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ORIGINAL PAPER

New method for assigning cardinal weights in multi-criteria decision-making: the constant weight ratio method

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Abstract
A new method is proposed to convert ordinal ranking of a number of criteria and an additional piece of information into numerical weights. A literature review of methods for assigning cardinal weights based on ordinal ranking is performed, as well as an analysis of their behaviour. The new method, called 'constant weight ratio' (CWR), enables better adjustment to the decision-maker's preferences than purely ordinal ranking methods. It also solves the problem of the excessive decrease in the weight of the most important criterion (or criteria) when the total number of criteria is large and the weight of the most important criterion (or criteria) must be high. It is achieved via three simple steps and flexible input data. The additional piece of information may be: (i) the relative importance of the criteria, i.e., the weight ratio, (ii) the total weight of the most important set of criteria, or (iii) the weight of the most important criterion. The proposed method is applied to two case studies in the cultural sector to illustrate that the resulting weights are equivalent to other methods requiring more input data from the decision maker.

Keywords Multi-criteria decision-making · Multi-attribute utility theory · Decision support systems · Weight assignment · Ordinal ranking

Mathematics Subject Classification 90B50 · 91B06 · 91B16

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1 Introduction

Evaluation and decision-making processes are common in people's daily living and work experiences (Wang and Yang 1998; Kitsios and Grigoroudis 2020; Basílio et al. 2022). The use of multi-criteria decision analysis (MCDA) is important for supporting strategic decision-making in organisations (Montibeller and Franco 2011). Decision makers are frequently required to choose between several alternatives or options with multiple, often conflicting, criteria or objectives. In such situations, it is normally impossible for them to achieve every objective to the degree they would like to at the same time (Reeves and Macleod 1999; Akpan and Morimoto 2022). The best alternative with respect to one criterion is not usually as good as another alternative with respect to another criterion. MCDA stands as one of the main decision-making analysis which aims to identify the best alternative by considering multiple criteria in the selection process (Taherdoost and Madanchian 2023).

A variety of interactive approaches has thus been developed to aid individual decision makers or decision-making groups in attempting to identify the best solution in complicated evaluations or decisions (Baucells and Sarin 2003; Rezaei 2015). One of the most widely used MCDA methods is the Multi-Attribute Utility Theory (MAUT) (Keeney and Raiffa 1979), which involves measuring two basic components: weight and value (Wang and Yang 1998; Phelps et al. 2018).

The weights are scaling constants that express the relative importance of each criterion to the decision maker and determine, to a large extent, which alternative is regarded as the best overall (Roberts and Goodwin 2002; Rossetto et al. 2015). Therefore, formulating appropriate methods for measuring weights is a key component and a major challenge in the development of a multi-criteria model for selecting the best alternative. The value is measured by means of value or utility functions that quantify how much value or satisfaction the decision maker associates with different levels of each attribute.

Although a multiplicative multi-criteria value model (Miyamoto et al. 1998) and other types of utility functions (Keeney 1996) are possible in some cases, the more common underlying evaluation model is an additive multi-criteria value model (Zhang et al. 2006) of the form:

$$V\left(a_j\right) = \sum_{i=1}^{n} w_i \cdot v_{ij}\ ; \quad i = 1, 2, \ldots, n\ ; j = 1, 2, \ldots, t \tag{1}$$

where $V\left(a_j\right)$ is the overall multi-criteria value of the jth alternative. a_j is the jth alternative. There are a total of t alternatives. w_i is the weight of the i th criterion. There are a total of n criteria. v_{ij} is the value associated with the i. th criterion of the j th alternative.

While weights play a crucial role in MAUT, their application extends to other MCDA methods. Outranking methods such as ELECTRE, PROMETHEE or TOPSIS also incorporate the use of weights.

There are many methods for assigning cardinal weights to criteria. These methods can be classified taking into account the pieces of information to be input by the decision maker and include:

(1) *Ordinal methods*, which assign cardinal weights based on ordinal ranking, which can be further broken down into:

    (1a) *Purely ordinal methods*, which only ask the decision maker to rank the criteria by priority. The rank-order centroid method (Solymosi and Dombi, 1985) is an example of this group
    (1a) Some other methods, which, in addition to the ranking of the criteria, ask the decision maker for an additional piece of information. The rank geometric method (Lootsma and Bots 1999) and the new method presented in this article are examples of this type

(2) *Cardinal methods*, which ask the decision maker for cardinal information. One widespread example of this class of methods is the AHP (Analytic Hierarchy Process) (Saaty 1980), which is used in many fields, including the evaluation of construction (Darko et al. 2019) and universities (Salimi and Rezaei 2015)

Weight assessment is a cognitively demanding task (Larichev 1992; Bregar 2022; Lauras et al. 2010) and may suffer on several points. First, weights are highly dependent on the elicitation method (Schoemaker and Waid 1982; Jaccard et al. 1986; Borcherding and von Winterfeldt 1988; Epple 1990; Riabacke et al. 2012; Gumus et al. 2016). Additionally, there is no consensus as to which method produces more accurate results since the 'true' weights remain unknown or, from a different point of view, are defined by the method. A more troubling aspect of this approach is that the elicitation of these exact weights imposes a precision that may be absent in the mind of the decision maker (Barron and Barrett 1996; Comes et al. 2011; Lin and Lu 2012; Zamani-Sabzi et al. 2016). The decision maker may be unavailable or unable or unwilling to specify sufficiently accurate weights, or there may not be a single decision maker and the decision-making group may only be able to reach agreement on a ranking of criterion weights.

According to Kirkwood and Sarin (1985), weights based on ranking, i.e. ordinal methods, may be necessary. There are many practical reasons to use ranking. It is a necessary first step in most procedures for more accurate weight elicitation (Johnson and Huber 1977; Watson and Buede, 1987; Buede 1988). It is also easier than more accurate assignments. Eckenrode (1965) stated that ranking was not only the easiest of several procedures but, in his view, the most reliable.

Consequently, to avoid the difficulties associated with detailed weight elicitation, the research presented in this paper focuses on the methods for assigning cardinal weights based on ordinal ranking. The objectives of this study are threefold: (1) the analysis of these methods and comparison of their advantages and disadvantages;

(2) the development of a new method able to overcome some of the disadvantages based on the ranking of the criteria and an additional piece of information; and (3) the implementation of the new method in two case studies of the programming at two cultural institutions: CaixaForum Barcelona and Palau de la Música Catalana, both located in Barcelona (Spain).

Regarding the purely ordinal methods, the analysis revealed that the relative importance of the criteria is determined by the method used (depending only on the method used and the total number of criteria) and that, when the number of criteria is large, the weight of the most important criterion decreases considerably. The analysed methods using ordinal ranking and an additional piece of information for assigning cardinal weights partially overcome these disadvantages, but the accuracy of the result depends on the accuracy of the only additional piece of information that the decision maker has to provide.

The newly proposed method consists of an ordinal ranking method with an additional piece of information which the decision maker can choose from among three possibilities, thereby overcoming the aforementioned disadvantages. The case studies illustrate that the results obtained with the proposed ordinal method are very close to those obtained with a cardinal method but involve a less demanding process.

The remainder of this paper is organised as follows: Sect. 2 analyses the main existing methods for assigning weights based on an ordinal ranking; Sect. 3 presents the new proposed method with a short example; Sect. 4 discusses the new method's behaviour, comparing it to the main existing methods and describing its limitations; Sect. 5 reports on two case studies using real data, highlighting the main results; and, finally, Sect. 6 presents the conclusions.

2 Analysis of the main methods for assigning cardinal weights based on ordinal ranking

This section analyses the features, advantages and disadvantages of the main existing methods for assigning weights based on ordinal ranking. These methods are based on a ranking of the criteria according to the decision maker's preferences. The only equations and inequalities to be fulfilled are as follows (Eq. (2) and inequalities (3) or Eq. (2) and inequalities (4)):

$$\sum_{i=1}^{n} w_i = 1 \tag{2}$$

If the ranking is from the most to the least important criterion, inequalities (3) have to be fulfilled:

$$w_i \geq w_j; i < j \tag{3}$$

If the criteria are ranked from the least to the most important criterion, inequalities (4) have to be fulfilled:

$$w_i \geq w_j; i > j \tag{4}$$

The foregoing equation and inequalities are insufficient to assess the weights.

