappa_2^4\right. \\ & \left.+a_{25}\kappa_2^5 - a_{26}\kappa_2^6 - \left(\Omega_2 - \sigma_2^2\right)\right]P_2 \\ & + \left(c_2 + \kappa_2 e_2\right)P_2^3 + \left(d_2 + \kappa_2 f_2\right)P_2 P_1^2 = 0, \end{aligned} \tag{5}$$

and

$$\begin{aligned} \Im_1 :\ & \left(a_{15} - 6\kappa_1 a_{16}\right)P_1^{(5)} + \left(a_{13} - 4\kappa_1 a_{14}\right. \\ & \left.-10\kappa_1^2 a_{15} + 20\kappa_1^3 a_{16}\right)P_1''' \\ & + \left[a_{11} - 2a_{12}\kappa_1 - 3a_{13}\kappa_1^2 + 4a_{14}\kappa_1^3\right. \\ & \left.+5a_{15}\kappa_1^4 - 6a_{16}\kappa_1^5 - V\right]P_1' \\ & + \left(e_1 + 2g_1\right)P_1^2 P_1' + f_1 P_2^2 P_1' + 2h_1 P_1 P_2 P_2' = 0, \end{aligned} \tag{6}$$

$$\Im_2 : \left(a_{25} - 6\kappa_2 a_{26}\right)P_2^{(5)} + \left(a_{23} - 4\kappa_2 a_{24} \right. \\ \left. -10\kappa_2^2 a_{25} + 20\kappa_2^3 a_{26}\right)P_2''' \\ + \left[a_{21} - 2a_{22}\kappa_2 - 3a_{23}\kappa_2^2 + 4a_{24}\kappa_2^3 \right. \\ \left. +5a_{25}\kappa_2^4 - 6a_{26}\kappa_2^5 - V\right]P_2' \\ + \left(e_2 + 2g_2\right)P_2^2 P_2' + f_2 P_1^2 P_2' + 2h_2 P_2 P_1 P_1' = 0. \tag{7}$$

Setting

$$P_2(z) = \beta P_1(z), \tag{8}$$

where β is a non zero constant, such that$\beta \neq 1$. Now, Eqs. (3)–(6) become

$$\Re_1 : a_{16}P_1^{(6)} + \left(a_{14} + 5\kappa_1 a_{15} - 15a_{16}\kappa_1^2\right)P_1^{(4)} \\ + \left(a_{12} + 3\kappa_1 a_{13} - 6\kappa_1^2 a_{14} - 10\kappa_1^3 a_{15} + 15\kappa_1^4 a_{16}\right)P_1'' \\ + \left[a_{11}\kappa_1 - a_{12}\kappa_1^2 - a_{13}\kappa_1^3 + a_{14}\kappa_1^4 \right. \\ \left. +a_{15}\kappa_1^5 - a_{16}\kappa_1^6 - \left(\Omega_1 - \sigma_1^2\right)\right]P_1 \\ + \left[c_1 + \kappa_1 e_1 + \beta^2\left(d_1 + \kappa_1 f_1\right)\right]P_1^3 = 0, \tag{9}$$

$$\Re_2 : a_{26}P_1^{(6)} + \left(a_{24} + 5\kappa_2 a_{25} - 15a_{26}\kappa_2^2\right)P_1^{(4)} \\ + \left(a_{22} + 3\kappa_2 a_{23} - 6\kappa_2^2 a_{24} - 10\kappa_2^3 a_{25} + 15\kappa_2^4 a_{26}\right)P_1'' \\ + \left[a_{21}\kappa_2 - a_{22}\kappa_2^2 - a_{23}\kappa_2^3 + a_{24}\kappa_2^4 \right. \\ \left. +a_{25}\kappa_2^5 - a_{26}\kappa_2^6 - \left(\Omega_2 - \sigma_2^2\right)\right]P_1 \\ + \left[\beta^2\left(c_2 + \kappa_2 e_2\right) + d_2 + \kappa_2 f_2\right]P_1^3 = 0, \tag{10}$$

