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RESEARCH ARTICLE

Degradation Modelling of and Optimising the Timing of Replacements for Batch Vacuum Pans

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Abstract
Replacing a pan involves the expenditure of significant capital for a sugar mill. Replacing a pan too late may result in excessive downtime, maintenance costs, and risk of catastrophic failure. On the other hand, replacing a pan too early will lead to wasting residual life and an unnecessary allocation of capital funds that may have been spent better elsewhere in the mill. This paper reports on the development of a replacement policy for batch vacuum pan components based on a stochastic model of degradation. Degradation data, principally wall-thickness measurements, were collected from the vacuum pans of an Australian sugar factory and used to develop component degradation models. Unlike the conventional approach of using a line of best fit to identify the end of life of the pan, the methods adopted account for the uncertainties due to seasonal operating conditions and inherent uncertainty in the degradation model parameters. The quantification of the uncertainty in identifying the end of life of a vacuum pan has shown that there is significant risk of a pan failing earlier than the straight-line prediction. Employing this quantification of the risk, a component replacement plan was developed by optimising the replacement of each component individually and subsequently optimising the replacement plan for the entire pan. This strategy is demonstrated using a case study with and without parametric uncertainty to evaluate its impact on maintenance optimisation. Including parametric uncertainty leads to the determination of greater risk earlier, proposing the replacement of components earlier than when parameters are considered as 'known'. It is, therefore, important to consider parametric uncertainty in the planning of pan component replacements to better manage risk.

Keywords Vacuum pans · Degradation · Failure risk · Uncertainty

Introduction

Batch pan functionality is of critical importance to sugar mill profitability, so asset maintainers aim to undertake maintenance and renewal actions at near-optimal times. Given the long lifetimes of the pans, scheduling is a difficult task, made more complicated by constrained capital budgets and limited condition inspections. Premature renewal wastes limited capital; whereas, late renewal may yield unacceptable failure risks. For assets that are inspected infrequently, a predictive approach is preferable, and making decisions based upon current asset condition allows for better balancing of costs and risks (Truong-Ba et al. 2019).

Pans are commonly subjected to corrosion of their internal surface, which increases stress on the surface, thereby increasing risk of failure (Cerit 2019). While this corrosion is a known physical characteristic, corrosion can occur in a highly localised and temporally stochastic process, making wall-thickness loss difficult to predict and the true risk of failure therefore difficult to quantify. This situation is particularly true when pan operating conditions vary significantly from season to season and limited historical condition data are available for modelling—both of which induce significant uncertainty on the degradation process. This uncertainty must be quantified to properly estimate the failure risks and balance them against the cost of component replacements.

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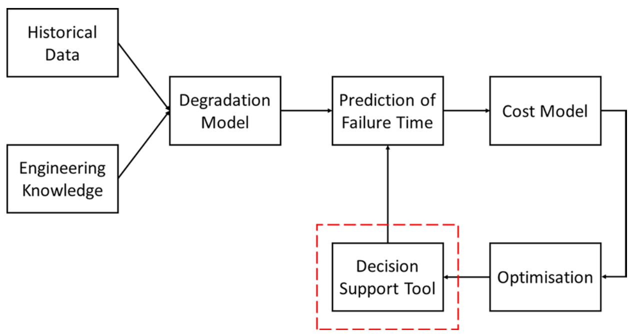

Fig. 1 Flowchart of high-level process to yield the decision support tool

Here, we present a methodology for optimising the renewal times of pan components using thickness loss data. The overall process of renewal optimisation is shown in Fig. 1. The degradation model uses a statistical method for modelling wall-thickness loss that incorporates both historical data and relevant engineering knowledge, and that acknowledges and accounts for uncertainty both inherent to the degradation process and due to poor knowledge of the parameters identified from "little" condition data. The degradation model yields a prediction of pan failure time, which can be combined with a pan cost model through optimisation to yield optimal times for preventive component replacement. The optimal replacement methodology thus provides a decision support approach for asset maintainers in creating renewal schedules. This process can be repeated when more condition data becomes available, to update the advice.

We firstly detail the cost model used for batch pans in the maintenance optimisation process, and then discuss the methodology for degradation modelling and maintenance optimisation with a full acknowledgement of parameter uncertainty induced by the limited data. Finally, we detail the results of a case study of these methods, undertaken on an example batch pan from a real Australian sugar mill.