For notational convenience, the convention $w_1 \geq w_2 \geq \cdots \geq w_n \geq 0$ is adopted. All the following formulae provide the normalised weights to total one.

2.1 Analysis of the main existing methods for assigning cardinal weights based only on ordinal ranking

2.1.1 Description of the methods

With these methods, the only piece of information given by the decision maker is the ranking of the criteria. These methods have the advantage of simplicity, as less information is asked of the decision maker and few calculations are required. Three methods based only on ordinal ranking have been found in the literature:

1. The rank sum (RS) method
2. The rank reciprocal (RR) method
3. The rank-order centroid (ROC) method

These methods were compared to each other by Barron and Barrett (1996), Roberts and Goodwin (2002), Ahn and Park (2008) and Alfares and Duffuaa (2008). The rank sum and rank reciprocal methods were examined by Stillwell et al. (1981). The rank-order centroid method was proposed by Solymosi and Dombi (1985). Each of these methods is explained below.

2.1.1.1 The rank sum (RS) method In the RS procedure, the weights w_i (RS) are the individual ranks as if the criteria were ranked from least to most important, normalised by dividing by the sum of the ranks. The formula yielding the weights can be written as (5). The relative importance between the $i+1$ criterion and the i criterion, k_i, varies according to i and is expressed in Eq. (6).

$$w_i(RS) = \frac{n+1-i}{\sum_{j=1}^{n} j} = \frac{2(n+1-i)}{n(n+1)}; \quad i = 1, \ldots, n \tag{5}$$

$$k_i(RS) = \frac{w_{i+1}}{w_i} = \frac{\frac{2(n+1-(i+1))}{n(n+1)}}{\frac{2(n+1-i)}{n(n+1)}} = \frac{n-i}{n-i+1}; \quad i = 1, \ldots, n \tag{6}$$

2.1.1.2 The rank reciprocal (RR) method This method uses the reciprocal of the ranks, which are normalised by dividing each term by the sum of the reciprocals. They are defined by formula (7). The relative importance between the $i+1$ criterion and the i criterion is the ratio of the inverse of their ranking, as expressed in Eq. (8).

$$w_i(RR) = \frac{\frac{1}{i}}{\sum_{j=1}^{n} \frac{1}{j}}; \quad i = 1, \dots, n \tag{7}$$

$$k_i(RR) = \frac{w_{i+1}}{w_i} = \frac{\frac{\frac{1}{i+1}}{\sum_{j=1}^{n} \frac{1}{j}}}{\frac{\frac{1}{i}}{\sum_{j=1}^{n} \frac{1}{j}}} = \frac{\frac{1}{i+1}}{\frac{1}{i}} = \frac{i}{i+1}; \quad i = 1, \dots, n \tag{8}$$

2.1.1.3 The rank-order centroid (ROC) method In the ROC procedure, the weight w_i (ROC) is the sum of the reciprocal of the ranks from the ith criterion to the nth criterion, normalised by dividing by the total number of criteria, n. The weights are defined by formula (9). The relative importance between the $i+1$ criterion and the i criterion varies according to i and can be written as expressed in Eq. (10).

$$w_i(ROC) = \frac{1}{n} \sum_{j=i}^{n} \frac{1}{j}; \; i = 1, \dots, n \tag{9}$$

$$k_i(ROC) = \frac{w_{i+1}}{w_i} = \frac{\frac{1}{n} \sum_{j=i+1}^{n} \frac{1}{j}}{\frac{1}{n} \sum_{j=i}^{n} \frac{1}{j}} = \frac{\sum_{j=i+1}^{n} \frac{1}{j}}{\sum_{j=i}^{n} \frac{1}{j}}; \; i = 1, \dots, n \tag{10}$$

2.1.2 Analysis of the methods

Some authors (Barron and Barret, 1996; Roberts and Goodwin 2002) have compared the three weight-assignment methods based only on ordinal ranking, as discussed in Sect. 1, in order to determine which is the most accurate by using the distributions of rank-order weights. Others (Ahn and Park 2008; Alfares and Duffuaa 2008) have done the same type of comparisons and proposed an empirically developed method. These studies are based on the notion that the only information available is the ranking of the criteria by importance and they are very useful in such cases.

The methods for assigning cardinal weights based only on ordinal ranking have two disadvantages:

a) The only information given by the decision maker is the ranking of the criteria by importance and, therefore, the intensity with which the $i+1$ criterion is preferred to i is unknown. As a result, the relative importance of a criterion with respect to the next criterion is determined by the method used. Figure 1 illustrates this phenomenon. For each of the methods presented above (Eqs. (6), (8) and (10)), the value of k_i (the ratio of the weight of the $i+1$ criterion to the weight of the i criterion $k_i = \frac{w_{i+1}}{w_i}$) depends only on the method used and the total number of

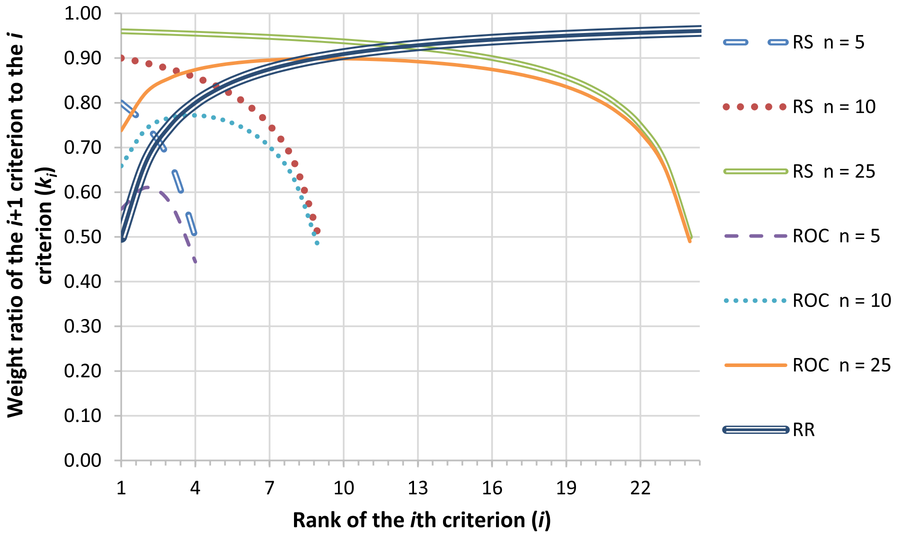

Fig. 1 Importance ratio k_i depending on the rank of the *i*th criterion for the different purely ordinal methods and different total numbers of criteria *n*

criteria, as shown in Fig. 1. The decision maker cannot change the value of k_i. For the different purely ordinal methods, the main values of k_i behave as follows:

1. The RS method. The following expressions were obtained replacing the values $i = 1$ and $i = n - 1$ in Eq. (6):

$$k_1 = \frac{n-1}{n} \tag{11}$$

$$k_{n-1} = \frac{1}{2} \tag{12}$$

Figure 1 shows that for the same value of i, k_i increases as n increases. For low i values, the value of k_i increases and approaches 1 as the value of n increases. For different values of n, when i increases, k_i decreases up to a value of 0.5 for $i = n$ - 1. Hence, the weights of the most important criteria are similar to each other and, as the least important criterion is approached, the weights of the least important criteria become more different from each other.

2. The RR method. The following expressions were obtained replacing the values $i = 1$ and $i = n - 1$ in Eq. (8):

$$k_1 = \frac{1}{2} \tag{13}$$

$$k_{n-1} = \frac{n-1}{n} \tag{14}$$

In this case, the value of k_i behaves contrary to how it does in the RS method. For $i = 1$, the value of k is 0.5. For high i values (up to n - 1) the value of k_{n-1} approaches 1 as n increases. Thus, the weights of the most important criteria are more different from each other than the weights of the least important criteria (which are more similar to each other). For the same value of i, k_i is constant for all values of n, i.e. k_i does not depend on n (see equation (8)).

