



LED diffused transmission freeform surface design for uniform illumination

Zhen-Min Zhu¹ · Jie Yuan¹ · Xiang Sun¹ · Bo Peng¹ · Xin Xu¹ · Quan-Xin Liu¹

Received: 31 May 2018 / Accepted: 14 December 2018 / Published online: 15 February 2019
© The Author(s) 2019

Abstract LED diffuse transmission has been widely applied in backlight display area. However, the application of diffuse transmission is mostly based on experimental method and lack of theoretical analysis. Based on the above problems, an approximate numerical method is proposed to design diffuse transmission freeform surfaces. First, based on the transmission surface and the property of the LED, a mathematical model is established according to the property of LED, and the nonlinear equation of the illuminance distribution on the diffuse transmission inner surface is deduced. Then, by solving the equation, the profiles of diffuse transmission freeform surface for an indirect illumination system are obtained. Finally, the non-sequential ray tracing is used to detect the performance of the illuminance distribution on the target plane. Experimental results show that the diffuse transmission freeform surface has a better lighting performance than traditional diffuse transmission lighting systems.

Keywords Diffuse transmission · Optical design · LED · Freeform surface · Uniform illumination

Introduction

LED has many advantages such as high efficiency, energy saving, environmental protection, long life and small volume [1–3]. It is rapidly replacing traditional light sources in many applications. However, LED has the disadvantages

of too large divergence angle and uneven distribution of illuminance, making it difficult to directly apply to lighting [2]. In order to improve the lighting performance in the effective area of the target plane, it is usually necessary to perform a secondary optical design to redistribute LED illuminance pattern [4, 5].

In recent years, many scholars have conducted secondary optical design of LEDs. Moreno had put forward an optical design method of direct lighting [6, 7]. Compared with direct lighting methods, redistribution of emitted light by catadioptric is an effective way to improve illuminance performance [8–13]. However, the traditional reflector or refractive lens has a high requirement for the processing precision and the material. In addition, in many cases, we need to use diffuse transmission to improve the uniformity of illumination system such as industrial inspections and/or real life [14, 15]. Ripoll J proposed the expression of the reflection and transmission coefficients at the diffusion–diffusion interface [16]. M. Pang proposed a novel measurement method for obtaining a large laser beam intensity distribution based on CCD diffuse transmission imaging [17]. Serra et al. [18] investigate the CW transmitted light profiles when an inhomogeneity is immersed in highly scattering media slabs. Wu et al. [19] through the analysis of single-layer and multilayer diffuse transmission plane influence on the luminous intensity distribution concluded that increasing the distance between the first diffuse screen and the target plane can improve the spot uniformity. Bressel and Reich [20] applied the Mie theory to calculate the scattering properties of the material. These scholars studied the characteristics of the illuminance distribution through the diffuse transmission plane. In the aspect of diffuse applications, Kim et al. [21] presented an integrated prism with a bridge pattern of quantum dot (QD) film to improve the efficiency of the LED backlight unit. Qin [22]

✉ Zhen-Min Zhu
zhuzhenmin1984@163.com

¹ School of Electrical and Automation Engineering, East China Jiaotong University, Nanchang 330013, China

enhanced the brightness of backlight by solving the related characteristics of prism film angle. Wang et al. [23] compared the distance–height ratio of different LED lighting systems and proposed a new reversing design method, and a kind of ultrathin direct-lit LED backlight system is designed. The above research on diffuse transmission has achieved desired results, but their design is only based on experiments or direct applications of diffuse transmission, with no direct design of diffuse surface to enhance performance of the whole system.

A mathematical model for diffuse transmission is presented in this paper, and the diffuse transmission surface is designed, which improved the illumination uniformity in the specific area of the target plane. Firstly, according to the illumination requirements of the transmission surface and the target plane, a series of nonlinear functions on profiles of the diffuse transmission freeform surface are established. Then, solve the equation numerically to get a set of point coordinates that describe the contour of the surface. Using the MATLAB data interaction and optical software TracePro, a solid model of the light source is obtained. Finally, the illumination performance of several lighting systems is compared through experiments, and the experimental results testify the validity of the design method.

The design of diffuse transmission of freeform surface

In practice, this dependence turns out to be a power law that mostly depends on the encapsulant and semiconductor region shapes. The irradiance distribution is given by [6, 7]

$$E = E_0 \cdot \cos^m(\phi) \tag{1}$$

E is the illuminance distribution on the light surface. E_0 is the distribution of illuminance on the freeform surface of LED light source when the $\angle\theta$ is 0° . $\angle\theta$ is the angle between the light from the LED and the direction of the vertical freeform surface. m , as Eq. (2), is generally provided by the LED manufacturer.

$$m = \frac{-\ln 2}{\ln(\cos \phi_{1/2})} \tag{2}$$

where $\phi_{1/2}$ is the angle between the direction of the normal direction and the 1/2 direction of the maximum luminous intensity, that is, the angle corresponding to half of the maximum luminous intensity value. In this article, the m value is 1 because of the most common Lambert body. The luminous intensity of the light emitted by the LED source in each direction of the receiver is $I(\theta)$ as Eq. (3).

