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Post-pandemic reflections: lessons from Chinese mathematics teachers about online mathematics instruction

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Abstract

This study investigated how teachers in China perceived the effects of online instruction on mathematics learning and examined the challenges they encountered when the country shifted to online instruction during the COVID-19 pandemic. We interviewed 152 mathematics teachers from 20 cities (municipalities) or provinces in China and adopted the four-component didactic tetrahedron model (teacher, technology, student, and mathematics) to identify their struggles with technology, teacher–student interactions, and delivery of mathematics instruction. Results showed that the teachers believed that the effectiveness of online teaching largely depends on student self-discipline. Analysis suggested a need to expand technology use during instruction, reshape the way teachers interact with students, and reorganize teaching methods in face-to-face classroom instruction. This research provided insights into integrating technology with instructional practice, the critical role of teachers in online learning, and other factors that may determine the effectiveness of online teaching.

Keywords Mathematics teacher · Online teaching · Teaching challenges · Pandemic

In March 2020, the COVID-19 pandemic forced many countries to close schools and universities. To help students continue learning, a common solution across the world was to conduct online classes at home. However, in February 2020, even before the pandemic was declared, Chinese schools and universities decided to remain closed after the winter holiday, which is traditionally from January to February. Students all over China were required to stay at home and continue their studies. Since then, online teaching and learning have continued nationwide.

In early February, the Ministry of Education (MoE), which is the governing body for all aspects of schooling in China, released several documents proposing the idea of "suspending classes without suspending school, without stopping teaching" (停课不停学, 停课不停教). This idea was communicated through government documents from February 4 to 10 and applied to both compulsory and higher education (Ministry of Education, MoE, 2020a, b). A later document (MoE, 2020b), which focused on compulsory education, stated that "local educational departments and schools should organize online teaching between teachers and students and clearly suggest the taught content, curriculum schedule, and the way to organize online teaching, such as with technical supports, platforms, and so on, which are all situated in local context" (MoE, 2020b). Starting mid-February, most schools in China began Internet-based instruction, which involved more than 300 million teachers and students according to official media (Zhao, Wang, & Cao, 2020).

As with many other countries, nationwide online teaching had never been set up in China before the pandemic. This undertaking would not have been possible until now, when the Internet and technology have already been relatively well developed and geared for it. However, this does not mean that students and teachers could automatically adapt

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or transition to online teaching from their normal in-person classrooms (Baran, Correia, & Thompson, 2011). In fact, face-to-face teaching has been such an entrenched practice that it seemed challenging for teachers and students to switch to online instruction. In this study, we aimed to portray how Chinese teachers engaged in online mathematics instruction during the pandemic and identify the challenges and difficulties they encountered while doing so. Through this study and similar observations from other parts of the world, we can analyze each community's response to the crisis and learn from each other to ensure minimal educational disruptions in the future.

Literature review and theoretical background

The past two decades have seen information technology slowly but increasingly change the way mathematics is taught and learned. Some researchers have focused on investigating mathematics instruction in classrooms with available technology, such as dynamic geometry software and spreadsheets (e.g., Drijvers, 2018; Niess, van Zee, & Gillow-Wiles, 2010). The Internet has enabled schools to offer online programs and courses that deliver education from a distance (Baran et al., 2011), but only a few studies have focused on online mathematics instruction. One reason for the gap in studies exploring how online instruction influences a student's understanding of mathematics (Hoyles et al., 2010) may be that face-to-face classroom teaching is the dominant form of instruction, and relatively few students need to or can participate in distance learning. However, some evidence has shown that online mathematics instruction could enable different connections between students and teachers and between students and the subject (Hoyles et al., 2010). Instead of being a mere substitute for face-to-face mathematics instruction, there may be value in developing the online medium into a discrete pedagogical tool, different from face-to-face instruction.

Using teachers' perceptions of online instruction during the pandemic, we will build a foundation for the current study in the following sections by discussing why we must consider the study from such a perspective. This is supported by existing studies that exemplify teachers' critical roles in classroom instruction. We will then discuss the conceptual framework we used to investigate teachers' perceptions of online teaching.

The role of teachers in Chinese mathematics classroom instruction

Research has shown that mathematics teachers play an influential role in traditional face-to-face Chinese classrooms. Based on data from the Learner's Perspective Study (Clarke, 2006), Ida Ah Chee Mok from the University of Hong Kong (2006) studied a mathematics teacher in Shanghai who taught a sequence of 14 seventh-grade lessons and found that the teacher thoroughly understood the subject matter, had designed the lessons based on his own understanding, and led the lesson with a clear rationale, helping his students understand the subject as he did. In another study, Xu and Clarke (2019) compared mathematics classrooms in Seoul, Shanghai, and Melbourne and found that in Melbourne, student–student interactions might involve discussions of mathematical terms, while in Shanghai, such discussions mainly occurred in whole-class situations and in teacher–student interactions. This means that in Shanghai, the teacher was almost always involved in discussions of mathematical terms.

