Ore grade estimation using the imperialist competitive algorithm (ICA)

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Abstract

In this paper, a hybrid inverse distance weighted (IDW) and the imperialist competitive algorithm (ICA) was proposed for grade estimations in mining projects. Indeed, the coefficients of the IDW equation, power, and the influence distance were modified by applying the ICA algorithm. For this, an example was solved by using IDW, the ordinary kriging, and IDW-ICA. After successfully validating the IDW-ICA, the iron grades were estimated in the Baba Ali iron mine, the most prominent iron mine in the west of Iran. For this, after gathering data from thirty-two boreholes, the grade estimations were conducted using IDW, kriging, and IDW-ICA. Some statistical parameters, including the mean absolute percentage error (MAPE), root-mean-squared error (RMSE), and correlation coefficient, were used to compare the results. The higher value of the R^2 , 94.41%, the lowest values of MAPE, 17.4%, and RMSE, 4.19, of the proposed algorithm, revealed that the IDW-ICA could be applied as a new-fangled method for grade estimations in the mining projects.

Keywords Ore grade estimation · Imperialist competitive algorithm · Ordinary Kriging · Inverse distance weighted

Introduction

Ore grade estimation is one of the most critical stages of ore reserve evaluation and plays a vital role in mining operations. In many cases, faults, dikes, fractures, and complex structures

make it particularly problematic to model the orebody. It is worth noting that geosciences' modeling is among the most complicated and difficult branches of mathematical geosciences' modeling due to heterogeneity of the earth structure and unique factors contributing to such these models. Different stages of a mining project, including the feasibility study, design, and planning, depend on the grade evaluation. Accordingly, one must obtain a reliable estimate of the ore grade to produce a realistic orebody reserve estimation. Generally, geostatistical methods represent the most common tools for predicting the grade variations and estimating the reserve of orebodies. Moreover, there are many pitfalls of using methods such as uncertainty of the assumptions made to determine the variogram parameters, time-intensiveness, and inability to provide accurate estimations if the regional variable spatial structure is inadequate. These disadvantages caused a great motivation for researchers toward using artificial intelligence-based (AI-based) estimators for grade evaluation instead of traditional geostatistical methods in recent years. For instance, Denby and Schofield (1995) applied the genetic algorithm (GA) to incorporate the grade variability into the planning process. In 2003, Ataei and Osanloo adopted the so-called golden-section search in optimizing the cutoff grade (Ataei and Osanloo 2003). Samanta et al. (2004) used multilayer neural networks (NNs) for modeling the grade

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across a gold orebody. Their study set the learning parameters through a trial-and-error procedure and proved the superiority of the built model over the geostatistical methods. Chatterjee et al. (2006) used a three-layer artificial neural network (ANN) to optimize grade evaluation in a limestone mine. The results revealed that the ANN method had higher efficiency compared to the geostatistical methods. In another research, Chatterjee et al. (2008) applied a genetic-neural hybrid algorithm for estimating the content of limestone in a limestone mine. The results highlighted the ability of the proposed algorithm to estimate the uncertainties. Tahmasebi and Hezarkhani (2010a) utilized an adaptive neuro-fuzzy inference system (ANFIS) to evaluate copper grade in Sarcheshmeh Copper Mine in southern Iran. Also, the obtained results were compared with other methods such as ANN and kriging. They found that the ANFIS had a better performance than the other methods in evaluating the grade. In another research in (2010b), Tahmasebi and Hezarkhani compared the performance of different learning algorithms on NN for iron ore grade estimation. Based on their results, it was found that the GA-based NN provides the best performance as a learning algorithm in their case. Their results showed that the GA-based NN could improve the network performance, lowering the mean squares error (MSE) while increasing the R^2 value. Li et al. (2013) used particle swarm optimization (PSO) to study copper grade on a hypothetical orebody. Soleimani and Jodeiri Shokri (2015) applied a combination of seismic and geostatistical methods to remove some ambiguities in heterogeneous oil reservoirs in the southwest of Iran. In another research, Soleimani and Jodeiri Shokri (2016) presented a mathematical-geological genetic model that was an advanced exploratory tool for better-evaluating chromite pods in the Sabzevar ophiolite complex in northeast of Iran. Gilani and Sattarvand (2016) evaluated the effect of grade uncertainties on the production planning for open-pit mines using the ant colony algorithm (OCA). Mohammadi et al. (2017) applied the imperialist competitive algorithm (ICA) to check for the influence of ore grade uncertainties on determining the cutoff grade. Jafrasteh et al. (2018) used the ANN, random forest algorithm (RFA), and gaussian processes to evaluate the copper grade. The results presented that the mentioned AI techniques could have better estimations than the traditional geostatistical methods. Mohammadi et al. (2018) utilized k-means and principal component analysis (PCA) methods to evaluate gold grade in the Khooni district in central Iran. They estimated the gold grade with a high correlation coefficient (91%). Zerzour et al. (2020) used the geostatistical interpolation to evaluate the iron ore mixtures in Wenza open cast mine, northeast Algeria. They applied semi-variograms and kriging techniques to interpolate the needed values for mineral resource estimation. Afeni et al. (2021) investigated the reserve estimation methods such as triangular block and geostatistical. Then, they used the mentioned methods to

