J Opt
https://doi.org/10.1007/s12596-024-01670-y

RESEARCH ARTICLE

Optical solitons for the dispersive concatenation model with power law of self-phase modulation and multiplicative white noise

Ahmed H. Arnous[1] · Anjan Biswas[2,3,4,5] · Yakup Yildirim[6,7] · Bharat S. Rawal[8] · Ali Saleh Alshomrani[3]

Received: 17 December 2023 / Accepted: 14 January 2024

Abstract This paper recovers optical solitons to the newly proposed dispersive concatenation model that comes with power law of self-phase modulation. The presence of white noise in the Itô sence makes the model stochastic. Two integration approaches retrieve bright and singular optical solitons. The intermediary Weierstrass' elliptic functions are implemented for this retrieval. It has been established that the effect of white noise stays confined to the phase component of the solitons.

Keywords Solitons · White noise · Concatenation · Power law · Kudryashov

OCIS Codes 060.2310 · 060.4510 · 060.5530 · 190.3270 · 190.4370

Introduction

The concepts of the concatenation model and dispersive concatenation model were sequentially conceived a decade ago [1–5]. Subsequently, extensive studies with these two models were carried out during the past couple of years. These range from the retrieval of soliton solutions and the conservation laws, studying the model with power law of self-phase modulation, addressing the Internet bottleneck effect and minimizing the slowdown of soliton transmission by introducing the spatiotemporal dispersion (STD) in addition to the pre-existing chromatic dispersion (CD). The numerical analysis of solitons for the concatenation model by the Laplace–Adomian decomposition was also conducted. Recently, the effect of white noise in soliton transmission for the concatenation model with Kerr law and power law of SPM yielded interesting results [6–15]. The current paper addresses the effect of white noise for the dispersive concatenation model that is studied in the current paper with power law of SPM. The results of the

✉ Anjan Biswas
biswas.anjan@gmail.com

1 Department of Physics and Engineering Mathematics, Higher Institute of Engineering, El–Shorouk Academy, Cairo, Egypt

2 Department of Mathematics and Physics, Grambling State University, Grambling, LA 71245–2715, USA

3 Mathematical Modeling and Applied Computation (MMAC) Research Group,Center of Modern Mathematical Sciences and their Applications (CMMSA), Department of Mathematics, King Abdulaziz University, Jeddah 21589, Saudi Arabia

4 Department of Applied Sciences, Cross–Border Faculty of Humanities, Economics and Engineering, Dunarea de Jos University of Galati, 111 Domneasca Street, 800201 Galati, Romania

5 Department of Mathematics and Applied Mathematics, Sefako Makgatho Health Sciences University, Medunsa 0204, South Africa

6 Department of Computer Engineering, Biruni University, Istanbul 34010, Turkey

7 Department of Mathematics, Near East University, 99138 Nicosia, Cyprus

8 Department of Computer Science and Digital Technologies, Grambling State University, Grambling, LA 71245–2715, USA

current paper follow a previously published prequel paper on dispersive optical solitons with the Kerr law of SPM, as recently reported [13].

The dispersive concatenation model is the combination of the Schrödinger–Hirota equation (SHE), Lakshmanan–Porsezian–Daniel (LPD) model and the dispersive nonlinear Schrödinger's equation (NLSE) that carries a fifth–order dispersive effect. Therefore, the model is truly dispersive as the name bears. The current paper will address this model with power law of SPM as a sequel to the Kerr law [13]. The white noise in Itô sense is included as a source of stochasticity to the model. Two integration approaches will lead to the soliton solutions. These are the enhanced Kudryashov's approach and the extended auxiliary equation scheme. These two algorithms would lead to the emergence of bright and singular optical solitons only. These approaches fail to recover the dark optical solitons as expected since it is well known, and experimentally proved in the past, that dark solitons are not supported by any model with power law of SPM unless the power law collapses to Kerr law. The details of the concepts and the derivation of the results are exhibited in the rest of the paper after a succinct recapitulation of the integration methodologies.

Governing model

In [1], the concatenation model with higher-order dispersion effects and power law nonlinearity was studied. In this study, we consider the dimensionless expression of this model by incorporating multiplicative white noise effect, which can be expressed as follows:

$$
\begin{aligned}
& iq_t + aq_{xx} + b|q|^{2n}q \\
& \quad - i\delta_1\left[\sigma_1 q_{xxx} + \sigma_2 |q|^{2n} q_x\right] + \sigma q\,\frac{dW(t)}{\mathrm{dt}} \\
& + \delta_2\left[\sigma_3 q_{xxxx} + \sigma_4 |q|^{2n} q_{xx} + \sigma_5 |q|^{2n+2} q \right. \\
& \quad \left. + \sigma_6 |q_x|^2 q + \sigma_7 q_x^2 q^* + \sigma_8 q_{xx}^* q^2\right] \\
& - i\delta_3\left[\sigma_9 q_{xxxxx} + \sigma_{10} |q|^{2n} q_{xxx} + \sigma_{11} |q|^{2n+2} q_x \right. \\
& \quad \left. + \sigma_{12} q q_x q_{xx}^* + \sigma_{13} q^* q_x q_{xx} + \sigma_{14} q q_x^* q_{xx} + \sigma_{15} q_x^2 q_x^*\right] = 0.
\end{aligned} \tag{1}
$$

Here, $q(x,\ t)$ represents a complex-valued function that describes the wave profile. In this context, x and t represent the spatial and temporal coordinates, respectively. The parameters a and b correspond to CD and SPM, respectively, of the NLSE with power law of SPM. The symbol $i = \sqrt{-1}$ is the imaginary unit. The coefficients δ_1, δ_2 and δ_3 are the nonzero parameters associated with SHE, LPD model and the fifth-order NLSE, respectively. The variable σ is used to represent a nonzero constant value and serves as the sign for indicating the intensity of white noise. Furthermore, the standard Wiener process, represented as $W(t)$, can be defined as the integral of the function $\Lambda(\eta) = dW(t)/dt$ concerning the Wiener process $W(\eta)$, where η is a variable that assumes values smaller than t. In the provided context, the symbol η represents a stochastic variable, whereas $\Lambda(\eta)$ is utilized to denote typical Gaussian white noise, popularly known as multiplicative white noise.

Mathematical analysis of the governing model

For investigating a model with a multiplicative white noise effect, the following wave structure is selected:

$$
q(x,t) = U(\xi)e^{i\left(-\kappa x + \omega t + \sigma W(t) - \sigma^2 t + \theta_0\right)}. \tag{2}
$$

The wave variable ξ is defined as

$$
\xi = k(x - vt), \tag{3}
$$

where k and v represent nonzero constants. In Eq. (2), $U(\xi)$ denotes a real-valued function representing the soliton solutions' amplitude components. The variable v corresponds to the speed of the soliton, while k represents the wave width. In the provided context, the symbol κ represents the frequency of the solitons. The symbol ω is used to describe the wave number. The symbol σ is employed to signify the noise coefficient. Lastly, the symbol θ_0 represents the phase constant. The following formulas are obtained by substituting Eq. (2) into Eq. (1) and subsequently decomposing them into their real component

$$
\begin{aligned}
& -k^2\left(a + 10\delta_3\kappa^3\sigma_9 - 6\delta_2\kappa^2\sigma_3 - 3\delta_1\kappa\sigma_1\right)U'' \\
& + \left(a\kappa^2 + \delta_3\kappa^5\sigma_9 - \delta_2\kappa^4\sigma_3 - \delta_1\kappa^3\sigma_1 - \sigma^2 + \omega\right)U \\
& \quad + \left(-b - \delta_3\kappa^3\sigma_{10} + \delta_2\kappa^2\sigma_4 + \delta_1\kappa\sigma_2\right)U^{2n+1} \\
& \quad + k^4\left(5\delta_3\kappa\sigma_9 - \delta_2\sigma_3\right)U^{(4)} + k^2\left(3\delta_3\kappa\sigma_{10} - \delta_2\sigma_4\right)U^{2n}U'' \\
& \quad + k^2\left(\delta_3\kappa\left(\sigma_{12} + \sigma_{13} - \sigma_{14}\right) - \delta_2\sigma_8\right)U^2U'' \\
& \quad + k^2\left(\delta_3\kappa\left(-2\sigma_{12} + 2\sigma_{13} + 2\sigma_{14} + \sigma_{15}\right) - \delta_2\left(\sigma_6 + \sigma_7\right)\right)UU'^2 \\
& \quad + \left(\delta_3\kappa\sigma_{11} - \delta_2\sigma_5\right)U^{2n+3} + \kappa^2\left(\delta_3\kappa\left(-\sigma_{12} - \sigma_{13} + \sigma_{14} + \sigma_{15}\right)\right. \\
& \quad \left. + \delta_2\left(-\sigma_6 + \sigma_7 + \sigma_8\right)\right)U^3 = 0,
\end{aligned} \tag{4}
$$