Methodology

In engineering contexts, pressure-vessel wall-thickness loss is commonly modelled with stochastic processes, particularly gamma processes that are random processes with gamma-distributed increments (Cholette et al. 2019; Haladuick and Dann 2016; Zhang and Zhou 2013). Their stochasticity accounts for some temporal uncertainty in degradation rate, and the monotonic progression of the gamma process is a natural fit for thickness loss. For this paper, a gamma process model structure for thickness loss was used which depends on the cumulative steam flow through the pan, which is an indicator of both operational time and relative load of the given pan. This framework allows for the assumption that pans with different steam flow (operating cycles) can be modelled by the same structure—with comparable parameters.

Consider component i over the time interval $(t_{j-1}, t_j]$. The change in thickness loss $\Delta x_{i,j} = x_{i,j} - x_{i,j-1}$ is proposed to follow a gamma distribution such that $\Delta x_{i,j} \sim G(a_j, \beta)$, where:

$$a_j = \alpha \sum_{j=j-1}^{j} \dot{m}_{\text{steam}}\left(t_{j-1}, t_j\right) \cdot \left(t_j - t_{j-1}\right) \tag{1}$$

where $\dot{m}_{steam}\left(t_{j-1}, t_j\right)$ is the (assumed constant) steam flow over the time interval and α and β are the gamma process parameters to be identified. The parameters α and β are the shape and rate parameters of the gamma process. In a practical sense, their values jointly control the rate of thickness loss, which is predicted by the degradation model. For example, Fig. 2 shows the impact on straight-line thickness loss prediction of a range of α values (when β is fixed) and a range of β values (when α is fixed).

Parameter Estimation

While the gamma process itself accounts for some degradation rate uncertainty, traditional techniques such as

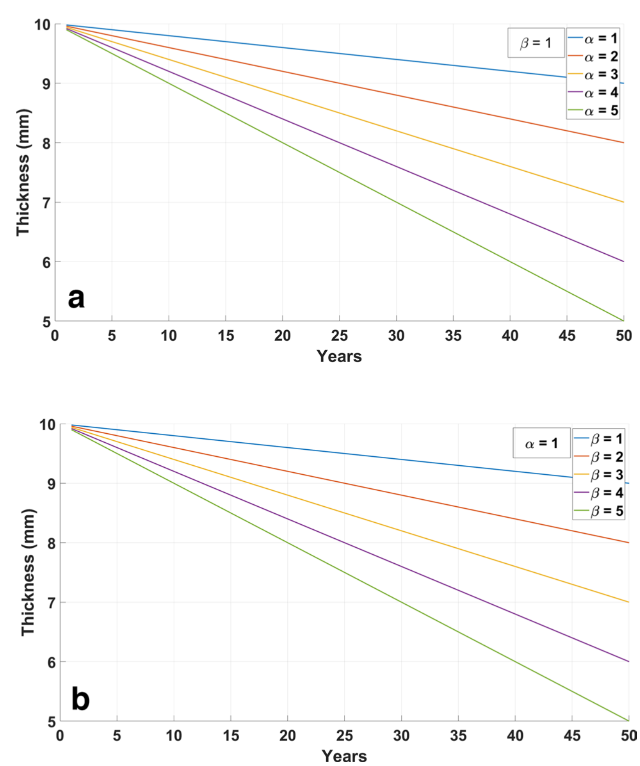

Fig. 2 Impact on straight-line-thickness loss prediction of different model parameter values

Maximum Likelihood Estimation (MLE) assume model parameters are known once estimated as point estimates (single parameter values)—something that is not assured under the condition of "little data". Moreover, MLE has no natural capability to exploit engineering knowledge in the parameter estimation. Given historical data and the proposed model structure, MLE yields the single point value of best fit for the model parameters.

On the other hand, Bayesian methods can be utilised to naturally acknowledge parametric uncertainty in limited data environments and can be used to incorporate engineering knowledge on the plausible values. Bayesian methods seek to compute the posterior distribution, which is the probability (of the value) of the model parameters given the observed historical data (Gelman et al. 2013). To do so, the methods leverage an expression based on the prior distribution and the likelihood function. The prior is the probability of the model parameters without "seeing" any data—this method is used to incorporate any existing knowledge of the degradation process. The likelihood function, or sampling distribution, is the probability of observing the historical data given the model parameters. Thus, we pursued Bayesian identification of the gamma process parameters to provide the capability to characterise the effect of limited data and potentially mitigate its effects by intelligent prior selection.