In the International Congress of Mathematics Education 13 (ICME-13), Guo and Cao (2012, 2015) reported on a large-scale quantitative study before the pandemic, which surveyed 65 junior high school mathematics teachers and their students (40–50 students per class) from three Chinese school districts. The study revealed that teachers' technological–pedagogical knowledge had a significantly positive effect on student achievements in both algebra and geometry in traditional face-to-face classes (see also Drijvers et al., 2016).

Mathematics teachers are also essential in organizing activities during classroom instruction, especially in Asian countries (Clarke, Keitel, & Shimizu, 2006). Shimizu (1999) explained that teachers provide instructions at student desks during classroom teaching (later referred to as "between-desk instruction" by Clarke, 2004), during which teachers evaluate individual problem-solving processes, make mental notes about the student's approaches and difficulties, and, in later teaching sessions, address these problems based on their observations.

Conceptualization of didactic relations in mathematics classrooms

To conceptualize didactic relationships in the use of computer technology in mathematics classrooms, several researchers developed a four-component model based on the sociodidactic tetrahedron (Olive et al., 2009; Freiman, Polotskaia, & Savard, 2017; Johnson, Coles, & Clarke, 2017; Tall, 1986). These intertwined components include the teacher, the student, technology, and mathematics. The mathematics component refers to mathematics knowledge shared by teachers and students, the teacher is the agent who delivers mathematics knowledge, the student constructs mathematical knowledge with the teacher, and the computer is used as a tool to deliver mathematics instruction.

Because of the critical role of teachers in enhancing the quality of classroom teaching, one must understand the challenges they face in online classrooms and help them overcome these difficulties. When applying the aforementioned model to online instruction, teachers must still deal with their relationships with students, mathematics, and technology, as shown in Fig. 1. The teacher–technology strand describes how teachers use technology, while the teacher–student strand refers to interactions between teachers and students during online instruction. Meanwhile, the teacher–mathematics instructions strand describes how teachers convey mathematics knowledge and provide instructions in the online context.

Researchers have recognized the importance of mathematics teachers' ability to identify and overcome the challenges surrounding the three other aspects. First, regarding technology use, Joubert (2013) reviewed 124 conference papers about technology, mathematics instruction, and learning to examine the issues, interests, and concerns within the mathematics education community. She found that teacher attitudes and beliefs significantly influenced the successful adoption of technology and that teachers may face practical difficulties in teaching, such as how to use technology or technological teaching resources such as videos. Meanwhile, Hoyles and Lagrange (2010) identified challenges with the use of digital technologies and the need to recognize their potential and increase their utilization to improve mathematics curriculum, instruction, and learning.

Chieu and Herbst (2016) examined teacher–student relationships in the online classroom. Teachers have found it difficult to increase student participation and monitor student learning, as interactions occurring in face-to-face classes, such as personal communication and whole-class discussions, are not as easily implemented in an online environment. Joubert (2013) also identified this problem, noting that teachers may encounter challenges in evaluating student learning and engaging them in classroom activities.

Finally, there is a notable consensus among researchers regarding the urgent need to rethink the nature of mathematics, how it could or should be taught, and how teachers might be supported in a technological environment (Artigue, 2010; Freiman et al., 2017; Pierce & Ball, 2009). For example, Mauricio Rosa (2008) examined the links between how identities are constructed in an online course and how integral concepts are taught and learned. His results suggest that, within the context of online learning, students might create different identities and use them to play different roles in constructing mathematical knowledge, indicating that online learning may open additional opportunities for students to acquire knowledge as a social construct in and of itself (Rosa & Lerman, 2011).

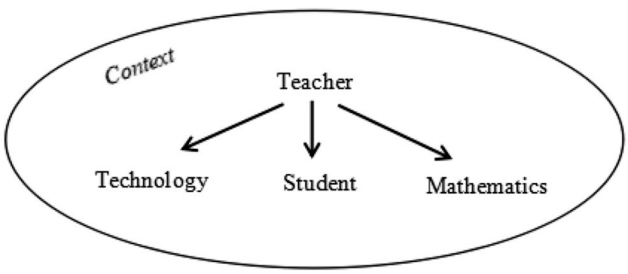

Fig. 1 Four-component model of online mathematics teaching

Research questions

In these studies, most of the challenges to mathematics instruction—whether in-person or online—were found to be specifically associated with the investigation of technology integration. Simply put, the teachers in these studies could decide on their own methods of using technology or not to use technology at all. These studies were often conducted in schools or experimental classrooms where instruction was carried out by one or more teachers who physically shared the same classroom space with students. These settings are different from a fully online environment, in which teachers and students have no choice but to interact remotely (Baran et al., 2011). Regarding online teaching, current research is mainly focused on university-level distance learning (e.g., Rosa & Lerman, 2011) or mathematics teacher training programs (e.g., Borba & Villarreal, 2005a, 2005b; Ekici, 2018). The common characteristic among these studies is that their participants are usually adults—either university mathematics students or mathematics teachers. Only a few studies have examined online mathematics instruction in places where educational resources are limited and online teaching has been used to share these resources (e.g., Borba & Villarreal, 2005a; Cady & Rearden, 2009). However, the extent of the shift to online teaching worldwide has been unprecedented after the pandemic.