estimate the Oyo-Iwa limestone deposit located in Nigeria. They concluded that the geostatistical method is more suitable, reliable, and preferable for grade estimation since it involves spatial modeling and cross-validated interpolation.

As found from the literature review, it seems that the AIs led to better results than conventional geostatistical methods. As a result, considering the importance of the uncertainty associated with the ore reserve estimation and its significant contribution to economic efficiency, a hybrid algorithm called IDW-ICA was proposed for the grade evaluation of the Baba-Ali Iron orebody in western Iran. For this purpose, the coefficients of the IDW equation, power, and the influence distance were modified by using the ICA.

Materials and methods

There are many methods for estimating a deposit reserve, which can be generally classified under two categories: geostatistical and classical methods. The classical methods employ classic statistics for grade evaluation. In contrast, the estimation in the geostatistical methods is based on the spatial structure of the medium. Indeed, geostatistical evaluation is a process where the value of a quantity at an unknown point can be estimated based on the value of the same quantity at known points. In general, geostatistics is a branch of statistics focusing on spatial or spatiotemporal datasets. It has initially been applied for predicting probability distributions of ore grades for mining operations.

Inverse-distance weighted method

As a conventional method for grade evaluation, the inverse distance weighted (IDW) method sets the contributions of surrounding grades into the grade of the central block dependent on the inverse of their distances to the central block. The closer blocks are likely to exhibit closer grades to that of the central block than the farther blocks. In order to place more emphasis on this inverse dependency, one may increase the weight in Eq. (1), which is done by changing the value of the power n (Bartier and Keller 1996).

$$g = \frac{\sum \frac{g_i}{d_i^n}}{\sum \frac{1}{d_i^n}} \quad (1)$$

where

g_i is the known grade of the ore;

d_i is the influence distance from the known grade to the estimation point.

Kriging method

The kriging is an estimation method that works based on a “weighted moving average” logic. It has been referred to as “the best unbiased linear estimation technique”. Among other techniques, the kriging provides the error associated with each estimation to calculate the confidence interval for each estimation provided by the kriging. Some of the most important distinctions of the kriging compared to other estimation methods are as follows:

- In kriging, estimation at an unknown point is fully controlled by the spatial structure of the domain, while similar methods most frequently consider only one geometrical factor (e.g., distance) to calculate the weights so that the result exhibits no sensitivity to spatial variations of the orebody.
- As the spatial structure becomes weak (increased nugget effect), the role of sample position fades. Accordingly, under the complete nugget effect, all samples are weighted identically.
- With increasing the radius of influence, the weight of known samples at far distances to the considered block, i.e., unknown point, increases. In the meantime, assuming zero nugget effect and a radius of influence equal to the side of the block, the central block will yet bear higher importance, which is translated to higher values of weight.
- Although the inverse-distance weighted method performs as strongly as the kriging under continuity condition ($C_0 = 0$), the presence of discontinuity (reflected as increased nugget effect) lowers the accuracy of the inverse-distance weighted method compared to the kriging.