Given historical data $\mathcal{D}$ of steam flow and thickness loss measurements, the Bayesian methods seek to compute the posterior distribution:

$$(\alpha, \beta) \sim p(\alpha, \beta|\mathcal{D}) = \frac{\ell(\mathcal{D}|\alpha, \beta) \cdot p(\alpha, \beta)}{\int\int \ell(\mathcal{D}|\alpha, \beta) \cdot p(\alpha, \beta) d\alpha d\beta} \tag{2}$$

where $p(\alpha, \beta)$ is the prior distribution of the parameters that encapsulates the prior knowledge and $\updownarrow(\mathcal{D}|\alpha, \beta)$ is the likelihood of the observed data given the model parameters. While the posterior $p(\alpha, \beta|\mathcal{D})$ can be analytically estimated in some cases, it is more common to sample from the posterior using Markov Chain Monte Carlo (MCMC) techniques. MCMC techniques utilise a Markov Chain (where the next sample is dependent on the existing sample) to randomly walk and draw samples from a region proportionate to, and thus effectively from, the parameter posterior distribution. Here, we used Hamiltonian Monte Carlo, which is a type of MCMC method, using the modelling software MATLAB Stan (Gelman et al. 2013; Stan 2022) to sample from, and, hence, to estimate statistical properties of, the posterior. In a practical sense, this process yielded distributions for the probability of a range of values for α and β comprised of parameter samples from the posterior.

In the degradation prediction space, there is a marked difference between using MLE point estimates and MCMC probability distributions for model parameters. Utilising the MLE point estimates for thickness loss prediction results in a single predicted thickness loss path—although confidence intervals can be included around this value. In comparison, altering model parameter values alters the predicted degradation rate (Fig. 2), so utilising the full distribution of MCMC parameter values therefore yields a predictive distribution of thickness loss paths. In practice this predictive distribution is commonly broader—more uncertain of prediction—than MLE point estimate confidence intervals.

Failure Time Distribution

The gamma process model detailed above can be utilised for the prediction of future wall-thickness loss and the distribution of the times when this loss hits the maximum allowable threshold—called *hitting time*. The Cumulative Distribution Function (CDF) of the hitting time ($F(m)$) represents the cumulative chance of the threshold being reached with steam flow less than or equal to 'm'. For homogeneous gamma processes with known (or estimated) parameters α and β, the CDF of the hitting time of threshold h can be found analytically (van Noortwijk and Klatter 1999):

$$F(m) = \frac{\Gamma(\alpha m\beta, h\beta)}{\Gamma(\alpha m\beta)} \tag{3}$$

where m is the cumulative steam flow and the numerator is equal to the 'upper incomplete gamma function' which is defined as $\Gamma(x, y) = \int_y^{\infty} t^{x-1} e^{-t} dy$. Note that hitting *cumulative steam flow* is considered rather than hitting time since steam flow is the independent variable in the degradation model in Eq. (3).

Renewal Optimisation

The timing of the renewal decision of the pan was optimised by minimising the total discounted cost C_T over a finite horizon of length T:

$$C_T = \sum_{t=0}^{T} r^t [c_{\text{renewal}}(\delta(t)) + c_{\text{risk}}(\delta(t))] \tag{4}$$

where $\delta(t)$ is a binary decision variable to indicate capital replacement ($\delta(t) = 1$ indicates that renewal occurs at time t), $c_{renewal}$ and c_{risk} are the expected cost of undertaking pan component/s renewal and the failure risk cost, respectively, and c_{risk}, is the failure risk cost, whose consequence is the sum of downtime, safety, and reactive maintenance costs incurred upon failure. A discount factor $0 < r \leq 1$ is applied to account for the time value of money. Sugar mills have two distinct seasons: growing and crushing. It is assumed that preventive renewal of pan component/s can be scheduled in the growing season, and that any failure occurs in the crushing season when pans are in use. Hence, failure incurs the downtime cost of loss of production. The only preventive maintenance action considered by this model is renewal of individual components. No costs are included for inspection, as the model predicts forward from current knowledge of condition, in this case study, from 'as new'.

The costs of maintenance and renewal must be computed according to the pan components specifications and reliability structure. A general reliability structure for the five pan components (top cone, main body, middle cone, calandria, and bottom cone) was developed based on discussions with sugar mill staff: if one component fails, the entire pan (all five components) must be replaced unless the failure occurs in the top or bottom cone. In this latter case, only the failed component must be replaced. When performing preventive renewals, it is assumed that any of the five components may be replaced individually.

