



# A Fuzzy Collaborative Intelligence Approach to Group Decision-Making: a Case Study of Post-COVID-19 Restaurant Transformation

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## Abstract

In a fuzzy group decision-making task, when decision makers lack consensus, existing methods either ignore this fact or force a decision maker to modify his/her judgment. However, these actions may be unreasonable. In this study, a fuzzy collaborative intelligence approach that seeks the consensus among experts in a novel way is proposed. Fuzzy collaborative intelligence is the application of biologically inspired fuzzy logic to a group task. The proposed methodology is based on the fact that a decision maker must make a choice even if he/she is uncertain. As a result, the decision maker's fuzzy judgment matrix may not be able to represent his/her judgment. To solve such a problem, the fuzzy judgment matrix of each decision maker is decomposed into several fuzzy judgment submatrices. From the fuzzy judgment submatrices of all decision makers, a consensus can be easily identified. The proposed fuzzy collaborative intelligence approach and several existing methods have been applied to the case of the post-COVID-19 transformation of a Japanese restaurant in Taiwan. Because such transformation was beyond the expectation of the Japanese restaurant, the employees lacked consensus if existing methods were applied to identify their consensus. The proposed methodology solved this problem. The optimal transformation plan involved increasing the distance between tables, erecting screens between tables, and improving air circulation. In a fuzzy group decision-making task, an acceptable decision cannot be made without the consensus among decision makers. Ignoring this or forcing decision makers to modify their preferences is unreasonable. Identifying the consensus among experts from another point of view is a viable treatment.

**Keywords** Fuzzy group decision-making · Fuzzy collaborative intelligence · Decomposition · Post-COVID-19 transformation · Restaurant

## Introduction

In a fuzzy group decision-making problem, the opinions, judgments, or preferences of decision makers are aggregated to make a joint decision. However, decision makers may be unsure of their preferences but nevertheless be required to express them definitely [4]. To address this issue, researchers have modeled decision makers' judgments or preferences by

using probabilistic or fuzzy sets [13, 18, 19]. Fuzzy sets have ranges that usually overlap to account for a decision maker's uncertainty [35, 37, 39]. Some recent studies have adopted advanced fuzzy numbers of different types with membership, nonmembership, and hesitation functions to provide more flexibility [2, 18, 31].

Another problem is that a decision maker may have multiple views but is forced to aggregate these views into a single preference [13, 20, 23]. As a result, the decision maker's fuzzy judgment matrix may not represent his/her judgment. Under such circumstances, aggregating the fuzzy judgment matrixes of decision makers is not only unreasonable, but also extremely challenging. To solve this problem, a fuzzy collaborative intelligence approach is proposed in this study. Fuzzy logic is a biologically inspired reasoning technique [34], and fuzzy collaborative intelligence is the application of fuzzy logic to a social context [28].

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In the proposed methodology, decision makers first express their judgments of the relative priorities of criteria. Then, fuzzy intersection (FI) is applied to check for the consensus among these judgments [13]. If no consensus has been reached, the multiple views of each decision maker are identified by decomposing the decision maker's fuzzy judgment matrix into several fuzzy judgment submatrices. All the views of each decision maker are considered in the aggregation process, thereby increasing the likelihood of reaching a consensus.

The novelty of the proposed methodology lies in the following aspects:

- (1) Unlike most fuzzy group decision-making methods [3, 14, 16, 17], the proposed methodology is not based on the similarity or proximity of decision makers' judgments or preferences.
- (2) When decision makers have no consensus, the proposed methodology does not force decision makers to modify their judgments or preferences [14] or eliminate decision makers to reach consensus [35]. Instead, it considers the multiple views of each decision maker to increase the possibility of reaching consensus.

The rest of this article is organized as follows. “Literature Review” presents a review of the literature on the subject. “Methodology” introduces the fuzzy collaborative intelligence approach, including the procedure for implementing the fuzzy collaborative intelligence approach, the measurement of consensus, the decomposition of a fuzzy judgment matrix, the derivation of fuzzy priorities, and the evaluation of the overall performance. “Case Study” details the application of the proposed methodology to a post-COVID-19 restaurant transformation problem. In “Background”, several existing methods have been applied to the same case for comparison. Finally, “Conclusions” concludes this study and presents recommendations for further research.

## Literature Review

The consensus among decision makers is typically measured by the similarity or proximity of their judgments [3, 16, 17, 19], for which the judgments of the decision makers are averaged as a baseline. Then, the decision maker whose judgment is furthest from the average is either excluded from the decision-making process [1] or asked to modify his/her judgment [14]. [38] expressed decision makers' preferences using interval fuzzy numbers. In addition, decision makers that contributed less to the consensus were asked to modify their preferences by considering those of nearby decision makers. For a similar purpose, [37] adopted fuzzy covering-based rough sets (or fuzzy rough coverings).

Some studies have proposed fuzzy collaborative intelligence methods to model the consensus among decision makers with the FI of their judgments [26, 35]. If the FI result is an empty set, no consensus exists. If decision makers' judgments are far from others', they do not overlap.

When no consensus can be reached by decision makers, partial consensus (i.e., the consensus among most decision makers) can be sought instead [7]. Then, the decision maker whose judgment is furthest from the partial consensus can be asked to modify his/her judgment.

Decision makers' preferences can be expressed in terms of utility values or rankings, which can be either multiplicative or additive. Aggregating preferences of different types is a challenging task [23].

A decision-making problem, such as one on an e-commerce application, e-marketplace, or social media platform, with numerous decision makers (e.g., more than 20) is considered a large-scale group decision-making problem [22, 30]. Two challenges must be overcome to solve such problems. First, decision makers usually have different backgrounds. Second, decision makers can express their opinions at different times and places [30]. Expressing the judgments or preferences of decision makers by using fuzzy numbers increases the possibility of reaching consensus.

In a large-scale group decision-making problem, decision makers exhibit bounded rationality and varying psychological behaviors, such as loss avoidance, sensitivity reduction, probability judgment distortion, and regret aversion [19]. Jin et al. [19] devised a regret–rejoice function. The greater the deviation between the utilities of two alternatives is, the less a decision maker regrets choosing the preferred alternative. In addition, a decision maker's authority level (or weight) is determined with respect to the consensus achieved with other decision makers.

## Methodology

In the proposed fuzzy collaborative intelligence approach, multiple decision makers collaborate with the assistance of a coordinator to evaluate and compare the overall performances of alternatives. The variables and notations used in the proposed methodology are defined as follows:

- (+): fuzzy addition
- (−): fuzzy subtraction
- (×): fuzzy multiplication
- $\tilde{\lambda}(k)$ : the fuzzy eigenvalue of  $\tilde{A}(k)$ ;  $k = 1-K$
- $\tilde{\lambda}_{\max}(k)$ : the maximal fuzzy eigenvalue of  $\tilde{A}(k)$ ;  $k = 1-K$
- $\tilde{p}_{qi}$ : the normalized value of  $\tilde{p}_{qi}$ ;  $q = 1-Q$ ;  $i \in [1, n]$
- $\tilde{A}^+$ : an ideal solution
- $\tilde{A}^-$ : an anti-ideal solution

- $\tilde{a}_{ij}(k)$ : the priority of criterion  $i$  relative to criterion  $j$  to decision maker  $k$ ;  $i, j \in [1, n]$ ;  $k = 1-K$
- $\tilde{\mathbf{A}}(k)$ : the fuzzy judgment matrix of decision maker  $k$ ;  $k = 1-K$
- $\tilde{\mathbf{A}}(k-m)$ : the fuzzy judgment submatrix of decision maker  $k$ ;  $k = 1-K$ ;  $m = 1-M$
- $COG()$ : the center-of-gravity function
- $\widetilde{CR}(k)$ : the fuzzy consistency ratio of  $\tilde{\mathbf{A}}(k)$ ;  $k = 1-K$
- $\tilde{d}()$ : a fuzzy distance function
- $\tilde{d}_q^+$ : the distance from alternative  $q$  to  $\tilde{\mathbf{A}}^+$ ;  $q = 1-Q$
- $\tilde{d}_q^-$ : the distance from alternative  $q$  to  $\tilde{\mathbf{A}}^-$ ;  $q = 1-Q$
- $\widetilde{det}()$ : the determinant function
- $\widetilde{FI}()$ : the FI function
- $\tilde{O}_q$ : the overall performance of alternative  $q$ ;  $q = 1-Q$
- $\tilde{p}_{qi}$ : the performance of alternative  $q$  in optimizing criterion  $i$ ;  $q = 1-Q$ ;  $i \in [1, n]$
- $RI$ : the random consistency index
- $\tilde{w}_i(k)$ : the fuzzy priority of criterion  $i$  to decision maker  $k$ ;  $i \in [1, n]$ ;  $k = 1-K$
- $\tilde{w}_i(k-m)$ : the fuzzy priority of criterion  $i$  to decision maker  $k$  according to the  $m$ th view;  $i \in [1, n]$ ;  $k = 1-K$ ;  $m = 1-M$
- $w_i^{(c)}(k)$ : the crisp priority of criterion  $i$  to decision maker  $k$ ;  $i \in [1, n]$ ;  $k = 1-K$
- $\tilde{\mathbf{x}}(k)$ : the fuzzy eigenvector of  $\tilde{\mathbf{A}}(k)$ ;  $k = 1-K$

Without loss of generality, all fuzzy variables and parameters used in the proposed approach are given in or approximated by triangular fuzzy numbers (TFNs).

**Procedure**

The proposed fuzzy collaborative intelligence approach comprises the following steps:

Step 1: (Each decision maker) Construct (or modify) a fuzzy judgment matrix  $\tilde{\mathbf{A}}(k)$ .

Step 2: (Each decision maker) If  $\tilde{\mathbf{A}}(k)$  is consistent, go to Step 3; otherwise, return to Step 1.

Step 3: (Each decision maker) Apply the calibrated fuzzy geometric mean (FGM) method [11] to derive the value of  $\tilde{w}_i(k)$ .

Step 4: (Coordinator) Apply  $FI()$  to aggregate the values of  $\{\tilde{w}_i(k) | k = 1-K\}$ .

Step 5: If  $\widetilde{FI}(\{\tilde{w}_i(k)\}) = \emptyset$ , go to Step 6; otherwise, go to Step 9.

Step 6: (Coordinator) Formulate and optimize a fuzzy nonlinear programming (FNLP) model to decompose  $\tilde{\mathbf{A}}(k)$  into  $\{\tilde{\mathbf{A}}(k-m) | m = 1 \sim M\}$ .