and

$$\Im_1 : \left(a_{15} - 6\kappa_1 a_{16}\right)P_1^{(5)} + \left(a_{13} - 4\kappa_1 a_{14} \right. \\ \left. -10\kappa_1^2 a_{15} + 20\kappa_1^3 a_{16}\right)P_1''' \\ + \left[a_{11} - 2a_{12}\kappa_1 - 3a_{13}\kappa_1^2 + 4a_{14}\kappa_1^3 \right. \\ \left. +5a_{15}\kappa_1^4 - 6a_{16}\kappa_1^5 - V\right]P_1' \\ + \left[e_1 + 2g_1 + \left(f_1 + 2h_1\right)\beta^2\right]P_1^2 P_1' = 0, \tag{11}$$

$$\Im_2 : \left(a_{25} - 6\kappa_2 a_{26}\right)P_1^{(5)} + \left(a_{23} - 4\kappa_2 a_{24} \right. \\ \left. -10\kappa_2^2 a_{25} + 20\kappa_2^3 a_{26}\right)P_1''' \\ + \left[a_{21} - 2a_{22}\kappa_2 - 3a_{23}\kappa_2^2 + 4a_{24}\kappa_2^3 \right. \\ \left. +5a_{25}\kappa_2^4 - 6a_{26}\kappa_2^5 - V\right]P_1' \\ + \left[\beta^2\left(e_2 + 2g_2\right) + f_2 + 2h_2\right]P_1^2 P_1' = 0. \tag{12}$$

Equations (10) and (11), when integrated with zero-integration constants, provide the following

$$\Im_1 : \left(a_{15} - 6\kappa_1 a_{16}\right)P_1^{(4)} + \left(a_{13} - 4\kappa_1 a_{14} \right. \\ \left. -10\kappa_1^2 a_{15} + 20\kappa_1^3 a_{16}\right)P_1'' \\ + \left[a_{11} - 2a_{12}\kappa_1 - 3a_{13}\kappa_1^2 + 4a_{14}\kappa_1^3 \right. \\ \left. +5a_{15}\kappa_1^4 - 6a_{16}\kappa_1^5 - V\right]P_1 \\ + \frac{1}{3}\left[e_1 + 2g_1 + \left(f_1 + 2h_1\right)\beta^2\right]P_1^3 = 0, \tag{13}$$

$$\Im_2 : \left(a_{25} - 6\kappa_2 a_{26}\right)P_1^{(4)} + \left(a_{23} - 4\kappa_2 a_{24} \right. \\ \left. -10\kappa_2^2 a_{25} + 20\kappa_2^3 a_{26}\right)P_1'' \\ + \left[a_{21} - 2a_{22}\kappa_2 - 3a_{23}\kappa_2^2 + 4a_{24}\kappa_2^3 \right. \\ \left. +5a_{25}\kappa_2^4 - 6a_{26}\kappa_2^5 - V\right]P_1 \\ + \frac{1}{3}\left[\beta^2\left(e_2 + 2g_2\right) + f_2 + 2h_2\right]P_1^3 = 0. \tag{14}$$

When we use Eqs. (12) and (13), which are linearly independent functions, and we set their coefficients to zero, we obtain

$$\kappa_l = \frac{a_{l5}}{6a_{l6}},, l = 1, 2, \tag{15}$$

$$a_{l3} = 4\kappa_l a_{l4} + 10\kappa_l^2 a_{l5} - 20\kappa_l^3 a_{l6},\ l = 1, 2, \tag{16}$$

also, we gain the velocity of the soliton

$$V = \left(a_{l1} - 2a_{l2}\kappa_l - 3a_{l3}\kappa_l^2 \right. \\ \left. +4a_{l4}\kappa_l^3 + 5a_{l5}\kappa_l^4 - 6a_{l6}\kappa_l^5\right), l = 1, 2, \tag{17}$$

and the constraints conditions

$$e_1 + 2g_1 + \left(f_1 + 2h_1\right)\beta^2 = 0, \tag{18}$$

$$\beta^2\left(e_2 + 2g_2\right) + f_2 + 2h_2 = 0, \tag{19}$$

where a_{15}, a_{16}, b_1, b_2 are nonzero constants. Under the constraint conditions, Eqs. (8) and (9) are equal:

$$\frac{a_{16}}{a_{26}} = \frac{a_{14} + 5\kappa_1 a_{15} - 15a_{16}\kappa_1^2}{a_{24} + 5\kappa_2 a_{25} - 15a_{26}\kappa_2^2} \\ = \frac{a_{12} + 3\kappa_1 a_{13} - 6\kappa_1^2 a_{14} - 10\kappa_1^3 a_{15} + 15\kappa_1^4 a_{16}}{a_{22} + 3\kappa_2 a_{23} - 6\kappa_2^2 a_{24} - 10\kappa_2^3 a_{25} + 15\kappa_2^4 a_{26}} \\ = \frac{a_{11}\kappa_1 - a_{12}\kappa_1^2 - a_{13}\kappa_1^3 + a_{14}\kappa_1^4 + a_{15}\kappa_1^5 - a_{16}\kappa_1^6 - \left(\Omega_1 - \sigma_1^2\right)}{a_{21}\kappa_2 - a_{22}\kappa_2^2 - a_{23}\kappa_2^3 + a_{24}\kappa_2^4 + a_{25}\kappa_2^5 - a_{26}\kappa_2^6 - \left(\Omega_2 - \sigma_2^2\right)} \\ = \frac{c_1 + \kappa_1 e_1 + \beta^2\left(d_1 + \kappa_1 f_1\right)}{\beta^2\left(c_2 + \kappa_2 e_2\right) + d_2 + \kappa_2 f_2}. \tag{20}$$

From (19), we gain the following:

$$a_{22} = \frac{15\left(\kappa_1^4 - \kappa_2^4\right)a_{16}a_{26} - 10\left(\kappa_1^3 a_{15}a_{26} - \kappa_2^3 a_{16}a_{25}\right) - 6\left(\kappa_1^2 a_{14}a_{26} - \kappa_2^2 a_{16}a_{24}\right) + 3\left(\kappa_1 a_{13}a_{26} - \kappa_2 a_{16}a_{23}\right) + a_{12}a_{26}}{a_{16}}, \tag{21}$$

provided $a_{16} \neq 0$. You may rewrite equation (8) as follows:

$$P_1^{(6)}(z) + \Delta_0 P_1^{(4)}(z) + \Delta_1 P_1''(z) + \Delta_2 P_1(z) + \Delta_3 P_1^3(z) = 0, \tag{22}$$

where

$$\Delta_0 = \frac{a_{14} + 5\kappa_1 a_{15} - 15 a_{16}\kappa_1^2}{a_{16}}, \Delta_1 = \frac{a_{12} + 3\kappa_1 a_{13} - 6\kappa_1^2 a_{14} - 10\kappa_1^3 a_{15} + 15\kappa_1^4 a_{16}}{a_{16}},$$
$$\Delta_2 = \frac{a_{11}\kappa_1 - a_{12}\kappa_1^2 - a_{13}\kappa_1^3 + a_{14}\kappa_1^4 + a_{15}\kappa_1^5 - a_{16}\kappa_1^6 - (\Omega_1 - \sigma_1^2)}{a_{16}}, \Delta_3 = \frac{c_1 + \kappa_1 e_1 + \beta^2(d_1 + \kappa_1 f_1)}{a_{16}}, \tag{23}$$

provided $a_{16} \neq 0$. The balance number $N = 3$ is obtained by balancing $P_1^{(6)}(z)$ and $P_1^3(z)$ in Eq. (21). The following methods are implemented in the next sections to discuss Eq. (21).

Integration approaches applied to the model

This section implements the basic mathematical foundations laid down in the previous section to integrate the governing model using four of the integration algorithms that are present in the literature.

Simplest equation method

Equation (21) enables the exact solution:

$$P_1(z) = A_0 + A_1 F(z) + A_2 F^2(z) + A_3 F^3(z), A_3 \neq 0, \tag{24}$$

and $F(z)$ fulfil the Bernoulli's equation

$$F'(z) = aF(z) + bF^2(z), \tag{25}$$

or the Riccati equation

$$F'(z) = \sigma + F^2(z), \tag{26}$$

in which A_0, A_1, A_2, A_3, a, b & σ are future-determined constants. Equation (24) has the solutions as:

$$F(z) = \frac{a \exp\left[a(z+z_0)\right]}{1 - b\exp\left[a(z+z_0)\right]}, \quad \text{if } a > 0, \quad b < 0, \tag{27}$$