We pursued a simulation–optimisation for minimising the total discounted cost (the expected value of Eq. (4)). A Monte Carlo simulation was used to estimate the expectation C_T using specified replacement ages and gamma process parameter distributions as inputs. For each simulation, a two-stage sampling was utilised: first of the parameter values (from the parameter posterior distributions) and then of the component failure times (via the CDF in Eq. (3) at the sampled parameter values). In the case of point estimates for parameters yielded from traditional techniques such as MLE, the first level of sampling is not necessary. For each component, a replacement occurred if the component failed or was preventively replaced within the horizon. Upon this replacement, the component age was reset to zero and the simulation was continued until the time horizon was achieved. The discounted total cost was computed by summing the (discounted) cost of each of the replacement events and the expectation of C_T was approximated by taking the average of N_{sample} sample path costs. This simulation was used as the fitness function in a Genetic Algorithm to find the five component replacement ages that minimized this (approximate) discounted total cost.

Case Study Results

A case study was undertaken on an example pan from a fleet of batch pans at an Australian sugar mill. The next section details the results of degradation model parameter estimation using both Bayesian techniques and the more traditional Maximum Likelihood Estimation (MLE) technique and assesses the impact of parameter uncertainty on the failure risks. We then present the results of renewal optimisation undertaken for the example pan.

Parameter Estimation and Degradation Modelling

Historical data available from an Australian sugar mill pan fleet was used to estimate the parameters for the degradation model. The data were comprised of historical cumulative steam-flow and thickness measurements. Using the degradation model framework detailed in the Methodology, change in thickness measurements for each of the five components were grouped across the fleet of seven pans. This approach yielded the small number of changes in thickness historical measurements identified in Table 1.

Point estimates for the degradation parameters α and β for each of the five components were firstly obtained via MLE; while, the Bayesian posteriors were sampled using Stan's MATLAB interface (Stan 2022). An example of the parameter estimation results for the top cone is shown in Fig. 3. Figure 3 shows the MLE point estimate, along with the probability distribution of values determined by the Bayesian techniques. The graphs show that there is quite high probability that the two parameters could be much higher or lower in value than the point estimate.

To then examine the impact of parameter uncertainty in the degradation predictions, we compared the hitting time distributions for the minimum thickness for both the MLE and Bayesian parameter identification methods. The maximum allowable thickness loss threshold was found by subtracting the minimum thickness from the original (as new) thickness of the components. Firstly, a straight-line prediction of hitting time was made using MLE point estimates.

Table 1 Number of historical thickness loss measurements available for each component for parameter estimation

Component	Number of measurements
Top cone	17
Main body	33
Middle cone	15
Calandria	48
Bottom cone	31

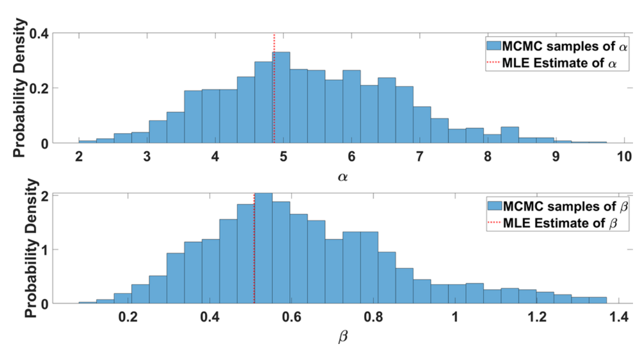

Fig. 3 Comparison of the parameter estimations for the top cone component

Secondly, the point estimates were used in Eq. (3) to yield a curve for the hitting time distribution which incorporates some uncertainty of future steam flow and operating conditions. Finally, the MCMC parameter distributions were utilised to simulate and store component hitting times, yielding a distribution of possible hitting times which also incorporates parametric uncertainty. An example of the hitting time distributions can be seen in Fig. 4 (middle cone). As can be seen in the figure, a significant portion of the MCMC simulated failure times fall outside the bounds of the 95% confidence interval of the MLE hitting time distribution, shown by the bell-shaped curve. This result indicates that the inclusion of parametric uncertainty (inherent in the Bayesian method) is important to properly evaluating risk of component failure since ignoring it under-estimates the probability of the tails (i.e. very early or very late failure times).