Step 7: (Coordinator) Derive  $\{\tilde{w}_i(k-m) | m = 1 \sim M\}$  from  $\tilde{\mathbf{A}}(k-m)$ .

Step 8: (Coordinator) Return to Step 4.

Step 9: (Coordinator) Evaluate the overall performance of each alternative by using the fuzzy technique for order

of preference by similarity to the ideal solution (TOPSIS) [12, 36].

Step 10: (All decision makers) Select the optimally performing alternative.

A flow chart is presented in Fig. 1 to illustrate the procedure.

**The Measurement of Consensus**

In the proposed methodology, each decision maker first performs a pairwise comparison of the relative priorities of criteria. Then, the pairwise comparison results by decision maker  $k$  are put in  $\tilde{\mathbf{A}}(k) = [\tilde{a}_{ij}(k)]$ , where

$$\tilde{a}_{ij}(k) = \begin{cases} 1 & \text{if } j = 1 \\ 1/\tilde{a}_{ji}(k) & \text{otherwise} \end{cases} \tag{1}$$

The fuzzy eigenvalue and eigenvector of  $\tilde{\mathbf{A}}(k)$  satisfy

$$\widetilde{det}(\tilde{\mathbf{A}}(k)(-\tilde{\lambda}(k)\mathbf{I})) = 0 \tag{2}$$

$$(\tilde{\mathbf{A}}(k)(-\tilde{\lambda}(k)\mathbf{I})(\times)\tilde{\mathbf{x}}(k) = 0 \tag{3}$$

Subsequently, the fuzzy priority of criterion  $i$  is derived as

$$\tilde{w}_i(k) = \frac{\tilde{x}_i(k)}{\sum_{h=1}^n \tilde{x}_h(k)} \tag{4}$$

From Eq. (4), the maximal fuzzy eigenvalue is derived as

$$\tilde{\lambda}_{\max}(k) = \frac{1}{n} \sum_{i=1}^n \sum_{j=1}^n \frac{\tilde{a}_{ij}(k)(\times)\tilde{w}_j(k)}{\tilde{w}_i(k)} \tag{5}$$

However, because Eqs. (2) and (3) involve many fuzzy multiplication operations, they are difficult to solve. Therefore, the calibrated FGM method [11] is applied to derive the approximate value of  $\tilde{w}_i(k)$ :

(The Calibrated FGM Method).

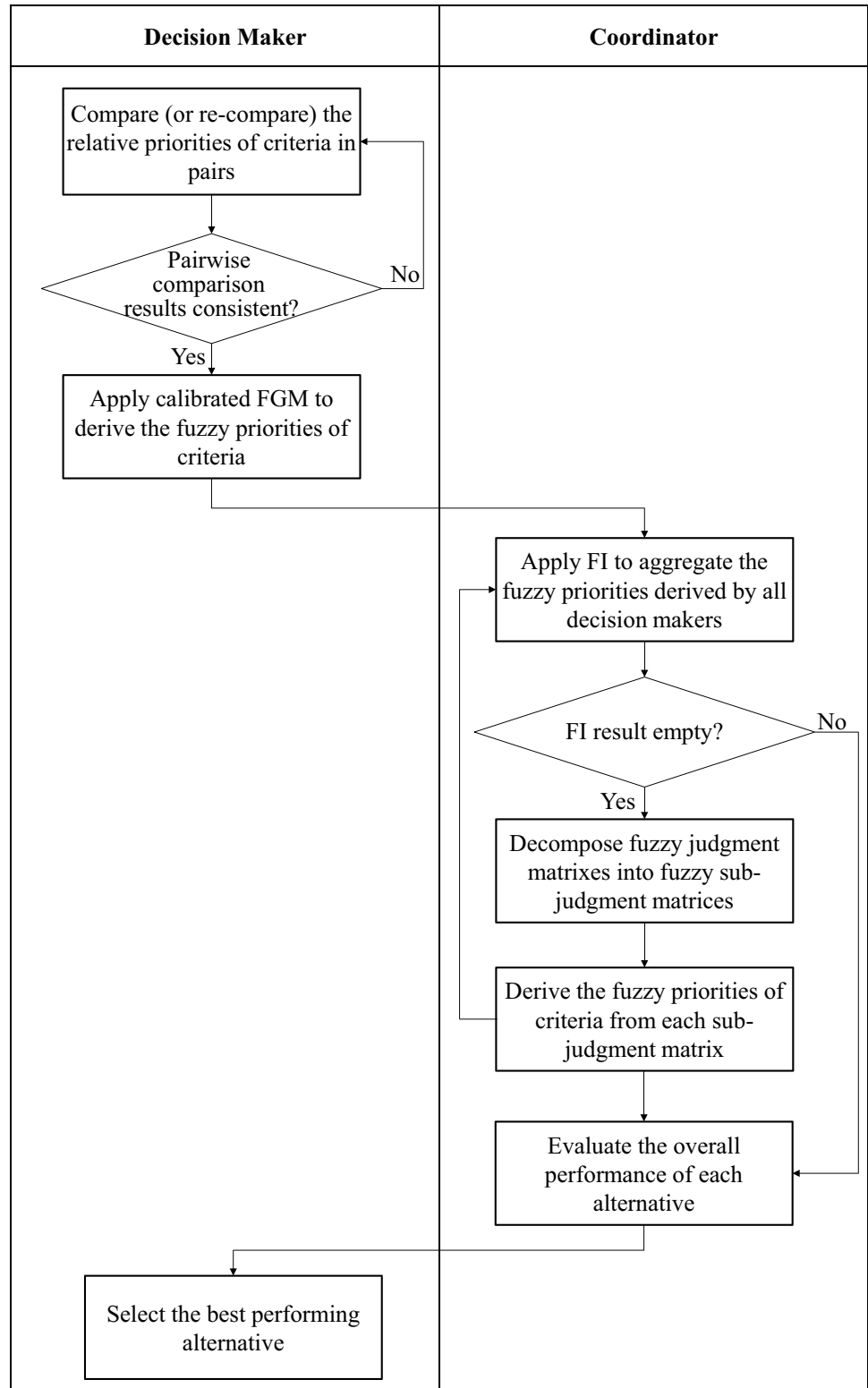
Step 1: Derive the approximate value of  $\tilde{w}_i(k)$  by using the traditional FGM method as follows [39]:

$$\tilde{w}_i(k) \cong \frac{\sqrt[n]{\prod_{j=1}^n \tilde{a}_{ij}(k)}}{\sum_{l=1}^n \sqrt[n]{\prod_{j=1}^n \tilde{a}_{lj}(k)}} \tag{6}$$

Step 2: Treat  $\tilde{\mathbf{A}}(k)$  as a crisp matrix by letting  $a_{ij1}(k) = a_{ij2}(k)$  and  $a_{ij3}(k) = a_{ij2}(k)$ ; then, derive the priority of criterion  $i$  using an eigen analysis [33].

Step 3: Calibrate the value of  $\tilde{w}_i$  in the following manner:

**Fig. 1** A flow chart of the fuzzy collaborative intelligence approach



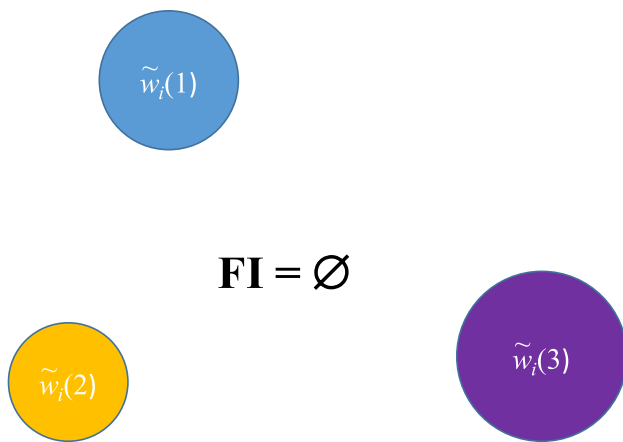


Fig. 2 The lack of consensus among decision makers

$$\begin{aligned} \tilde{w}_i(k) &\rightarrow \tilde{w}_i(k) + w_i^{(c)}(k) - w_{i2}(k) \\ &= (w_{i1}(k) + w_i^{(c)}(k) - w_{i2}(k), w_i^{(c)}(k), w_{i3}(k) + w_i^{(c)}(k) - w_{i2}(k)) \end{aligned} \tag{7}$$

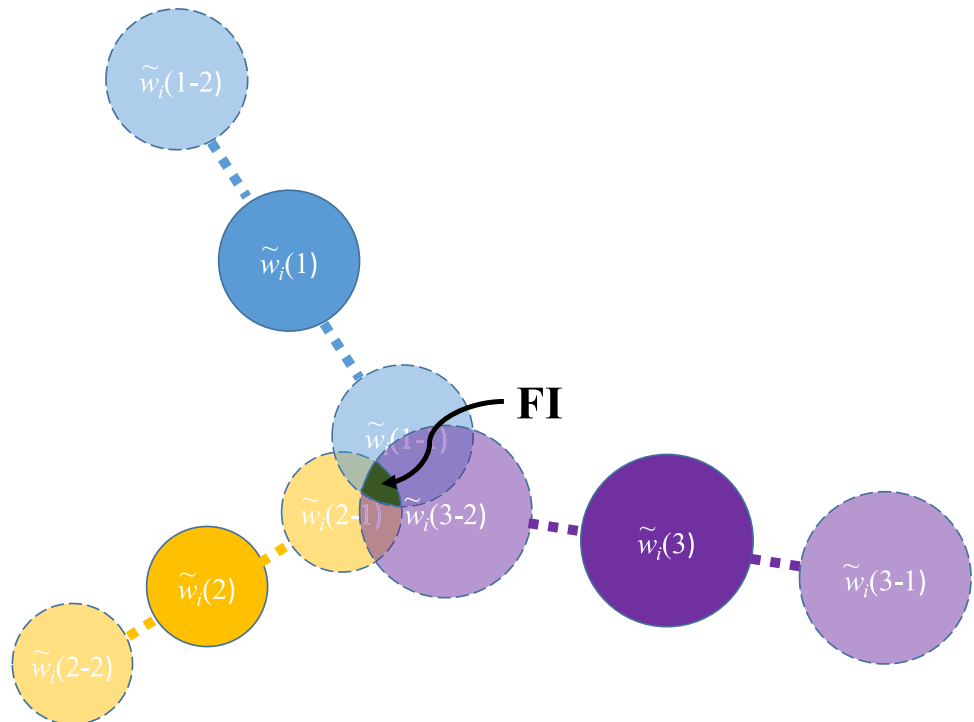
The consistency of  $\tilde{\mathbf{A}}(k)$  can be evaluated in terms of  $\tilde{CR}$  as

$$\tilde{CR}(k) = \frac{\tilde{\lambda}_{\max}(k) - n}{n - 1} \frac{1}{RI} \tag{8}$$

$\tilde{CR}(k)$  should be less than 0.1; otherwise, the decision maker must modify his/her pairwise comparison results to rederive the values of  $\tilde{\lambda}(k)$ ,  $\tilde{\mathbf{x}}(k)$ , and  $\tilde{w}_i(k)$ .