3. The ROC method. The following expressions were obtained replacing the values $i=1$ and $i=n-1$ in Eq. (10):

$$k_1 = \frac{\sum_{j=2}^{j=n} \frac{1}{j}}{\sum_{j=1}^{j=n} \frac{1}{j}} \tag{15}$$

$$k_{n-1} = \frac{\frac{1}{n}}{\frac{1}{n} + \frac{1}{n-1}} \tag{16}$$

For the same value of i, k_i increases as n increases. For high i values (up to n - 1), k_i takes a value slightly lower than 0.5: the greater n is, the closer to 0.5 the value of k_i is. Unlike the other two methods, with this one, higher values of k_i are obtained for intermediate values of i. Hence, the weights of the criteria placed in the intermediate

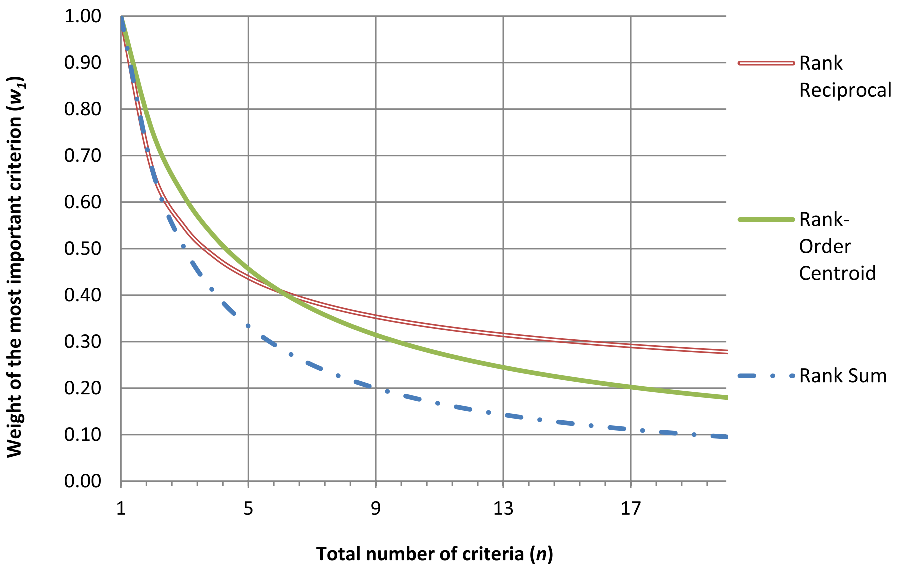

Fig. 2 Weight of the most important criterion w_1 versus the total number of criteria n for the different purely ordinal methods

zone of the ranking are more similar to each other than the weights of the most or least important criteria are to each other.

b) When the number of criteria is large (over 15), the weight of the most important criterion decreases considerably. For example, when n is equal to 16, the weight of the most important criterion is never greater than 30% in any of the three methods and is only around 11% in the rank sum method. Figure 2 illustrates this phenomenon.

2.2 Analysis of the existing methods for assigning cardinal weights based on ordinal ranking and an additional piece of information

2.2.1 Description of the methods

Two methods for assigning weights using ordinal ranking and an additional piece of information were found in the literature:

1. The rank geometric (RG) method
2. The rank exponent (RE) method

Both methods are presented below and the required additional piece of information is discussed.

2.2.1.1 The rank geometric (RG) method This method was proposed by Lootsma and Bots (1999) and was compared with other methods by Alfares and Duffuaa (2008). Although an explicit formula in Lootsma and Bots (1999) was not found, it has been deduced from their explanations. Apart from the ranking of the criteria, the only additional piece of information needed is the ratio of the weight of the least important criterion to the weight of the most important criterion $\left(b = \frac{w_n}{w_1}\right)$. In the absence of any further information, Lootsma and Bots (1999) assumed that the ratios of the successive output criteria are equal. With this information, it is easy to deduce the formula for obtaining the weights (17) and the importance ratio of the $i+1$ criterion to the i criterion (19).

$$w_i(RG) = \frac{b^{\frac{i-1}{n-1}}}{\sum_{j=1}^{n} b^{\frac{j-1}{n-1}}}; \quad i = 1, \dots, n \tag{17}$$

In this case, k may be obtained directly from the datum b as follows:

$$\left.\begin{array}{c} b = \frac{w_n}{w_1} = \frac{w_n}{w_1} \cdot \frac{w_{n-1}}{w_{n-1}} \cdot \frac{w_{n-2}}{w_{n-2}} \cdot \ldots \cdot \frac{w_2}{w_2} \cdot \frac{w_1}{w_1} = \frac{w_n}{w_{n-1}} \cdot \frac{w_{n-1}}{w_{n-2}} \cdot \ldots \cdot \frac{w_2}{w_1} = \prod_{i=1}^{i=n-1} \frac{w_{i+1}}{w_i} \\ k_i = \frac{w_{i+1}}{w_i} = k \end{array}\right\} \rightarrow b = k^{n-1} \rightarrow k = \sqrt[n-1]{b} \tag{18}$$

Or, as it is usually expressed:

$$k_i(RG) = \frac{w_{i+1}}{w_i} = \frac{\frac{b^{\frac{(i+1)-1}{n-1}}}{\sum_{j=1}^{n} b^{\frac{j-1}{n-1}}}}{\frac{b^{\frac{i-1}{n-1}}}{\sum_{j=1}^{n} b^{\frac{j-1}{n-1}}}} = \frac{b^{\frac{i}{n-1}}}{b^{\frac{i-1}{n-1}}} = b^{\frac{1}{n-1}} = \sqrt[n-1]{b}; \quad i = 1, \ldots, n \tag{19}$$

Alfares and Duffuaa (2008) use the particular case $k = \frac{1}{\sqrt{2}}$ for their study.

2.2.1.2 The rank exponent (RE) method This method was examined by Stillwell et al. (1981). The rank exponent method requires specific knowledge of the exact weight of the most important criterion. This weight is entered into formula (20), for the case $i = 1$:

$$w_i(RE) = \frac{(n-i+1)^z}{\sum_{j=1}^{n} (n-j+1)^z}; \quad i = 1, \ldots, n \tag{20}$$

which may then be solved for z via an iterative process. Once z is known, the rest of the weights are determined. The importance ratio of the $i+1$ criterion to the i criterion depends on i and is shown in Eq. (21).

$$k_i(RE) = \frac{w_{i+1}}{w_i} = \frac{\frac{(n-(i+1)+1)^z}{\sum_{j=1}^{n} (n-j+1)^z}}{\frac{(n-i+1)^z}{\sum_{j=1}^{n} (n-j+1)^z}} = \frac{(n-i)^z}{(n-i+1)^z}; \quad i = 1, \ldots, n \tag{21}$$

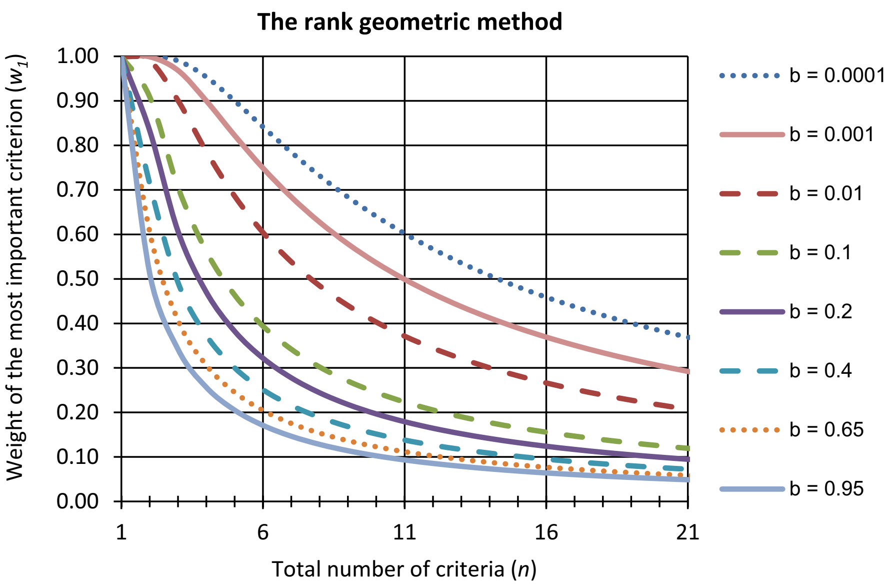

Fig. 3 Weight of the most important criterion w_1 for the rank geometric method versus the total number of criteria n for different values of the b parameter