COVID-19 has profoundly affected how people learn and work (Bakker & Wagner, 2020), with education becoming particularly challenging because of school closures and students and teachers being required to stay at home. To continue education, many countries developed digital pedagogical tools and/or conducted online teaching (Cerna, 2020). In China, the nationwide transition effort was carried out from mid-February to mid-May. At the time of this writing (June 2020), schools were gradually reopening, with teachers and students returning to physical classrooms. As schools slowly returned to regular face-to-face classroom teaching, it became important to reflect on and learn from the changes that had taken place. Hence, we propose a comprehensive investigation of mathematics teachers' experiences with online teaching that will allow us to learn from

the crisis situation and offer insights into the improvement of instruction in both environments. This study adopted the four-component model to examine various aspects of online instruction from the teachers' perspectives and aimed to address the following research questions:

(1) How do teachers perceive online mathematics instruction affect students' mathematics learning?
(2) What are the difficulties that mathematics teachers perceive regarding online instruction during the pandemic?

Method

To answer these research questions, this study administered a large-scale survey involving mathematics teachers in 20 of the 34 provinces in China. The interview transcripts were analyzed quantitatively and qualitatively based on statistical analysis and content analysis, respectively. A preliminary statistical analysis facilitated our understanding of how the online environment was generally implemented as reported by the teachers. The interview transcripts then underwent content analysis for a thorough investigation of the teachers' online pedagogical practices.

Teacher interviews

We recruited 168 mathematics teachers from primary and secondary schools from March 27, 2020, to May 27, 2020. We first approached a few expert teachers and educational research specialists (who are usually in charge of local district in-service teacher training programs) in 20 different locations and asked them about the online teaching situations in their respective schools. They were the first group of teachers invited to participate in the study, and most of them were willing to ask other teachers at their schools or within their districts to do the same. Since the first recruits were expert teachers or teaching research specialists, they had a positive reputation among the groups of teachers they approached, which helped the research team collect further data.

Next, we created chat groups via WeChat (a communication app in China), which enabled us to interact with each participant. We sent interview questions to each teacher and allowed them to send back their responses. The collected data from the interviews served as the primary data source. Rather than dictate a specific timeline, we encouraged the first group of teachers to help us collect all the feedback to the interview questions by the end of May 2020, when most of the schools in the country announced they would reopen for at least 2 to 3 weeks. This collection method alleviated the teachers' burden of participating in the study, especially during the crisis. Questions included basic information about the participants. To investigate how teachers worked during the pandemic, we asked them about their experiences with and challenges concerning online teaching and how they coped with them. We also included questions about how teachers viewed the effects of online teaching on students' abilities to learn mathematics. "Appendix" shows the detailed list of interview questions.

Of the interviews with the 168 mathematics teachers, 152 were regarded as valid, as the teachers explained their answers in detail to some extent. These participants, who came from urban areas, were numbered from T1 to T152. We contacted them in case we needed to collect any further information; 126 participants were approached over the phone through WeChat, 22 were approached via audio or video calls (mainly expert teachers and teaching research specialists), and 4 were approached face-to-face, as they were based in the same city as the researcher. The complementary audio and video calls and face-to-face conversations were recorded with permission from participants, transcribed for analysis, and paired with the written documents collected from each participant. Table 1 and Fig. 2 show detailed participant information.

Classification of teachers' comments on the effects of online teaching

The interview data analysis was divided into two stages. The first stage involved classifying the participants' responses regarding the effects of online teaching on learning outcomes to understand how they viewed the overall quality of online instruction. Informal conversations with

Table 1 Participants' basic information

Teaching experience	Primary school (grades 1 to 6)	Lower-secondary school (grades 7 to 9)	Higher-secondary school (grades 10 to 12)	Total
1 to 10 years	12	20	10	42
11 to 20 years	38	17	15	70
21 to 30 years	10	16	7	33
More than 30 years	1	3	3	7
Total	61	56	35	152

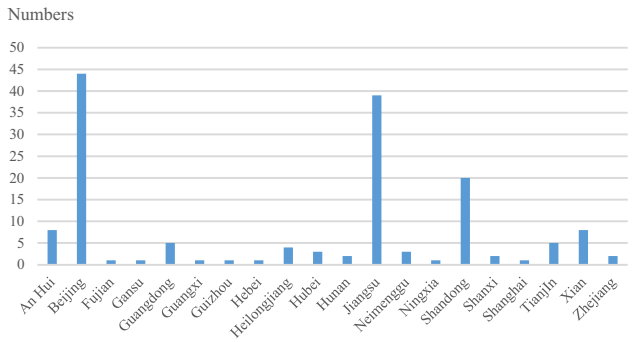

Fig. 2 Participants' districts

some teachers suggested that most schools have organized mathematics tests after school reopened; therefore, in the interview questions, we asked teachers to confirm if their schools had conducted mathematics tests and how they thought students performed on the tests. Also worth noting is that we particularly focused on teachers' reactions toward online instruction during the pandemic; therefore, their responses to student learning during online instruction helped shape our understanding of their thoughts about the method compared to in-person instruction in general. Rather than collecting mathematics test results from students, which would be time-consuming and meaningless because different schools have different tests, we placed value on teachers' responses and their overall assessments of student performance, especially when coupled with written explanations and interpretations.