To check for the consensus among decision makers, the FI of the priorities derived by them,  $FI(\{\tilde{w}_i(k)\})$ , is derived:

Fig. 3 A consensus achieved by overlapping fuzzy judgment submatrices



$$\mu_{\tilde{FI}(\{\tilde{w}_i(k)\})}(x) = \sup_i \min(\{\mu_{\tilde{w}_i(k)}(x)\}) \tag{9}$$

If  $FI(\{\tilde{w}_i(k)\}) = \emptyset$  for any  $i$ , then a consensus does not exist, as illustrated in Fig. 2. To solve this problem, the fuzzy judgment matrices of decision makers are decomposed into fuzzy judgment submatrices to derive priorities that may overlap, as illustrated in Fig. 3.

### The Decomposition of a Fuzzy Judgment Matrix

#### The FNL Model

Lin and Chen [21] decomposed a crisp judgment matrix into several judgment submatrices, making these judgment submatrices more consistent than the original judgment matrix and far from each other. Subsequently, Chen and Lin [10] extended this method to decompose a fuzzy judgment matrix. However, these studies involved only a single decision maker.

In the proposed methodology, the fuzzy judgment matrix of decision maker  $k$  is decomposed into several fuzzy judgment submatrices, for which the fuzzy arithmetic average operator [10] is applied to active pairwise comparison results:

$$\tilde{\mathbf{A}}(k) := \frac{\sum_{m=1}^M \tilde{\mathbf{A}}(k - m)}{M} \tag{10}$$

All fuzzy judgment submatrices meet the following basic requirements:

$$\det(\tilde{\mathbf{A}}(k-m)(-\tilde{\lambda}(k-m)\mathbf{I}) = 0 \tag{11}$$

$$(\tilde{\mathbf{A}}(k-m)(-\tilde{\lambda}(k-m)\mathbf{I})(\times)\tilde{\mathbf{x}}(k-m) = 0 \tag{12}$$

$$\tilde{w}_i(k-m) = \frac{\tilde{x}_i(k-m)}{\sum_{h=1}^n \tilde{x}_h(k-m)} \tag{13}$$

In addition, a fuzzy judgment submatrix should not differ considerably from the original fuzzy judgment matrix:

$$\tilde{d}(\tilde{\mathbf{A}}(k), \tilde{\mathbf{A}}(k-m)) \leq \xi \tag{14}$$

After decomposition, the fuzzy priorities of criteria derived by all decision makers should overlap:

$$\tilde{FI}(\{\tilde{w}_i(k-m)\}) \neq \emptyset \tag{15}$$

This requirement can be decomposed into the requirement for each pair:

$$\tilde{FI}(\tilde{w}_i(k-m), \tilde{w}_i(l-m)) \neq \emptyset \tag{16}$$

A fuzzy judgment matrix can be decomposed in numerous ways. The fuzzy judgment submatrices of decision makers can overlap in even more ways, as illustrated in Fig. 4. In the proposed methodology, the decomposition of fuzzy judgment matrices is optimized by maximizing the overlap among the fuzzy priorities derived from the fuzzy judgment submatrices. For this purpose, the average width of  $\tilde{FI}(\{\tilde{w}_i(k-m)\})$  is maximized:

$$\text{Max } Z = \frac{1}{n} \sum_{i=1}^n (\max(\tilde{FI}(\{\tilde{w}_i(k-m)\})) - \min(\tilde{FI}(\{\tilde{w}_i(k-m)\}))) \tag{17}$$

Finally, the following FNLP model is optimized: (The FNLP model)

$$\text{Max } Z = \frac{1}{n} \sum_{i=1}^n (\max(\tilde{FI}(\{\tilde{w}_i(k-m)\})) - \min(\tilde{FI}(\{\tilde{w}_i(k-m)\}))) \tag{18}$$

subject to

$$\tilde{FI}(\tilde{w}_i(k-m), \tilde{w}_i(l-m)) \neq \emptyset \forall i, k, l, \text{ and } m; k \neq l \tag{19}$$

$$\tilde{d}(\tilde{\mathbf{A}}(k), \tilde{\mathbf{A}}(k-m)) \leq \xi \forall k \text{ and } m \tag{20}$$

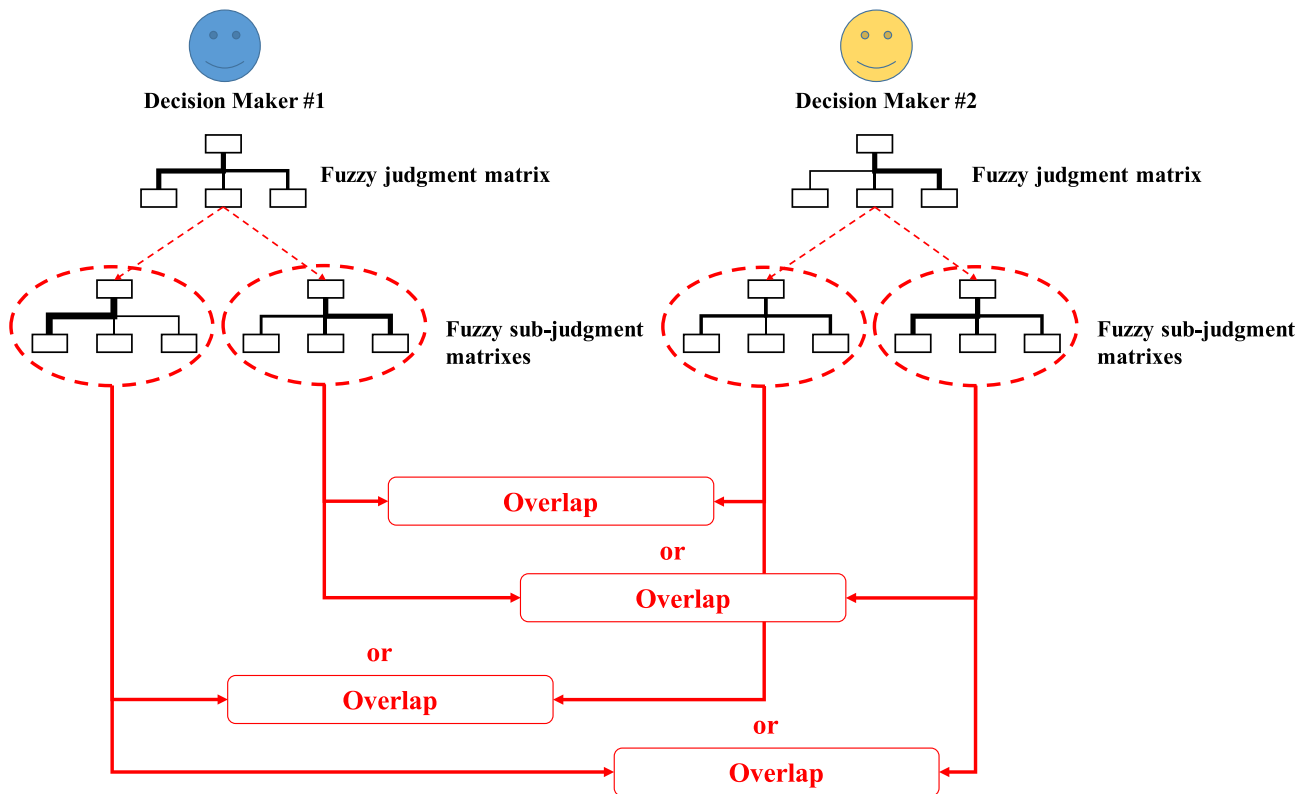


Fig. 4 Possible ways of overlapping fuzzy judgment submatrices

$$\tilde{A}(k) := \frac{\sum_{m=1}^M \tilde{A}(k-m)}{M} \forall k \tag{21}$$

$$\det(\tilde{A}(k-m)(-) \tilde{\lambda}(k-m)I) = 0 \forall k \text{ and } m \tag{22}$$

$$(\tilde{A}(k-m)(-) \tilde{\lambda}(k-m)I)(\times) \tilde{x}(k-m) = 0 \forall k \text{ and } m \tag{23}$$

$$\tilde{w}_i(k-m) = \frac{\tilde{x}_i(k-m)}{\sum_{h=1}^n \tilde{x}_h(k-m)} \forall k \text{ and } m \tag{24}$$

However, the FNLP model is cumbersome. Therefore, in the next section, the FNLP model is converted into an equivalent nonlinear programming (NLP) problem.

**An Equivalent NLP Model**

The two variables in the objective function are equivalent to (Chen and Lin, 2008)

$$\max(\tilde{FI}(\{\tilde{w}_i(k-m)\})) = \min_k(\max(\{\tilde{w}_i(k-m)\})) \tag{25}$$

$$\min(\tilde{FI}(\{\tilde{w}_i(k-m)\})) = \max_k(\min(\{\tilde{w}_i(k-m)\})) \tag{26}$$

Equations (25) and (26) are equivalent to

$$\max(\tilde{FI}(\{\tilde{w}_i(k-m)\})) \leq w_{i3}(k-m) \forall k \tag{27}$$

$$\min(\tilde{FI}(\{\tilde{w}_i(k-m)\})) \geq w_{i1}(k-m) \forall k \tag{28}$$

To optimize Z, the two sides of these equations must be equal.

The following theorem helps to convert Constraint (19).