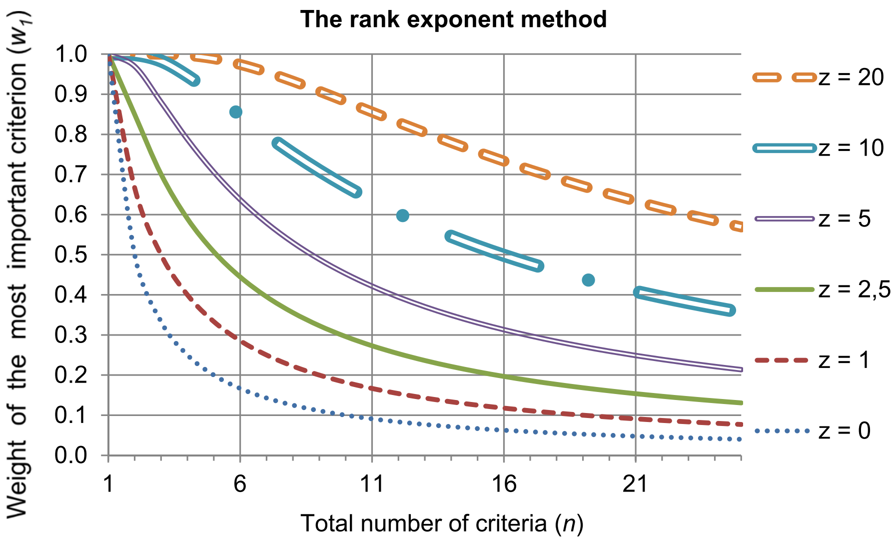

Fig. 4 Weight of the most important criterion w_1 versus the total number of criteria n for the rank exponent method depending on the value of the z parameter

The rank exponent method exhibits several interesting characteristics: $z=0$ defines the equal weights case and $z=1$ defines the rank sum method.

2.2.2 Analysis of the methods

Both the rank geometric method and the rank exponent method have certain advantages over purely ordinal methods. As can be seen in Figs. 3 and 4, depending on the value assigned to the variable b in the rank geometric method and the value of z obtained in the rank exponent method, these methods can accommodate many situations.

For example, using either of these two methods, if there are 16 criteria, the weight of the most important criterion can vary from 6% to over 40% in the rank geometric method and over 70% in the rank exponent method.

However, both methods pose some problems. The decision maker may not have an accurate idea of the initial additional data: the value of $b = \frac{w_n}{w_1}$ in the rank geometric method and the value of the weight of the most important criterion, from which z is obtained in the rank exponent method. Flexibility is considered important in weight-assignment methods, i.e. for the decision maker to be able to choose what further information he or she wants to give apart from the ranking of criteria to minimise the lack of accuracy in the initial information.

In addition, when using the rank geometric method, giving an accurate value of b (the ratio of the weight of the least important criterion to the weight of the most important criterion) can be difficult for the decision maker when b is lower than $\frac{1}{9}$ because the human mind is not considered to be able to compare two values precisely when this ratio is greater than 9 or lower than $\frac{1}{9}$. Based on this fact, Saaty (1977) discussed what scale of comparison is most appropriate when comparing the importance of two criteria. The scale used by Saaty thus includes only values from $\frac{1}{9}$ to 9.

For instance, if there are 15 criteria and assuming $k_i = 0.8$, for $i = 1,\ldots, 14$, then, $w_1 = 20.7\%$ and $w_{15} = 0.9\%$. The ratio of the weight of the least important criterion to the weight of the most important criterion is $b = 0.044 < \frac{1}{9}$, a value that could not easily be provided by the decision maker.

3 New method: the constant weight ratio

In order to overcome some of the disadvantages of the existing methods explained in Sect. 2, a new method was designed and is presented in this paper. This new method is intended to make the approximation of the weights more accurate than in methods based only on a single ranking, while at the same time preserving the simplicity for the decision maker. Asking the decision maker for a single piece of information is preferable to establishing randomly or by a predetermined method the relative importance of criteria (the value of $k_i = \frac{w_{i+1}}{w_i}$). Moreover, the decision maker can choose which additional piece of information he or she wants to provide from three possibilities. This new method can also solve the excessive decrease in the weight of the most important criterion (or criteria) when the total number of criteria is large (Liu et al. 2017) and the weight of the most important criterion (or criteria) must be high. Like the rank geometric and rank exponent methods, this method may be classified between purely ordinal methods and cardinal methods.

3.1 Design of the new method

Given that the shape and tendency of the function k_i is unknown, the simplest function of k_i, a constant k, is considered. Hence, the main characteristic of the constant weight ratio (CWR) method is that the importance ratio of the $i+1$ criterion to the i criterion (value of k_i) is constant for any value of i. The condition to be imposed for satisfying this property is shown in Eq. (22):

$$w_{i+1} = k \cdot w_i; \quad \forall i = 1, \ldots, n \tag{22}$$

Equation (23) shows the value of the weight of any criterion in terms of k and of the weight of the most important criterion:

$$w_i = k^{i-1} \cdot w_1 \tag{23}$$

If it is imposed that the sum of all weights is equal to 1 (Eq. (24)) and the weight of each criterion is replaced with Eq. (23), then the value of k is directly linked to w_1 (Eq. (25)).

$$\sum_{i=1}^{n} w_i = w_1 + w_2 + w_3 + \cdots + w_n = 1 \tag{24}$$

$$\sum_{i=1}^{n} w_i = w_1 + kw_1 + k^2 w_1 + \ldots + k^{n-1} w_1 = w_1 \sum_{i=0}^{n-1} k^i = 1 \tag{25}$$

Since the value of the summation of Eq. (26) is known (Spiegel, 2005), it is possible to find a simple equation that relates the value of k to w_1 (Eqs. (27) and (28)).

$$\sum_{i=0}^{n-1} k^i = \frac{1-k^n}{1-k} \tag{26}$$

$$w_1 \frac{1-k^n}{1-k} = 1 \tag{27}$$

$$w_1 = \frac{1-k}{1-k^n} \tag{28}$$

The value of k is still undefined. There are several options to find it:

(a) If the decision maker knows the value of k, weights are immediately determined by Eqs. (28) and (23).
(b) In most cases, the decision maker will not know k or providing the value of k will be difficult for him or her. In these cases, he or she has to decide which is the set of the most important criteria and its total weight (P). The set of the most important criteria consists of m criteria, where m is any value between 1 and n—1. A particular case is when the decision maker provides the weight of the most important criterion (case $m = 1$).

In order to find the value of k in terms of the total weight of the first m criteria (P), it is necessary to solve Eq. (30). Equation (30) is Eq. (29) replacing the different weights of the criteria with the value appearing in Eq. (23):

$$\sum_{i=1}^{m} w_i = w_1 + w_2 + \ldots + w_m = P \tag{29}$$

$$\sum_{i=1}^{m} w_i = w_1 + k \cdot w_1 + \cdots + k^{m-1} \cdot w_1 = P \tag{30}$$

A direct relationship among the value of P, k and w_1 is obtained (Eqs. (31) and (32)) by doing the same operations as previously (replacing the value of the summation with the value shown in Eq. (23)).

$$\sum_{i=1}^{m} w_i = \sum_{i=0}^{m-1} w_1 k^i = w_1 \sum_{i=0}^{m-1} k^i = w_1 \frac{1-k^m}{1-k} = P \quad (31)$$

$$P \frac{1-k}{1-k^m} = w_1 \quad (32)$$

From Eq. (28), in which w_1 depends on k and n, it is possible to replace the value of w_1 in Eq. (32), and Eq. (33) is obtained. Finally, the value of P is obtained (sum of the weights of the m most important criteria) in terms of m, n and k (Eq. (34)).

$$P \frac{1-k}{1-k^m} = \frac{1-k}{1-k^n} \quad (33)$$

$$P = \frac{1-k^m}{1-k^n} \quad (34)$$

3.2 Applying the new method for assigning weights: step by step

The CWR method is implemented through the three simple steps presented in Fig. 5, which are explained below.

Step 1 Ranking of the n criteria from most to least important. If certain criteria are considered to be of equal importance to each other, they will be ranked randomly among themselves, as it will not change the result. This is discussed in step 3.