Analysis of teacher interviews

In the second stage, we employed both concept- and data-driven coding processes to examine teachers' perceptions of the challenges to online teaching based on the four-component model (Kuckartz, 2019). We first reviewed the transcriptions and highlighted responses describing online teaching difficulties while two members of our team collaboratively documented any relevant information and possible ideas based on existing literature and our prior knowledge. Then, we conducted open (data-driven) coding, where we identified and coded specific incidents referred to by the teachers. In the next step, we adopted the three main categories (teacher–technology, teacher–student, and teacher–mathematics) and conducted subcategory (concept-driven) coding. Codes such as those identified in the previous stage were classified and constructed as one specific subcategory. We checked whether all the challenge subcategories could be classified under the three main categories. In case we held different opinions regarding the categorization of some codes, we conducted coding meetings to reach consensus. For each category, we re-sorted the collected codes and related texts into different subcategories. Similarly, in the sorting process, we resolved our differences until we achieved a final agreement. Table 2 presents the final subcategories.

We performed a frequency count for each constructed subcategory to obtain a general picture of participants' responses. This refers to how many times the participants mentioned each subcategory. We considered these considered alongside the quotes from the interviews, which were selected to explain each online teaching challenge subcategory.

Results

First, we will provide context by briefly introducing how teachers delivered online instruction in China. Because the teachers we approached were from 20 different areas, they may have employed a variety of teaching styles. Most of them did not have prior experience in online teaching, so our summary here is based on the participants' most common responses. Following the introduction, we will present results based on the classifications of teachers' responses about the quality of online teaching, which then guided us when we asked additional questions about how teachers viewed their experiences in terms of online teaching challenges during the pandemic.

A brief description of participants' online teaching methods

During the online teaching period, many Internet-based platforms were available for free use in China. Schools and teachers could choose from many different platforms for their instruction, with the school or local department usually suggesting one or two. The participants identified a variety of online instruction platforms, which allowed them to deliver teaching in different ways, such as live broadcasts, and which also influenced their classroom management.

For example, Chinese media reported Ding Class as a popular online teaching platform in China, widely used in more than 300 cities by more than 50 million students who registered with their identification credentials as active users (Chen, 2020). Of all the study participants, more than 80% (123 of 152) had used Ding Class for mathematics instruction at some point during the pandemic. Ding Class allows students or their parents to provide information regarding their daily health status, which local educational institutions then collect to ensure students were safe and to further determine any possible infections. The platform also supports various teaching functions, such as video conferences, in which all participants had to turn on their cameras to attend the class, and live broadcasts, which are used for lectures

Table 2 Themes and subcategories

Categories	Subcategories	Sample responses
Teacher–Technology	Difficulties in combining digital pedagogical resources in teaching	"I tried to play videos [prerecorded videos of instruction by teachers who had released the video online],...but I feel a bit helpless when delivering tutorials as I am not sure if students learned or not" (T15, primary school mathematics teacher)
	Difficulties in using the different functions of the online teaching platform	"I can't get used to the technology stuff. I prefer to use a blackboard or whiteboard to [teach] the lesson" (T4, primary school mathematics teacher)
Teacher–Student	Inability to monitor student participation during lessons	"Some students might watch a teacher's live broadcast [during which students might not need to turn on their cameras] while eating snacks or watching TV shows. We just don't know, and teachers cannot track student participation time" (T9, primary school mathematics teacher)
	Problems with teacher–student interactions during online instruction	"The interaction form became very dull, as you may only be able to talk to one student each time" (T25, secondary school mathematics teacher) "Sometimes when you ask questions, you can't get a response instantly, since you need to ask a student to turn on their video or microphone first, and then maybe repeat the questions and so on...his slows the teaching pace" (T35, primary school mathematics teacher)
Teacher–Mathematics	Difficulties in delivering various mathematics teaching activities for different mathematics content as in normal classroom instruction	"It is hard to organize some teaching activities, for example, like collaborative problem-solving, group discussion, student handwork and so on, in online teaching" (T1, primary school mathematics teacher)
	Difficulties following mathematics teaching procedures as in normal classroom instruction	"In normal classroom instruction, you could easily come down from the platform and approach each student during the lesson, so you know whether they solved the problem following the proper norms, but it becomes hard in online teaching" (T30, secondary school mathematics teacher)