A similar comparison was performed for each of the five components in the example pan. The results are presented in Table 2, which quantifies the added risk of failure identified by including parametric uncertainty in degradation modelling of the example pan. The results show significant risk of failure occurring both before and after the analytical modelling predicts. This identified risk has important implications for renewal optimisation but is difficult to use directly for decision making. In the next section the impact of this earlier identified risk is demonstrated on optimal replacement ages, which can be used as a decision support tool. Note that if ignoring parameter uncertainty is acceptable, the percentage of samples falling outside the bounds of the confidence interval of the analytical distribution would be 2.5% on either end.

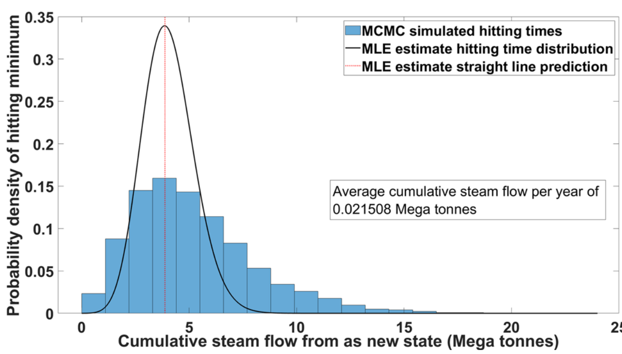

Fig. 4 Comparison of the analytical estimation of failure time distribution to the Monte Carlo simulated times

Renewal Optimisation

The renewal optimisation of the example pan was undertaken to identify optimal ages for the component renewals. The downtime cost was computed based on the fraction of lost productivity due to the pan's failure. Individual component renewal costs were estimated as $\frac{V_{\text{component}}}{V_{\text{pan}}} C_{\text{capital}}$ where V is the volume of material (either for the component or the pan) and C_{capital} is the capital cost of the pan. Safety risk cost was considered to be AUD50,000.

MATLAB's inbuilt Genetic Algorithm (GA) was utilised to solve for the optimal replacement ages for the components of the example pan. The components were considered concurrently, with the reliability structure as noted in the Methodology. The optimisation took place with a finite horizon of $T = 50$ years, and a discount rate of $r = 0.95$ was used. To assess the impact of including parametric uncertainty in the modelling, optimisation was undertaken twice: firstly, utilising the Bayesian posterior parameter samples (and two-stage sampling), and secondly using the MLE point estimates and no parameter sampling. The optimal preventive replacement ages for the components are presented in Table 3.

When utilising the full parameter marginal distributions in the simulation–optimisation based on the Bayesian approach, all five components are found to be optimally preventively replaced prior to the 50-year horizon—from their as new state. Whereas when utilising the parameter point estimates based on the MLE approach, none of the components are scheduled for preventive replacement. This result indicates that when parameter uncertainty is included in the process the model identifies potential failure earlier than when the parameters are 'known' using point estimates. Including parameter uncertainty results in the earlier scheduled preventive replacements of components.

Table 2 Percentage of Monte Carlo failure times outside the bounds of the point estimate driven analytical distribution confidence interval for components of the example pan. If ignoring the parameter uncertainty is acceptable, the percentage of samples outside the lower/upper CI would be 2.5%. Clearly, the true probability is higher if one considers parameter uncertainty

Component	Percentage of simulated failure times before lower bound analytical CI	Percentage of simulated failure times after upper bound analytical CI	Total failure risk outside bounds of analytical CI
Top cone	4.0	7.0	11.0
Main body	5.5	9.0	14.5
Middle cone	10.0	25.0	35.0
Calandria	6.7	13.6	20.3
Bottom cone	5.0	10.0	15.0

Table 3 Genetic algorithm optimised replacement ages, generated using full set of MCMC parameters and MLE point estimates

Component	MCMC parameters	MLE parameters
Top cone	44	-*
Main body	47	-
Middle cone	42	-
Calandria	47	-
Bottom cone	44	-

*A dash indicates that the solver found the optimal action was to plan for no preventive replacement of the component

Following the simulation–optimisation of the replacement ages, the Monte Carlo algorithm was utilised to find the expected discounted yearly costs when following the optimal replacement schedule. The yearly costs are comprised of the expected amount of component replacement and failure costs. Each Monte Carlo simulation that experienced component failure or preventive replacement recorded the appropriate indicative costs of the reliability model as the yearly costs of the simulation. The expected yearly costs are; thus, the mean of the yearly costs of all the simulations. They are a portion of the cost of a failure event. The expected yearly costs, for each of the components and for the pan, are presented in Fig. 5. Note that the yearly costs peak and begin to reduce in the second half of the finite horizon, this is due to the effect of the discount rate on the costs.