and imaginary component

$$
\begin{aligned}
& k\left(2a\kappa + 5\delta_3\kappa^4\sigma_9 - 4\delta_2\kappa^3\sigma_3 - 3\delta_1\kappa^2\sigma_1 + v\right)U' \\
& \quad + \delta_3k^5\sigma_9U^{(5)} + \delta_3k^3\sigma_{10}U^{(3)}U^{2n} \\
& \quad + k^3\left(2\kappa\left(2\delta_2\sigma_3 - 5\delta_3\kappa\sigma_9\right) + \delta_1\sigma_1\right)U^{(3)} \\
& \quad + \delta_3k^3\sigma_{15}U'^3 + \delta_3k^3\left(\sigma_{12} + \sigma_{13} + \sigma_{14}\right)UU'U'' \\
& \quad + k\left(\kappa\left(2\delta_2\sigma_4 - 3\delta_3\kappa\sigma_{10}\right) + \delta_1\sigma_2\right) \\
& \quad U^{2n}U' + \delta_3k\sigma_{11}U^{2n+2}U' \\
& \quad + \kappa kU^2\left(\delta_3\kappa\left(\sigma_{12} - 3\sigma_{13} + \sigma_{14} + \sigma_{15}\right)\right. \\
& \quad \left. + 2\delta_2\left(\sigma_7 - \sigma_8\right)\right)U' = 0.
\end{aligned} \tag{5}
$$

From the imaginary part, the soliton speed reads as

$$v = -2a\kappa - 5\delta_3\kappa^4\sigma_9 + 4\delta_2\kappa^3\sigma_3 + 3\delta_1\kappa^2\sigma_1, \tag{6}$$

and the soliton frequency as

$$\kappa = -\frac{\delta_2(\sigma_8 - \sigma_7)}{2\delta_3\sigma_{13}}, \tag{7}$$

with the following parametric restrictions

$$\begin{cases} \sigma_1 = -\frac{2\delta_2^2\sigma_3(\sigma_7-\sigma_8)}{\delta_1\delta_3\sigma_{13}}, \\ \sigma_4 = -\frac{\delta_1\delta_3\sigma_2\sigma_{13}}{\delta_2^2(\sigma_7-\sigma_8)}, \\ \sigma_{12} + \sigma_{13} + \sigma_{14} = 0, \\ \sigma_9 = \sigma_{10} = \sigma_{11} = \sigma_{15} = 0. \end{cases} \tag{8}$$

Under the previous conditions, Eq. (1) will have the following form

$$\begin{aligned} &iq_t + aq_{xx} + b|q|^{2n}q \\ &\quad - i\delta_1\left[\sigma_1 q_{xxx} + \sigma_2|q|^{2n}q_x\right] \\ &\quad + \sigma q\,\frac{dW(t)}{dt} \\ &\quad + \delta_2\left[\sigma_3 q_{xxxx} + \sigma_4|q|^{2n}q_{xx}\right. \\ &\quad + \sigma_5|q|^{2n+2}q \\ &\quad \left.+\sigma_6|q_x|^2 q + \sigma_7 q_x^2 q^* + \sigma_8 q_{xx}^* q^2\right] \\ &\quad - i\delta_3\left[\sigma_{12} q q_x q_{xx}^* + \sigma_{13} q^* q_x q_{xx}\right. \\ &\quad \left.+\sigma_{14} q q_x^* q_{xx}\right] = 0. \end{aligned} \tag{9}$$

Additionally, Eq. (4) will take on the following form

$$\begin{aligned} k^2U^{(4)} &+ w_8U^{2n}U'' + w_7U^2U'' + w_6U'' + w_5UU'^2 \\ &+ w_4U^{2n+1} + w_3U^{2n+3} + w_2U^3 + w_1U = 0, \end{aligned} \tag{10}$$

with

$$\begin{cases} w_1 = \frac{-a\kappa^2+\delta_2\kappa^4\sigma_3+\delta_1\kappa^3\sigma_1+\sigma^2-\omega}{\delta_2k^2\sigma_3}, \\ w_2 = -\frac{\kappa^2\left(2\delta_3\kappa\sigma_{14}+\delta_2(-\sigma_6+\sigma_7+\sigma_8)\right)}{\delta_2k^2\sigma_3}, \\ w_3 = \frac{\sigma_5}{k^2\sigma_3}, \\ w_4 = \frac{b-\delta_2\kappa^2\sigma_4-\delta_1\kappa\sigma_2}{\delta_2k^2\sigma_3}, \\ w_5 = \frac{\delta_2(\sigma_6+\sigma_7)-4\delta_3\kappa(\sigma_{13}+\sigma_{14})}{\delta_2\sigma_3}, \\ w_6 = \frac{a-6\delta_2\kappa^2\sigma_3-3\delta_1\kappa\sigma_1}{\delta_2\sigma_3}, \\ w_7 = \frac{2\delta_3\kappa\sigma_{14}+\delta_2\sigma_8}{\delta_2\sigma_3}, \\ w_8 = \frac{\sigma_4}{\sigma_3}. \end{cases} \tag{11}$$

Provided that $\sigma_3 \neq 0$. Using the transformation

$$U = V^{\frac{1}{n}},$$

Equation (10) collapses to

$$\begin{aligned} &k^2n^3V^{(4)}V^3 - 3k^2(n-1)n^2V^2V''^2 \\ &\quad - 4k^2(n-1)n^2V^{(3)}V^2V' + 6k^2n\left(2n^2-3n+1\right)VV'^2V'' \\ &\quad + k^2\left(-6n^3+11n^2-6n+1\right)V'^4 + n^4w_4V^6 \\ &\quad + n^4w_1V^4 + n^3w_8V^5V'' + n^3w_6V^3V'' \\ &\quad + V^{\frac{2}{n}}\left(n^4w_3V^6 + n^4w_2V^4 + n^3w_7V^3V''\right. \\ &\quad \left.+n^2\left(w_5-(n-1)w_7\right)V^2V'^2\right) \\ &\quad - (n-1)n^2w_8V^4V'^2 - (n-1)n^2w_6V^2V'^2 = 0. \end{aligned} \tag{12}$$

In terms of integrability, it is observed that we possess

$$w_2 = w_3 = w_5 = w_7 = 0 \tag{13}$$

This assumption leads $\sigma_5 = 0$. In this case, Eq. (12) reads

$$\begin{aligned} &k^2n^3V^{(4)}V^3 - 3k^2(n-1)n^2V^2V''^2 \\ &\quad - 4k^2(n-1)n^2V^{(3)}V^2V' + 6k^2n\left(2n^2-3n+1\right)VV'^2V'' \\ &\quad + k^2\left(-6n^3+11n^2-6n+1\right)V'^4 \\ &\quad + n^4w_4V^6 + n^4w_1V^4 + n^3w_8V^5V'' + n^3w_6V^3V'' \\ &-(n-1)n^2w_8V^4V'^2 - (n-1)n^2w_6V^2V'^2 = 0. \end{aligned} \tag{14}$$

Balancing V^3V'''' with V^5V'' or V^6 in Eq. (14) gives $N = 1$ or $N = 2$.

An outline of the integration algorithms

We could include a governing model which has the structure of

$$F(u, u_x, u_t, u_{xt}, u_{xx}, \ldots) = 0, \tag{15}$$

where $u = u(x, t)$ represents a wave profile, where t and x describe the time and space variables, respectively.