$$I(\phi) = I_0 \cdot \cos^m(\phi) \tag{3}$$

I_0 is the luminous intensity of the LED light source on the freeform surface when the $\angle\theta$ is 0° . According to the inverse square law of illuminance, as shown in Fig. 1, the illuminance produced by a LED on the receiving surface is like Eq. (4).

$$E_0 = \frac{I_0}{R^2} \tag{4}$$

The R is the vertical distance between the LED and the target plane. From Eq. (4), we can get the illuminance distribution at any point on the freeform surface, and its illuminance is obtained from Eq. (5).

$$E_i(r, \phi) = I_0 \cdot \cos^m(\phi) \cdot r^{-2} \tag{5}$$

E represents the illuminance distribution of any point in the target plane.

Figure 2a, b shows the Cartesian rectangular coordinate system. Substitute Eq. (1) into Eq. (6).

$$E_i(x, y, z) = \sum_{s=1}^n \frac{I_0 \cdot \cos^m(\phi)}{(x_i - x_s)^2 + (y_i - y_s)^2 + z_i^2} \tag{6}$$

$E_i(x, y, z)$ represents the illuminance of arbitrary coordinate positions of diffuse transmission freeform surface. (x_i, y_i, z_i) represents the coordinates on the freeform surface, (x_s, y_s) represents the position of the LED light source, and n represents the number of discrete points on the freeform surface. The illuminance generated by all LED array light sources is superimposed on the diffuse transmission freeform surface.

According to Fig. 2a, Eq. (6) can also be expressed as Eq. (7).

$$E_i(x, y, z) = \sum_{s=1}^n z_i^m \cdot I_0 \cdot \left[(x_i - x_s)^2 + (y_i - y_s)^2 + z_i^2 \right]^{-\frac{m+2}{2}} \tag{7}$$

To simplify the calculation, a number of sample points $(x_1, 0, z_1), (x_2, 0, z_2), \dots, (x_u, 0, z_u)$ which can be regarded as Lambertian secondary point sources are selected to approximate the outline of the freeform surface. Suppose that an outgoing light L_i of one LED $(x_i, y_i, 0)$ is $L_i(x_i, 0, z_i)$ on the transmission surface. As the Lambertian secondary point sources on the freeform surface, the illuminance of L_i on the target plane is obtained from Eq. (8).

$$E_j(z_j, \phi_j) = E_{0j}(z_j) \cdot \cos(\phi_j) \tag{8}$$

Fig. 1 LED at (x, y) point of illuminance produced by the schematic

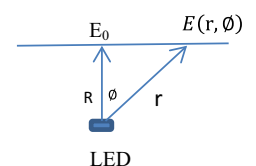
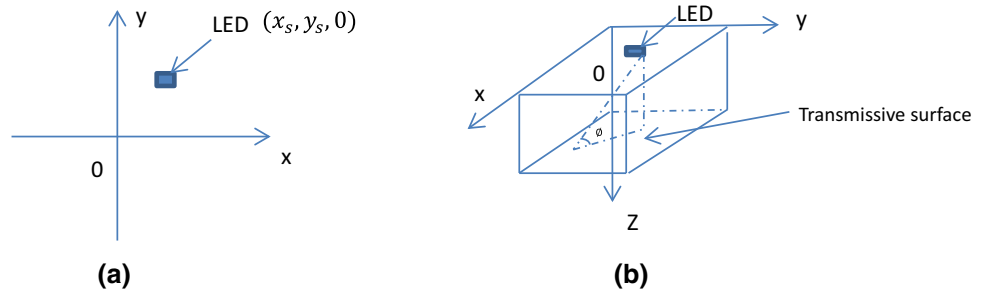


Fig. 2 LED in a Cartesian coordinate system in the position



where z_j is the distance between LED and the target plane, θ_j is the viewing angle and E_{0j} is the illuminance distribution at a z_j distance and $\angle\theta_j = 0^\circ$. As shown in Fig. 3, we establish the mathematical model of the surface profile of the diffuse transmission freeform surface. Considering that the diffuse transmission freeform surface model we have established is a rotational symmetry model, it is only necessary to design a curve in the freeform in this paper. Therefore, we simplify a three-dimensional problem to a two-dimensional problem; that is, only one of the contour lines in the freeform surface can be obtained through rotation to obtain the whole freeform surface model.