Conclusions

Here, we have presented a model for predicting thickness loss of sugar mill batch pans while acknowledging and quantifying uncertainty. This model was compared to a baseline version wherein model parameters were considered 'known' after (point) estimation. A methodology was presented to utilise the uncertain model in optimising preventive replacement scheduling of pans in accordance with their presented reliability structure. A case study was undertaken on an example pan from a real Australian sugar mill, utilising historical data from the fleet of pans at the mill. We found that treating the model parameters as known led to overconfidence in the model and under evaluation of failure risk. Maintaining parametric uncertainty led to identification of this risk, and hence earlier scheduled preventive replacements of the pan's components. We recommend that asset

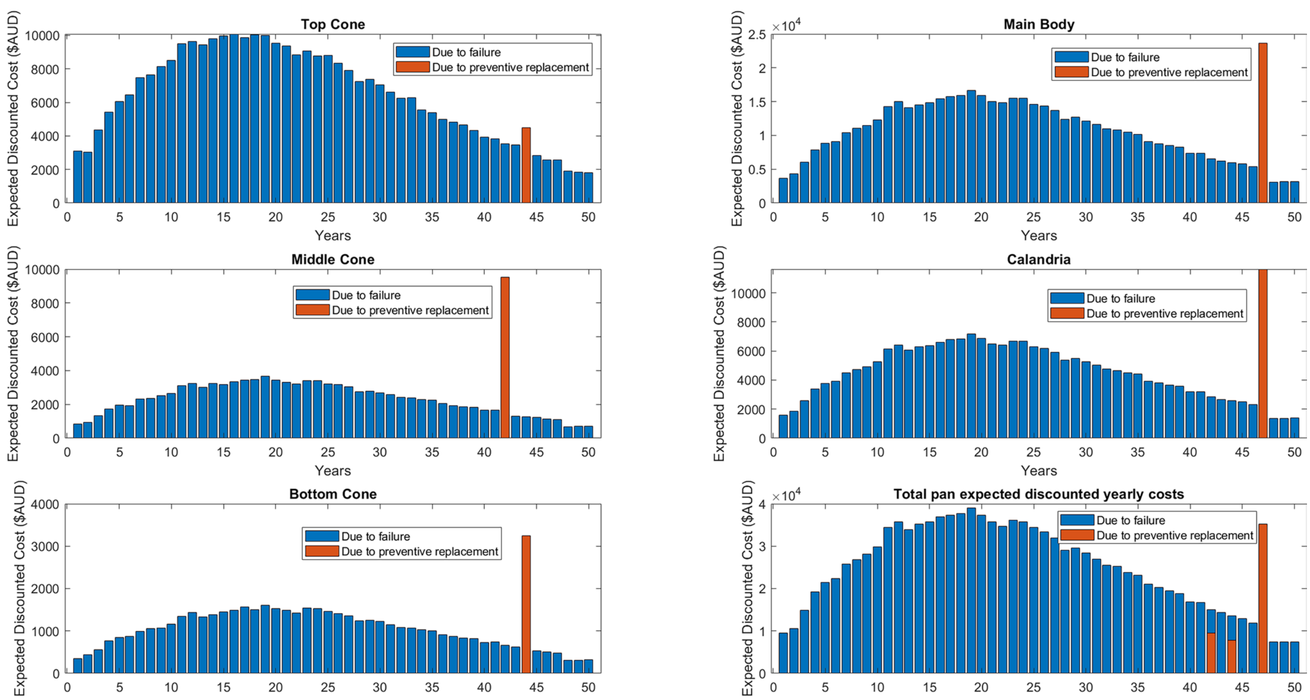

Fig. 5 Expected discounted yearly costs when following the optimised preventive replacement schedule

maintainers utilise this method when predicting future wall thickness loss and scheduling preventive replacement.

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Declarations

Conflict of interest This manuscript was presented at ISSCT Congress 2023 then subsequently invited to submit to Sugar Tech, with permission from ISSCT.There are no conflicts of interest.

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