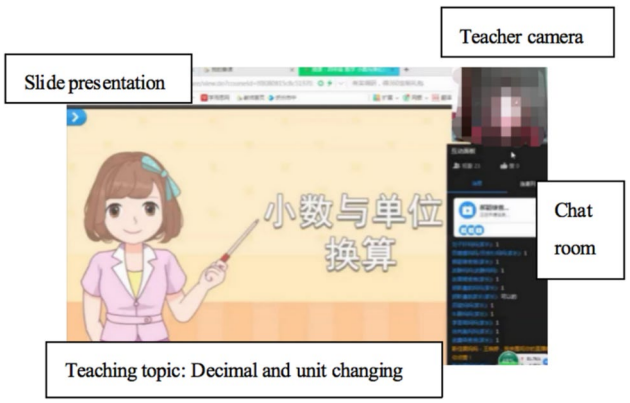

Fig. 3 Live broadcast class (grade 3 mathematics lesson)

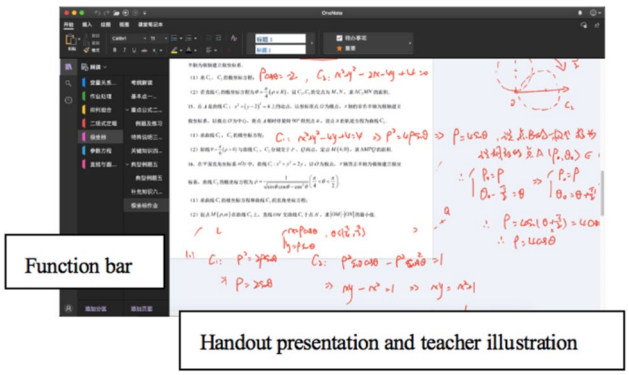

Fig. 4 Teacher handout and annotations during online teaching (grade 11 mathematics lesson)

and do not require students to use their cameras unless the teacher asked them to do so. In live broadcasts, students could respond to their teachers in the chatroom without turning on their cameras (see Fig. 3). Additional functions include grading homework, making document revisions, and writing notes on handouts (teachers could share their handouts on screen and write on them in real time; see Fig. 4).

During the pandemic, some schools used different timetables for lessons than those used during in-person classes. Five primary school teachers from City P stated that their teaching time decreased with the online platform compared to in-class instruction. For instance, before the pandemic, mornings usually consisted of four 45-min lessons, with 10- to 20-min breaks in between. However, during online

teaching, each morning consisted of only two lessons, each lasting for approximately an hour, with a 10-min break between the two.

Teachers' perceptions of online teaching effects

After reviewing the data, we classified the participants' responses regarding their assessment of the effects of online mathematics instruction into three categories, shown in Table 3 (see "Appendix", interview question 7). A total of 46 mathematics teachers (30.3%) stated that online teaching did not promote student learning the same way as a real classroom does. Most of these teachers explained that their students' mathematics grades were worse than expected, and the instruction was not effective. Only seven teachers said their students maintained the same level of learning achievement as in a normal classroom environment. One teacher reported administering the same mathematics test she had used previously, and her students still achieved an average score of approximately 92/100, which was the norm for her classes.

Besides the two groups of teachers who answered either "good" or "bad" to the questions on whether online instruction had a positive or negative effect on student learning, most of the other participants commented that the students themselves could easily influence the effects of online instruction, and it was difficult to provide a direct answer to the questions. Ninety-nine teachers (65.1%) suggested that the main issue that affects online instruction effectiveness was *how* students learn mathematics. These teachers identified key issues in determining the effects of online learning, including whether students were self-motivated, whether they could manage their own learning well, and whether they were self-disciplined. For these teachers, their students were clearly divided into two: one group could maintain their mathematics test grades, while the other group experienced a significant decline in scores after online instruction. For example, more than 40 participants reported teaching a group of students that had achieved similar test scores in the last semester before they were required to teach online. They expected all these students to achieve similar scores after online instruction, but instead they were split into two groups. Group A achieved similar test scores, but Group B attained lower test scores, which teachers attributed to a lack of self-discipline.

Overall, the initial results from Table 2 suggest that, for most teachers, the effectiveness of online teaching is not guaranteed and that online teaching introduces challenges not found in face-to-face classrooms. In line with the study's conceptual framework (Fig. 1), the following sections present these difficulties across three dimensions from a mathematics teacher's perspective: difficulties coping with technology, challenges in connecting with students, and problems organizing instructional activities.

Difficulties in the teacher–technology category

Difficulties in combining digital pedagogical resources in teaching

Digital pedagogical resources refer to teaching materials, including slides, textbooks, and prerecorded videos that are usually prepared by outside educators, expert teachers, or educational research teams from institutions or publishing companies. These materials show how some teachers, including experts, teach a lesson and prepare slides. A local educational department may purchase these materials and provide them to teachers so they can learn from them or use them in practice.