Equation (8) is transformed into Cartesian coordinate system and then converted to Eq. (9).

$$E_j(x, 0, R) = \sum_{i=1}^u z_j^m \cdot I_j \cdot [(x_j - x_i)^2 + R^2]^{-\frac{m+2}{2}} \quad (9)$$

u is the number of sampling points of the freeform surface in the x -axis direction. Through ray tracing, the vector from point $(x_i, 0, z_i)$ to point $(x_j, 0, R)$ is emergent ray $\overrightarrow{O\text{U}} = (x_j - x_i, R - z_i)$, x_j is the coordinates of the target plane along the x -axis, and R is a constant. The normal vector on the diffuse transmission freeform surface is $\overrightarrow{\text{NOR}} = (dz, dx)$. Here dz and dx represent the differential of z and x , respectively. Assume that the included angle between $\overrightarrow{O\text{U}}$ and $\overrightarrow{\text{NOR}}$ is $\angle\delta$, and Eq. (10) is obtained.

$$\cos \delta = \frac{(x_j - x_i) \cdot dz + (R - z_i) \cdot dx}{\sqrt{(x_j - x_i)^2 + (R - z_i)^2} \cdot \sqrt{(dz)^2 + (dx)^2}} \quad (10)$$

The ideal diffuse transmission freeform surface with the cosine characteristic can be regarded as a continuous distributed point light source, and its illuminance on the target plane is obtained from Eq. (11). The transmission light emitted from this freeform inner surface is scattered equally at random direction.

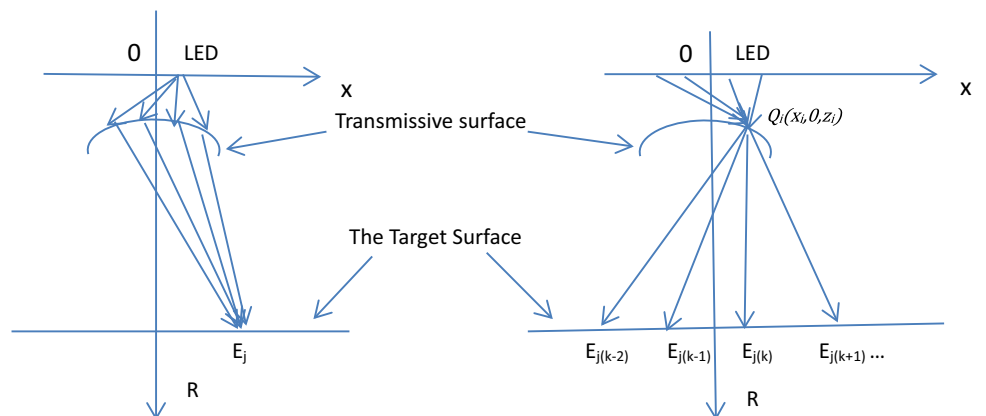
$$I(\delta) = I_{0j} \cdot \cos \delta \quad (11)$$

where I_{0j} is the central luminous intensity corresponding to $\angle\delta = 0^\circ$, the whole surface illuminance distribution as in Eq. (12).

$$E_s = \int \frac{BTDF \cdot \sum_{i=1}^u E_i(x, y, z) \cdot \cos \delta_i \cdot da}{r_s^2} \quad (12)$$

where “ a ” is the length of the outline of the freeform surface in the x -axis direction and r_s is the distance between the freeform surface to the target plane. da represents the area element on the freeform surface. $BTDF$ is a bidirectional transmittance distribution function, which is used to represent the transmission characteristics of the transmission surface.

Fig. 3 A mathematical model for two-dimensional curve profile of diffuse transmission freeform surface



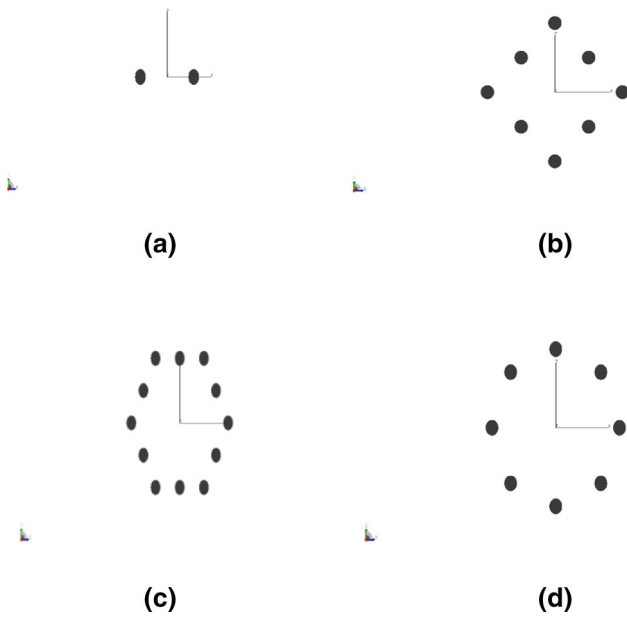


Fig. 4 **a** Linear array of 2 LEDs; **b** a square LED array; **c** a hexagonal LED array; and **d** a circular LED array