and

$$F(z) = -\frac{a \exp\left[a(z+z_0)\right]}{1 + b\exp\left[a(z+z_0)\right]}, \quad \text{if } a < 0, \quad b > 0, \tag{28}$$

in which z_0 is an integration constant, and if $\sigma < 0$, the following soliton solution structures emerge:

$$F(z) = -\sqrt{-\sigma}\coth\left(\sqrt{-\sigma} z\right), \tag{29}$$

or

$$F(z) = -\sqrt{-\sigma}\tanh\left(\sqrt{-\sigma} z\right), \tag{30}$$

which are singular and dark soliton solutions respectively. The remaining cases when $\sigma > 0$ and $\sigma = 0$ are excluded since tey do not yield soliton solutions.

Bernoulli's equation approach

The results are obtained by inserting (24) and (25) into Eq. (21), collecting all the coefficients of each power $F^s(z), (s = 0, 1, .., 9)$,and setting these coefficients to zero:

$$A_0 = 0, A_1 = 12a^2 b\sqrt{-\frac{35}{\Delta_3}}, A_2 = 36ab^2\sqrt{-\frac{35}{\Delta_3}}, A_3 = 24b^3\sqrt{-\frac{35}{\Delta_3}}, \Delta_0 = 0, \Delta_1 = -21a^4, \Delta_2 = 20a^6, \tag{31}$$

provided $\Delta_3 < 0$.

(I) If $a > 0, b < 0$, one gains

$$u(x,t) = 12a^3 b\sqrt{-\frac{35}{\Delta_3}}\left[\frac{\exp\left[a(z+z_0)\right]}{1 - b\exp\left[a(z+z_0)\right]}\right]\left[1 + \frac{3b\exp\left[a(z+z_0)\right]}{1 - b\exp\left[a(z+z_0)\right]} + \frac{2b^2\exp\left[2a(z+z_0)\right]}{\left(1 - b\exp\left[a(z+z_0)\right]\right)^2}\right] e^{i\left[-\kappa_1 x + \Omega_1 t + \sigma_1 W_1(t) - \sigma_1^2 t\right]}, \tag{32}$$

$$v(x,t) = 12a^3 b\beta\sqrt{-\frac{35}{\Delta_3}}\left[\frac{\exp\left[a(z+z_0)\right]}{1 - b\exp\left[a(z+z_0)\right]}\right]\left[1 + \frac{3b\exp\left[a(z+z_0)\right]}{1 - b\exp\left[a(z+z_0)\right]} + \frac{2b^2\exp\left[2a(z+z_0)\right]}{\left(1 - b\exp\left[a(z+z_0)\right]\right)^2}\right] e^{i\left[-\kappa_2 x + \Omega_2 t + \sigma_2 W_2(t) - \sigma_2^2 t\right]}. \tag{33}$$

(II) If $a < 0, b > 0$, we obtain

$$u(x,t) = -12a^3 b\sqrt{-\frac{35}{\Delta_3}}\left[\frac{\exp\left[a(z+z_0)\right]}{1 + b\exp\left[a(z+z_0)\right]}\right]\left[1 - \frac{3b\exp\left[a(z+z_0)\right]}{1 + b\exp\left[a(z+z_0)\right]} + \frac{2b^2\exp\left[2a(z+z_0)\right]}{\left(1 + b\exp\left[a(z+z_0)\right]\right)^2}\right] e^{i\left[-\kappa_1 x + \Omega_1 t + \sigma_1 W_1(t) - \sigma_1^2 t\right]}, \tag{34}$$

$$v(x,t) = -12a^3 b\beta\sqrt{-\frac{35}{\Delta_3}}\left[\frac{\exp\left[a(z+z_0)\right]}{1+b\exp\left[a(z+z_0)\right]}\right]\left[1-\frac{3b\exp\left[a(z+z_0)\right]}{1+b\exp\left[a(z+z_0)\right]}+\frac{2b^2\exp\left[2a(z+z_0)\right]}{\left(1+b\exp\left[a(z+z_0)\right]\right)^2}\right] e^{i\left[-\kappa_2 x+\Omega_2 t+\sigma_2 W_2(t)-\sigma_2^2 t\right]}. \tag{35}$$