The kriging calculations can be simply explored by referring to a handful of geostatistical textbooks, where the solution and determination of the estimated grades are well discussed. Given that such equations and associated challenges are beyond the scope of the present article, we herein ignore them and instead only refer to the main features of this method:

- The weights are determined in such a way not only to keep the solution unbiased but also to minimize the variance.
- The level of error associated with each estimation is also provided.
- In the estimation at an actual sampled point, the estimated value must be equal to the measured value, making a variance of zero at such a point.
- The kriging error is a function of the variogram properties (spatial structure), the geometry of the estimated blocks, and the block used to obtain the estimations rather than the actual data values.

- Kriging results are additive, meaning that kriging in some small blocks can be added up to obtain estimations over a larger block composed of all the small blocks. The blocks were graded using the kriging based on the parameters of the obtained variogram and the kriging.

Imperialist competitive algorithm (ICA)

Searching for an appropriate solution is a challenging inevitable task to address in many engineering problems yet. Accordingly, many search algorithms have been proposed for finding a solution for such problems. Evolutionary algorithms are a significant class of search algorithms where the evolutionary rules of nature are utilized to find the optimal solution. Some of the most applicable evolutionary algorithms in geosciences are the GA, ACO, ICA, bat algorithm (BA), and biogeography-based optimization (BBO). Human social behaviors have been used in the ICA. In 2007, this algorithm was introduced by Atashpaz-Gargari and Lucas. ICA is a robust optimization algorithm where socio-politic phenomena are mathematically modeled (Atashpaz-Gargari and Lucas 2007; Biabangard-Oskouyi et al. 2009; Jodeiri Shokri et al. 2020). In the ICA, an initial population is generated by selecting several countries randomly at the first step. Subsequently, a cost function is established, and the entire pool of the countries is classified into imperialists and colonies. Indeed, the more powerful countries are classified as imperialists while the other countries are grouped as colonies. Once the imperialists are recognized, recognizing their powers, each of the other countries is captured by an imperialist. An imperialist and its captured colony(ies) are collectively referred to as an empire. Another approach to the assignment of the colonies to the imperialists is to do it depending on their normalized cost (Eq. (2) (Atashpaz-Gargari and Lucas 2007):

$$C_n = \max_i \{c_i\} - c_n \quad (2)$$

where

c_n is the cost of the n^{th} imperialist;
 $\max_i \{c_i\}$ is the maximum cost among the imperialists;
 C_n is the normalized cost on the n^{th} imperialist.

The normalized relative power of each imperialist, which is, indeed, the ratio of the number of its colonies to the total number of colonies, can be determined through Eq. (3) (Atashpaz-Gargari and Lucas 2007).

$$P_n = \left| \frac{C_n}{\sum_{i=1}^{N_{\text{imp}}} C_i} \right| \quad (3)$$

Moreover, the initial number of colonies of an empire is calculated by Eq. (4):

Fig. 1 Different steps of the proposed algorithm

$$N.C._n = \text{round}\{P_n.N_{col}\} \quad (4)$$

where

N_{col} : is the total number of colonies in the initial population of countries.

Considering the value of $N.C._n$ for each empire, some initial colonies are selected at random and captured by the n th imperialist. Then, an attempt is made to move the imperialist as close to the respective colonies as possible to analyze their culture and social structure, followed by moving the colonies toward the respective imperialist (Atashpaz-Gargari and Lucas 2007). It is worth noting that there may be cases where a colony reaches a point in the cost function at which

the associated cost is lower than that of the imperialist. In such cases, the imperialist and the colony are swapped. The algorithm proceeded with the imperialist at the new position from which it applies the assimilation policy to its colonies. In order to evaluate the cost function, total empire cost is calculated through Eq. (5) (Atashpaz-Gargari and Lucas 2007).

$$T.C._n = \text{Cost}(\text{imperialist}_n) + \xi \times \text{mean}\{\text{Cost}(\text{colonies of empire}_n)\} \quad (5)$$

where

ξ : is a constant value between 0 and 1 (close to 0).