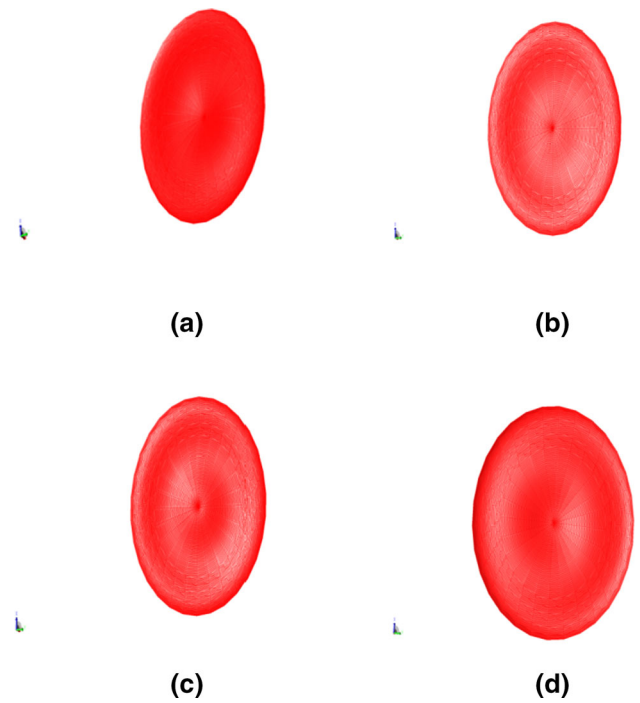


Fig. 6 Four different diffuse transmission surfaces: **a** transmission surface of 2-LED array, **b** transmission surface of the square array, **c** transmission surface of hexagonal array, **d** transmission surface of the circular array

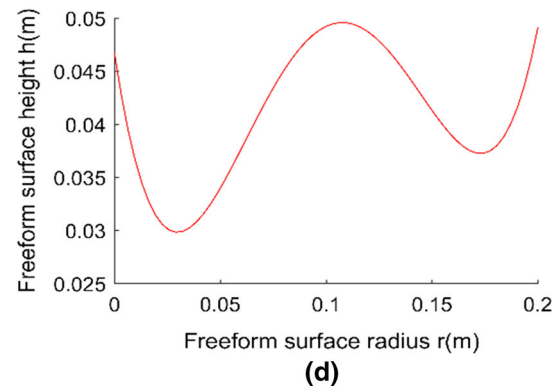
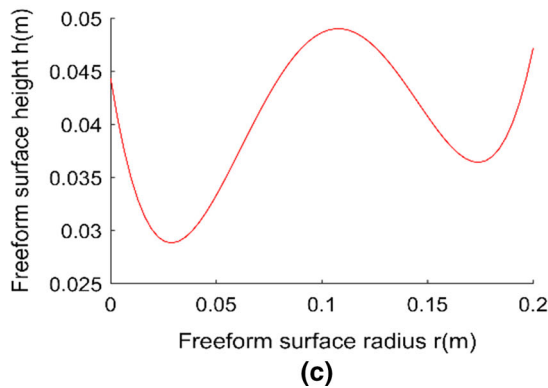
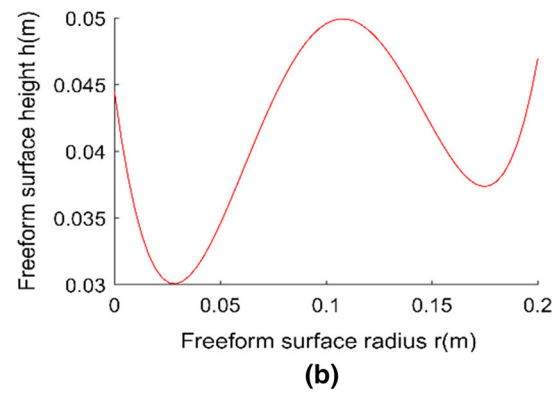
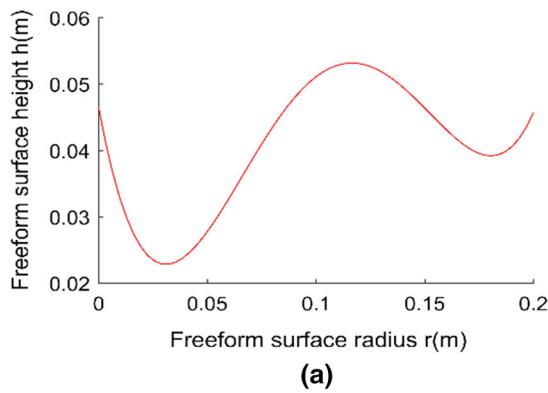


Fig. 5 Diffuse transmission freeform surface 2D contours for the four LED arrays: **a** 2-LED array; **b** square array; **c** hexagonal array; **d** circular array