For example, if $a=1, b=-1$ or $a=-1, b=1$, the combo dark soliton solutions are available:

$$u(x,t) = 3\sqrt{-\frac{35}{\Delta_3}}\left[\tanh\left(\frac{z+z_0}{2}\right)-\tanh^3\left(\frac{z+z_0}{2}\right)\right] e^{i\left[-\kappa_1 x+\Omega_1 t+\sigma_1 W_1(t)-\sigma_1^2 t\right]}, \tag{36}$$

$$v(x,t) = 3\beta\sqrt{-\frac{35}{\Delta_3}}\left[\tanh\left(\frac{z+z_0}{2}\right)-\tanh^3\left(\frac{z+z_0}{2}\right)\right] e^{i\left[-\kappa_2 x+\Omega_2 t+\sigma_2 W_2(t)-\sigma_2^2 t\right]}. \tag{37}$$

Riccati equation scheme

The following results are obtained by inserting (24) and (26) into Eq. (21), accumulating the coefficients of each power $F^s(z)$, $(s=0,1,2,..,9)$, and then setting each of these coefficients to zero:

$$A_0=0, A_1=\frac{18}{83}\sqrt{-\frac{35\Delta_0^2}{\Delta_3}}, A_2=0, A_3=24\sqrt{-\frac{35}{\Delta_3}}, \Delta_1 = \frac{946}{6889}\Delta_0^2, \Delta_2=-\frac{1260}{571787}\Delta_0^3, \sigma=\frac{\Delta_0}{332}, \tag{38}$$

provided $\Delta_3<0$. Now, one finds the soliton solutions to Equations (1) and (2) outlined below for $\Delta_0<0$, after ignoring the remaining cases when $\Delta_0>$) or $\Delta_0=0$, which do not give way to soliton solutions:

The straddled singular solitons are:

$$u(x,t) = -\frac{3\Delta_0}{83}\sqrt{\frac{35\Delta_0}{83\Delta_3}}\left[3\coth\left(\sqrt{-\frac{\Delta_0}{332}}z\right)-\coth^3\left(\sqrt{-\frac{\Delta_0}{332}}z\right)\right] e^{i\left[-\kappa_1 x+\Omega_1 t+\sigma_1 W_1(t)-\sigma_1^2 t\right]}, \tag{39}$$

$$v(x,t) = -\frac{3\beta\Delta_0}{83}\sqrt{\frac{35\Delta_0}{83\Delta_3}}\left[3\coth\left(\sqrt{-\frac{\Delta_0}{332}}z\right)-\coth^3\left(\sqrt{-\frac{\Delta_0}{332}}z\right)\right] e^{i\left[-\kappa_2 x+\Omega_2 t+\sigma_2 W_2(t)-\sigma_2^2 t\right]}, \tag{40}$$

and the straddled dark soliton solutions are:

$$u(x,t) = -\frac{3\Delta_0}{83}\sqrt{\frac{35\Delta_0}{83\Delta_3}}\left[3\tanh\left(\sqrt{-\frac{\Delta_0}{332}}z\right)-\tanh^3\left(\sqrt{-\frac{\Delta_0}{332}}z\right)\right] e^{i\left[-\kappa_1 x+\Omega_1 t+\sigma_1 W_1(t)-\sigma_1^2 t\right]}, \tag{41}$$

$$v(x,t) = -\frac{3\beta\Delta_0}{83}\sqrt{\frac{35\Delta_0}{83\Delta_3}}\left[3\tanh\left(\sqrt{-\frac{\Delta_0}{332}}z\right)-\tanh^3\left(\sqrt{-\frac{\Delta_0}{332}}z\right)\right] e^{i\left[-\kappa_2 x+\Omega_2 t+\sigma_2 W_2(t)-\sigma_2^2 t\right]}. \tag{42}$$