**Theorem 1:** Two TFNs  $\tilde{A}$  and  $\tilde{B}$  overlap if  $\min(A_3, B_3) \geq \max(A_1, B_1)$ .

**Proof:** The ranges of  $\tilde{A}$  and  $\tilde{B}$  can be represented by  $[A_1, A_3]$  and  $[B_1, B_3]$ , respectively, which overlap if the upper bound of the smaller one is greater than the lower bound of the larger one:

$$\max(\min([A_1, A_3], [B_1, B_3])) \geq \min(\max([A_1, A_3], [B_1, B_3])) \tag{29}$$

which becomes

$$\max(\min(A_1, B_1), \min(A_3, B_3)) \geq \min(\max(A_1, B_1), \max(A_3, B_3)) \tag{30}$$

Therefore,

$$\min(A_3, B_3) \geq \max(A_1, B_1) \tag{31}$$

Hence, Theorem 1 is proved.

Applying Theorem 1 to Eq. (19) gives

$$\min(w_{i3}(k-m), w_{i3}(l-m)) \geq \max(w_{i1}(k-m), w_{i1}(l-m)) \tag{32}$$

In addition, constraint (20) can be simplified as

$$d(\mathbf{A}_2(k), \mathbf{A}_2(k-m)) \leq \xi \tag{33}$$

because the lower and upper bounds of  $\tilde{a}_{ij}(k)$  are usually dependent on the core [39]. Therefore, considering only the core of  $\tilde{a}_{ij}(k)$  in measuring the distance is sufficient. Then, the Frobenius distance [15] is used as  $d()$ :

$$d(\mathbf{A}_2(k), \mathbf{A}_2(k-m)) = \sqrt{\sum_{i=1}^n \sum_{j=1}^n (a_{ij2}(k) - a_{ij2}(k-m))^2} \tag{34}$$

Equation (21) is equivalent to

$$\tilde{a}_{ij}(k) = \frac{\sum_{m=1}^M \tilde{a}_{ij}(k-m)}{M} \forall \tilde{a}_{ij}(k) \geq 1 \tag{35}$$

To convert Eqs. (22)–(24), the following theorem is helpful.

**Theorem 2:** In the traditional FGM method, a fuzzy priority is derived as

$$w_{i1}(k) \cong \frac{1}{1 + \sum_{h \neq i} \frac{\sqrt[n]{\prod_{j=1}^n a_{hj3}(k)}}{\sqrt[n]{\prod_{j=1}^n a_{ji1}(k)}}} \tag{36}$$

$$w_{i2}(k) \cong \frac{1}{1 + \sum_{h \neq i} \frac{\sqrt[n]{\prod_{j=1}^n a_{hj2}(k)}}{\sqrt[n]{\prod_{j=1}^n a_{ij2}(k)}}} \tag{37}$$

$$w_{i3}(k) \cong \frac{1}{1 + \sum_{h \neq i} \frac{\sqrt[n]{\prod_{j=1}^n a_{hj1}(k)}}{\sqrt[n]{\prod_{j=1}^n a_{ij3}(k)}}} \tag{38}$$

**Proof:** The proof refers to Chen and Wang [5].

Applying Theorem 2 to Eq. (7) yields

$$w_{i1}(k) = \frac{1}{1 + \sum_{h \neq i} \frac{\sqrt[n]{\prod_{j=1}^n a_{hj3}(k)}}{\sqrt[n]{\prod_{j=1}^n a_{ji1}(k)}}} + w_i^{(c)}(k) - \frac{1}{1 + \sum_{h \neq i} \frac{\sqrt[n]{\prod_{j=1}^n a_{hj2}(k)}}{\sqrt[n]{\prod_{j=1}^n a_{ij2}(k)}}} \tag{39}$$

$$w_{i2}(k) = w_i^{(c)}(k) \tag{40}$$

$$w_{i2}(k - m) = w_i^{(c)}(k - m) \tag{52}$$

$$w_{i3}(k) = \frac{1}{1 + \sum_{h \neq i} \frac{\sqrt[n]{\prod_{j=1}^n a_{hj1}(k)}}{\sqrt[n]{\prod_{j=1}^n a_{ij3}(k)}}} + w_i^{(c)}(k) - \frac{1}{1 + \sum_{h \neq i} \frac{\sqrt[n]{\prod_{j=1}^n a_{hj2}(k)}}{\sqrt[n]{\prod_{j=1}^n a_{ij2}(k)}}} \tag{41}$$

$$w_{i1}(k - m) = \frac{1}{1 + \sum_{h \neq i} \frac{\sqrt[n]{\prod_{j=1}^n a_{hj1}(k-m)}}{\sqrt[n]{\prod_{j=1}^n a_{ij3}(k-m)}}} + w_i^{(c)}(k - m) - \frac{1}{1 + \sum_{h \neq i} \frac{\sqrt[n]{\prod_{j=1}^n a_{hj2}(k-m)}}{\sqrt[n]{\prod_{j=1}^n a_{ij2}(k-m)}}} \tag{53}$$

Therefore, Eqs. (22)–(24) are equivalent to

$$w_{i1}(k - m) = \frac{1}{1 + \sum_{h \neq i} \frac{\sqrt[n]{\prod_{j=1}^n a_{hj3}(k-m)}}{\sqrt[n]{\prod_{j=1}^n a_{ij1}(k-m)}}} + w_i^{(c)}(k - m) - \frac{1}{1 + \sum_{h \neq i} \frac{\sqrt[n]{\prod_{j=1}^n a_{hj2}(k-m)}}{\sqrt[n]{\prod_{j=1}^n a_{ij2}(k-m)}}} \tag{42}$$

$$w_{i2}(k - m) = w_i^{(c)}(k - m) \tag{43}$$

$$w_{i3}(k - m) = \frac{1}{1 + \sum_{h \neq i} \frac{\sqrt[n]{\prod_{j=1}^n a_{hj1}(k-m)}}{\sqrt[n]{\prod_{j=1}^n a_{ij3}(k-m)}}} + w_i^{(c)}(k - m) - \frac{1}{1 + \sum_{h \neq i} \frac{\sqrt[n]{\prod_{j=1}^n a_{hj2}(k-m)}}{\sqrt[n]{\prod_{j=1}^n a_{ij2}(k-m)}}} \tag{44}$$

Finally, the following NLP problem can be solved instead: (The NLP Model)

$$\text{Max } Z = \frac{1}{n} \sum_{i=1}^n (\max(\tilde{F}I(\{\tilde{w}_i(k - m)\})) - \min(\tilde{F}I(\{\tilde{w}_i(k - m)\}))) \tag{45}$$

subject to

$$\max(\tilde{F}I(\{\tilde{w}_i(k - m)\})) \leq w_{i3}(k - m) \forall i, k, \text{ and } m \tag{46}$$

$$\min(\tilde{F}I(\{\tilde{w}_i(k - m)\})) \geq w_{i1}(k - m) \forall i, k, \text{ and } m \tag{47}$$

$$\min(w_{i3}(k - m), w_{i3}(l - m)) \geq \max(w_{i1}(k - m), w_{i1}(l - m)) \forall i, k, l, \text{ and } m; k \neq l \tag{48}$$

$$\sqrt{\sum_{i=1}^n \sum_{j=1}^n (a_{ij2}(k) - a_{ij2}(k - m))^2} \leq \xi \forall i, k, \text{ and } m \tag{49}$$

$$\tilde{a}_{ij}(k) = \frac{\sum_{m=1}^M \tilde{a}_{ij}(k - m)}{M} \forall \tilde{a}_{ij}(k) \geq 1 \tag{50}$$

$$w_{i1}(k - m) = \frac{1}{1 + \sum_{h \neq i} \frac{\sqrt[n]{\prod_{j=1}^n a_{hj3}(k-m)}}{\sqrt[n]{\prod_{j=1}^n a_{ij1}(k-m)}}} + w_i^{(c)}(k - m) - \frac{1}{1 + \sum_{h \neq i} \frac{\sqrt[n]{\prod_{j=1}^n a_{hj2}(k-m)}}{\sqrt[n]{\prod_{j=1}^n a_{ij2}(k-m)}}} \tag{51}$$

The NLP model can be solved using approaches such as the outer approximation/generalized Benders decomposition method (Costa, 2005) or a branch-and-bound method (Cacchiani and D’Ambrosio, 2017).

The FI result is a polygonal fuzzy number [9]. To facilitate the subsequent evaluation process, the FI result is approximated by an equivalent TFN as [27, 32, 35]

$$\begin{aligned} \tilde{F}I(\{\tilde{w}_i(k - m)\}) &\cong (\min(\tilde{F}I(\{\tilde{w}_i(k - m)\})), \\ &3COG(\tilde{F}I(\{\tilde{w}_i(k - m)\})) - \min(\tilde{F}I(\{\tilde{w}_i(k - m)\})) - \max(\tilde{F}I(\{\tilde{w}_i(k - m)\})), \\ &\max(\tilde{F}I(\{\tilde{w}_i(k - m)\}))) \end{aligned} \tag{54}$$

where

$$COG(\tilde{F}I(\{\tilde{w}_i(k - m)\})) = \frac{\int_{all\ x} x \mu_{\tilde{F}I}(x) dx}{\int_{all\ x} \mu_{\tilde{F}I}(x) dx} \tag{55}$$

In this manner, the lower bound, center of gravity (COG), and upper bound of the equivalent TFN are the same as those of the FI result.

### The Evaluation of the Overall Performance

Based on the FI result, decision makers evaluate the overall performance of an alternative using fuzzy TOPSIS [12, 36]. Other fuzzy evaluation methods such as the fuzzy weighted average (FWA) method [8, 27], the fuzzy *visekriterijumska optimizacija i kompromisno resenje* (VIKOR) method [25], and the fuzzy measurement alternatives and ranking according to compromise solution (MARCOS) method [24] are also applicable.