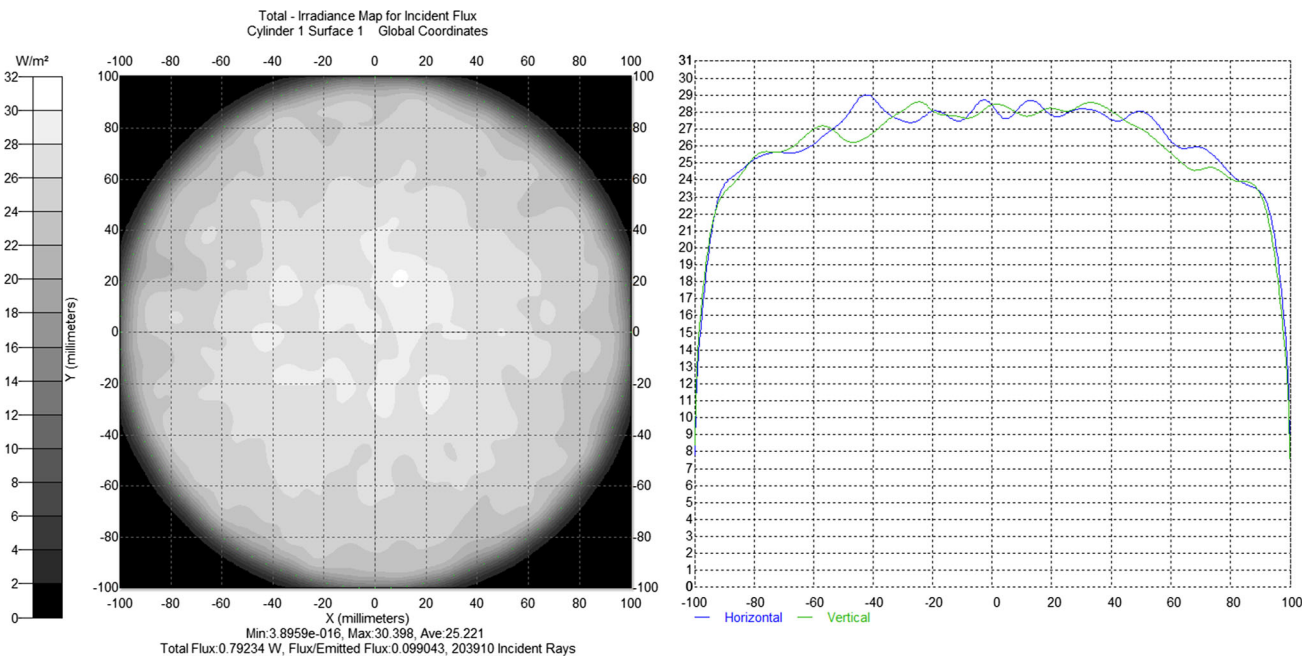


Fig. 7 Simulation results of the illumination uniformity over the target plane at the distance $R = 300$ mm, while the radius of the target plane is $r = 100$ mm

$$\begin{cases} Q = \int LdS \cdot \cos \phi d\omega = \pi LS \\ M = Q/S = \lambda \cdot E \end{cases} \quad (13)$$

where M is the radiation emission, Q represents the radiant flux, L represents the radiance, ϕ represents the solid angle and λ represents the transmittance of the diffuse

transmission material. It is not difficult to derive from Eq. (13) that the expression of BTDF is

$$BTDF = \lambda/\pi \quad (14)$$

Table 1 Illumination uniformity in four lighting systems

LED arrays	Size of system (mm)	Freeform surface	Hemispherical surface	Plane	Direct illumination
	Uniformity				
2-LED array	$R = 1000, D = 250;$	70.12	63.79	66.19	65.64
	$R = 900, D = 200;$	67.77	58.92	63.42	63.98
	$R = 800, D = 250;$	71.31	64.69	70.95	68.14
	$R = 600, D = 100;$	59.10	56.43	58.70	57.59
Circular array	$R = 1000, D = 250;$	80.59	75.33	78.18	75.33
	$R = 1000, D = 200;$	80.23	70.98	78.09	75.72
	$R = 800, D = 200;$	82.05	79.04	78.80	73.44
	$R = 600, D = 100;$	77.20	70.12	74.76	74.97
Square array	$R = 1000, D = 200;$	79.17	76.88	77.95	76.44
	$R = 800, D = 150;$	78.81	71.64	76.85	73.64
	$R = 600, D = 150;$	81.41	76.38	77.73	79.64
Hexagonal array	$R = 1000, D = 200;$	81.26	79.20	80.19	80.68
	$R = 800, D = 200;$	84.65	79.66	79.86	83.63
	$R = 800, D = 150;$	83.13	80.94	76.44	80.53
	$R = 600, D = 100;$	80.96	77.46	78.12	78.01