Table 1 Coordinates and ore grades of the bores shown in Fig. 1 (Hughes and Davey 1979)

Borehole	Coordinates		Grade (Cu %)	Borehole	Coordinates		Grade (Cu %)
	East	North			East	North	
C1	1100	1200	0.719	C27	900	1200	0.453
C2	1500	1200	0.893	C28	600	700	0.409
C3	1100	800	0.638	C29	1300	800	1.615
C4	1100	400	0.188	C30	1700	800	0.465
C5	1500	400	0.395	C31	1100	600	0.063
C6	1500	800	0.765	C32	900	400	0.224
C7	500	900	0.644	C33	1300	400	0.027
C8	900	1000	1.365	C34	1100	200	0.225
C9	1300	1000	1.335	C35	1300	1400	0.475
C10	1700	1000	0.072	C36	700	1400	0.717
C11	500	1600	0.396	C37	500	1400	0.320
C12	900	1600	0.377	C38	300	1400	0.392
C13	1100	1600	0.427	C39	400	700	0.476
C14	1500	1600	0.140	C40	400	1050	0.102
C15	900	1400	0.806	C41	700	1000	0.023
C16	1100	1400	0.889	C42	1100	1000	0.915
C17	1300	1200	1.009	C43	1500	1000	0.519
C18	1700	1200	0.089	C44	1900	1000	0.040
C19	1900	1200	0.092	C45	1900	800	0.034
C20	600	1900	0.175	C46	900	800	0.258
C21	1100	1800	0.489	C47	900	600	0.165
C22	800	1800	0.417	C48	1300	600	0.406
C23	200	1600	0.215	C49	1500	600	0.996
C24	700	1600	0.685	C50	1700	600	0.012
C25	500	1200	0.230	C51	500	500	0.228
C26	800	1200	0.833				

Fig. 2 Locations of the bores at the bench 5140 (Hughes and Davey 1979)

Fig. 3 Locations of estimation bores

Once finished with calculating the cost of each empire, the colonies belonging to weaker empires are eliminated and captured by the more powerful empires.

The attraction or competition process is iterated among the imperialists. At each iteration, the least potent imperialist loses a colony. The separate colony from the weakest imperialist is the weakest among the colonies of that imperialist. Subsequently, all empires start a competition for capturing the colonies. The empire that lost all of its colonies was eliminated and captured by another imperialist to serve as a colony. This process is continued until the single most powerful imperialist is achieved (Atashpaz-Gargari and Lucas 2007). In order to have the algorithm stopped somewhere, some stopping criterion is considered. This may be achieving a particular number of iterations or processing time. Accordingly, the algorithm is terminated once either the stopping criterion is met or a single final imperialist is reached.

Fig. 4 Cumulative Distribution Function (CDF) of the database utilized for pattern recognition

Proposed algorithm

Regarding the mentioned problems and the importance of providing an appropriate and accurate estimation of ore grade for reserve design, the present paper attempts to present a hybrid IDW-ICA algorithm to attenuate the ore grade evaluation error. In order to demonstrate the performance of the hybrid algorithm, firstly, its different steps were applied on a hypothetical case, followed by using the algorithm to evaluate the ore grade in an actual mine. Different steps of the proposed algorithm are shown in Fig. 1.

Step 1: data gathering In this section, different steps of grade evaluation are described using the data presented by Hughes and Davey (1979). Locations of the bores are presented in Fig. 2 and Table 1.

Step 2: determining the evaluation points In order to investigate the performance of the proposed algorithm, seven boreholes, including C4, C7, C10, C12, C17, C37, and C49, were selected for copper grade evaluation. These seven boreholes were selected among all 51 boreholes in such a way to cover the entire model. Locations of the selected boreholes are marked with red squares in Fig. 3.

Step 3: running the model and grade evaluation In this step, the ore grade at the selected bores was evaluated using the two conventional ore evaluation techniques, the kriging and the IDW methods, and the ICA algorithm.

Kriging

The grade-count fractal method is a technique for population identification in geoscience data, which works based on the cumulative frequency of the data. The discontinuities seen on

Fig. 5 Two perpendicular variograms prepared for determining the search ellipsoid

the respective histogram mark the changes in the populations (Fig. 4). As seen on this histogram, three distinct populations (i.e., dimensions) were identified on the data from the studied area. This classification was then used for scaling the original data and the estimation results.