The use of the wave transformation

$$u(x,t) = U(\xi), \quad \xi = k(x - \upsilon t), \tag{16}$$

causes a reduction of Eq. (15) to

$$P(U, -k\upsilon U', kU', k^2U'', \ldots) = 0. \tag{17}$$

In that expression, k represents the wave width, ξ represents the wave variable, and υ represents the wave velocity.

The enhanced Kudryashov's method

This subsection presents a thorough overview of the basic procedures with the enhanced Kudryashov technique.

Step 1: The explicit solution for the reduced model Eq. (17) is provided as follows

$$U(\xi)=\sigma_0+\sum_{i=1}^{N}\left\{\sigma_i R(\xi)^i+\rho_i\left(\frac{R'(\xi)}{R(\xi)}\right)^i\right\}, \tag{18}$$

along with the auxiliary equation

$$R'(\xi)^2=R(\xi)^2(1-\chi R(\xi)^2). \tag{19}$$

The constants σ_0, χ, σ_i, and ρ_i (where $i=1,...,N$) will be provided, with N determined by the balancing procedure in Eq. (17).

Step 2: Eq. (19) gives the soliton waves

$$R(\xi)=\frac{4c}{4c^2e^{\xi}+\chi e^{-\xi}}, \tag{20}$$

where c is nonzero constant.

Step 3: By inserting Eq. (18) into Eq. (17), together with Eq. (19), we can derive the requisite constants for Eq. (16) and (18). In order to incorporate the identified parametric restrictions, they can be substituted into Eq. (18) together with Eq. (20). Consequently, straddled solitons are obtained, which can be further classified as bright, dark, or singular solitons.

The extended auxiliary equation scheme

This subsection presents a thorough overview of the basic procedures with the extended auxiliary equation technique.

Step 2: We assume that the solution of Eq. (17) can be expressed in the form

$$U(\xi)=\alpha_0+\sum_{i=1}^{N}\left\{\alpha_i\theta(\xi)^i+\beta_i\theta(\xi)^{-i}\right\}, \tag{21}$$

where $\theta(\xi)$ satisfies

$$\theta'(\xi)^2=\sum_{l=0}^{4}\tau_l\theta(\xi)^l. \tag{22}$$

This equation gives various kinds of solutions as follows

Case 1 $\tau_0=\tau_1=\tau_3=0$. Bright and singular soliton solutions are obtained:

$$\theta(\xi)=\sqrt{-\frac{\tau_2}{\tau_4}}\operatorname{sech}\left[\sqrt{\tau_2}\,\xi\right],\quad \tau_2>0,\ \tau_4<0, \tag{23}$$

and

$$\theta(\xi)=\sqrt{\frac{\tau_2}{\tau_4}}\operatorname{csch}\left[\sqrt{\tau_2}\,\xi\right],\quad \tau_2>0,\ \tau_4>0. \tag{24}$$

Case 2 $\tau_0=\frac{\tau_2^2}{4\tau_4}$, $\tau_1=\tau_3=0$.

Dark and singular soliton solutions are obtained:

$$\theta(\xi)=\sqrt{-\frac{\tau_2}{2\tau_4}}\tanh\left[\sqrt{-\frac{\tau_2}{2}}\,\xi\right],\quad \tau_2<0,\ \tau_4>0, \tag{25}$$

and

$$\theta(\xi)=\sqrt{-\frac{\tau_2}{2\tau_4}}\coth\left[\sqrt{-\frac{\tau_2}{2}}\,\xi\right],\quad \tau_2<0,\ \tau_4>0. \tag{26}$$

Case 3 $\tau_1=\tau_3=0$. A Weierstrass elliptic doubly periodic type solution is obtained:

$$\theta(\xi)=\frac{3\wp'(\xi;g_2,g_3)}{\sqrt{\tau_4}\left[6\wp(\xi;g_2,g_3)+\tau_2\right]}, \tag{27}$$

where $g_2=\frac{\tau_2^2}{12}+\tau_0\tau_4$ and $g_3=\frac{\tau_2(36\tau_0\tau_4-\tau_2^2)}{216}$ are called invariants of the Weierstrass elliptic function.

Case 4 $\tau_0=\tau_1=0$, τ_2, $\tau_4>0$, $\tau_3\neq\pm2\sqrt{\tau_2\tau_4}$ Straddled soliton solutions are obtained:

$$\theta(\xi)=\frac{-\tau_2\operatorname{sech}^2\left[\frac{1}{2}\sqrt{\tau_2}\,\xi\right]}{\pm2\sqrt{\tau_2\tau_4}\tanh\left[\frac{1}{2}\sqrt{\tau_2}\,\xi\right]+\tau_3}, \tag{28}$$

and

$$\theta(\xi)=\frac{\tau_2\operatorname{csch}^2\left[\frac{1}{2}\sqrt{\tau_2}\,\xi\right]}{\pm2\sqrt{\tau_2\tau_4}\coth\left[\frac{1}{2}\sqrt{\tau_2}\,\xi\right]+\tau_3}. \tag{29}$$

Step 3: Determine the positive integer number N in Eq. (8) by balancing the highest order derivatives and the nonlinear terms in Eq. (17).

Step 4: Substitute (21) into (17) along with (22). As a result of this substitution, we get a polynomial of $\theta(\xi)$. In this polynomial we gather all terms of same powers and equating them to be zero, we get an over-determined system of algebraic equations which can be solved together to get the unknown parameters k, v, α_0, α_i and β_i $(i=1,2,...)$. Consequently, we obtain the exact solutions of (15).

Application to the governing model

This section uses the two preceding techniques discussed in this work. Using these methods, we predict the emergence of bright and dark soliton solutions. This section will be divided into two subsections. At first, we will study the governing model in the case when the balance constant $N=1$. Subsequently, we will investigate the governing model under the issue where the balance constant $N=2$.

Case 1 $N=1$

The enhanced Kudryashov method

In accordance with the enhanced Kudryashov technique, the solution is expressed in the following structure

$$V(\xi) = a_0 + a_1 R(\xi) + b_1 \left(\frac{R'(\xi)}{R(\xi)}\right). \tag{30}$$

Plugging Eq. (30) together with Eq. (19) into Eq. (14), we get a system of algebraic equations. Solving these equations together yields the following result:

$$\begin{aligned} &a_0 = 0,\ a_1 = \pm\sqrt{-\frac{w_6\left(6n^3+11n^2+6n+1\right)\chi}{w_8\left(4n^3+2n^2+n+1\right)-w_4n^2\left(6n^2+5n+1\right)}},\\ &b_1 = 0,\ k = n\sqrt{\frac{w_8w_6(n+1)}{w_4n^2\left(6n^2+5n+1\right)-w_8\left(4n^3+2n^2+n+1\right)}},\\ &w_1 = \frac{w_6(2n+1)\left(w_4(3n+1)-2w_8\right)}{w_8\left(4n^3+2n^2+n+1\right)-w_4n^2\left(6n^2+5n+1\right)}. \end{aligned} \tag{31}$$

As a consequence, we obtain the exact solutions of Eq. (1) as follows

$$q(x,t) = \left\{\frac{\pm 4c\sqrt{-\frac{w_6\left(6n^3+11n^2+6n+1\right)\chi}{w_8\left(4n^3+2n^2+n+1\right)-w_4n^2\left(6n^2+5n+1\right)}}}{4c^2 e^{n\sqrt{\frac{w_8w_6(n+1)}{w_4n^2\left(6n^2+5n+1\right)-w_8\left(4n^3+2n^2+n+1\right)}}(x-vt)} + \chi e^{-n\sqrt{\frac{w_8w_6(n+1)}{w_4n^2\left(6n^2+5n+1\right)-w_8\left(4n^3+2n^2+n+1\right)}}(x-vt)}}\right\}^{\frac{1}{n}} \times e^{i\left(\left\{\frac{\delta_2(\sigma_8-\sigma_7)}{2\delta_3\sigma_{13}}\right\}x+\omega t+\sigma W(t)-\sigma^2 t+\theta_0\right)}. \tag{32}$$