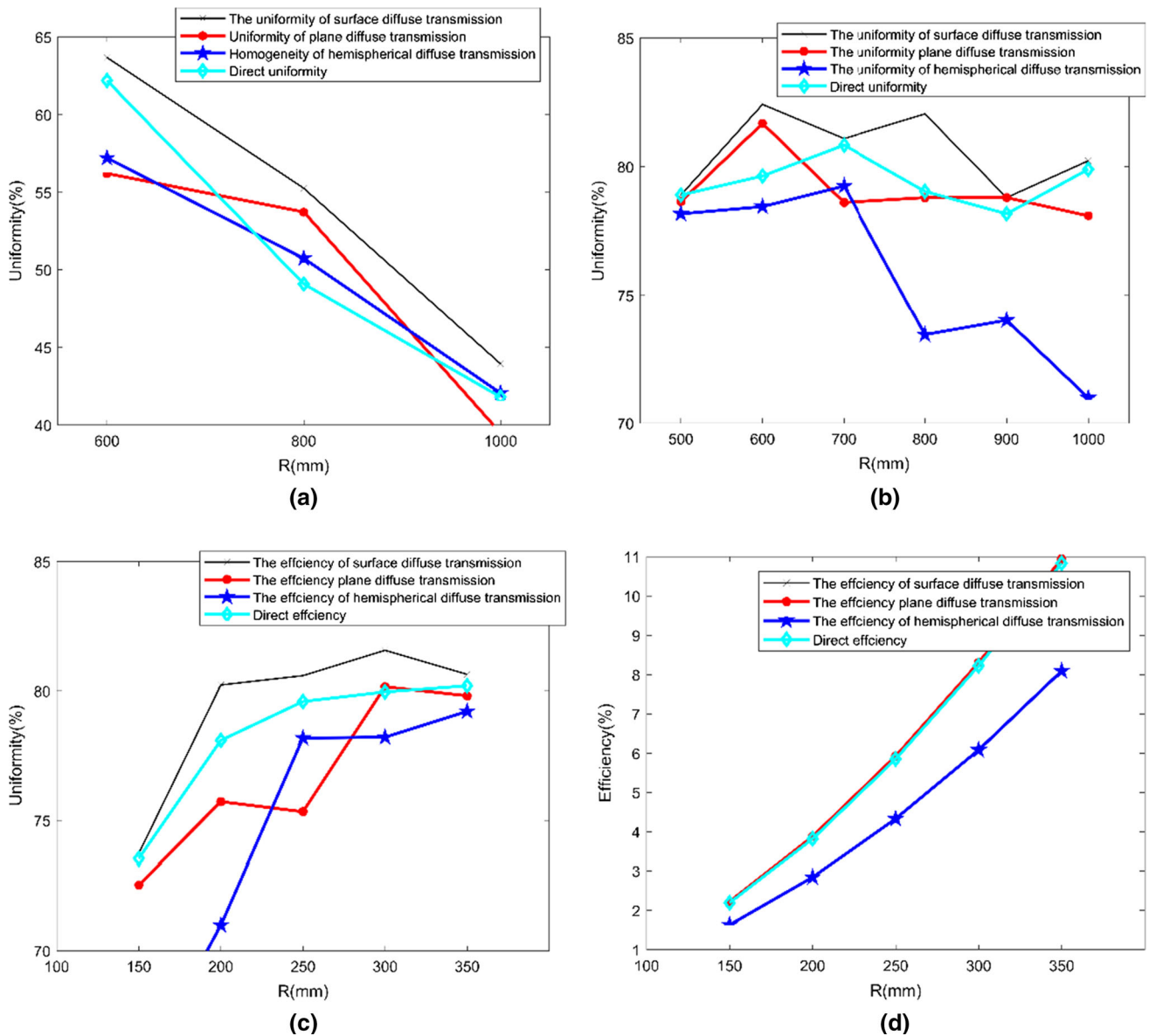


Fig. 8 Under the conditions of different circular LED array illumination uniformity: **a** when the target plane radius is 50 mm, the uniformity of the system is illuminated at different target plane distances; **b** when the target plane radius is 200 mm, the uniformity of

the system is illuminated at different target plane distances; **c** uniformity under different target illumination surfaces when the illumination distance is 1000 mm; and **d** the system lighting efficiency under the same conditions

The Eq. (12) is integrated on the target plane, and the expression of illuminance on the whole target plane is obtained, such as Eq. (15).

$$E_s = \iint \frac{BTDF \cdot E(x, y, z) \cdot \cos \vartheta \cdot da \cdot Er}{r_s^2} \quad (15)$$

where r_s is the length of the target plane in the x -axis direction. As in Fig. 3, a number of feature points are selected on the target lighting surface to calculate the structure parameters that can make the $d^2E_s/dx^2 = 0$ get minimal value at each feature point. In this way, the coordinates of every point q_i on the two-dimensional curve

on the freeform surface can be calculated by numerical integration iteration. According to the discrete data obtained from MATLAB programming, it is fitted to a closed curve. As shown in Fig. 4, in order to prove the validity of this design, this paper selects four kinds of LED arrays, namely linear array of 2 LEDs, square LED array, hexagonal LED array and circular array. The coordinate position of the LED in the 2D (x, y) and the unit of axis x and axis y were both in millimeters:

- (a) 2-LED array. $(-20, 0), (20, 0)$;