The geostatistical studies and variogram development were performed using a geostatistical software called Data Mine 3.21. This software is capable of developing three-dimensional variograms. The input data for variography included the (x, y, z) coordinates, the considered variable (S_i),

Fig. 6 Anisotropic search ellipsoid

Table 2 Cross-validation statistics for grade estimation

Cross-validation statistics or grade	
Number of estimated samples	50
Number of not estimated samples	1
Mean of actual values	0.47404
Mean of estimated values	0.51214977
Mean difference (act – est)	–0.03810977
Mean difference (as % of actual)	–8.039
Mean absolute difference	0.25186146
The variance of actual values	0.1387426
The variance of estimated values	0.0871345
Correlation coefficient	0.543
<u>Kriging variance</u>	
Mean of estimated KV from the model	370.310999
Mean of squared differences	0.10781973
Ratio	3434.538
<u>Regression equation</u>	
Actual	$0.12281932 + 0.68577729 \times \text{estimate}$
Standard error	0.31267247

the azimuth of the vector h (Φ), the respective tolerances, bandwidth, number of lags, and the lag spacing. This software could further be used to perform the kriging to obtain grade estimates and distribute the resultant estimation error.

Variogram is used to express the spatial relationship between the grade (or any other property) at different points across a region, making it an essential tool in geostatistical studies. The variogram is of particular interest for mining engineers, as it can be utilized to investigate essential properties of the orebody that affect its estimation. These include the mineralization continuity, radius of effect, and isotropic nature of the orebody. These properties can be studied by analyzing the similarities and dissimilarities of the grades at different points of the orebody at a predefined distance h to one another. In general, in the framework of the inherent hypothesis, the variogram is a second-reorder momentum defined as follows:

$$2\gamma(h) = \text{Var}[Z(x+h)-Z(x)] = E[Z(x+h)-Z(x)]^2. \quad (6)$$

For a general case, it can be written as follows (Eq. (7)):

$$2\gamma(h) = \frac{1}{V} \oint_V [Z(x+h)-Z(x)]^2 dx \quad (7)$$

Table 3 Result of Jack-knife kriging (grade versus estimated grade)

NAME	Grade	GradeEST
Minimum	0.012	0.040
Maximum	1.615	1.312
Mean	0.474	0.512
Correlation coefficient	0.543	

where

V : is the geometric field (e.g., the volume of the block to be estimated) of the point x . Generally, the symbol * is used to indicate the estimated (calculated) values in geostatistics.

Therefore, the empirical variogram can be evaluated with Eq. (8):

$$2\gamma^* = \frac{1}{N(h)} \sum [Z(x+h)-Z(x)]^2 \quad (8)$$

where

$N(h)$ is the number of sample pairs used in the calculations for each lag, h . That is, the number of pairs is a function of h . Typically, the larger the value of h , the lower will be the number of pairs.

Fig. 7 Result of Jack-knife kriging (actual data versus estimated data)

Fig. 8 Histogram of residual distribution of the Jack-knife kriging algorithm (the results almost show the estimator is unbiased)

Once finished with the variography at different directions, two normal variograms were found and used to establish the searching ellipse (Fig. 5).

Figure 6 shows the searching ellipse, which is a basis of geostatistical estimations through kriging. The results of variography processing such as the anisotropy, axis rotation direction, anisotropy radius, the nugget and ceiling effects have been involved in this searching ellipse.

In the kriging estimations, the importance of this control is even doubled. It is due to the significant dependence of the accuracy of the results to the considered model. In addition, variogram modeling is a sensitive process. The exact shape of the variogram is not previously known at all. The model provides no more than an approximation of the actual variogram function. Validity control refers to the estimation at each of the sampled points across a region using the value at neighboring points (without considering the actual value at the considered point) using the kriging method. The estimated values are compared to the actual data to find whether the variogram model of the neighborhood search parameters considers the distance-deponent variations appropriately. Usually referred

to as the jackknife method, this kriging-based estimation method evaluates a particular property at an unknown point using the neighboring samples (Fig. 7). By using the jackknife method, the grade was estimated at the selected boreholes, with the results presented in Fig. 7 and Tables 2 and 3. The results revealed that the variogram parameters were 55% valid for block grade evaluation, a desirable level of validity. This justified the use of the variogram parameters for the grade evaluation of exploratory blocks.

Residuals analysis is used to investigate biases of the estimator in geostatistical modeling. Accordingly, an estimator can be recognized as unbiased if the histogram of the residuals (difference between actual data and estimated values) is distributed as a normal function around a mean value of zero with an adequately low standard deviation (Fig. 8).