The fuzzy TOPSIS method is applied to evaluate the overall performance of alternative *q* as follows:

$$\tilde{O}_q = \frac{\tilde{d}_q^-}{\tilde{d}_q^+ (+) \tilde{d}_q^-} \tag{56}$$

where

$$\tilde{d}_q^+ = \sqrt{\sum_{i=1}^n (\tilde{\Lambda}_i^+(-)(\tilde{F}I(\{\tilde{w}_i(k - m)\}))(\times) \tilde{\rho}_{qi})^2} \tag{57}$$



$$\tilde{d}_q^- = \sqrt{\sum_{i=1}^n (\tilde{\Lambda}_i^-(\cdot)(\tilde{FI}(\{\tilde{w}_i(k-m)\})(\times)\tilde{\rho}_{qi}))^2} \quad (58)$$

in which

$$\tilde{\Lambda}_i^+ \geq \tilde{FI}(\{\tilde{w}_i(k-m)\})(\times)\tilde{\rho}_{qi} \forall q \quad (59)$$

$$\tilde{\Lambda}_i^- \leq \tilde{FI}(\{\tilde{w}_i(k-m)\})(\times)\tilde{\rho}_{qi} \forall q \quad (60)$$

and

$$\begin{aligned} \tilde{\rho}_{qi} &= \frac{\tilde{p}_{qi}}{\sqrt{\sum_{\phi=1}^Q \tilde{p}_{\phi i}^2}} \\ &= \frac{1}{\sqrt{1 + \sum_{\phi \neq q} \left(\frac{\tilde{p}_{\phi i}}{\tilde{p}_{qi}}\right)^2}} \end{aligned} \quad (61)$$

## A Case Study

### Background

The COVID-19 pandemic heavily affected the hospitality industry. Most restaurants were forced to close to avoid the spread of the virus through air circulation in indoor spaces. In Taiwan, according to the level-three alert restrictions implemented on May 19, 2021, restaurants could not provide dine-in services [5]. Only takeout and delivery services were permitted, which were insufficient to compensate for the sharp decline in dine-in sales revenue. After several weeks under level-three restrictions, many restaurants were forced to close down. The negative implications for Taiwanese restaurants included the following. A restaurant could be

open for only a very short period each day. Even if a restaurant could reopen, few customers would eat there. Finally, recovering to the previous level of sales revenue could take 6 months or more.

Considering such a future, restaurant owners must decide, on the basis of a detailed cost–benefit analysis, whether to close their restaurants. If the answer was no, then the restaurant owner must develop a business plan for operating during the COVID-19 pandemic, possibly including offering take-out or delivery meals, applying for government bailout funds, negotiating lower rents, and restructuring human resources. However, as the virus continued to mutate, whether restaurant operations could return to normal was unclear. The slow vaccination rate in Taiwan exacerbated such concerns. Therefore, restaurant owners must consider changes.

In this case, the owner of a Japanese restaurant in Taichung City, Taiwan, wished to transform the restaurant after the COVID-19 pandemic. During the pandemic, many Japanese restaurants in Taiwan, such as Sono, Kikumodo, Ranmaru, and Ito, were forced to close down because of the government ban on indoor dining. The following factors were critical in decision-making for restaurant owners wishing to operate during the pandemic: (1) estimated expenses, (2) the approximate time required, (3) the attractiveness to customers, (4) changes to the image of the restaurant, and (5) the compatibility with current operations.

The following alternatives were considered for the transformation of the Japanese restaurant in this case:

- (1) Alternative I: Reducing the indoor dining area and increasing the outdoor dining area;
- (2) Alternative II: Dividing the indoor dining area into smaller booths;
- (3) Alternative III: Increasing the distance between tables, erecting screens between tables, and improving air circulation;

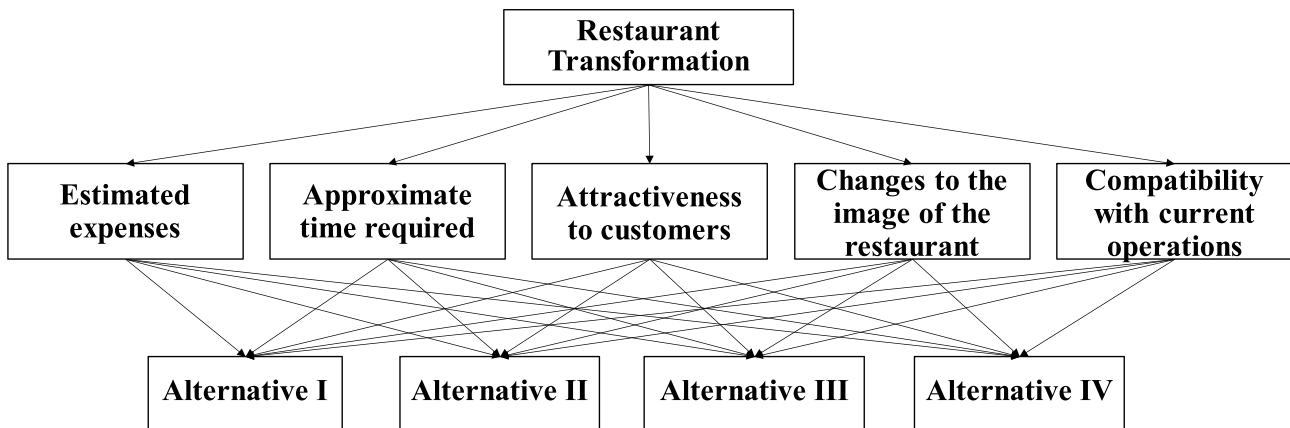


Fig. 5 The restaurant transformation decision-making problem

(4) Alternative IV: Reducing the number of indoor tables to limit the number of simultaneous diners.

that of the approximate time required and was represented by the TFN (3, 5, 7). The consistency ratios of these fuzzy judgment matrices were evaluated as

The restaurant transformation decision-making problem is illustrated in Fig. 5.

$$\widetilde{CR}(1) = (0.00, 0.09, 6.61)$$

$$\widetilde{CR}(2) = (0.00, 0.07, 10.26)$$

$$\widetilde{CR}(3) = (0.00, 0.10, 11.26)$$

### The Application of the Proposed Methodology

Three decision makers, namely, the owner, manager, and chef of the Japanese restaurant, made a joint decision by applying the proposed methodology.

All showed certain levels of consistency.

First, the decision makers made judgments about the relative priorities of criteria. The results are summarized in Table 1. For example, according to Decision Maker 1, the priority of estimated expenses was approximately five times

To check for the consensus among the three decision makers, the FI of the fuzzy priorities derived by them was obtained. The result is shown in Fig. 6. These decision makers lacked a consensus on the fuzzy priority of the

**Table 1** Fuzzy judgment matrices constructed by decision makers

(a) Decision Maker 1					
	Estimated expenses	Approximate time required	Attractiveness to customers	Changes to restaurant image	Compatibility with current operations
Estimated expenses	1	(3, 5, 7)	1/(3, 5, 7)	1/(1, 3, 5)	1/(1, 3, 5)
Approximate time required	1/(3, 5, 7)	1	1/(5, 7, 9)	1/(2, 4, 6)	1/(2, 4, 6)
Attractiveness to customers	(3, 5, 7)	(7, 9, 9)	1	(2, 4, 6)	(1, 3, 5)
Changes to restaurant image	(1, 3, 5)	(3, 5, 7)	1/(2, 4, 6)	1	1/(1, 3, 5)
Compatibility with current operations	(1, 3, 5)	(2, 4, 6)	1/(1, 3, 5)	(1, 3, 5)	1
(b) Decision Maker 2					
	Estimated expenses	Approximate time required	Attractiveness to customers	Changes to restaurant image	Compatibility with current operations
Estimated expenses	1	(1, 3, 5)	(1, 3, 5)	(1, 3, 5)	(1, 1, 3)
Approximate time required	1/(1, 3, 5)	1	1/(1, 3, 5)	1/(1, 3, 5)	1/(3, 5, 7)
Attractiveness to customers	1/(1, 3, 5)	(1, 3, 5)	1	(1, 3, 5)	(1,1, 3)
Changes to restaurant image	1/(1, 3, 5)	(1, 3, 5)	1/(1, 3, 5)	1	1/(2, 4, 6)
Compatibility with current operations	1/(1, 1, 3)	(3, 5, 7)	1/(1, 1, 3)	(2, 4, 6)	1
(c) Decision Maker 3					
	Estimated expenses	Approximate time required	Attractiveness to customers	Changes to restaurant image	Compatibility with current operations
Estimated expenses	1	1/(1, 3, 5)	1/(2, 4, 6)	(1, 3, 5)	(1, 3, 5)
Approximate time required	(1, 3, 5)	1	1/(2, 4, 6)	(1, 3, 5)	(1, 3, 5)
Attractiveness to customers	(1, 3, 5)	(1, 3, 5)	1	(3, 5, 7)	(2, 4, 6)
Changes to restaurant image	1/(1, 3, 5)	1/(1, 3, 5)	1/(2, 4, 6)	1	1/(2, 4, 6)
Compatibility with current operations	1/(1, 3, 5)	1/(1, 3, 5)	1/(2, 4, 6)	(2, 4, 6)	1

approximate time required. Therefore, MATLAB R2017a was used to formulate and optimize an NLP model on a PC with an i7-7700 3.6-GHz CPU and 8 GB of RAM to decompose the fuzzy judgment matrices of decision makers into fuzzy judgment submatrices.

The threshold of the distance from a fuzzy judgment submatrix to the original fuzzy judgment matrix,  $\xi$ , was set to  $\sqrt{90}$ . The optimal solution is presented in Table 2. For example, in the first view of Decision Maker 1, the priority of estimated expenses was (1, 3, 5) times that of the approximate time required. In the second view of the decision maker, the relative priority was (5, 7, 9). The optimal objective function value was 0.123. Table 3 presents the results of the consensus achieved based on the fuzzy judgment submatrices.

FI was employed again to aggregate the fuzzy priorities of criteria derived by the decision makers. The results are summarized in Fig. 7.