The decision maker then chooses between step 2a or step 2b:

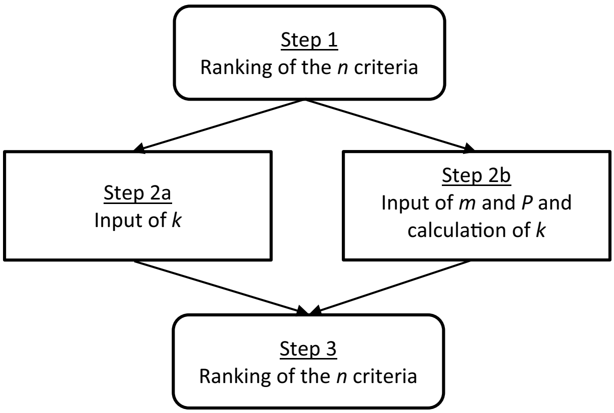

Fig. 5 Steps for implementing the CWR method

Step 2a The decision maker provides the value of k.

Step 2b The decision maker chooses m and provides P (the sum of the weights of the m most important criteria). As P, m and n are known, the value of k can be calculated from Eq. (35) by using numerical methods such as the bisection method (Hoffman, 1992).

$$P = \frac{1 - k^m}{1 - k^n} \tag{35}$$

Step 3 Knowing k and n, the weight of each criterion can be immediately determined with Eqs. (36) and (37):

$$w_1 = \frac{1 - k}{1 - k^n} \tag{36}$$

$$w_i = k^{i-1} \cdot w_1 \tag{37}$$

If 2 or more criteria are of equal importance, the method proposed by Kendall (1970) is used: calculating the average weight of the criteria that are equally important. That is, if the criteria c, $c+1$, …, $c+$v have the same importance, the weight of each one of these criteria will be:

$$w'_c = w'_{c+1} = \cdots = w'_{c+v} = \frac{\sum_{i=c}^{c+v} w_i}{v + 1} \tag{38}$$

The following example reflects this calculation.

3.3 Example of application

Consider that there are 12 criteria, the first 4 criteria have a total importance of 60%, criteria 1 and 2 are equally important, and criteria 7, 8 and 9 are also of equal importance.

Step 1

$$n = 12 \tag{39}$$

$$w_1 = w_2 \geq w_3 \geq w_4 \geq w_5 \geq w_6 \geq w_7 = w_8 = w_9 \geq w_{10} \geq w_{11} \geq w_{12} \geq 0 \tag{40}$$

Step 2b

$$m = 4 \tag{41}$$

$$P = 0.6 \tag{42}$$

By replacing the values of n, m and P in Eq. (35), Eq. (43) is obtained.

$$0.6 = \frac{1 - k^4}{1 - k^{12}} \tag{43}$$

By applying a numerical method, k is obtained:

$$k = 0.8224 \tag{44}$$

Step 3

The weight of the most important criterion is obtained by replacing the values in Eq. (36):

$$w_1 = \frac{1 - k}{1 - k^{12}} = \frac{1 - 0.8224}{1 - 0.8224^{12}} = 0.196 \tag{45}$$

The rest of the weights are obtained by means of Eq. (37), still without taking into account that there are criteria with the same importance.

$$\begin{aligned} & w_2 = 0.162; w_3 = 0.133; w_4 = 0.109; w_5 = 0.090; w_6 = 0.074; \\ & w_7 = 0.061; w_8 = 0.050; w_9 = 0.041; w_{10} = 0.034; w_{11} = 0.028; w_{12} = 0.023 \end{aligned} \tag{46}$$

The adjustment proposed by Kendall (1970) is made (Eq. (38)):

$$w'_1 = w'_2 = \frac{w_1 + w_2}{2} = \frac{0.196 + 0.162}{2} = 0.179 \tag{47}$$

$$w'_7 = w'_8 = w'_9 = \frac{w_7 + w_8 + w_9}{3} = \frac{0.061 + 0.050 + 0.041}{3} = 0.051 \tag{48}$$

The final weights are as follows:

$$\begin{aligned} & w'_1 = 0.179; w'_2 = 0.179; w_3 = 0.133; w_4 = 0.109; w_5 = 0.090; \\ & w_6 = 0.074; w'_7 = 0.051; w'_8 = 0.051; w'_9 = 0.051; w_{10} = 0.034; w_{11} = 0.028; w_{12} = 0.023 \end{aligned} \tag{49}$$

4 Discussion

4.1 Behaviour of the new method

Figure 6 illustrates the weights obtained using the CWR method for various values of k and number of criteria (n). Figure 7 provides examples of the function P (weight of the first m criteria) for different combinations of m and n. The behaviour of these weights and the P function is studied next:

- The weight of the most important criterion, w_1, has a value of 1 when $k=0$ for any n, according to Eq. (36). Consequently, the weight of all the other criteria will be $w_i = k^{i-1} \cdot w_1 = 0 \cdot 1 = 0$ according to Eq. (37).
- Similarly, the function P takes a value of 1 when $k=0$ (see Eq. (35) and Fig. 7).

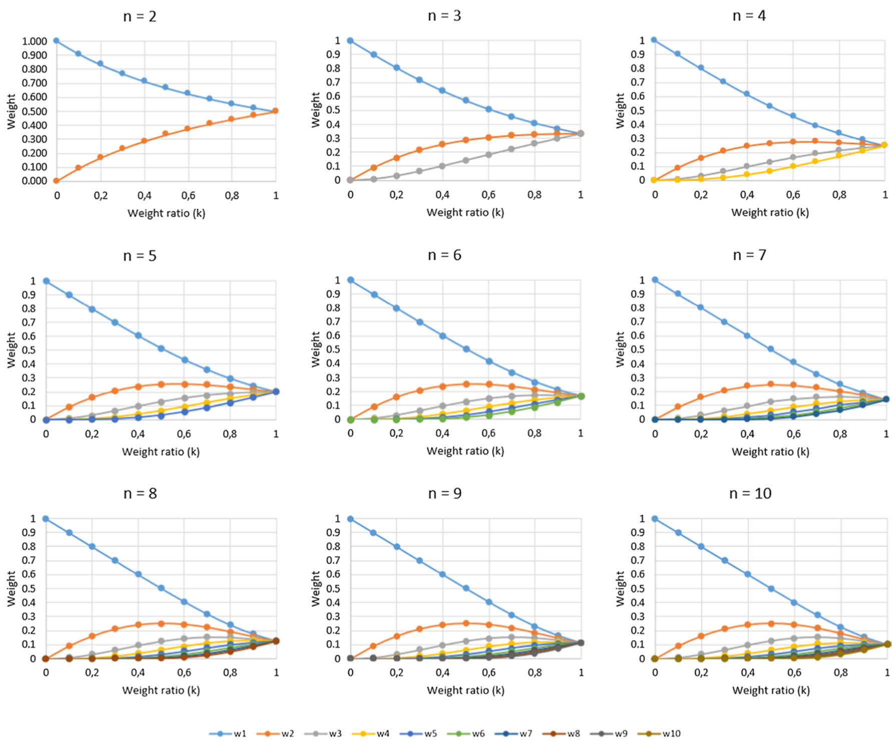

Fig. 6 Distribution of weights using the CWR method for various numbers of criteria and values of k

- For $k = 1$, all the weights must have the same value since k is the weight ratio of two consecutive criteria. Therefore, the weight of every criterion is equal to $\frac{1}{n}$ and the sum of the weights of the m most important criteria will be $\frac{m}{n}$.