Extended simplest equation algorithm

Equation (21), which relies on the explicit solution:

$$P_1(z) = \chi_0+\chi_1\left(\frac{\phi'(z)}{\phi(z)}\right)+\chi_2\left(\frac{\phi'(z)}{\phi(z)}\right)^2+\chi_3\left(\frac{\phi'(z)}{\phi(z)}\right)^3+B_0\left(\frac{1}{\phi(z)}\right) +B_1\left(\frac{\phi'(z)}{\phi(z)}\right)\left(\frac{1}{\phi(z)}\right)+B_2\left(\frac{\phi'(z)}{\phi(z)}\right)^2\left(\frac{1}{\phi(z)}\right), \tag{43}$$

where $\chi_0, \chi_1, \chi_2, \chi_3, B_0, B_1$ and B_2 are constants, $\chi_3^2+B_2^2\neq 0$ and the function $\phi(z)$ presumes the auxiliary equation

$$\phi''(z)+\delta\phi(z)=\upsilon_0, \tag{44}$$

in where δ and υ_0 are integers. The case when $\delta<0$ is considered here since the other two cases, namely when $\delta=0$ or $\delta>0$ are discarded since they do not yield soliton solutions.

Here, we replace (43) for Eq.(21) and apply Eq.(43) together with the connection

$$\left(\frac{\phi'(z)}{\phi(z)}\right)^2 = L_1\left(\frac{1}{\phi(z)}\right)^2-\delta+\frac{2\upsilon_0}{\phi(z)}, \tag{45}$$

where $L_1=\delta\left(\rho_1^2-\rho_2^2\right)-\frac{\upsilon_0^2}{\delta}$, yields the following solutions even when ρ_1 and ρ_2 are constants.

Solution–1:

$$\begin{aligned}
&\chi_0=\delta\chi_2, \chi_1=\delta\left(9\sqrt{-\frac{35}{\Delta_3}}-\frac{B_1}{2\upsilon_0}\right),\\
&\chi_2=\chi_2, \chi_3=\left(3\sqrt{-\frac{35}{\Delta_3}}-\frac{B_1}{2\upsilon_0}\right), B_0=-2\chi_2\upsilon_0,\\
&B_1=B_1, B_2=0, \Delta_0=83\delta, \Delta_1=946\delta^2,\\
&\Delta_2=-1260\delta^3, \rho_1=\sqrt{\rho_2^2+\frac{\upsilon_0^2}{\delta^2}},
\end{aligned} \tag{46}$$

provided $\Delta_3<0, \left(\rho_2^2+\frac{\upsilon_0^2}{\delta^2}\right)>0$ and $\upsilon_0\neq 0$. As a result, the solitary solutions to Eqs.(1) and (2) are as follows:

$$u(x,t) = \left[\begin{array}{c} \left(\delta\chi_2+\delta\left(3\sqrt{\frac{35\delta}{\Delta_3}}-\frac{\sqrt{-\delta}B_1}{2\upsilon_0}\right)\Theta_1(z)\right)\left[1-\Theta_1^2(z)\right]+6\delta\sqrt{\frac{35\delta}{\Delta_3}}\Theta_1(z) \\ -2\chi_2\upsilon_0\Theta_2(z)+B_1\sqrt{-\delta}\Theta_1(z)\Theta_2(z) \end{array}\right] e^{i\left[-\kappa_1 x+\Omega_1 t+\sigma_1 W_1(t)-\sigma_1^2 t\right]}, \tag{47}$$

and

$$v(x,t)=\beta\begin{bmatrix}\left(\delta\chi_2+\delta\left(3\sqrt{\frac{35\delta}{\Delta_3}}-\frac{\sqrt{-\delta}B_1}{2\upsilon_0}\right)\Theta_1(z)\right)\left[1-\Theta_1^2(z)\right]+6\delta\sqrt{\frac{35\delta}{\Delta_3}}\Theta_1(z)\\ -2\chi_2\upsilon_0\Theta_2(z)+B_1\sqrt{-\delta}\Theta_1(z)\Theta_2(z)\end{bmatrix} e^{i\left[-\kappa_2x+\Omega_2t+\sigma_2W_2(t)-\sigma_2^2t\right]}, \tag{48}$$

where

$$\Theta_1(z)=\frac{\sqrt{\rho_2^2+\frac{\upsilon_0^2}{\delta^2}}\sinh\left[\sqrt{-\delta}z\right]+\rho_2\cosh\left[\sqrt{-\delta}z\right]}{\sqrt{\rho_2^2+\frac{\upsilon_0^2}{\delta^2}}\cosh\left[\sqrt{-\delta}z\right]+\rho_2\sinh\left[\sqrt{-\delta}z\right]+\frac{\upsilon_0}{\delta}},$$

$$\Theta_2(z)=\frac{1}{\sqrt{\rho_2^2+\frac{\upsilon_0^2}{\delta^2}}\cosh\left[\sqrt{-\delta}z\right]+\rho_2\sinh\left[\sqrt{-\delta}z\right]+\frac{\upsilon_0}{\delta}}. \tag{49}$$