Set $\chi = \pm 4c^2$ in solution (32). Consequently, for $w_4n^2\left(6n^2+5n+1\right) - w_8\left(4n^3+2n^2+n+1\right)$ and $w_8w_6(n+1) > 0$, we have bright soliton with $w_6 > 0$ and singular soliton with $w_6 < 0$

$$q(x,t) = \left\{\pm\sqrt{-\frac{w_6\left(6n^3+11n^2+6n+1\right)}{w_8\left(4n^3+2n^2+n+1\right)-w_4n^2\left(6n^2+5n+1\right)}} \times \operatorname{sech}\left[n\sqrt{\frac{w_8w_6(n+1)}{w_4n^2\left(6n^2+5n+1\right)-w_8\left(4n^3+2n^2+n+1\right)}}(x-vt)\right]^{\frac{1}{n}}\right\} \times e^{i\left(\left\{\frac{\delta_2(\sigma_8-\sigma_7)}{2\delta_3\sigma_{13}}\right\}x+\omega t+\sigma W(t)-\sigma^2 t+\theta_0\right)} \tag{33}$$

and

$$q(x,t) = \left\{\pm\sqrt{\frac{w_6\left(6n^3+11n^2+6n+1\right)}{w_8\left(4n^3+2n^2+n+1\right)-w_4n^2\left(6n^2+5n+1\right)}} \times \operatorname{csch}\left[n\sqrt{\frac{w_8w_6(n+1)}{w_4n^2\left(6n^2+5n+1\right)-w_8\left(4n^3+2n^2+n+1\right)}}(x-vt)\right]\right\}^{\frac{1}{n}} e^{i\left(\left\{\frac{\delta_2(\sigma_8-\sigma_7)}{2\delta_3\sigma_{13}}\right\}x+\omega t+\sigma W(t)-\sigma^2 t+\theta_0\right)}. \tag{34}$$

The extended auxiliary equation method

In accordance with the extended auxiliary equation technique, the solution is expressed in the following structure

$$V(\xi)=\alpha_0+\alpha_1\theta(\xi)+\frac{\beta_1}{\theta(\xi)}. \tag{35}$$

Plugging Eq. (35) together with Eq. (22) into Eq. (14), we get a system of algebraic equations. Solving these equations together yields the following results:

Case 1 $\tau_0=\tau_1=\tau_3=0$.

$$\begin{aligned}&\alpha_0=\beta_1=0,\ \tau_2=\frac{n^2\left(w_4k^2\left(6n^2+5n+1\right)-w_8w_6(n+1)\right)}{w_8k^2\left(4n^3+2n^2+n+1\right)},\ \tau_4=-\frac{\alpha_1^2w_8n^2}{k^2\left(6n^2+5n+1\right)},\\&w_1=-\frac{(2n+1)\left(w_4k^2(3n+1)+2w_8w_6n^2\right)\left(w_4k^2(n(6n+5)+1)-w_8w_6(n+1)\right)}{w_8^2k^2\left(4n^3+2n^2+n+1\right)^2}.\end{aligned} \tag{36}$$

For this case, the solution of (1) reads

$$q(x,t)=\left\{\pm\sqrt{\frac{(2n+1)(3n+1)\left(w_4k^2(n(6n+5)+1)-w_8w_6(n+1)\right)}{w_8^2(4n^3+2n^2+n+1)}}\times\operatorname{sech}\left[n\sqrt{\frac{w_4k^2(6n^2+5n+1)-w_8w_6(n+1)}{w_8(4n^3+2n^2+n+1)}}(x-vt)\right]\right\}^{\frac{1}{n}} e^{i\left(\left\{\frac{\delta_2(\sigma_8-\sigma_7)}{2\delta_3\sigma_{13}}\right\}x+\omega t+\sigma W(t)-\sigma^2t+\theta_0\right)}, \tag{37}$$

and

$$\begin{aligned}q(x,t)=&\left\{\pm\sqrt{-\frac{(2n+1)(3n+1)\left(w_4k^2(n(6n+5)+1)-w_8w_6(n+1)\right)}{w_8^2\left(4n^3+2n^2+n+1\right)}}\right.\\&\left.\times\operatorname{csch}\left[n\sqrt{\frac{w_4k^2\left(6n^2+5n+1\right)-w_8w_6(n+1)}{w_8\left(4n^3+2n^2+n+1\right)}}(x-vt)\right]\right\}^{\frac{1}{n}}\\&e^{i\left(\left\{\frac{\delta_2(\sigma_8-\sigma_7)}{2\delta_3\sigma_{13}}\right\}x+\omega t+\sigma W(t)-\sigma^2t+\theta_0\right)}.\end{aligned} \tag{38}$$

These solitons with $\tau_2>0$ are bright for $\tau_4<0$ and singular for $\tau_4>0$.

Case 2 $\tau_1=0,\ \tau_3=0,\ \tau_0=\frac{\tau_2^2}{4\tau_4}$

$$\begin{aligned}&\alpha_0=0,\ \alpha_1=\frac{\sqrt{-k^2w_8^3\left(6n^2+5n+1\right)\left(4n^3+2n^2+n+1\right)^2\tau_4}}{w_8^2n\left(4n^3+2n^2+n+1\right)},\\&\beta_1=\frac{n\sqrt{6n^2+5n+1}\left(w_4k^2\left(6n^2+5n+1\right)-w_8w_6(n+1)\right)}{4\sqrt{w_8^3\left(-k^2\right)\left(4n^3+2n^2+n+1\right)^2\tau_4}},\\&\tau_2=-\frac{n^2\left(w_4k^2\left(6n^2+5n+1\right)-w_8w_6(n+1)\right)}{2w_8k^2\left(4n^3+2n^2+n+1\right)},\\&w_1=-\frac{(2n+1)\left(w_4k^2(3n+1)+2w_8w_6n^2\right)\left(w_4k^2(n(6n+5)+1)-w_8w_6(n+1)\right)}{w_8^2k^2\left(4n^3+2n^2+n+1\right)^2}.\end{aligned} \tag{39}$$

For this case, the solution of (1) reads

$$\begin{aligned}q(x,t)=&\left\{\pm\sqrt{\frac{(2n+1)(3n+1)\left(w_8w_6(n+1)-w_4k^2(n(6n+5)+1)\right)}{w_8^2\left(4n^3+2n^2+n+1\right)}}\right.\\&\left.\times\operatorname{csch}\left[n\sqrt{\frac{w_4k^2(n(6n+5)+1)-w_8w_6(n+1)}{w_8\left(4n^3+2n^2+n+1\right)}}(x-vt)\right]\right\}^{\frac{1}{n}}\\&\times e^{i\left(\left\{\frac{\delta_2(\sigma_8-\sigma_7)}{2\delta_3\sigma_{13}}\right\}x+\omega t+\sigma W(t)-\sigma^2t+\theta_0\right)}.\end{aligned} \tag{40}$$

The obtained soliton is singular with $\tau_2 < 0$ and $\tau_4 > 0$.

Case 3 $\tau_1 = 0,\ \tau_3 = 0$

$$\alpha_0 = 0,\ \tau_0 = 0,\ \beta_1 = 0,\ \tau_2 = \frac{n^2\left(w_4k^2\left(6n^2+5n+1\right) - w_8w_6(n+1)\right)}{w_8k^2\left(4n^3+2n^2+n+1\right)},$$
$$\tau_4 = -\frac{\alpha_1^2 w_8 n^2}{k^2\left(6n^2+5n+1\right)},$$
$$w_1 = -\frac{(2n+1)\left(w_4k^2(3n+1)+2w_8w_6n^2\right)\left(w_4k^2(n(6n+5)+1)-w_8w_6(n+1)\right)}{w_8^2k^2\left(4n^3+2n^2+n+1\right)^2}. \tag{41}$$