Table 2 Lighting efficiency of four lighting methods

LED arrays	Size of system (mm)	Freeform surface	Hemispherical surface	Plane	Direct illumination
	Efficiency/%				
2-LED array	$R = 1000, D = 250;$	5.81	4.32	5.90	5.82
	$R = 900, D = 200;$	4.65	3.50	4.75	4.65
	$R = 800, D = 250;$	9.03	6.74	9.03	8.83
	$R = 600, D = 100;$	2.72	2.12	2.82	2.66
Circular array	$R = 1000, D = 250;$	5.87	4.33	5.92	5.85
	$R = 1000, D = 200;$	3.84	2.84	3.88	3.81
	$R = 800, D = 200;$	5.85	4.42	5.99	5.80
	$R = 600, D = 100;$	2.70	2.11	2.80	2.68
Square array	$R = 1000, D = 200;$	3.82	2.28	3.87	3.81
	$R = 800, D = 150;$	3.41	2.57	3.48	3.37
	$R = 600, D = 150;$	6.01	4.65	6.09	5.85
Hexagonal array	$R = 1000, D = 200;$	3.86	2.82	3.78	3.82
	$R = 800, D = 200;$	5.85	4.42	5.99	5.80
	$R = 800, D = 150;$	3.42	3.37	3.47	3.37
	$R = 600, D = 100;$	2.70	2.11	2.80	2.68

- (b) square LED array. $(-20, 0), (20, 0), (0, 20), (0, -20), (10, 10), (-10, 10), (10, -10), (-10, -10);$
- (c) hexagonal LED array. $(20, 0), (-20, 0), (17.32, 10), (-17.32, -10), (10, 17.32), (-10, -17.32), (0, 17.32), (-10, 17.32), (0, -17.32), (10, -17.32), (-17.32, 10), (17.32, -10);$
- (d) circular array. $(20, 0), (-20, 0), (0, 20), (0, -20), (14.14, 14.14), (-14.14, 14.14), (14.14, -14.14), (-14.14, -14.14).$

Figure 5 shows the diffuse transmission freeform surface 2D contours required for the four LED arrays. Then the complete freeform surfaces are obtained by rotating 360° around the z -axis as shown in Fig. 6. Here (a), (b), (c) and (d) represent four different diffuse transmission freeform surfaces of 2-LED array, square array, hexagonal array and circular array.

Results

As shown in Fig. 5, based on the above design method of diffuse transmission freeform surface, we draw the surface profile of the diffuse transmission freeform surface and use the Monte Carlo ray tracing principle of TracePro software to simulate the experimental results. Select the LED with a power of 1 W. For the convenience of experiment, we set each LED as point source, and the emergent ray is 250,000. In this paper, the distance between LED and target plane is

600–1500 mm, with 100 mm to 500 mm as the effective radius of illumination area.

In order to prove the validity of the design of the diffuse transmission freeform surface, the experimental results of the diffuse transmission freeform surface are compared with the common diffuse transmission hemispherical surface, the diffuse transmission sheet and the direct illumination. Figure 7 shows the spot of light under a diffuse transmission freeform surface. Table 1 shows the illumination uniformity of the four lighting systems.

Figure 8 shows the uniformity and efficiency of the illumination of a circular LED array under different conditions. It is obvious that with the increase in the distance between the light source and the target plane, the lighting conditions of each system have changed. In this paper, the uniformity of the designed diffuse transmission freeform surface is higher than that of other lighting systems.

Table 1 shows that the uniformity of the designed freeform illumination system is about 3% higher than other illumination systems, which is an improvement for the diffuse transmission illumination system. Table 2 shows the lighting efficiency of the four lighting systems in the near field. Compared with other diffuse transmission surfaces, the efficiency of the transmission hemispherical surface is significantly lower than that of the other three lighting systems. In addition, the diffuse freeform surface is slightly better than the direct exposure but is slightly worse than the transmission sheet. In this regard, our explanation is that two times of the transmission of the LED rays from the freeform surface to the target plane

have taken place so that the efficiency of the lighting is slightly reduced.

Conclusion

In this paper, a method of transmission freeform surface design for LED is presented. Based on several LED arrays and diffuse transmission characteristics, a set of nonlinear equations for solving the contour of freeform surface is derived by establishing an approximate mathematical model in a Cartesian coordinate system. The illuminance distribution of the diffuse transmission freeform surfaces are compared with direct illumination, transmission sheet and transmission hemispherical surface on the effective illumination area. Experimental results show that the illumination performance of the diffuse transmission freeform surface lighting system is superior to others. The rationality and effectiveness of this design are further verified. This method proposed in this paper also expanded the prospect of the indirect lightings in practical.

Acknowledgements The authors thank the Key Laboratory of Advanced Control and Optimization of Jiangxi Province for providing the experimental platform and engineer Danny Carulla's observation.

Open Access This article is distributed under the terms of the Creative Commons Attribution 4.0 International License (<http://creativecommons.org/licenses/by/4.0/>), which permits unrestricted use, distribution, and reproduction in any medium, provided you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license, and indicate if changes were made.

Funding This research is supported by the Key Project of Industrialization of Invention Patents in Jiangxi Province (Grant No. 20161BBM26037), the Program of Jiangxi Outstanding Youth Talent Assistance (Grant No. 20162BCB23047), and the Science and Technology Pillar Program of Jiangxi Province (Grant No. 20151BBE50116).