<u>Solution–2</u>:

$$\begin{aligned}&\chi_0=\chi_1=\chi_2=\chi_3=0, B_0=120\delta\sqrt{-\frac{35\delta\left(\rho_1^2-\rho_2^2\right)}{289\Delta_3}},\\ &B_1=0, B_2=408\sqrt{-\frac{35\delta\left(\rho_1^2-\rho_2^2\right)}{289\Delta_3}}, \Delta_0=-\frac{581\delta}{17},\\ &\Delta_1=\frac{92659\delta^2}{289}, \Delta_2=\frac{102825\delta^3}{289}, \upsilon_0=0,\end{aligned} \tag{50}$$

provided $\left(\rho_1^2-\rho_2^2\right)\Delta_3>0$. In light of this, we arrive at the subsequent solitary solutions to Eqs. (1) and (2):

$$u(x,t)=24\delta\sqrt{-\frac{35\delta\left(\rho_1^2-\rho_2^2\right)}{289\Delta_3}}\left[5\tau_2(z)-17\tau_1^2(z)\tau_2(z)\right] e^{i\left[-\kappa_1x+\Omega_1t+\sigma_1W_1(t)-\sigma_1^2t\right]}, \tag{51}$$

and

$$v(x,t)=24\delta\beta\sqrt{-\frac{35\delta\left(\rho_1^2-\rho_2^2\right)}{289\Delta_3}}\left[5\tau_2(z)-17\tau_1^2(z)\tau_2(z)\right] e^{i\left[-\kappa_2x+\Omega_2t+\sigma_2W_2(t)-\sigma_2^2t\right]}, \tag{52}$$

where

$$\tau_1(z)=\frac{\rho_1\sinh\left[\sqrt{-\delta}z\right]+\rho_2\cosh\left[\sqrt{-\delta}z\right]}{\rho_1\cosh\left[\sqrt{-\delta}z\right]+\rho_2\sinh\left[\sqrt{-\delta}z\right]},$$

$$\tau_2(z)=\frac{1}{\rho_1\cosh\left[\sqrt{-\delta}z\right]+\rho_2\sinh\left[\sqrt{-\delta}z\right]}. \tag{53}$$

We get the combo-singular soliton solutions if we put $\rho_1=0, \rho_2\neq 0$, in (51) and (52):

$$u(x,t)=24\delta\sqrt{\frac{35\delta}{289\Delta_3}}\left(12\operatorname{csch}\left[\sqrt{-\delta}z\right]+17\operatorname{csch}^3\left[\sqrt{-\delta}z\right]\right) e^{i\left[-\kappa_1x+\Omega_1t+\sigma_1W_1(t)-\sigma_1^2t\right]}, \tag{54}$$

and

$$v(x,t)=24\delta\beta\sqrt{\frac{35\delta}{289\Delta_3}}\left(12\operatorname{csch}\left[\sqrt{-\delta}z\right]+17\operatorname{csch}^{33}\left[\sqrt{-\delta}z\right]\right) e^{i\left[-\kappa_2x+\Omega_2t+\sigma_2W_2(t)-\sigma_2^2t\right]}, \tag{55}$$

provided $\Delta_3<0$. The combo-bright soliton solutions are available if we put $\rho_1\neq 0, \rho_2=0$:

$$u(x,t)=-24\delta\sqrt{-\frac{35\delta}{289\Delta_3}}\left(12\operatorname{sech}\left[\sqrt{-\delta}z\right]-17\operatorname{sech}^3\left[\sqrt{-\delta}z\right]\right) e^{i\left[-\kappa_1x+\Omega_1t+\sigma_1W_1(t)-\sigma_1^2t\right]}, \tag{56}$$

and

$$\begin{aligned}v(x,t)=&-24\delta\beta\sqrt{-\frac{35\delta}{289\Delta_3}}\left(12\operatorname{sech}\left[\sqrt{-\delta}z\right]\right.\\ &\left.-17\operatorname{sech}^3\left[\sqrt{-\delta}z\right]\right)e^{i\left[-\kappa_2x+\Omega_2t+\sigma_2W_2(t)-\sigma_2^2t\right]},\end{aligned} \tag{57}$$

provided $\Delta_3>0$.