A total of 123 participants (80.9%) reported applying digital pedagogical resources during their online instruction, with 29 suggesting they had good-quality digital resources they could directly use or draw from to create their own materials. However, some participants were concerned about using the available resources and the extent to which they should be used in online teaching. One fourth-grade teacher stated,

> At our school, teachers were required to play videos [from digital pedagogical resources] during the online class and then ask students to learn through watching videos by themselves—you know, videos provided by the People's Education Press. After watching the videos, teachers were available to answer questions from

Table 3 Teachers' responses about the effects of online teaching

	Number of teachers	Percentage of all participating teachers
Mathematics learning during online instruction is worse than during normal instruction	46	30.3
Mathematics learning during online instruction is as good as (or even better than) normal instruction	7	4.6
Mathematics learning during online instruction is highly dependent on self-learning skills	99	65.1
Total	152	100

> students. I am certainly confused about this, because the students are not engaging in it, ...watching videos cannot be a way to learn mathematics, but I don't know how can we teach online without videos either. (T8)

Difficulties in using the various functions of the online teaching platform

As noted earlier, online teaching platforms usually support several functions for teachers. Since most of the participants taught online for the first time during the pandemic, the technology may have been completely new to them. A total of 147 participants (59.2%) mentioned that they had experience using the different functions of the platforms. Among them, 90 reported using the digital tools with no specific difficulties, while 57 said they had difficulty getting accustomed to the digital teaching environment. These teachers, especially the ones with the longest teaching careers, felt helpless when using the online teaching platforms. Because they were used to teaching while writing on a blackboard or a whiteboard, they seemed to be burdened by having to learn the new technology. One teacher who has taught for 25 years (T4, a sixth-grade teacher) said she followed a traditional teaching style because she could not get used to the online settings and functions. She set up a whiteboard at home and pointed the camera toward it so students could see what she was writing and hear what she was teaching.

Teachers' difficulties in the teacher–student category

Inability to monitor student participation during lessons

The participants also noted difficulties with classroom management, as some said they were unable to monitor student participation. A total of 106 participants (69.7%) stated that the problems with classroom management could be attributed to the lack of a mechanism to track student participation during online instruction, primarily when teachers used the live broadcast function. This became a particular problem for students who felt unenthusiastic about participating. As reported by T9, a fifth-grade mathematics teacher,

> In my class, a small group of students could not engage in learning during the whole period of online teaching. Students have different learning habits. Some of their learning habits are really not good, but we are not able to manage and note that. For example, some students might watch teachers' live broadcasts [during which students might not need to turn on their cameras] while eating snacks or watching TV shows. We just don't know, and teachers cannot track students' participation in time. (T9)

Problems with teacher–student interactions

A total of 90 participants (59.2%) reported difficulties in adapting to the change in teacher–student interactions during online instruction. Some mentioned that, in a physical classroom, interaction can include verbal communication, eye contact, and other nonverbal cues that help teachers establish connections with students and monitor their learning. However, many such forms of communication became particularly challenging during online instruction, as the teachers and students did not share the same physical space. This complicated interactions among the whole class and made it difficult to monitor the reactions or attention levels of other students while responding to an individual. For example, one middle-school mathematics teacher stated that

> it is difficult to get a sense of whether students are listening to you or not. Even interacting with children is difficult, because you cannot initiate interactions as usual, like choral interaction [asking a question that is supposed to be answered by all the class chorally]. Instead, you have to pick one student to talk with, and at the same time, you are not able to see whether other students are listening or not. I tried my best to encourage students to interact with me, for example, using random drawing activities, but it is not a long-term strategy, I think. (T3)

Teachers' difficulties in the teacher–instruction category

Difficulties in the delivery of various mathematics teaching activities for different mathematics content

A total of 124 mathematics teachers noted that online teaching restricted the way they taught mathematics, especially for the lower grade levels. For primary students, some topics must be taught by combining classroom activities, discussions, and peer collaboration. In online teaching, since each student attends class via their personal devices, teachers find it difficult to group students and organize classroom activities. T4, for example, reported her strategy of changing the order of topics being taught during online teaching:

> I cannot teach *shapes* at this moment, because teaching shapes requires some classroom activities to be organized; for example, students can use scissors to cut shapes. I also need to prepare some teaching models and to show them to students to let them understand the nature of shapes. But if teaching online, it is difficult to allow students to feel the spatial sense of the shapes, so I would prefer to postpone this part of teaching until later. (T4)

However, the organization of various teaching activities seemed better for secondary school teachers; 14 of 91 secondary school teachers (lower- and higher-secondary school) reported that their use of technology in online classes provided more opportunities for students to visualize concepts.