Mathematically, for the value $k = 1$, an indefiniteness is obtained in the function P. Applying L'Hôpital's rule to the limit of P when k tends to 1 yields the expected value, $\frac{m}{n}$ (Eq. (50)).

$$\lim_{k\to 1} P = \lim_{k\to 1} \frac{1-k^m}{1-k^n} = \lim_{k\to 1} \frac{\frac{d(1-k^m)}{dk}}{\frac{d(1-k^n)}{dk}} = lim_{k\to 1} \frac{-mk^{m-1}}{-nk^{n-1}} = \frac{m}{n} \quad (50)$$

- For intermediate cases where $0 < k < 1$, both w_1 and P decrease as k increases. The decrease of w_1 appears to be more linear for higher the values of n.
- When the number of criteria is $n \geq 4$, the use of very low values of k (e.g., $k = 0.1$ or 0.2) results in criteria with weights below 0.01 (1%).
- According to Eqs. (36) and (37), the values of n and k determine the weights. Therefore, for the same k and n the set of weights remains the same even though

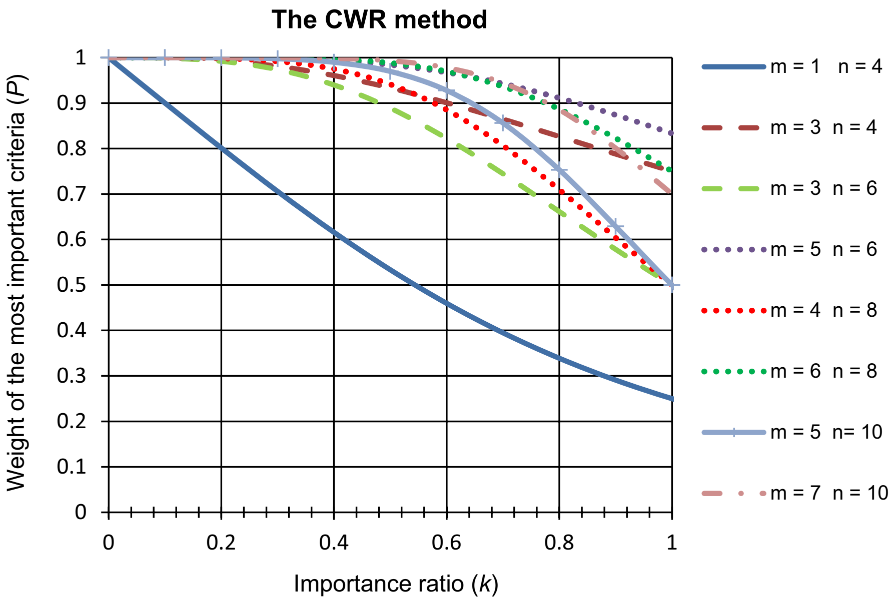

Fig. 7 Total weight P of the most important criteria versus the importance ratio k for different numbers of the most important criteria m and total number of criteria n in the CWR method

different values of m yield different values of P resulting in different lines in the graphical representation (Fig. 7). For instance, for $n=4$ and $k=0.6$, the set of weights is $w_1=0.460$, $w_2=0.276$, $w_3=0.165$, and $w_4=0.099$. In this case, $P=0.460$ for $m=1$, while $P=w_1+w_2+w_3=0.901$ for $m=3$.

4.2 Comparison of the new method with the purely ordinal methods

The advantages of the new method over purely ordinal methods are as follows:

(1) The relative importance between the different criteria is determined by the information that the decision maker provides and not by the chosen method.

(2) If the decision maker chooses to provide the weight of the most important criterion or the sum of the weights of the m most important criteria, they are more accurately known than in purely ordinal methods in which that weight is determined by the chosen method (assuming that the data provided by the decision maker are accurate). In the proposed method, the result of the analysis is more dependent on the weights of the most important criteria and less dependent on the weights of the least important criteria. Consequently, since the weights of the most important criteria are more accurate and the result largely depends on them, the result will also be more accurate.

(3) As justified below, it is possible to avoid the problem of the excessive decrease in the weight of the most important criterion when the total number of criteria

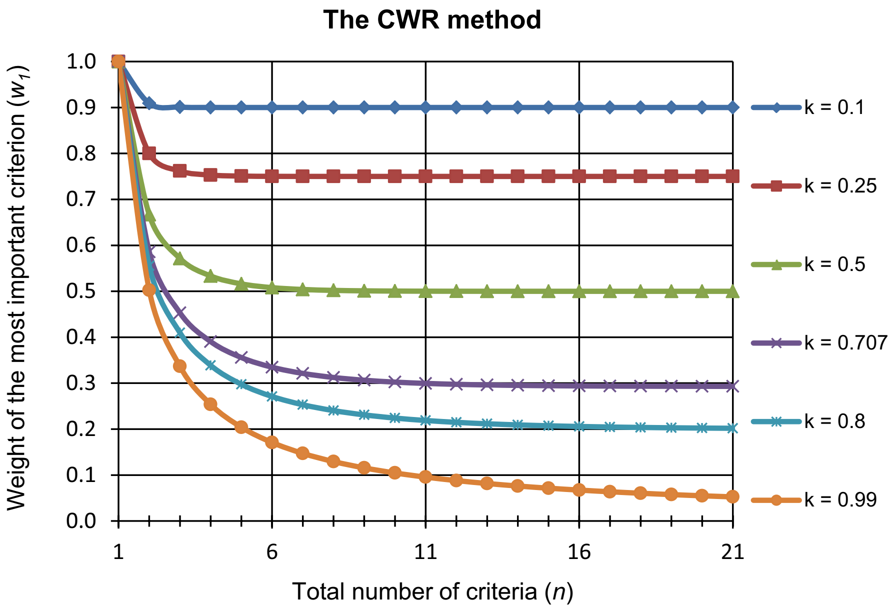

Fig. 8 Weight of the most important criterion w_1 versus the total number of criteria n with the new method and for different values of importance ratio k

considered is very high and the weight of the most important criterion must be high.

If the number of criteria tends to infinity, the weight of the most important criterion is the one shown in Eq. (51). That is to say, using the CWR method, however much the number of criteria grows, it is possible to define a value of k that makes it possible for the weight of the most important criterion not to decrease excessively:

$$\lim_{n\to\infty} w_1 = \lim_{n\to\infty} \frac{1-K}{1-K^n} = 1-k \tag{51}$$

As can be seen in Fig. 8, the weight of the most important criterion can be highly dependent on the value of k even with a large number of criteria (see cases $k=0.1$; $k=0.25$ or $k=0.5$).

The new method is highly adaptable to different cases depending on the preferences of the decision maker. Compared to the purely ordinal methods, the CWR method makes it possible to obtain a wide range of solutions simply by varying the value of k, as shown in Fig. 9.

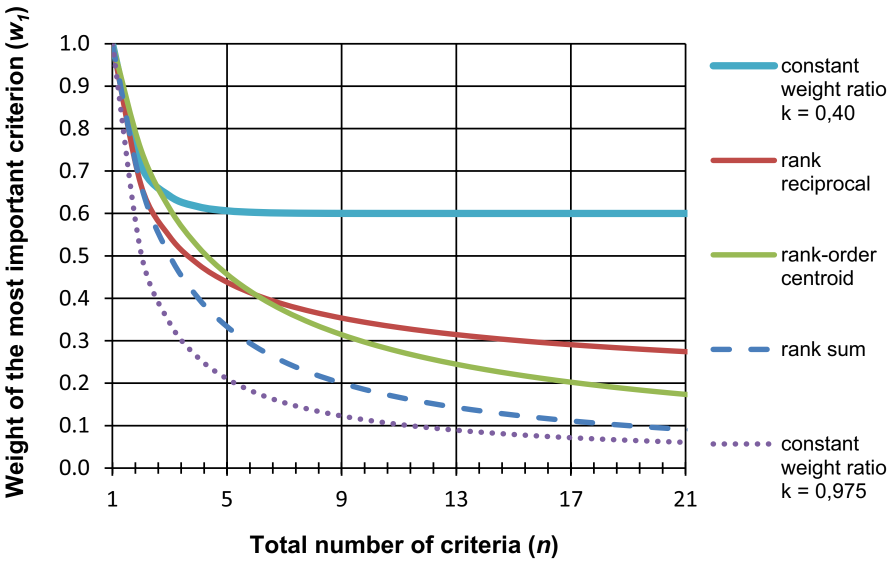

Fig. 9 Weight of the most important criterion w_1 according to the total number of criteria n for the purely ordinal methods and two variations of the new method

4.3 Comparison of the new method with the ordinal methods requiring an additional piece of information

The three ordinal methods requiring an additional piece of information, namely RG, RE, and the proposed CWR method, can be adapted to various scenarios. Figure 10 shows that by adjusting the specific parameter in each method (b in RG, w_1 leading to z in RE, and k, w_1 or P in CWR) it is possible to assign a high weight to the most important criterion, if this is the preference of the decision maker.

The advantage of the new method over the rank geometric and rank exponent methods (the two methods based on the ranking of the criteria and an additional piece of information) is as follows:

Greater flexibility in the information requested of the decision maker. While in the rank geometric method, the additional data requested of the decision maker is the ratio of the weight of the most important criterion to the weight of the least important criterion, and in the rank exponent method, it is the weight of the most important criterion, the new method allows the decision maker to choose which information to provide from the following: (1) k, the weight or importance ratio, (2) P, the total weight of the set of the most important criteria or, as a particular case of the last one, (3) w_1, the weight of the most important criterion.