<u>Solution–3</u>:

$$\begin{aligned}&\chi_0=0, \chi_1=72\delta\sqrt{-\frac{35}{\Delta_3}}, \chi_2=0, \chi_3\\ &=24\sqrt{-\frac{35}{\Delta_3}}, B_0=B_1=B_2=0,\\ &\Delta_0=332\delta, \Delta_1=15136\delta^2,\\ &\Delta_2=-80640\delta^3, \upsilon_0=0,\end{aligned} \tag{58}$$

provided $\Delta_3<0$. As a result, we arrive at the solitary solutions of equations (1) and (2) as follows:

$$u(x,t)=24\delta\sqrt{\frac{35\delta}{\Delta_3}}\left[3\tau_1(z)-\tau_1^3(z)\right] e^{i\left[-\kappa_1x+\Omega_1t+\sigma_1W_1(t)-\sigma_1^2t\right]}, \tag{59}$$

and

$$v(x,t)=24\delta\beta\sqrt{\frac{35\delta}{\Delta_3}}\left[3\tau_1(z)-\tau_1^3(z)\right] e^{i\left[-\kappa_2x+\Omega_2t+\sigma_2W_2(t)-\sigma_2^2t\right]}. \tag{60}$$

We get the combo-singular soliton solutions in (59) and (60), specifically if we put $\rho_1=0, \rho_2\neq 0$:

$$u(x,t) = 24\delta\sqrt{\frac{35\delta}{\Delta_3}}\left(3\coth\left[\sqrt{-\delta}z\right] - \coth^3\left[\sqrt{-\delta}z\right]\right) e^{i\left[-\kappa_1 x+\Omega_1 t+\sigma_1 W_1(t)-\sigma_1^2 t\right]}, \tag{61}$$

and

$$v(x,t) = 24\delta\beta\sqrt{\frac{35\delta}{\Delta_3}}\left(3\coth\left[\sqrt{-\delta}z\right] - \coth^3\left[\sqrt{-\delta}z\right]\right) e^{i\left[-\kappa_2 x+\Omega_2 t+\sigma_2 W_2(t)-\sigma_2^2 t\right]}, \tag{62}$$

while in (59) and (60), the combo-dark soliton solutions are obtained if we set $\rho_1 \neq 0, \rho_2 = 0$:

$$u(x,t) = 24\delta\sqrt{\frac{35\delta}{\Delta_3}}\left(3\tanh\left[\sqrt{-\delta}z\right] - \tanh^3\left[\sqrt{-\delta}z\right]\right) e^{i\left[-\kappa_1 x+\Omega_1 t+\sigma_1 W_1(t)-\sigma_1^2 t\right]}, \tag{63}$$

and

$$v(x,t) = 24\delta\beta\sqrt{\frac{35\delta}{\Delta_3}}\left(3\tanh\left[\sqrt{-\delta}z\right] - \tanh^3\left[\sqrt{-\delta}z\right]\right) e^{i\left[-\kappa_2 x+\Omega_2 t+\sigma_2 W_2(t)-\sigma_2^2 t\right]}. \tag{64}$$

Note that, when $\Delta_0 = 332\delta$, the solutions (61)-(64) are similar to the solutions (39)-(42).

Conclusions

The current paper retrieved a full spectrum of highly dispersive optical solitons for SSE in birefringent fibers in presence of white noise when the SPM is of Kerr type. A wide range of integration algorithms has made this retrieval possible. It has been observed that the effect of white noise stays confined to the phase component of the solitons and never enters the amplitude portion of such pulses. The results are thus overwhelming and stand strong for future activities in this field. The model is to be next considered with additional forms of SPM that would produce further interesting results. Additionally, later the model would be generalized to dispersion-flattened fibers. That's when the studies are going to get more interesting. The results of such research activities will be disseminated across the board with time after they are colineared with the pre-existing ones [26–54].

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