Difficulties following mathematics teaching procedures

A total of 113 participants (74.3%) discussed the challenges of following mathematics teaching procedures as they would in a normal classroom. Some realized that each teaching activity for a lesson in a normal classroom played a role in various aspects of student learning. For example, when students were practicing on their own, teachers could walk around and observe how they solved problems, getting a sense of each student's learning status. However, this was not possible during online teaching. As one lower-secondary school (grade 7) teacher commented,

> [F]or some problem-solving teaching, it was essential to follow students' problem-solving processes during instruction, to make them follow the mathematics norms (the reasoning process). Since we were far from students, especially since we could not see them doing mathematics problem-solving, we were not able to directly notice their faults and help them to revise. (T5)

In addition, most teachers viewed exercises and tests administered after classroom instruction as vital pieces of analysis, as the manner in which students complete these exercises helps them obtain a general picture of what the students have learned. However, during online instruction, since students could not hand in their homework on paper, teachers may not know how they solved the problems and finished their homework, which can usually be determined by their handwriting and calculation drafts. As one lower-secondary mathematics teacher stated,

> I assign homework or organize tests sometimes, but as students were at home, they had smartphones and iPads with them. Students could easily search the test items online or from other apps, which made it difficult to assess whether they really learned or not, especially if you [the teacher] don't have many chances to talk to the students. (T23)

Discussion

Before presenting a broad discussion of the study, we first reiterate the study's research questions: (1) How do teachers perceive online mathematics instruction affect students' mathematics learning? (2) What are the difficulties that mathematics teachers perceive regarding online instruction during the pandemic? Regarding the first research question, the teachers' responses suggested that most of them could not guarantee that online teaching would have an effect, which more than 60% of teachers attributed to the students' lack of self-learning skills. The first question helped us search for additional answers and raised concerns about our current educational system's ability to foster students' self-learning skills. Meanwhile, in the second research question, we inquired about the challenges to online teaching to explore teachers' experiences in online instruction. As shown in Table 3, the three dimensions developed from the sociodidactic model guided our analysis of teachers' difficulties with technology, teacher–student interaction, and mathematics instruction during their online teaching during the pandemic. The issues addressed based on these results can be summarized in two points. First, despite the growth of technology integration as a common topic in educational research, this study suggested that many problems continue to occur in practice, especially when teachers are unprepared to incorporate technology into their online instructional practice. Second, after reviewing teachers' reports and hearing their online instruction experiences, we noticed that coping with issues in teacher–student interaction and mathematical instruction during online teaching requires substantial wisdom from teachers. However, teachers lack such wisdom because they have insufficient experience with online instruction; hence, investigating teachers' experiences in online teaching and providing suggestions on how they can help enhance its effectiveness have become urgent. While some participants in our study could maintain their instructional quality, others could not. Examining the two research questions facilitated our understanding of how mathematics teachers perceived mathematics instruction when nationwide teaching modes required them to shift to online instruction. The following lessons are presented in relation to our research questions.

Lesson One: integrating technology with online mathematics instruction requires practice

The interviewees frequently mentioned that improving technology use during online teaching is a challenge. First, they felt overwhelmed by being exposed to a great number of digital pedagogical resources. It is worth noting that in China, the Full-Time Compulsory Education Mathematics Curriculum Standard is a national-level document that outlines what students are expected to learn about a subject. School textbooks and corresponding instructor materials are developed within the standard's framework and are considered the main teaching guidelines. For example, mathematics textbooks published by People's Education Press are used in approximately 70% to 80% of mainland China (Zhang et al., 2019). For most Chinese mathematics teachers, the

curriculum standard, school textbooks, and teacher materials are the main resources for normal classroom teaching (Cao & Wu, 2016). However, when they became required to conduct online teaching during the pandemic, teachers suddenly had many choices of pedagogical resources, which consisted of different materials in many forms, including lesson plans, prerecorded lesson videos, and teaching slides. Teachers' lack of experience with different forms of digital resources may have made it difficult for them to decide which would be suitable for the context of online instruction.

Most participants also cited difficulties adapting to online teaching because of technology-related issues during lessons. For example, some teachers, such as T4, did not adjust to the technological environment and rather insisted on teaching in a more traditional way with a whiteboard. Although different scholars have noted the advantages of using technology in mathematics classrooms (e.g., Calder, 2012), based on the results of this study, we suggest that mathematics teachers be provided more support before they use it in everyday teaching. Such support must include factual knowledge about how to use technical tools, directions as to how to promote technology integration with practical teaching, and how to address any potential problems (troubleshooting). As also suggested by Joubert (2013), sufficiently addressing these challenges would likely promote the integration of technology with practical teaching.

Lesson Two: the critical role of mathematics teachers in online instruction

In an online mathematics teaching context, teachers must address different issues related to the six subcategories identified in this study. To adapt to an online environment, teachers must adjust their teaching strategies, be flexible with lesson design, integrate certain resources (e.g., digital pedagogical resources), and respond to unpredictable situations (e.g., tracking student participation) during instruction. This analysis echoes Johnson, Coles and Clarke's (2017) recommendation that in a mathematics classroom, teachers need to adapt their planned activities to the demands of each circumstance and situation. We found that despite experiencing difficulties, some of the teachers responded to such challenges and came up with solutions to cope. For example, one primary school teacher (T1) said, "I feel we did not need to rush in online teaching, we needed to leave enough time for students to communicate with us and present themselves. Otherwise, there was no way to understand how well students learned and no possibility to achieve the teaching goal. Online teaching requires much more attention to student learning than the normal classroom, as students are far from you."