That the decision maker can give P as a starting datum enables, in a simple way, the assignment of weights following a Pareto (power-law) distribution. The Pareto distribution represents the wealth distribution among individuals and quantifies the level of wealth inequalities (Klass et al. 2006). A minority of the people owns the majority of the wealth. This law can be applied to many

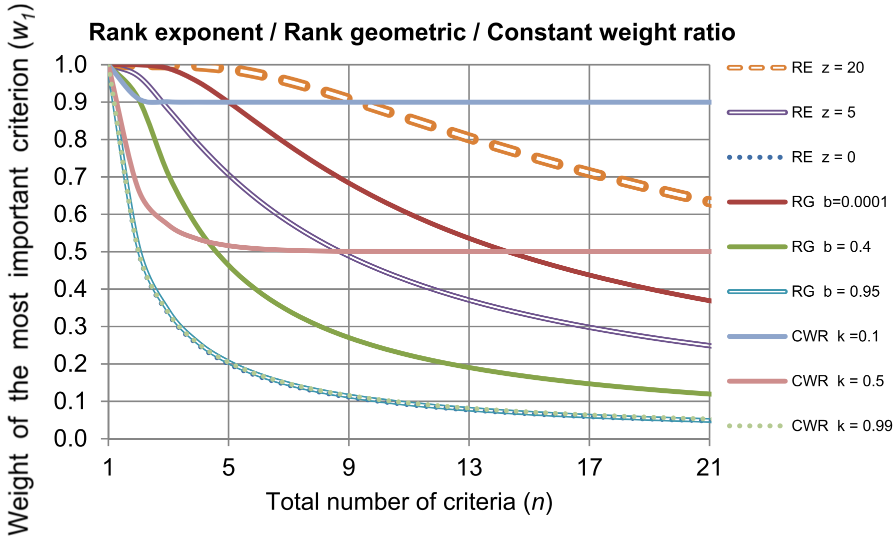

Fig. 10 Weight of the most important criterion w_1 according to the total number of criteria n for the ordinal methods with an additional piece of information

areas. If individuals are changed to criteria and wealth is changed to weight, the Pareto distribution can be applied to weight-assignment methods as follows: a minority of the criteria (e.g., 20% of the criteria) account for the majority of the weight (e.g. 80% of the weight). For example, in an instance of decision-making involving 40 criteria, the 8 most important criteria would have a total weight of 0.8 ($n = 40$; $m = 8$; $P = 0.8$). These percentages can be modified according to the needs or preferences of the decision maker. The proposed method makes it possible to keep the total weight of the set of the most important criteria high when the total number of criteria is large and the total weight of the set of the most important criteria must be high.

4.4 Comparison of the new method with the cardinal methods

Since it asks for less and simple information, the advantages of the new method over the cardinal methods are as follows:

(1) The information required by the new method does not involve as cognitively demanding of a task for the decision maker as the cardinal methods.
(2) Consequently, the decision maker is less likely to be unavailable or unable or unwilling to specify sufficiently accurate weights.

4.5 Limitations of the new method

The CWR method is appropriate for most situations. It accurately adjusts to cases in which there are many or few criteria and in which the most important criteria have high weights (low k values) or weights that are not so high (k values close to 1). The only case in which it does not adapt accurately is when the importance ratio should vary greatly depending on whether the most or least important criteria are being analysed, for instance, when the most important criteria have very similar weights (k value close to 1) and the least important criteria have very different weights (k value near 0) or vice versa. This limitation is due to the fact that the value of k is constant for all weights.

5 Case studies

5.1 Introduction and methods

Multi-criteria analysis is useful for strategic decisions, but also for tactical or operational ones. It can be applied to the service sector, e.g. hotels (Montibeller and Franco 2011), or, as shown more recently, to cultural institutions (Casanovas-Rubio et al. 2020; Imbernon et al. 2022). In this section, the proposed method has been applied to two case studies from the cultural sector. Specifically, a season value index (SVI) was calculated for the programming of the Palau de la Música Catalana (Casanovas-Rubio et al. 2020), a musical institution, and CaixaForum Barcelona (Imbernon et al. 2022), a cultural institution, both located in Barcelona (Spain). The SVI in those two studies is based on the Multi-Attribute Utility Theory with an additive model (Eq. (1)) and indicates the value provided by a specific season programme according to a set of criteria. The weights of the criteria were determined by direct assignment, i.e. a cardinal method, which means 9–10 value judgements for each study as they have 10 criteria each.

The objective of the present case study is to reproduce the evaluation of the two institutions' season programmes using the new method proposed here, i.e. the CWR method. To this end, the following two steps were taken: (1) determination of the weights using the ordinal methods with an additional piece of information, i.e. the RG, RE, and CWR methods, and their comparison with the original weights determined by direct assignment; and (2) determination of the SVI obtained with the ordinal methods and comparison thereof with the SVI from the original studies. These case studies aim to show that the weights obtained with the CWR method, with less input data, and the resulting SVI are very similar or equivalent to those obtained with other methods requiring the decision maker to input more data.

In order to determine the weights with the ordinal methods, including the proposed CWR, the following considerations were taken into account:

1. The ranking of the criteria was the same as in the original study. The additional piece of information required by each method was chosen to reproduce the original weights.
2. The weights presented in the table are the result of directly rounding the weights to two decimal places without making any adjustments. This is why, with some of the methods, the weights do not add up to exactly 100.00%.
3. The absolute error between the original weight and the weight calculated with the ordinal methods was calculated for each weight and method and, then, the mean absolute error (MAE) as presented in Eq. (52), where w_i is the weight of criterion i calculated with an ordinal method, wo_i is the original weight, and n is the total number of criteria (10).

$$MAE = \frac{\sum_{i=1}^{n} \left| w_i - wo_i \right|}{n} \tag{52}$$

5.2 Palau de la Música Catalana

The first case study refers to the SVI of the 2015–2016 programme for the Palau de la Música Catalana (Casanovas-Rubio et al. 2020). Table 1 shows the criteria and original weights used in that study, as well as the weights calculated applying the RG, RE and CWR methods. In order to calculate the weights with these methods, the following considerations were taken into account:

Table 1 Weights for the Palau de la Música Catalana case study including the weights from the original study determined by direct assignment and those of the CWR (proposed method)

Criterion		Direct assignment (original weights) (%)	RE $z=1.870$		CWR (proposed method)					
					Option 1 k = 0.8363 (equivalent to RG with b = 0.2)		Option 2 m = 1 $P=0.25$ k = 0.7678		Option 3 m = 3 $P=0.55$ k = 0.7975	
			Weight (%)	Error (%)	Weight (%)	Error (%)	Weight (%)	Error (%)	Weight (%)	Error (%)
1	Quality	25	25.00	0.00	19.66	5.34	25.00	0.00	22.60	2.40
2	Audience	15	18.50	3.50	15.10	0.10	16.97	1.97	16.20	1.20
3	Attractiveness	15	18.50	3.50	15.10	0.10	16.97	1.97	16.20	1.20
4	Dose of risk	10	11.23	1.23	10.56	0.56	10.00	0.00	10.30	0.30
5	Singularity	10	11.23	1.23	10.56	0.56	10.00	0.00	10.30	0.30
6	Locality	5	3.11	1.89	5.81	0.81	4.21	0.79	4.88	0.12
7	Internationality	5	3.11	1.89	5.81	0.81	4.21	0.79	4.88	0.12
8	Education	5	3.11	1.89	5.81	0.81	4.21	0.79	4.88	0.12
9	Social commitment	5	3.11	1.89	5.81	0.81	4.21	0.79	4.88	0.12
10	Efficient management	5	3.11	1.89	5.81	0.81	4.21	0.79	4.88	0.12
Mean absolute error (MAE)				1.89		1.07		0.79		0.60
Maximum error				3.50		5.34		1.97		2.40
Minimum error				0.00		0.10		0.00		0.12

1. In the RE, z = 1.870 was determined as the four-significant-digits figure with the lowest error, so that $w_1 = 25.00\%$ with an absolute error lower than 0.00001%.
2. The weights of Option 1 were obtained considering the ratio of the least important to the most important criterion ($w_{10}/w_1 = 0.2$) and, then, k = 0.2^(1/9) = 0.8363 (Step 2a).
3. The weights obtained with RG ($b = w_{10}/w_1 = 0.2$) coincide with those of Option 1 of the CWR method when inputting the value k = 0.8363.
4. The weights of Option 2 were calculated considering that the most important criterion was the same as in the original study ($w_1 = 25\%$), meaning that P = 0.25 and m = 1 (Step 2b). Equation (35) was then solved, yielding the value k = 0.7678 (Step 3).
5. The weights of Option 3 were calculated considering that the weights of the three most important criterion add up to $w_1 + w_2 + w_3 = 55\%$, as in the original study, which means that P = 0.55 and m = 3 (Step 2b). Equation (35) was then solved, yielding the value k = 0.7975 (Step 3).
6. As certain criteria share equal weights (2 and 3, 4 and 5, and 6–10), the method proposed by Kendall (1970), as described in Eq. (38), was used for both the RE and CWR methods.