For this case, the solution of (1) reads

$$q(x,t) = \left\{\frac{3\wp'(\xi;g_2,g_3)}{\sqrt{-\frac{w_8n^2}{k^2(6n^2+5n+1)}}\left[6\wp(\xi;g_2,g_3) + \frac{n^2\left(w_4k^2(6n^2+5n+1)-w_8w_6(n+1)\right)}{w_8k^2(4n^3+2n^2+n+1)}\right]}\right\}^{\frac{1}{n}} \times e^{i\left(\left\{\frac{\delta_2(\sigma_8-\sigma_7)}{2\delta_3\sigma_{13}}\right\}x+\omega t+\sigma W(t)-\sigma^2 t+\theta_0\right)}, \tag{42}$$

where

$$g_2 = \frac{\tau_2^2}{12},\ g_3 = -\frac{\tau_2^3}{216}. \tag{43}$$

The Weierstrass elliptic doubly periodic type solution (42) with its restricted invariants (43) can be converted to a singular soliton solution

$$q(x,t) = \left\{\pm\sqrt{\frac{(2n+1)(3n+1)\left(w_8w_6(n+1) - w_4k^2(n(6n+5)+1)\right)}{w_8^2\left(4n^3+2n^2+n+1\right)}} \times \operatorname{csch}\left[n\sqrt{\frac{w_4k^2(n(6n+5)+1) - w_8w_6(n+1)}{w_8\left(4n^3+2n^2+n+1\right)}}(x-vt)\right]\right\}^{\frac{1}{n}} \times e^{i\left(\left\{\frac{\delta_2(\sigma_8-\sigma_7)}{2\delta_3\sigma_{13}}\right\}x+\omega t+\sigma W(t)-\sigma^2 t+\theta_0\right)}. \tag{44}$$

Case 4 $\tau_0 = 0,\ \tau_1 = 0$

$$\alpha_0 = \beta_1 = 0,\ \tau_4 = -\frac{\alpha_1^2 w_8 n^2}{k^2\left(6n^2+5n+1\right)},$$
$$\tau_2 = \frac{n^2\left(w_4k^2(n(6n+5)+1) - w_8w_6(n+1)\right)}{w_8k^2\left(4n^3+2n^2+n+1\right)},\ \tau_3 = 0,$$
$$w_1 = -\frac{(2n+1)\left(w_4k^2(3n+1)+2w_8w_6n^2\right)\left(w_4k^2(n(6n+5)+1)-w_8w_6(n+1)\right)}{w_8^2k^2\left(4n^3+2n^2+n+1\right)^2}. \tag{45}$$

For this case, the obtained solitons are bright for $\tau_2 > 0$ and $\tau_4 < 0$ and singular for $\tau_2 > 0$ and $\tau_4 > 0$

and

$$q(x,t) = \left\{\pm\sqrt{-\frac{(2n+1)(3n+1)\left(w_4k^2(n(6n+5)+1) - w_8w_6(n+1)\right)}{w_8^2\left(4n^3+2n^2+n+1\right)}} \times \operatorname{csch}\left[n\sqrt{\frac{w_4k^2\left(6n^2+5n+1\right) - w_8w_6(n+1)}{w_8\left(4n^3+2n^2+n+1\right)}}(x-vt)\right]\right\}^{\frac{1}{n}} e^{i\left(\left\{\frac{\delta_2(\sigma_8-\sigma_7)}{2\delta_3\sigma_{13}}\right\}x+\omega t+\sigma W(t)-\sigma^2 t+\theta_0\right)}. \tag{47}$$

Case $2N = 2$

The enhanced Kudryashov method

In accordance with the enhanced Kudryashov technique, the solution is expressed in the following structure

$$V(\xi) = a_0 + a_1R(\xi) + b_1\left(\frac{R'(\xi)}{R(\xi)}\right) + a_2R(\xi)^2 + b_2\left(\frac{R'(\xi)}{R(\xi)}\right)^2. \tag{48}$$

Insert (48) together with (19) into Eq. (14) to get a system of algebraic equations. Solving these equations together yields the following result:

$$a_0 = -b_2,\ a_2 = b_2\chi \pm \sqrt{\frac{(n+1)(n+2)(3n+2)w_6\chi^2}{n^2(n(n+2)+2)w_4}},\quad k = \frac{1}{2}\sqrt{-\frac{n^2w_6}{n(n+2)+2}},$$
$$w_1 = -\frac{4(n+1)^2w_6}{n^2\left(n^2+2n+2\right)},\ w_8 = 0. \tag{49}$$

As a consequence, the solution of Eq. (1) reaches

$$q(x,t) = \left\{\pm\sqrt{\frac{(2n+1)(3n+1)\left(w_4k^2(n(6n+5)+1) - w_8w_6(n+1)\right)}{w_8^2\left(4n^3+2n^2+n+1\right)}} \times \operatorname{sech}\left[n\sqrt{\frac{w_4k^2\left(6n^2+5n+1\right) - w_8w_6(n+1)}{w_8\left(4n^3+2n^2+n+1\right)}}(x-vt)\right]\right\}^{\frac{1}{n}} \times e^{i\left(\left\{\frac{\delta_2(\sigma_8-\sigma_7)}{2\delta_3\sigma_{13}}\right\}x+\omega t+\sigma W(t)-\sigma^2 t+\theta_0\right)}, \tag{46}$$

$$q(x,t)=\left\{\chi\left(b_2\pm\sqrt{\frac{(n+1)(n+2)(3n+2)w_6}{n^2(n(n+2)+2)w_4}}\right)\left(\frac{4c}{4c^2e^{\frac{1}{2}\sqrt{-\frac{n^2w_6}{n(n+2)+2}}(x-vt)}+\chi e^{-\frac{1}{2}\sqrt{-\frac{n^2w_6}{n(n+2)+2}}(x-vt)}}\right)^2\right.$$
$$\left.+b_2\left(\frac{\chi-4c^2e^{\sqrt{-\frac{n^2w_6}{n(n+2)+2}}(x-vt)}}{\chi+4c^2e^{\sqrt{-\frac{n^2w_6}{n(n+2)+2}}(x-vt)}}\right)^2-b_2\right\}^{\frac{1}{n}}\quad e^{i\left(\left\{\frac{\delta_2(\sigma_8-\sigma_7)}{2\delta_3\sigma_{13}}\right\}x+\omega t+\sigma W(t)-\sigma^2t+\theta_0\right)}. \tag{50}$$

Set $\chi=\pm4c^2$ to recover bright and singular solitons for $w_4<0$ and $w_6<0$

$$q(x,t)=\left\{\pm\sqrt{\frac{(n+1)(n+2)(3n+2)w_6}{n^2(n(n+2)+2)w_4}}\operatorname{sech}^2\left[\frac{1}{2}\sqrt{-\frac{n^2w_6}{n(n+2)+2}}(x-vt)\right]\right\}^{\frac{1}{n}}\times e^{i\left(\left\{\frac{\delta_2(\sigma_8-\sigma_7)}{2\delta_3\sigma_{13}}\right\}x+\omega t+\sigma W(t)-\sigma^2t+\theta_0\right)}, \tag{51}$$

and

$$q(x,t)=\left\{\mp\sqrt{\frac{(n+1)(n+2)(3n+2)w_6}{n^2(n(n+2)+2)w_4}}\operatorname{csch}^2\left[\frac{1}{2}\sqrt{-\frac{n^2w_6}{n(n+2)+2}}(x-vt)\right]\right\}^{\frac{1}{n}}\times e^{i\left(\left\{\frac{\delta_2(\sigma_8-\sigma_7)}{2\delta_3\sigma_{13}}\right\}x+\omega t+\sigma W(t)-\sigma^2t+\theta_0\right)}. \tag{52}$$

The extended auxiliary equation method

In accordance with the extended auxiliary equation technique, the solution is expressed in the following structure

$$V(\xi)=\alpha_0+\alpha_1\theta(\xi)+\frac{\beta_1}{\theta(\xi)}+\alpha_2\theta(\xi)^2+\frac{\beta_2}{\theta(\xi)^2}. \tag{53}$$

Plugging Eq. (53) together with Eq. (22) into Eq. (14), we get a system of algebraic equations. Solving these equations together yields the following results:

Case 1 $\tau_0=\tau_1=\tau_3=0$.