IDW method

As mentioned, this method estimates the grade using two parameters: power and radius of influence. In the following example, the power and the radius of influence were 2 and 250 ft, respectively (Eq. (9)).

$$g = \frac{\sum \frac{g_i}{d_i^2}}{\sum \frac{1}{d_i^2}} \quad (9)$$

ICA

In the ICA, to minimize the RMSE, Eq. (10) was taken as the objective function. The coding was performed in <https://www.mathworks.com/products/matlab/student.html>.

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (P_i - O_i)^2} \quad (10)$$

Fig. 9 A decreasing trend of RMSE during the course of performing the ICA

- Grade evaluation by suggesting the proposed algorithm, the IDW-ICA;
- Presenting the results.

The decreasing trend of RMSE during performing the ICA is shown in Fig. 9. The distribution of the boreholes, the power, and the influence distance of the estimated grade have been illustrated in Fig. 10.

After 200 iterations of the ICA in Matlab, the results showed that the most suitable influence distance and power were 281.85 ft, 131.05, respectively. Also, indeed, the most appropriate modified coefficients of the IDW equation have been suggested using ICA. As a result, the following IDW-ICA equation has been proposed for grade evaluation:

$$g = \frac{\sum \frac{0.9319 \times g_i}{d_i^{131.0518}}}{\sum \frac{0.7469}{d_i^{131.0518}}} \quad (11)$$

Step 4: Comparing the results In order to validate the proposed method, the estimation error was calculated using RMSE (Eq. (10)), coefficient of determination (Eq. (12)), and mean absolute percentage error (MAPE) (Eq. (13)).

$$R^2 = 1 - \frac{\sum_{i=1}^n (P_i - O_i)^2}{\sum_{i=1}^n (x_i - \bar{x})^2} \quad (12)$$

$$MAPE = \frac{1}{n} \sum_{i=1}^n \frac{|P_i - O_i|}{P_i} \quad (13)$$

Fig. 12 Coordinates of the bores along with section #2390

Fig. 13 Cumulative Distribution Function (CDF) of the database utilized for pattern recognition

where

$\bar{x}$: is mean value of actual x_i values.

The estimated grade using the IDW, the kriging, and the IDW-ICA have been presented in Table 2. The boreholes, including C4, C7, C10, C12, C17, C37, and C24, were chosen for analyzing the IDW, the kriging, and the ICA methods. According to Hughes and Davey (1979), in the IDW, the power was 2 while the efficient distance was 250 ft. Also, the Datamine studio software has been utilized for grade estimation by applying the kriging technique. Along with these two methods, the ICA algorithm for finding the best coefficients of the IDW equation. After running the ICA codes in the <https://www.mathworks.com/products/matlab/student.html>, the radius of influence distance and power were found 277 and 2 in the IDW equation, respectively. The real data refers to the actual copper grade in the boreholes. Some statistical parameters such as determination of coefficients (R -squared),

Fig. 14 Two perpendicular variograms ((a) azimuth 75° and (b) azimuth 165°) prepared for determining the search ellipsoid

RMSE, and MAPE were considered for evaluating the performance of the methods. Indeed, it was an attempt to find out that if the IDW-ICA estimations were close to other convectional estimations methods such as the IDW and the kriging or not. As found from the table, the R -squared values in the

IDW, the kriging, and the IDW-ICA were 22.83, 48.02, and 44.34, respectively. Moreover, the RMSE values in the methods were 0.31, 0.25, and 0.26. The results revealed that the proposed algorithm, IDW-ICA, had a better result than IDW. Moreover, it could estimate the grade close to the kriging technique.

Fig. 15 Anisotropic search ellipsoid

Table 4 The obtained results

Parameter	Real grade of Cu	IDW	Kriging	IDW-ICA
Borehole C4	0.188	0.1347	0.144	0.1681
Borehole C7	0.644	0.2346	0.2432	0.3776
Borehole C10	0.072	0.2783	0.277	0.3471
Borehole C12	0.377	0.5925	0.6105	0.5202
Borehole C17	1.009	0.8555	0.905	1.0673
Borehole C37	0.320	0.4338	0.313	0.5411
Borehole C49	0.996	0.3945	0.58	0.4922
The influence distance (ft)	-	250	-	281.85
Power	-	2	-	131.05
RMSE	-	0.31	0.25	0.26
R-squared coefficient (%)	-	22.83	48.02	44.43
MAPE (%)	-	85.35	69.51	78.11