The FI results were approximated with TFNs as

$$\tilde{w}_1 \cong (0.17, 0.21, 0.24)$$

$$\tilde{w}_2 \cong (0.10, 0.11, 0.13)$$

$$\tilde{w}_3 \cong (0.22, 0.34, 0.46)$$

$$\tilde{w}_4 \cong (0.08, 0.11, 0.14)$$

$$\tilde{w}_5 \cong (0.15, 0.24, 0.32)$$

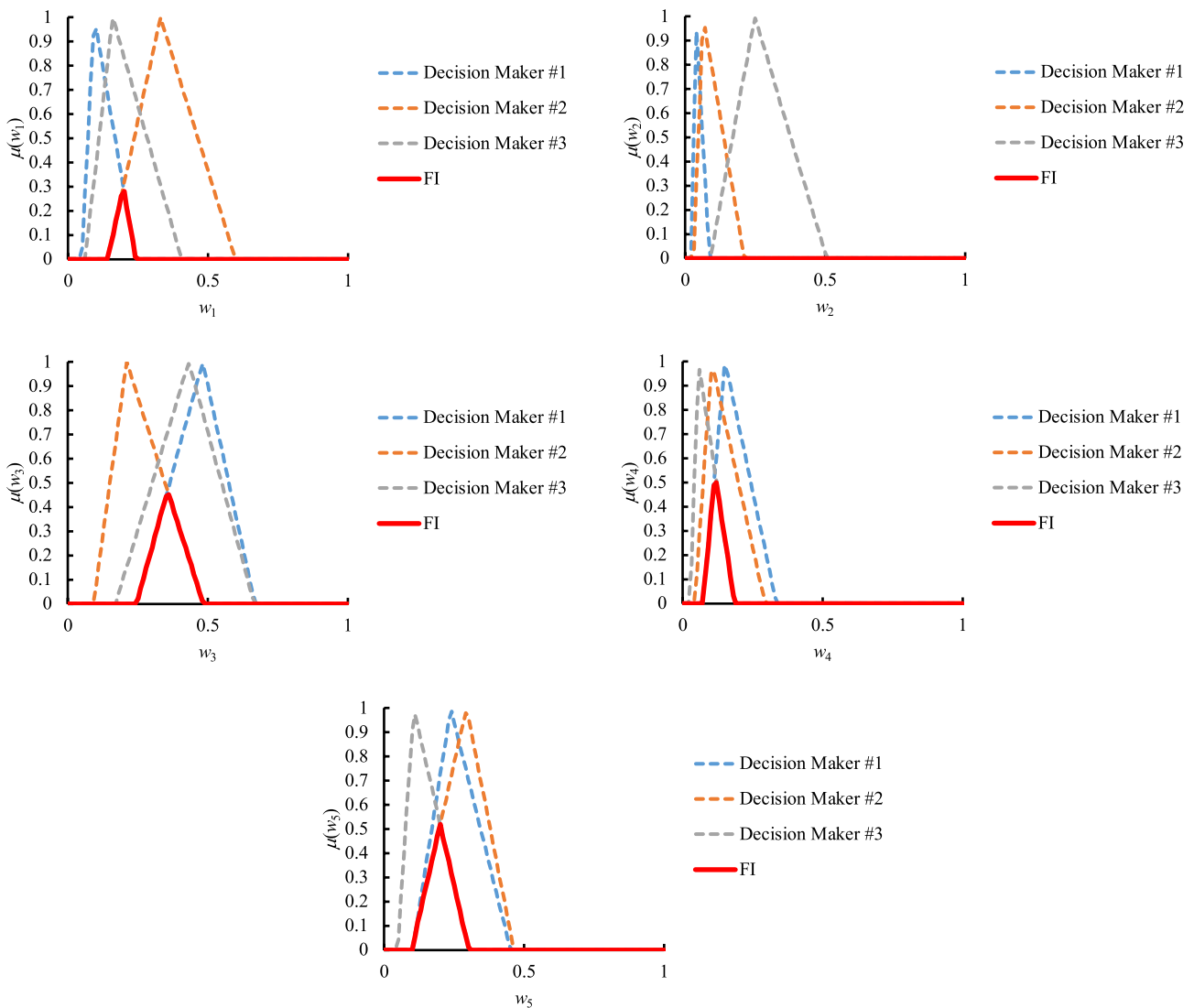


Fig. 6 The FI of the fuzzy priorities derived by decision makers

**Table 2** Fuzzy judgment submatrices of decision makers

$k$	$\tilde{A}^*(k-1)$	$\tilde{A}^*(k-2)$
1	$\begin{pmatrix} 1 & (1, 3, 5) & - & - & - \\ - & 1 & - & - & - \\ (1, 2, 4) & (7, 9, 9) & 1 & (1, 1, 3) & (1, 1, 3) \\ (1, 1, 3) & (1, 2, 4) & - & 1 & - \\ (3, 5, 7) & (1, 3, 5) & - & (1, 1, 3) & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & (5, 7, 9) & - & - & - \\ - & 1 & - & - & - \\ (6, 8, 9) & (7, 9, 9) & 1 & (5, 7, 9) & (3, 5, 7) \\ (3, 5, 7) & (6, 8, 9) & - & 1 & - \\ (1, 1, 3) & (3, 5, 7) & - & (3, 5, 7) & 1 \end{pmatrix}$
2	$\begin{pmatrix} 1 & (1, 2, 4) & (1, 2, 4) & (2, 4, 6) & (1, 1, 3) \\ - & 1 & - & - & - \\ - & (3, 5, 7) & 1 & (2, 4, 6) & (1, 1, 3) \\ - & (1, 1, 3) & - & 1 & - \\ - & (6, 8, 9) & - & (4, 6, 8) & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & (2, 4, 6) & (2, 4, 6) & (1, 2, 4) & (1, 1, 3) \\ - & 1 & - & - & - \\ - & (1, 1, 3) & 1 & (1, 2, 4) & (1, 1, 3) \\ - & (3, 5, 7) & - & 1 & - \\ - & (1, 2, 4) & - & (1, 2, 4) & 1 \end{pmatrix}$
3	$\begin{pmatrix} 1 & - & - & (1, 1, 3) & (1, 1, 3) \\ (1, 2, 4) & 1 & - & (1, 1, 3) & (1, 1, 3) \\ (2, 4, 6) & (1, 2, 4) & 1 & (6, 8, 9) & (1, 2, 4) \\ - & - & - & 1 & - \\ - & - & - & (3, 5, 7) & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & - & - & (3, 5, 7) & (3, 5, 7) \\ (2, 4, 6) & 1 & - & (3, 5, 7) & (3, 5, 7) \\ (1, 2, 4) & (2, 4, 6) & 1 & (1, 2, 4) & (4, 6, 8) \\ - & - & - & 1 & - \\ - & - & - & (1, 3, 5) & 1 \end{pmatrix}$

The most critical factor was the attractiveness to customers, followed by the compatibility with current operations and estimated expenses.

Subsequently, the data of the four alternatives on the critical factors were collected or evaluated by the decision makers jointly, as summarized in Table 4.

The performance of each alternative was evaluated in accordance with the rules in Table 5. The evaluation results are summarized in Table 6. All are given in TFNs. No alternative was perfect.

Fuzzy TOPSIS was applied to evaluate the overall performance of each alternative with a TFN. The result is shown in Table 7.

In this case, the optimal alternative was Alternative III, which included increasing the distance between tables, erecting screens between tables, and improving air circulation. In addition to maintaining a moderate level of the attractiveness to customers, Alternative III incurred extremely low estimated costs. The second-best alternative was Alternative II, which included dividing the indoor dining area into smaller booths. This alternative was the most attractive to customers.

Although Alternative II was not optimal, its most likely performance  $O_{q2}$  was the highest among all alternatives.

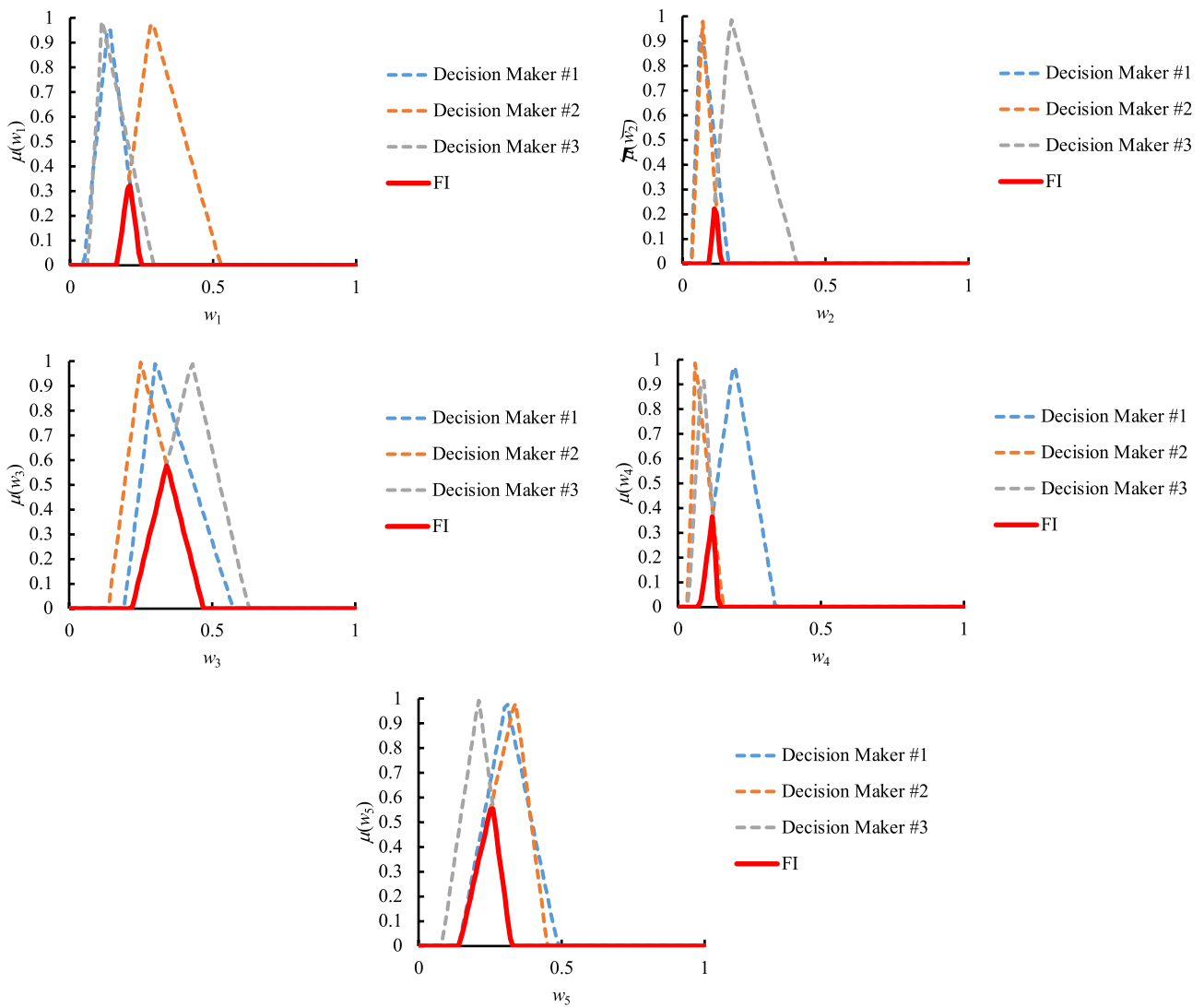
**Table 3** The consensus achieved based on fuzzy judgment submatrices

Fuzzy sub-judgment matrixes	Existence of overall consensus
$\tilde{A}^*(1-1), \tilde{A}^*(2-1), \tilde{A}^*(3-1)$	Yes
$\tilde{A}^*(1-1), \tilde{A}^*(2-1), \tilde{A}^*(3-2)$	No
$\tilde{A}^*(1-1), \tilde{A}^*(2-2), \tilde{A}^*(3-1)$	No
$\tilde{A}^*(1-2), \tilde{A}^*(2-1), \tilde{A}^*(3-1)$	No
$\tilde{A}^*(1-1), \tilde{A}^*(2-2), \tilde{A}^*(3-2)$	No
$\tilde{A}^*(1-2), \tilde{A}^*(2-1), \tilde{A}^*(3-2)$	No
$\tilde{A}^*(1-2), \tilde{A}^*(2-2), \tilde{A}^*(3-1)$	No
$\tilde{A}^*(1-2), \tilde{A}^*(2-2), \tilde{A}^*(3-2)$	No

Alternative IV required the fewest adjustments and seemed to be the most economical. However, without major renovations, customers might think that dining in the restaurant remains unsafe, possibly resulting in a loss of revenue.