$$\begin{aligned}\alpha_0&=\alpha_1=\beta_1=\beta_2=0,\\ \alpha_2&=\pm\frac{2k\sqrt{3n^3+11n^2+12n+4}\tau_4}{\sqrt{-w_4}n^2},\\ \tau_2&=-\frac{w_6n^2}{4k^2\left(n^2+2n+2\right)},\\ w_8&=0,\ w_1=\frac{w_6^2(n+1)^2}{k^2\left(n^2+2n+2\right)^2}.\end{aligned} \tag{54}$$

For this case, the solution of (1) reads

$$q(x,t)=\left\{\pm\frac{w_6\sqrt{(n+1)(n+2)(3n+2)}}{2\sqrt{-w_4}k(n(n+2)+2)}\operatorname{sech}^2\left[\frac{n}{2}\sqrt{-\frac{w_6}{n^2+2n+2}}(x-vt)\right]\right\}^{\frac{1}{n}}\times e^{i\left(\left\{\frac{\delta_2(\sigma_8-\sigma_7)}{2\delta_3\sigma_{13}}\right\}x+\omega t+\sigma W(t)-\sigma^2t+\theta_0\right)}, \tag{55}$$

and

$$q(x,t)=\left\{\mp\frac{w_6\sqrt{(n+1)(n+2)(3n+2)}}{2\sqrt{-w_4}k(n(n+2)+2)}\text{csch}^2\left[\frac{n}{2}\sqrt{-\frac{w_6}{n^2+2n+2}}(x-vt)\right]\right\}^{\frac{1}{n}}\times e^{i\left(\left\{\frac{\delta_2(\sigma_8-\sigma_7)}{2\delta_3\sigma_{13}}\right\}x+\omega t+\sigma W(t)-\sigma^2 t+\theta_0\right)}. \tag{56}$$

These solutions are bright and singular solitons with $w_4 < 0$ and $w_6 < 0$.

Case 2 $\tau_0 = \frac{\tau_2^2}{4\tau_4},\ \tau_1 = \tau_3 = 0.$

$$\begin{aligned}&\alpha_1=\beta_1=\beta_2=0,\ \alpha_0=\pm\frac{w_6}{2}\sqrt{-\frac{(n+1)(n+2)(3n+2)}{w_4k^2(n(n+2)+2)^2}},\\&\tau_2=\frac{w_6n^2}{2k^2(n(n+2)+2)},\\&\tau_4=\mp\frac{\alpha_2\sqrt{-w_4}n^2}{2k\sqrt{3n^3+11n^2+12n+4}},\\&w_8=0,\ w_1=\frac{w_6^2(n+1)^2}{k^2\left(n^2+2n+2\right)^2}.\end{aligned} \tag{57}$$

For this case, the solution of (1) reads

$$q(x,t)=\left\{\pm\frac{w_6\sqrt{(n+1)(n+2)(3n+2)}}{2\sqrt{-w_4}k(n(n+2)+2)}\text{sech}^2\left[\frac{n}{2}\sqrt{-\frac{w_6}{n^2+2n+2}}(x-vt)\right]\right\}^{\frac{1}{n}}\times e^{i\left(\left\{\frac{\delta_2(\sigma_8-\sigma_7)}{2\delta_3\sigma_{13}}\right\}x+\omega t+\sigma W(t)-\sigma^2 t+\theta_0\right)}, \tag{58}$$

and

$$q(x,t)=\left\{\mp\frac{w_6\sqrt{(n+1)(n+2)(3n+2)}}{2\sqrt{-w_4}k(n(n+2)+2)}\text{csch}^2\left[\frac{n}{2}\sqrt{-\frac{w_6}{n^2+2n+2}}(x-vt)\right]\right\}^{\frac{1}{n}}\times e^{i\left(\left\{\frac{\delta_2(\sigma_8-\sigma_7)}{2\delta_3\sigma_{13}}\right\}x+\omega t+\sigma W(t)-\sigma^2 t+\theta_0\right)}. \tag{59}$$

Case 3 $\tau_1 = \tau_3 = 0.$

$$\begin{aligned}&\alpha_0=0,\ \alpha_1=\beta_1=\beta_2=0,\ \tau_0=0,\\&\tau_2=-\frac{w_6n^2}{4k^2\left(n^2+2n+2\right)},\\&\tau_4=\pm\frac{\alpha_2\sqrt{w_4}n^2}{2\sqrt{-k^2\left(3n^3+11n^2+12n+4\right)}},\\&w_8=0,\ w_1=\frac{w_6^2(n+1)^2}{k^2\left(n^2+2n+2\right)^2}.\end{aligned} \tag{60}$$

For this case, the solution of (1) reads

$$q(x,t)=\left\{\mp\frac{92\sqrt{-k^2\left(3n^3+11n^2+12n+4\right)}\left(\wp'\left(\xi;g_2,g_3\right)\right)^2}{n^2\sqrt{w_4}\left[6\wp\left(\xi;g_2,g_3\right)-\frac{w_6n^2}{4k^2\left(n^2+2n+2\right)}\right]^2}\right\}^{\frac{1}{n}}\times e^{i\left(\left\{\frac{\delta_2(\sigma_8-\sigma_7)}{2\delta_3\sigma_{13}}\right\}x+\omega t+\sigma W(t)-\sigma^2 t+\theta_0\right)}, \tag{61}$$

where

$$g_2=\frac{\tau_2^2}{12},\ g_3=-\frac{\tau_2^3}{216}. \tag{62}$$

The Weierstrass elliptic doubly periodic type solution (61) with its restricted invariants (62) can be converted to a singular soliton solution

$$q(x,t)=\left\{\mp\frac{23w_6\sqrt{-k^2(n+1)(n+2)(3n+2)}}{9\sqrt{w_4}k^2(n(n+2)+2)}\text{csch}^2\left[\frac{n}{2}\sqrt{-\frac{w_6}{n^2+2n+2}}(x-vt)\right]\right\}^{\frac{1}{n}}\\ timese^{i\left(\left\{\frac{\delta_2(\sigma_8-\sigma_7)}{2\delta_3\sigma_{13}}\right\}x+\omega t+\sigma W(t)-\sigma^2 t+\theta_0\right)}. \tag{63}$$

Case 4 $\tau_0 = \tau_1 = 0.$

$$\begin{aligned}&\alpha_0=0,\ \alpha_1=\beta_1=\beta_2=0,\ \tau_3=0,\\&\tau_2=-\frac{w_6n^2}{4k^2\left(n^2+2n+2\right)},\\&\tau_4=\pm\frac{\alpha_2\sqrt{w_4}n^2}{2\sqrt{-k^2\left(3n^3+11n^2+12n+4\right)}},\\&w_8=0,\ w_1=\frac{w_6^2(n+1)^2}{k^2\left(n^2+2n+2\right)^2}.\end{aligned} \tag{64}$$

For this case, the solution of (1) reads

$$q(x,t)=\left\{\pm\frac{w_6\sqrt{(n+1)(n+2)(3n+2)}}{2\sqrt{-w_4}k(n(n+2)+2)}\operatorname{sech}^2\left[\frac{n}{2}\sqrt{-\frac{w_6}{n^2+2n+2}}(x-vt)\right]\right\}^{\frac{1}{n}}\times e^{i\left(\left\{\frac{\delta_2(\sigma_8-\sigma_7)}{2\delta_3\sigma_{13}}\right\}x+\omega t+\sigma W(t)-\sigma^2 t+\theta_0\right)}, \tag{65}$$

and

$$q(x,t)=\left\{\mp\frac{w_6\sqrt{(n+1)(n+2)(3n+2)}}{2\sqrt{-w_4}k(n(n+2)+2)}\operatorname{csch}^2\left[\frac{n}{2}\sqrt{-\frac{w_6}{n^2+2n+2}}(x-vt)\right]\right\}^{\frac{1}{n}}\times e^{i\left(\left\{\frac{\delta_2(\sigma_8-\sigma_7)}{2\delta_3\sigma_{13}}\right\}x+\omega t+\sigma W(t)-\sigma^2 t+\theta_0\right)}. \tag{66}$$

These solutions are bright and singular solitons with $w_4 < 0$ and $w_6 < 0$.

The enhanced Kudryashov's approach and the extended auxiliary equation technique were unsuccessful in retrieving dark solitons within the constraints of the governing model.