Results and discussions

Case study

Baba-Ali Iron Ore Mine (Hamedan Province, Iran) is located 35 km to the northwest of Hamedan. The mine includes two orebody parts, namely Baba-Ali 1 and Baba-Ali 2. Although the proven reserve of Baba-Ali 1 has been estimated to reach some 6 million tons of iron ore with an average grade of 59.14%, the Baba-Ali 2 is being explored and evaluated deposit estimation. Figure 11 shows the geographical map of the Baba Ali iron mine in Iran. The deposit comprises iron ore with moderate to high breakage resistance, with the waste

being mainly composed of diorite and calcsilicate with moderate to high breakage resistance. The deposit is part of the Sanandaj-Sirjan belt with a height of about 1950 m above sea level. The oldest rocks in the Baba-Ali deposit are limestone with the Cretaceous in age. The host rock of Baba-Ali iron mineralization is volcanic-sedimentary rocks of the Songhor series with Permian-Triassic. The petrologic composition of these series includes felsic to intermediate volcanic rocks, often as lava and rhyolitic tuff, within interbedded carbonate layers and little tuffaceous sandstones. Emplacement and translocation of intrusions and subvolcanic rocks with the composition of gabbro-diorite, quartz-monzodiorite, granodiorite, syenite, syenogranite, and granite in this sequence during tectonic activities caused

Table 5 Cross-validation statistics for grade estimation

Cross-validation statistics for grade	
Number of estimated samples	32
Number of not estimated samples	0
Mean of actual values	38.0863636
Mean of estimated values	37.458668
Mean difference (act – est)	0.62769566
Mean difference (as % of actual)	1.648
Mean absolute difference	10.2783342
The variance of actual values	359.090041
The variance of estimated values	184.889017
Correlation coefficient	0.65
<u>Kriging variance</u>	
Mean of estimated KV from the model	319.076626
Mean of squared differences	209.03403
Ratio	1.526
<u>Regression equation</u>	
Actual	$4.11638099 + 0.90686574 \times \text{estimate}$
Standard error	14.3887561

Fig. 16 Result of Jack-knife kriging (actual data versus estimated data)

deformation and sometimes metamorphism of the Baba Ali region and iron ores (Rostami Paydar et al. 2010).

In the recent past, isolated exploration activities have been performed on this orebody. Among others, geological and geophysical maps of the mine have been prepared, a total of 21 magnetic sections have been surveyed, trenches have been excavated, surface and subsurface samples have been collected, and exploratory bores have been drilled. At the current time, a total of 120 bores were drilled in Baba-Ali Iron Mine. This study focused on section #2390 because it hosted the most significant number of bores. Of the 32 bores drilled along this section, 27 bores were used for training the algorithm, and five bores were utilized for model validation. The estimation bores were selected in such a way to cover the model almost entirely. Figure 12 demonstrates the distribution of the bores along with section #2390.

Modeling results

Kriging

Exploring the data gathered from the study area, three populations (with three different dimensions) were identified and used to scale the original data and estimation results (Fig. 13).

Once finished with the variography at different directions, two normal variograms along with two different azimuths 75

and 165° were found and used to establish the searching ellipse (Fig. 14).

Once finished with variography, including the anisotropy angles, axis rotation direction, anisotropy radius (Fig. 15), and the nugget and ceiling effects, the searching ellipse was developed and used the basis for geostatistical estimations via kriging. Therefore, validation of these parameters was essential.

As is evident from the results, the validity of the variogram parameters for block grade evaluation was about 65%, a desirable level of validity (Fig. 16 and Tables 4 and 5). This could be considered as a justification using the variogram parameters for the grade evaluation of exploratory blocks.

Residual analysis is used to investigate biases of the estimator in geostatistical modeling. Accordingly, an estimator can be recognized as unbiased if the histogram of the residuals (difference between actual data and estimated values) has a normal distribution around a mean value of zero with an adequately low standard deviation (Fig. 17).

IDW

For the considered reserve, the power and the radius of influence were set to 2 and 80 m, respectively, and the grades were evaluated through Eq. (9).