It is worth mentioning how the participants compared online teaching with normal classroom teaching and discussed the difficulties in changing the context of a teaching practice, especially teacher–student interaction and the organization of online instruction. The restrictions brought by the shift to online instruction made many teachers feel helpless in terms of shaping their classes. For instance, when comparing mathematics classrooms across 13 countries, Clarke (2004) identified that between-desk instruction was a crucial lesson event through which teachers provided targeted instruction during seatwork and drew upon those insights to develop additional lessons. In this study, the participants reported that a lack of between-desk instruction during teaching rendered them unable to monitor student learning, causing teaching difficulties. This reflected the value this lesson event holds in enhancing the quality of teaching as identified by previous studies. It also raised questions about whether there can be an equivalent lesson event in an online teaching context and, if so, how teachers might identify such an event during online instruction.

Lesson Three: implications about fostering independent study skills among students

Existing studies agree that teachers play a critical role in educational outcomes, especially in an online learning environment (e.g., Freiman et al., 2017). However, in this study—and contrary to expectations—teachers in China recognized the important role of students in determining the success of their own learning, particularly when they engage in online learning. The participants' responses showed that they were concerned about students' ability to advance, as they found that success depended on whether students could manage their learning activities at home. These concerns might also reflect the participants' difficulties in monitoring student learning during online instruction, as student learning was determined primarily by the students themselves. The broader question may be whether mathematics education sufficiently fosters independent study skills.

The experience of online instruction during the pandemic provided teachers, researchers, educators, and the mathematics education community with an opportunity to rethink school-based mathematics education. One of the core questions pertains to how we would expect our students to continue learning if schools have to be closed for an extended period. Most of the interviewed teachers stated that, for those willing to learn, online teaching was as efficient as, or even better than, traditional teaching because students had more time to study and learn independently. However, for those who lacked learning motivation or self-management abilities, online teaching tended to be less effective. Based on the participants' responses, this issue emerged as extremely important. Students' intentions toward and motivations for learning are essential to ensure satisfactory teaching and learning outcomes (Seah, 2019). We would say that this is

not only a challenge for teachers to address, but it is also a burden for the educational system itself.

Conclusion

The results point to the need to reconsider the level of importance of developing students' independent learning skills in mathematics. In China, for more than three months during the COVID-19 pandemic, some mathematics teachers viewed online teaching practices as an experiment in the technology era and/or a learning experience/technology test. During this time, various members of society were involved in the battle against the virus, while the educational community assumed the responsibility of attending to children's learning. Although it was difficult in the beginning, as reported by most of the teacher participants, they eventually got used to online teaching, and their anxiety about it declined.

However, this study's results are far from available findings in terms of lessons from online learning experiences during the pandemic. While we investigated teaching challenges from educators' perspectives, we did not examine students' responses and engagement. Also, despite identifying certain challenges, we found that some teachers were able to develop effective teaching strategies to cope with the difficulties of the online environment. Further investigation into potentially effective teaching strategies is needed, and we will report results in future studies. We also wondered whether these strategies could be used in other classrooms or by teachers of different areas of mathematics. This study only aimed to draw attention to the specific online instruction setup in China and provide lessons we could learn from the unique teaching experiences there. A more in-depth exploration will help us better understand the nature of online mathematics instruction in various cultural contexts.

Appendix: Teacher interview questions

Please indicate your name:
Your teaching grade:
Your teaching experience (in years and/or months):
Your school and its location:

1. Did you have online teaching during the crisis time? How long did it last? From when to when?
2. If you delivered online instruction, was this your first time conducting online classes?
3. Compared to normal classroom teaching, what do you think of online instruction, especially for teaching mathematics? Did you identify any questions or challenges during online teaching? Please use examples.
4. If you discovered challenges during online teaching, how did you overcome these challenges? Can you illustrate your strategy of overcoming difficulties in your teaching practice with examples?
5. During the time of online mathematics instruction, did you feel that you had any chance to learn? Did you have any changes in your mathematics pedagogical knowledge, teaching skills, and beliefs? If so, please give examples.
6. If your school reopens, do you think your teaching practice will change after the period of online instruction? If so, please use examples to illustrate.
7. Comparing student learning before and after the period of online instruction, how do you think of the effect of online instruction on students' mathematics learning?
8. What do you think you should pay attention to after this period of online instruction (e.g., student learning, teaching adjustment, personal tutoring, etc.)? After school reopens, do you think you will make any changes in your teaching practice in terms of technology use, teaching methods, etc.? If so, please give examples.

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