Conclusions

The paper studied the dispersive concatenation model with power law of SPM in presence of white noise. The bright and singular soliton solutions to the model were retrieved using a couple of integration algorithms. They are the enhanced Kudryashov's approach and the extended auxiliary equation scheme. These methodologies failed to furnish the dark soliton solutions to the model. This is because of the fact that the model with power law of SPM can recover dark solitons only if the power law collapses to Kerr law, a fact that has been experimentally proven earlier. There is another observation that has been made in the work. The effect of white noise is confined to the phase component of the bright and singular soliton solutions and does not affect the amplitude components of such solitons.

The results of the paper are thus indeed promising. The studies with the current model can be extended further. For example, this research can be carried out with additional optoelectronic devices such as optical couplers, optical metamaterials, PCF, gap solitons and many others. Later, the studies can be extended to fibers with differential group delay as well as dispersion–flattened fibers. The numerical scheme such as variational iteration method and/or Laplace–Adomian decomposition schemes will be implemented to achieve a numerical picture to the studies. The bifurcation analysis and additional analytical aspects are yet to be covered. These results will be disseminated after they are available and aligned with the pre–existing ones [16–58]

References

1. A. Ankiewicz, N. Akhmediev, Higher-order integrable evolution equation and its soliton solutions. Phys. Lett. A **378**, 358–361 (2014)
2. A. Ankiewicz, Y. Wang, S. Wabnitz, N. Akhmediev, Extended nonlinear Schrödinger equation with higher-order odd and even terms and its rogue wave solutions. Phys. Rev. E **89**, 012907 (2014)
3. A. Chowdury, D.J. Kedziora, A. Ankiewicz, N. Akhmediev, Soliton solutions of an integrable nonlinear Schrödinger equation with quintic terms. Phys. Rev. E **91**, 032922 (2014)
4. A. Chowdury, D.J. Kedziora, A. Ankiewicz, N. Akhmediev, Breather-to-soliton conversions described by the quintic equation of the nonlinear Schrödinger hierarchy. Phys. Rev. E **91**, 032928 (2015)
5. A. Chowdury, D.J. Kedziora, A. Ankiewicz, N. Akhmediev, Breather solutions of the integrable quintic nonlinear Schrödinger equation and their interactions. Phys. Rev. E **91**, 022919 (2015)
6. A. H. Arnous, A. Biswas, A. H. Kara, Y. Yıldırım & A. Asiri. Optical solitons and conservation laws for the dispersive concatenation model with power–law nonlinearity. J. Opt. (2023) https://doi.org/10.1007/s12596-023-01453-x
7. A.H. Arnous, A. Biswas, Y. Yildirim, A. Asiri, Quiescent optical solitons for the concatenation model having nonlinear chromatic dispersion with differential group delay. Contemp. Math. **4**(4), 877–904 (2023)
8. A. H. Arnous, M. Mirzazadeh, A. Biswas, Y. Yildirim, H. Triki & A. Asiri. A wide spectrum of optical solitons for the dispersive concatenation model. J. Opti. https://doi.org/10.1007/s12596-023-01383-8
9. E. M. E. Zayed, A. H. Arnous, A. Biswas, Y. Yildirim & A. Asiri. Optical solitons for the concatenation model with multiplicative white noise. J. Opt. (2023) https://doi.org/10.1007/s12596-023-01381-w
10. A.H. Arnous, Optical solitons with Biswas–Milovic equation in magneto-optic waveguide having Kudryashov's law of refractive index''. Optik **247**, 167987 (2021). https://doi.org/10.1016/j.ijleo.2021.167987
11. A.M. Elsherbany, A.H. Arnous, A.J.M. Jawad, A. Biswas, Y. Yildirim, L. Moraru, A.S. Alshomrani, Quiescent optical solitons for the dispersive concatenation model with Kerr law of