Four existing fuzzy group decision-making methods were also applied to this case for comparison. The first was the FGM–FGM–FWA method, in which the decision makers’ judgments were aggregated using FGM. Then, the fuzzy priorities of the criteria were determined using FGM. Finally, and the overall performance of each alternative was evaluated using FWA. The second method was the FGM–fuzzy extent analysis (FEA)–FWA method, wherein FEA [6] was applied to derive the priorities of criteria in place of the FGM method used by the aforementioned method. The third method was the FGM–FGM–fuzzy TOPSIS method, which was also similar to the FGM–FGM–FWA method except that fuzzy TOPSIS was employed to compare the overall performances of alternatives. The fourth method was the linguistic ordered weighted average (LOWA)–FGM–FWA method, in which an LOWA operator [16] was applied to aggregate the fuzzy judgment matrices of decision makers. One advantage of the LOWA method is that the aggregation result can also be represented by original linguistic terms. Here, a moderately optimistic decision strategy was adopted. In all methods, the COG method was applied to defuzzify the evaluation result and generate the absolute rankings. The results obtained using these methods are summarized in Table 8. The FGM–FGM–FWA method, the FGM–FEA–FWA method, and the FGM–FGM–fuzzy TOPSIS method recommended Alternative II, whereas the LOWA–FGM–FWA method recommended Alternative I. All the four methods returned results different from that (Alternative III) obtained using the proposed methodology. This difference was due to the inability of the existing methods to handle the lack of consensus.

The top two alternatives recommended by the existing methods, Alternatives I and II, are similar. By contrast, the



**Fig. 7** The FI result of the fuzzy priorities of the decision makers after decomposition

top two alternatives recommended by the proposed methodology, Alternatives III and II, are considerably different because of the diversification mechanism of the proposed methodology.

Most fuzzy collaborative intelligence methods cannot solve this problem because of the lack of consensus. However, an exception is the partial-consensus fuzzy analytic hierarchy process (FAHP) method proposed by [20], in

which the consensus among a subset of decision makers is sought using the partial-consensus FI (PCFI). The PCFI result is also approximated by a TFN. As a result, the priorities of criteria can be derived as

$$\tilde{w}_1 \cong (0.07, 0.17, 0.4)$$

$$\tilde{w}_2 \cong (0.03, 0.08, 0.21)$$

**Table 4** The data of the four alternatives

<i>q</i>	Estimated expenses (NTD)	Approximate time required (days)	Attractiveness to customers	Changes to restaurant image	Compatibility with current operations
I	250,000	35	High	Very high	High
II	220,000	55	Very high	Moderate	Very high
III	120,000	21	Moderate	Low	Moderate
IV	115,000*	3	Low	Very low	Moderate

\*Including lost monthly revenue

**Table 5** Rules for evaluating the performance of alternatives

Critical factor	Rule
Estimated expenses	$\tilde{p}_{q1}(x_{q1}) = \begin{cases} (0, 0, 1) & \text{if } 0.1 \cdot \min_r x_{r1} + 0.9 \cdot \max_r x_{r1} \leq x_{q1} \text{ or data not available} \\ (0, 1, 2) & \text{if } 0.35 \cdot \min_r x_{r1} + 0.65 \cdot \max_r x_{r1} \leq x_{q1} < 0.1 \cdot \min_r x_{r1} + 0.9 \cdot \max_r x_{r1} \\ (1.5, 2.5, 3.5) & \text{if } 0.65 \cdot \min_r x_{r1} + 0.35 \cdot \max_r x_{r1} \leq x_{q1} < 0.35 \cdot \min_r x_{r1} + 0.65 \cdot \max_r x_{r1} \\ (3, 4, 5) & \text{if } 0.9 \cdot \min_r x_{r1} + 0.1 \cdot \max_r x_{r1} \leq x_{q1} < 0.65 \cdot \min_r x_{r1} + 0.35 \cdot \max_r x_{r1} \\ (4, 5, 5) & \text{if } x_{q1} < 0.9 \cdot \min_r x_{r1} + 0.1 \cdot \max_r x_{r1} \end{cases}$ <p>where <math>x_{q1}</math> is the estimated expenses of the <math>q</math>th alternative</p>
Approximate time required	$\tilde{p}_{q2}(x_{q2}) = \begin{cases} (0, 0, 1) & \text{if } 0.1 \cdot \min_r x_{r2} + 0.9 \cdot \max_r x_{r2} \leq x_{q2} \text{ or data not available} \\ (0, 1, 2) & \text{if } 0.35 \cdot \min_r x_{r2} + 0.65 \cdot \max_r x_{r2} \leq x_{q2} < 0.1 \cdot \min_r x_{r2} + 0.9 \cdot \max_r x_{r2} \\ (1.5, 2.5, 3.5) & \text{if } 0.65 \cdot \min_r x_{r2} + 0.35 \cdot \max_r x_{r2} \leq x_{q2} < 0.35 \cdot \min_r x_{r2} + 0.65 \cdot \max_r x_{r2} \\ (3, 4, 5) & \text{if } 0.9 \cdot \min_r x_{r2} + 0.1 \cdot \max_r x_{r2} \leq x_{q2} < 0.65 \cdot \min_r x_{r2} + 0.35 \cdot \max_r x_{r2} \\ (4, 5, 5) & \text{if } x_{q2} < 0.9 \cdot \min_r x_{r2} + 0.1 \cdot \max_r x_{r2} \end{cases}$ <p>where <math>x_{q2}</math> is the approximate time required for the <math>q</math>th alternative</p>
Attractiveness to customers	$\tilde{p}_{q3}(x_{q3}) = \begin{cases} (0, 0, 1) & \text{if } x_{q3} = \text{''Very low'' or data not available} \\ (0, 1, 2) & \text{if } x_{q3} = \text{''Low''} \\ (1.5, 2.5, 3.5) & \text{if } x_{q3} = \text{''Moderate''} \\ (3, 4, 5) & \text{if } x_{q3} = \text{''High''} \\ (4, 5, 5) & \text{if } x_{q3} = \text{''Very high''} \end{cases}$ <p>where <math>x_{q3}</math> is the attractiveness of the <math>q</math>th alternative to customers</p>
Changes to restaurant image	$\tilde{p}_{q4}(x_{q4}) = \begin{cases} (0, 0, 1) & \text{if } x_{q4} = \text{''Very low'' or data not available} \\ (0, 1, 2) & \text{if } x_{q4} = \text{''Low''} \\ (1.5, 2.5, 3.5) & \text{if } x_{q4} = \text{''Moderate''} \\ (3, 4, 5) & \text{if } x_{q4} = \text{''High''} \\ (4, 5, 5) & \text{if } x_{q4} = \text{''Very high''} \end{cases}$ <p>where <math>x_{q4}</math> is the changes to restaurant image brought about by the <math>q</math>th alternative</p>
Compatibility with current operations	$\tilde{p}_{q5}(x_{q5}) = \begin{cases} (0, 0, 1) & \text{if } x_{q5} = \text{''Very low'' or data not available} \\ (0, 1, 2) & \text{if } x_{q5} = \text{''Low''} \\ (1.5, 2.5, 3.5) & \text{if } x_{q5} = \text{''Moderate''} \\ (3, 4, 5) & \text{if } x_{q5} = \text{''High''} \\ (4, 5, 5) & \text{if } x_{q5} = \text{''Very high''} \end{cases}$ <p>where <math>x_{q5}</math> is the compatibility with current operations of the <math>q</math>th alternative</p>

$\tilde{w}_3 \cong (0.17, 0.45, 0.66)$

$\tilde{w}_4 \cong (0.05, 0.11, 0.29)$

$\tilde{w}_5 \cong (0.01, 0.25, 0.45)$

**Table 6** The performances of alternatives

$q$	$\tilde{p}_{q1}$	$\tilde{p}_{q2}$	$\tilde{p}_{q3}$	$\tilde{p}_{q4}$	$\tilde{p}_{q5}$
I	(0, 0, 1)	(1.5, 2.5, 3.5)	(3, 4, 5)	(4, 5, 5)	(3, 4, 5)
II	(0, 1, 2)	(0, 0, 1)	(4, 5, 5)	(1.5, 2.5, 3.5)	(4, 5, 5)
III	(4, 5, 5)	(3, 4, 5)	(1.5, 2.5, 3.5)	(0, 1, 2)	(1.5, 2.5, 3.5)
IV	(4, 5, 5)	(4, 5, 5)	(0, 1, 2)	(0, 0, 1)	(1.5, 2.5, 3.5)

Fuzzy TOPSIS is then applied to compare the overall performances of the alternatives. The ranking result is presented in Table 9. The result of the FAHP-PCFI-fuzzy TOPSIS method was different from that generated using the proposed methodology because the partial-consensus FAHP method aggregated fuzzy judgment matrices when no consensus existed. By contrast, the proposed methodology aggregated fuzzy judgment submatrices to reach consensus.