References

1. Z.M. Zhu, X. Sun, B. Peng, The design of diffuse reflective off-axis surface for non-circular LED arrays. *IEEE. Photon. J.* **PP**, 1 (2017)
2. L. Gui, X.H. Sun, S.Y. Yin, C.L. Du, Y. Xiang, Modeling error analysis and compensation design for free-form uniform illumination lens. *Acta Opt. Sin.* **35**, 1022008 (2015)
3. C.W. Chiang, Y.K. Hsu, J.W. Pan, Design and demonstration of high efficiency anti-glare LED luminaires for indoor lighting. *Opt. Exp.* **23**, 12–23 (2015)
4. H.C. Chen, J.Y. Lin, H.Y. Chiu, Rectangular illumination using a secondary optics with cylindrical lens for LED street light. *Opt. Exp.* **21**, 3201–3212 (2013)
5. J.W. Pan, Y.C. Su, Y.S. Chen, Secondary optical element design for intracorporeal LED illumination system. *Opt. Lett.* **39**, 224–227 (2014)
6. I. Moreno, M. Avendañoalejo, R.I. Tzonchev, Designing light-emitting diode arrays for uniform near-field irradiance. *Appl. Opt.* **45**, 2265–2272 (2006)
7. I. Moreno, J. Muñoz, Uniform illumination of distant targets using a spherical light-emitting diode array. *Opt. Eng.* **46**, 033001–033001–033001–033007 (2007)
8. H. Ries, J.A. Muschaweck, Tailoring freeform lenses for illumination. *Proc. SPIE* **4442**, 43–50 (2001)
9. C.H. Tsuei, J.W. Pen, W.S. Sun, Simulating the illuminance and the efficiency of the led and fluorescent lights used in indoor lighting design. *Opt. Exp.* **16**, 18692–18701 (2008)
10. C.H. Tsuei, W.S. Sun, C.C. Kuo, Hybrid sunlight/led illumination and renewable solar energy saving concepts for indoor lighting. *Opt. Exp.* **18**, A640 (2010)
11. C.H. Tsuei, W.S. Sun, Momentary adjusting methods for simulating the color temperature, hues and brightness of daylight illumination with RGB LEDs for indoor lighting. *Opt. Exp.* **19**, A908 (2011)
12. M.A. Moiseev, S.V. Kravchenko, L.L. Doskolovich, Design of efficient LED optics with two free-form surfaces. *Opt. Exp.* **22**, A1926 (2014)
13. R. Wu, C.Y. Huang, X. Zhu, H.N. Cheng, R. Liang, Direct three-dimensional design of compact and ultra-efficient freeform lenses for extended light sources. *Optica* **3**, 1 (2016)
14. J.A. Wheatley, G.J. Benoit, J.E. Anderson, R.W. Biernath, D.G. Freier, T.R. Hoffend, C.D. Hoyle, T.T. Liu, J.D. Lu, M.A. Meis, V.V. Savvateev, C.R. Schardt, M.E. Sousa, M.F. Weber, T.J. Nevitt, Efficient led light distribution cavities using low loss, angle-selective interference transfectors. *Opt. Exp.* **17**, 10612–10622 (2009)
15. A. Travis, T. Large, N. Emerton, S. Bathiche, Collimated light from a waveguide for a display backlight. *Opt. Exp.* **17**, 19714 (2009)
16. J. Ripoll, M. Nieto-Vesperinas, Reflection and transmission coefficients for diffuse photon density waves. *Opt. Lett.* **24**, 796 (1999)
17. M. Pang, J. Rong, X. Yuan, X. Gao, X. Hu, S. Zhou, Research on the measurement method for a large laser beam profile based on CCD diffuse transmission imaging. *Meas. Sci. Technol.* **24**, 5202 (2013)
18. M.W. Serra, N. Carbone, H.D. Rocco, H. García, D. Iriarte, J. Pomarico, H. Ranea-Sandoval, Diffuse light transmission profiles obtained using CW: a comparative analysis with time resolved experiments. *Optik Int. J. Light Electron Opt.* **125**, 3507–3513 (2014)
19. J. Wu, S. Zhou, M. Pang, X. Gao, X. Hu, Influence of diffuse transmission screen on intensity distribution of incident laser. *High Power Laser Part. Beams* **24**, 1780–1784 (2012)
20. L. Bressel, O. Reich, Theoretical and experimental study of the diffuse transmission of light through highly concentrated absorbing and scattering materials: part I: Monte-Carlo simulations. *J. Quant. Spectrosc. Radiat. Transf.* **146**, 190–198 (2014)
21. S.E. Kim, J.Y. Lee, M.H. Shin, H.J. Kim, Y.J. Kim, Integration of prism sheet on quantum dot film with bridge patterns to enhance luminance of LED backlight unit. *IEEE. Trans. Electron. Dev.* **PP**, 1–8 (2017)
22. Z. Qin, Luminance enhancement without sacrificing the viewing angle in a direct-lit backlight by addressing the angle-dependent characteristic of the prism film. *Displays* **50**, 31 (2017)
23. K. Wang, D. Wu, Z. Qin, F. Chen, X. Luo, S. Liu, New reversing design method for LED uniform illumination. *Opt. Exp.* **4**, A830 (2011)