Fig. 17 Histogram of residual distribution of the Jack-knife kriging algorithm (the results almost show the estimator is unbiased)

Table 6 Result of Jack-knife kriging (grade versus estimated grade)

NAME	Grade	GradeEST
Minimum	3.010	9.821
Maximum	66.000	60.848
Mean	38.086	37.459
Correlation coefficient	0.651

Fig. 18 A decreasing trend of RMSE while performing the ICA

IDW-ICA results

In the model, 50 countries, 9 imperialists, and 300 iterations were considered. The decreasing trend of RMSE during the

Fig. 19 Distribution of the boreholes along with section #2390 and the radius of influence of the validation boreholes **Table 7** The obtained results for Baba-Ali Iron Mine

Parameter	Real data	IDW	Kriging	IDW-ICA
Borehole BH1	38.96	43.13	43.25	36.23
Borehole BH2	16.32	36.71	36.71	16.50
Borehole BH3	21.90	39.23	28.54	29.89
Borehole BH4	7.15	11.22	11.33	9.80
Borehole BH5	57.04	61.46	60.85	53.97
The influence distance (m)	-	80	-	77.76
Power	-	2	-	-181.34
RMSE	-	12.41	10.10	4.19
R-squared coefficient (%)	-	51.02	67.53	94.41
MAPE (%)	-	57	46.28	17.4

performing the ICA is shown in Fig. 18. Figure 19 shows the distribution of the boreholes along with section #2390 and the radius of influence of the estimated boreholes (Table 6).

The ICA results showed that the most suitable radius of influence was 77.76 m using this algorithm; Eq. (14) was developed for grade evaluation.

$$g = \frac{\sum \frac{0.45828 \times g_i}{d_i^{-181.3350}}}{\sum \frac{0.52237}{d_i^{-181.3350}}} \quad (14)$$

The boreholes, including BH1, BH2, BH3, BH4, and BH5 in the Baba Ali iron mine, were selected for the iron grade

estimation by applying the IDW, the kriging, and the IDW-ICA methods. The power and the radius of influence values in the IDW were supposed to 2 and 80 m, respectively (Hughes and Davey 1979). Also, the grade estimation by using the kriging technique was conducted by employing the Datamine studio software. Along with these two methods, the modifications of IDW's coefficients, power, and influence distance, d_i , were done by performing the ICA algorithm in the <https://www.mathworks.com/products/matlab/student.html>. As a result, the effective distance and power were calculated 77.6 m and -181.34 in the IDW equation, respectively. The estimated iron grades in the Baba Ali iron mine have been estimated by using the IDW, the kriging, and the IDW-ICA. The results have been presented in Table 7. The real data referred to the actual iron grade in the boreholes. Some statistical parameters such as *R*-squared coefficients, RMSE, and MAPE were considered for evaluating the performance of the methods. As found from the table, the *R*-squared values in the IDW, the kriging, and the IDW-ICA were 51.02%, 67.53%, and 94.41%, respectively. Moreover, comparing the residual statistics, including RMSE and MAPE, revealed that the proposed technique, IDW-ICA, had the lowest values than the IDW and the kriging methods. By comparing the results, It is found that the IDW-ICA technique could estimate the grade values better than the IDW and the kriging.

Conclusions

In the present research, a new hybrid algorithm called IDW-ICA was proposed for grade estimation, and the following results were obtained:

- Considering the more accuracy, less time-consuming, and user-friendly use of the ICA algorithm, this technique was well capable of being used for grade estimation of the mineral orebodies.
- The classic IDW method with no accurate control on the evaluation parameters, including the influence distance and power, may lead to unrealistic results. Such unrealistic results may distract the designers and mining engineers in the design stage, leading to inappropriate mineral project assessment. For modifying the IDW, the coefficients were modified and found with the ICA algorithm. The higher values of *R*-squared coefficients and lower values of RMSE and MAPE revealed that the proposed method, IDW-ICA, had better estimations.
- For validating the proposed method, IDW-ICA, an example was solved by associating the IDW, the kriging, and IDW-ICA. The results showed that the IDW-ICA had better results than IDW, close to the kriging technique. Indeed, when the number of data points was low, it

dropped drastically below the kriging as the number of data points increased.

- The obtained results of the grade estimations in the case study showed that the proposed hybrid algorithm, IDW-ICA, could be applied as a new-fangled method for grade estimation. It may lead to deciding with a high degree of confidence in mining projects.

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Declarations

Conflict of interest The author(s) declare that they have no competing interests..

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