**Table 7** The overall performances of alternatives

Alternative	$\tilde{O}_q$	$COG(\tilde{O}_q)$	Rank
I	(0.046, 0.541, 0.893)	0.494	3
II	(0.000, 0.596, 0.941)	0.512	2
III	(0.109, 0.509, 1.000)	0.540	1
IV	(0.109, 0.412, 0.939)	0.487	4

**Table 8** Ranking results using various fuzzy group decision-making methods

Alternative	Rank (FGM-FGM-FWA)	Rank (FGM-FEA-FWA)	Rank (FGM-FGM-FTOPSIS)	Rank (LOWA-FGM-FWA)	Rank (proposed methodology)
I	2	2	2	1	3
II	1	1	1	2	2
III	3	3	3	3	1
IV	4	4	4	4	4

**Table 9** Comparing the ranking results using the FAHP-PCFI-fuzzy TOPSIS method and the proposed methodology

Alternative	Rank (partial-consensus FAHP)	Rank (proposed methodology)
I	2	3
II	1	2
III	3	1
IV	4	4

## Conclusions

When decision makers lack consensus, existing methods either ignore this fact or force a decision maker to modify his/her judgment. To better address this issue, a novel fuzzy collaborative intelligence approach is proposed in this research. In the proposed methodology, the fuzzy judgment matrix of each decision maker is decomposed into several fuzzy judgment submatrices that are more likely to overlap. To optimize the decomposition result, an FNLP problem is formulated and solved. Subsequently, the fuzzy judgment submatrices of the decision makers are aggregated using FI to derive the fuzzy priorities of criteria. On the basis of the FI result, the overall performances of alternatives are evaluated and compared using fuzzy TOPSIS.

The proposed methodology has been applied to the case of planning the transformation of a Japanese restaurant after the COVID-19 pandemic. According to the experimental results, the following conclusions were drawn:

- (1) Such transformation was beyond the expectation of the Japanese restaurant employees. Therefore, no consensus was discovered among them. Nevertheless, the proposed fuzzy collaborative intelligence method successfully solved this problem.
- (2) The optimal transformation plan involved increasing the distance between tables, erecting folding screens between tables, and improving air circulation.
- (3) Four prevalent fuzzy group decision-making methods were also applied to this case. Three of these methods recommended that the indoor dining area be divided into smaller booths, and one suggested reducing the indoor dining area and expanding the outdoor dining

area. However, these methods could not account for the lack of consensus among decision makers.

An obvious drawback of the proposed methodology is the difficulty associated with solving an NLP problem. In addition, unlike existing methods [18, 19], the proposed method does not interpret the multiple views of a decision maker, thus limiting its further application.

In future research, the efficiency of decomposing a fuzzy judgment matrix can be improved. In addition, in the proposed methodology, a fuzzy judgment matrix is the arithmetic mean of its fuzzy judgment submatrices. Other operators can be adopted to define this relationship.

## Declarations

**Ethical Approval** This study did not perform any procedure on human or animal participants.

**Conflict of Interest** The authors declare no competing interests.

## References

1. Abadi SS, Widyarto S. The model of determining quality of management private higher education using FAHP (Fuzzy Analytic Hierarchy Process) method. *ICESIA*. 2016;1:166–72.
2. Atanassov K. Intuitionistic fuzzy sets *International Journal Bio-automation*. 2016;20:1.
3. Cabrerizo FJ, Moreno JM, Pérez JJ, Herrera-Viedma E. Analyzing consensus approaches in fuzzy group decision making: advantages and drawbacks. *Soft Comput*. 2010;14(5):451–63.
4. Capuano N, Chiclana F, Fujita H, Herrera-Viedma E, Loia V. Fuzzy group decision making with incomplete information guided by social influence. *IEEE Trans Fuzzy Syst*. 2017;26(3):1704–18.
5. CGTN (2021). COVID-19 in Taiwan: border closure and tests urged amid surging cases. <https://news.cgtn.com/news/2021-05-18/COVID-19-in-Taiwan-Border-closure-and-tests-urged-amid-surging-cases-10mE0nPVGp2/index.html>.
6. Chang DY. Applications of the extent analysis method on fuzzy AHP. *Eur J Oper Res*. 1996;95(3):649–55.
7. Chen T. A hybrid fuzzy and neural approach with virtual experts and partial consensus for DRAM price forecasting. *Int J Innov Comput Info Control*. 2012;8(1):583–97.
8. Chen T, Chuang YH. Fuzzy and nonlinear programming approach for optimizing the performance of ubiquitous hotel recommendation. *J Ambient Intell Humaniz Comput*. 2018;9(2):275–84.
9. Chen T, Lin YC. A fuzzy-neural system incorporating unequally important expert opinions for semiconductor yield

- forecasting. *Int J Uncertain Fuzziness Knowledge-Based Systems*. 2008;16(01):35–58.
10. Chen TCT, Lin YC. Diverse three-dimensional printing capacity planning for manufacturers. *Robot Comput Integr Manuf*. 2021;67:102052.
  11. Chen T, Wang Y-C. A calibrated piecewise-linear FGM approach for travel destination recommendation amid the COVID-19 pandemic. *Appl Soft Comput*. 2021;109:107535.
  12. Chen T, Wang Y-C, Wu H-C. Analyzing the impact of vaccine availability on alternative supplier selection amid the COVID-19 pandemic: a cFGM-FTOPSIS-FWI approach. *Healthcare*. 2021;9(1):71.
  13. Chiu MC, Chen T. Assessing mobile and smart technology applications for active and healthy aging using a fuzzy collaborative intelligence approach. *Cogn Comput*. 2021;13(2):431–46.
  14. Gao H, Ju Y, Gonzalez EDS, Zhang W. Green supplier selection in electronics manufacturing: an approach based on consensus decision making. *J Clean Prod*. 2020;245:118781.
  15. Golub GH, Van Loan CF. *Matrix computations*. Baltimore, MD: Johns Hopkins; 1996.
  16. Herrera-Viedma E, Pasi G, Lopez-Herrera AG, Porcel C. Evaluating the information quality of web sites: a methodology based on fuzzy computing with words. *J Am Soc Inform Sci Technol*. 2006;57(4):538–49.
  17. Herrera F, Herrera-Viedma E. A model of consensus in group decision making under linguistic assessments. *Fuzzy Sets Syst*. 1996;78(1):73–87.
  18. Jin F, Liu J, Chen H, Langari R. Interval type-2 trapezoidal fuzzy decision-making method with consistency-improving algorithm and DEA model. *IEEE Access*. 2020;8:120456–72.
  19. Jin F, Liu J, Zhou L, Martínez L. Consensus-based linguistic distribution large-scale group decision making using statistical inference and regret theory. *Group Decis Negot*. 2021;1–33.
  20. Lin CW, Chen T. 3D printing technologies for enhancing the sustainability of an aircraft manufacturing or MRO company—a multi-expert partial consensus-FAHP analysis. *Int J Adv Manuf Technol*. 2019;105(10):4171–80.
  21. Lin YC, Chen T. A multibelief analytic hierarchy process and nonlinear programming approach for diversifying product designs: Smart backpack design as an example. *Proc Inst Mech Eng Part B J Eng Manuf*. 2020;234(6–7):1044–56.
  22. Liu B, Shen Y, Chen X, Chen Y, Wang X. A partial binary tree DEA-DA cyclic classification model for decision makers in complex multi-attribute large-group interval-valued intuitionistic fuzzy decision-making problems. *Info Fusion*. 2014;18:119–30.
  23. Misitano G. Interactively learning the preferences of a decision maker in multi-objective optimization utilizing belief-rules. 2020 IEEE Symposium Series on Computational Intelligence; 2020. pp. 133–140.
  24. Mitrović Simić J, Stević Ž, Zavadskas EK, Bogdanović V, Subotić M, Mardani A. A novel CRITIC-Fuzzy FUCOM-DEA-Fuzzy MARCOS model for safety evaluation of road sections based on geometric parameters of road. *Symmetry*. 2020;12(12):2006.
  25. Opricovic S. Fuzzy VIKOR with an application to water resources planning. *Expert Syst Appl*. 2011;38(10):12983–90.
  26. Pan NF. Fuzzy AHP approach for selecting the suitable bridge construction method. *Autom Constr*. 2008;17(8):958–65.
  27. Pavlacka O, Talašová J. The fuzzy weighted average operation in decision making models. *Proceedings of the 24th International Conference Mathematical Methods in Economics*; 2016. pp. 419–426.
  28. Pedrycz W. Collaborative architectures of fuzzy modeling. *IEEE World Congress on Computational Intelligence*; 2008. pp. 117–139.
  29. Saaty TL. Decision making with the analytic hierarchy process. *Int J Serv Sci*. 2008;1(1):83–98.
  30. Tang M, Liao H. From conventional group decision making to large-scale group decision making: what are the challenges and how to meet them in big data era? A state-of-the-art survey. *Omega*. 2021;100:102141.
  31. Torra V. Hesitant fuzzy sets. *Int J Intell Syst*. 2010;25(6):529–39.
  32. Van Broekhoven E, De Baets B. Fast and accurate center of gravity defuzzification of fuzzy system outputs defined on trapezoidal fuzzy partitions. *Fuzzy Sets Syst*. 2006;157(7):904–18.
  33. Volmer C, Weber J, Stephan R, Blau K, Hein MA. An eigenanalysis of compact antenna arrays and its application to port decoupling. *IEEE Trans Antennas Propag*. 2008;56(2):360–70.
  34. Wang, L., Yang, S. X., & Biglarbegian, M. (2012). A fuzzy logic based bio-inspired system for mobile robot navigation. In 2012 IEEE International Conference on Multisensor Fusion and Integration for Intelligent Systems, pp. 219–224.
  35. Wu HC, Wang YC, Chen TCT. Assessing and comparing COVID-19 intervention strategies using a varying partial consensus fuzzy collaborative intelligence approach. *Mathematics*. 2020;8(10):1725.
  36. Yong D. Plant location selection based on fuzzy TOPSIS. *Int J Adv Manuf Technol*. 2006;28(7–8):839–44.
  37. Zhan J, Alcantud JCR. A novel type of soft rough covering and its application to multicriteria group decision making. *Artif Intell Rev*. 2019;52(4):2381–410.
  38. Zhang Z, Gao Y, Li Z. Consensus reaching for social network group decision making by considering leadership and bounded confidence. *Knowl-Based Syst*. 2020;204:106240.
  39. Zheng G, Zhu N, Tian Z, Chen Y, Sun B. Application of a trapezoidal fuzzy AHP method for work safety evaluation and early warning rating of hot and humid environments. *Saf Sci*. 2012;50(2):228–39.

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