nonlinearity having nonlinear chromatic dispersion. Ukr. J. Phys. Opt. **25**(1), 01054–01064 (2024)
12. A. H. Arnous, A. Biswas, Y. Yildirim, A. S. Alshomrani. Stochastic perturbation of optical solitons for the concatenation model with power–law of self–phase modulation having multiplicative white noise. Contemp. Math. (2024) https://doi.org/10.37256/cm.5120244107
13. A. H. Arnous, A. Biswas, Y. Yildirim & A. S. Alshomrani. Optical solitons with dispersive concatenation model having multiplicative white noise by the enhanced direct algebraic method. Submitted
14. A. Biswas, J.M. Vega-Guzman, Y. Yildirim, S.P. Moshokoa, M. Aphane, A.A. Alghamdi, Optical solitons for the concatenation model with power-law nonlinearity: undetermined coefficients. Ukr. J. Phys. Opt. **24**(3), 185–192 (2023)
15. O. Gonzalez-Gaxiola, A. Biswas, J.R. de Chavez, A. Asiri, Bright and dark optical solitons for the concatenation model by Laplace-Adomian decomposition scheme. Ukr. J. Phys. Opt. **24**(3), 222–234 (2023)
16. A.J.M. Jawad, M.J.A. Al-Shaeer, Highly dispersive optical solitons with cubic law and cubic-quintic-septic law nonlinearities by two methods. Al-Rafidain J. Eng. Sci. **1**(1), 1–8 (2023)
17. A.J.M. Jawad, A. Biswas, Solutions of resonant nonlinear Schrödinger's equation with exotic non-Kerr law nonlinearities. Al-Rafidain J. Eng. Sci. **2**(1), 43–50 (2024)
18. N. Jihad, M.A.A. Almuhsan, Evaluation of impairment mitigations for optical fiber communications using dispersion compensation techniques. Al-Rafidain J. Eng. Sci. **1**(1), 81–92 (2023)
19. Z. Li & E. Zhu. Optical soliton solutions of stochastic Schrödinger-Hirota equation in birefringent fibers with spatiotemporal dispersion and parabolic law nonlinearity". J. Opt. https://doi.org/10.1007/s12596-023-01287-7
20. S. Nandy, V. Lakshminarayanan, Adomian decomposition of scalar and coupled nonlinear Schrödinger equations and dark and bright solitary wave solutions. J. Opt. **44**, 397–404 (2015)
21. A. Secer, Stochastic optical solitons with multiplicative white noise via Itô calculus. Optik **268**, 169831 (2022)
22. L. Tang. Bifurcations and optical solitons for the coupled nonlinear Schrödinger equation in optical fiber Bragg gratings. J. Opt. **52**, 1388–1398. https://doi.org/10.1007/s12596-022-00963-4
23. L. Tang, Phase portraits and multiple optical solitons perturbation in optical fibers with the nonlinear Fokas–Lenells equation. J. Opt. **52**(4), 2214–2223 (2023)
24. Y. Zhong, H. Triki, Q. Zhou, Analytical and numerical study of chirped optical solitons in a spatially inhomogeneous polynomial law fiber with parity-time symmetry potential. Commun. Theor. Phys. **75**, 025003 (2023)
25. Q. Zhou, Influence of parameters of optical fibers on optical soliton interactions. Chin. Phys. Lett. **39**(1), 010501 (2022)
26. E.M. Zayed, R.M. Shohib, A.G. Al-Nowehy, On solving the (3+ 1)-dimensional NLEQZK equation and the (3+ 1)-dimensional NLmZK equation using the extended simplest equation method. Comput. Math. Appl. **78**(10), 3390–3407 (2019)
27. E.M. Zayed, R.M. Shohib, A.G. Al-Nowehy, Solitons and other solutions for higher-order NLS equation and quantum ZK equation using the extended simplest equation method. Comput. Math. Appl. **76**(9), 2286–2303 (2018)
28. E.M. Zayed, M.E. Alngar, R.M. Shohib, Cubic-quartic embedded solitons with χ (2) and χ (3) nonlinear susceptibilities having multiplicative white noise via Itô calculus. Chaos Solitons Fract. **168**, 113186 (2023)
29. E.M. Zayed, M.E. Alngar, R.M. Shohib, Dispersive optical solitons to stochastic resonant NLSE with both spatio-temporal and inter-modal dispersions having multiplicative white noise. Mathematics **10**(17), 3197 (2022)
30. E.M. Zayed, R.M. Shohib, M.E. Alngar, Cubic-quartic optical solitons in Bragg gratings fibers for NLSE having parabolic nonlocal law nonlinearity using two integration schemes. Opt. Quant. Electron. **53**(8), 452 (2021)
31. E.M. Zayed, K.A. Gepreel, R.M. Shohib, M.E. Alngar, Solitons in magneto-optics waveguides for the nonlinear Biswas–Milovic equation with Kudryashov's law of refractive index using the unified auxiliary equation method. Optik **235**, 166602 (2021)
32. E.M. Zayed, K.A. Gepreel, R.M. Shohib, M.E. Alngar, Y. Yıldırım, Optical solitons for the perturbed Biswas–Milovic equation with Kudryashov's law of refractive index by the unified auxiliary equation method. Optik **230**, 166286 (2021)
33. Zayed, E. M. E., & Shohib, R. M. A. (2019). Solitons and other solutions for two higher-order nonlinear wave equations of KdV type using the unified auxiliary equation method. Acta Physica Polonica A 136(1)
34. E.M. Zayed, R.M. Shohib, K.A. Gepreel, M.M. El-Horbaty, M.E. Alngar, Cubic-quartic optical soliton perturbation Biswas–Milovic equation with Kudryashov's law of refractive index using two integration methods. Optik **239**, 166871 (2021)
35. E.M. Zayed, R.M. Shohib, M.E. Alngar, A. Biswas, S. Khan, Y. Yıldırım, H. Triki, A.K. Alzahrani, M.R. Belic, Cubic-quartic optical solitons with Kudryashov's arbitrary form of nonlinear refractive index. Optik **238**, 166747 (2021)
36. E. Zayed, R. Shohib, M. Alngar, A. Biswas, M. Ekici, S. Khan, A.K. Alzahrani, M. Belic, Optical solitons and conservation laws associated with Kudryashov's sextic power-law nonlinearity of refractive index. Ukr. J. Phys. Opt. **22**, 38–49 (2021)
37. E.M. Zayed, R.M. Shohib, M.E. Alngar, Y. Yıldırım, Optical solitons in fiber Bragg gratings with Radhakrishnan–Kundu–Lakshmanan equation using two integration schemes. Optik **245**, 167635 (2021)
38. E.M. Zayed, R.M. Shohib, M.M. El-Horbaty, A. Biswas, M. Asma, M. Ekici, A.K. Alzahrani, M.R. Belic, Solitons in magneto-optic waveguides with quadratic-cubic nonlinearity. Phys. Lett. A **384**(25), 126456 (2020)
39. S.A. AlQahtani, R.M. Shohib, M.E. Alngar, A.M. Alawwad, High-stochastic solitons for the eighth-order NLSE through Itô calculus and STD with higher polynomial nonlinearity and multiplicative white noise. Opt. Quant. Electron. **55**(14), 1227 (2023)
40. S.A. AlQahtani, M.S. Al-Rakhami, R.M.A. Shohib, M.E.M. Alngar, P. Pathak, Dispersive optical solitons with Schrödinger-Hirota equation using the -model expansion approach. Opt. Quant. Electron. **55**, 701 (2023)
41. A.R. Adem, T.J. Podile, B. Muatjetjeja, A generalized (3+ 1)-dimensional nonlinear wave equation in liquid with gas bubbles: symmetry reductions; exact solutions; conservation laws. Int. J. Appl. Comput. Math. **9**(5), 82 (2023)
42. I. Humbu, B. Muatjetjeja, T.G. Motsumi, A.R. Adem, Solitary waves solutions and local conserved vectors for extended quantum Zakharov–Kuznetsov equation. Eur. Phys. J. Plus **138**(9), 873 (2023)
43. M.C. Sebogodi, B. Muatjetjeja, A.R. Adem, Exact solutions and conservation laws of a (2+ 1)-dimensional combined potential kadomtsev–petviashvili-b-type kadomtsev–petviashvili equation. Int. J. Theor. Phys. **62**(8), 165 (2023)
44. I. Humbu, B. Muatjetjeja, T.G. Motsumi, A.R. Adem, Periodic solutions and symmetry reductions of a generalized Chaffee–Infante equation. Partial Diff. Equ. Appl. Math. **7**, 100497 (2023)
45. A.R. Adem, T.S. Moretlo, B. Muatjetjeja, A generalized dispersive water waves system: Conservation laws; symmetry reduction; travelling wave solutions; symbolic computation. Partial Diff. Equ. Appl. Math. **7**, 100465 (2023)
46. A.R. Adem, B. Muatjetjeja, T.S. Moretlo, An extended (2+ 1)-dimensional coupled burgers system in fluid mechanics:

symmetry reductions; Kudryashov method; conservation laws. Int. J. Theor. Phys. **62**(2), 38 (2023)
47. A.R. Adem, B. Muatjetjeja, Conservation laws and exact solutions for a 2D Zakharov–Kuznetsov equation. Appl. Math. Lett. **48**, 109–117 (2015)
48. A.R. Adem, The generalized (1+ 1)-dimensional and (2+ 1)-dimensional Ito equations: multiple exp-function algorithm and multiple wave solutions. Comput. Math. Appl. **71**(6), 1248–1258 (2016)
49. A.R. Adem, X. Lü, Travelling wave solutions of a two-dimensional generalized Sawada–Kotera equation. Nonlinear Dyn. **84**, 915–922 (2016)
50. A.R. Adem, Solitary and periodic wave solutions of the Majda–Biello system. Mod. Phys. Lett. B **30**(15), 1650237 (2016)
51. Adem, A. R. A (2+ 1)-dimensional Korteweg-de Vries type equation in water waves: Lie symmetry analysis; multiple exp-function method; conservation laws. Int. J. Modern Phys. B **30**, 1640001 (2016)
52. E.M. Zayed, M.E. Alngar, R.M. Shohib, A. Biswas, Y. Yıldırım, L. Moraru, S. Moldovanu, P.L. Georgescu, Dispersive optical solitons with differential group delay having multiplicative white noise by ito calculus. Electronics **12**(3), 634 (2023)
53. A. Biswas, J. Vega-Guzman, A.H. Kara, S. Khan, H. Triki, O. González-Gaxiola, L. Moraru, P.L. Georgescu, Optical solitons and conservation laws for the concatenation model: undetermined coefficients and multipliers approach. Universe **9**(1), 15 (2022)
54. A. H. Arnous, A. Biswas, A. H. Kara, Y. Yıldırım, L. Moraru, S. Moldovanu, P. L. Georgescu, A. A. Alghamdi, Dispersive optical solitons and conservation laws of Radhakrishnan–Kundu–Lakshmanan equation with dual-power law nonlinearity. Heliyon (2023) https://doi.org/10.1016/j.heliyon.2023.e14036
55. E.M. Zayed, M. El-Horbaty, M.E. Alngar, R.M. Shohib, A. Biswas, Y. Yıldırım, L. Moraru, C. Iticescu, D. Bibicu, P.L. Georgescu, A. Asiri, Dynamical system of optical soliton parameters by variational principle (super-Gaussian and super-sech pulses). J. Eur. Opt. Soc. **19**(2), 38 (2023)
56. R.M.A. Shohib, M.E.M. Alngar, A. Biswas, Y. Yildirim, H. Triki, L. Moraru, C. Iticescu, P.L. Georgescu, A. Asiri, Optical solitons in magneto-optic waveguides for the concatenation model. Ukr. J. Phys. Opt. **24**, 248–261 (2023)
57. A.H. Arnous, A. Biswas, Y. Yildirim, L. Moraru, C. Iticescu, P.L. Georgescu, A. Asiri, Optical solitons and complexitons for the concatenation model in birefringent fibers. Ukr. J. Phys. Opt. **24**, 04060–04086 (2023)
58. E.M.E. Zayed, M.E.M. Alngar, R.M.A. Shohib, A. Biswas, Y. Yildirim, L. Moraru, P.L. Georgescu, C. Iticescu, Siri, A., Highly dispersive solitons in optical couplers with metamaterials having Kerr law of nonlinear refractive index. Ukr. J. Phys. Opt. **25**, 1001–1019 (2024)