Next, a comparison between the original SVI and the SVI calculated with the ordinal methods with an additional piece of information (i.e. RE, RG and CWR - the proposed method) was made and is presented in Table 2. The calculations were performed using only the published values to two decimal places, although the calculations in the paper may have used more. That is why some of the original weighted values and the SVI presented here (0.830) differ slightly from the ones in the original publication (SVI = 0.829) (Casanovas et al., 2020).

5.3 CaixaForum Barcelona

The second case study refers to the SVI of the 2015 and 2016 programmes of CaixaForum Barcelona (Imbernon et al. 2022). Table 3 shows the criteria and original weights used in that study, as well as the weights calculated applying RG, RE and the CWR method. As in the previous case, the following was considered:

1. In the RE method, z = 1.226 was determined to be the figure with four significant digits with the lowest error, such that $w_1 = 20.00\%$ with an absolute error of less than 0.00002%.
2. The weights of Option 1 were obtained considering the ratio of the least important to the most important criterion ($w_{10}/w_1 = 0.25$) and, then, k = 0.25^(1/9) = 0.8572 (Step 2a).
3. The weights with RG ($b = w_{10}/w_1 = 0.25$) coincide with those of Option 1 of the CWR method when the value k = 0.8572 is provided.
4. The weights of Option 2 were calculated considering that the most important criterion was the same as in the original study ($w_1 = 20\%$), meaning that P = 0.20

Table 2 SVI for different weight-assignment methods for Palau de la Música Catalana

Criterion		Value	Weighted value (value × weight) with:				
			Direct assignment (original weights)	RE $z = 1.870$	CWR (proposed method)		
					Option 1 k = 0.8363 (equivalent to RG with $b = 0.2$)	Option 2 m = 1 $P = 0.25$ k = 0.7678	Option 3 m = 3 $P = 0.55$ k = 0.7975
1	Quality	0.97	0.243	0.243	0.191	0.2425	0.2192
2	Audience	0.83	0.125	0.154	0.125	0.1408	0.1345
3	Attractiveness	0.83	0.125	0.154	0.125	0.1408	0.1345
4	Dose of risk	0.98	0.098	0.110	0.103	0.0980	0.1010
5	Singularity	0.81	0.081	0.091	0.086	0.0810	0.0835
6	Locality	0.30	0.015	0.009	0.017	0.0126	0.0146
7	Internationality	0.83	0.042	0.026	0.048	0.0350	0.0405
8	Education	1.00	0.050	0.031	0.058	0.0421	0.0488
9	Social commitment	1.00	0.050	0.031	0.058	0.0421	0.0488
10	Efficient management	0.06	0.003	0.002	0.003	0.0025	0.0029
SVI			**0.830**	**0.850**	**0.816**	**0.838**	**0.828**
Relative error			–	2.38%	− 1.74%	0.91%	− 0.22%

Table 3 Weights for the case study of CaixaForum Barcelona including the weights from the original study determined by direct assignment and those of the CWR (proposed method)

Criterion		Direct assignment (original weights) (%)	RE $z=1.226$		CWR (proposed method)					
					Option 1 k = 0.8572 (equivalent to RG with $b=0.25$)		Option 2 m = 1 $P=0.20$ k = 0.8317		Option 3 m = 2 $P=0.35$ k = 0.8460	
			Weight (%)	Error (%)	Weight (%)	Error (%)	Weight (%)	Error (%)	Weight (%)	Error (%)
1	Internal complementarity	20	20.00	0.00	18.17	1.83	20.00	0.00	18.96	1.04
2	Novelty	15	17.58	2.58	15.58	0.58	16.63	1.63	16.04	1.04
3	Acquired knowledge	10	10.78	0.78	10.05	0.05	9.90	0.10	9.99	0.01
4	Institutional visibility	10	10.78	0.78	10.05	0.05	9.90	0.10	9.99	0.01
5	Local complementarity	10	10.78	0.78	10.05	0.05	9.90	0.10	9.99	0.01
6	Opportunity	10	10.78	0.78	10.05	0.05	9.90	0.10	9.99	0.01
7	Touring	10	10.78	0.78	10.05	0.05	9.90	0.10	9.99	0.01
8	Budget ratio	5	2.85	2.15	5.34	0.34	4.63	0.37	5.02	0.02
9	Conceptual accessibility	5	2.85	2.15	5.34	0.34	4.63	0.37	5.02	0.02
10	Length of the show	5	2.85	2.15	5.34	0.34	4.63	0.37	5.02	0.02
Mean absolute error (MAE)				1.29		0.37		0.33		0.22
Maximum error				2.58		1.83		1.63		1.04
Minimum error				0.00		0.05		0.00		0.01

Table 4 SVI for different weight-assignment methods for CaixaForum Barcelona

Criterion		Value		Weighted value (value × weight) with:									
				Direct assignment (original weights)		RE $z = 1.226$		CWR (proposed method)					
								Option 1 k = 0.8572 (equivalent to RG with $b = 0.25$)			Option 2 m = 1 $P = 0.20$ k = 0.8317	Option 3 m = 2 $P = 0.35$ k = 0.8460	
		2015	2016	2015	2016	2015	2016	2015	2016	2015	2016	2015	2016
1	Internal complementarity	0.60	0.80	0.120	0.160	0.120	0.160	0.109	0.145	0.120	0.160	0.114	0.152
2	Novelty	0.64	0.65	0.096	0.098	0.113	0.114	0.100	0.101	0.106	0.108	0.103	0.104
3	Acquired knowledge	0.47	0.45	0.047	0.045	0.051	0.048	0.047	0.045	0.047	0.045	0.047	0.045
4	Institutional visibility	0.58	0.55	0.058	0.055	0.063	0.059	0.058	0.055	0.057	0.054	0.058	0.055
5	Local complementarity	0.78	0.68	0.078	0.068	0.084	0.073	0.078	0.068	0.077	0.067	0.078	0.068
6	Opportunity	1.00	0.67	0.100	0.067	0.108	0.072	0.100	0.067	0.099	0.066	0.100	0.067
7	Touring	0.75	0.5	0.075	0.050	0.081	0.054	0.075	0.050	0.074	0.049	0.075	0.050
8	Budget ratio	0.44	0.50	0.022	0.025	0.013	0.014	0.023	0.027	0.020	0.023	0.022	0.025
9	Conceptual accessibility	0.64	0.73	0.032	0.037	0.018	0.021	0.034	0.039	0.030	0.034	0.032	0.037
10	Length of the show	0.84	0.71	0.042	0.036	0.024	0.020	0.045	0.038	0.039	0.033	0.042	0.036
SVI				**0.670**	**0.640**	**0.673**	**0.637**	**0.671**	**0.637**	**0.670**	**0.640**	**0.670**	**0.638**
Relative error				–	–	0.44%	− 0.45%	0.14%	− 0.46%	− 0.06%	0.07%	0.05%	− 0.24

and m = 1 (Step 2b). Equation (35) was then solved, yielding the value k = 0.8317 (Step 3).

5. The weights of Option 3 were calculated considering that the weights of the two most important criterion added up to $w_1 + w_2 = 35\%$, as in the original study, meaning that P = 0.35 and m = 2 (Step 2b). Equation (35) was then solved, yielding the value k = 0.8460 (Step 3).
6. As certain criteria share equal weights (3–7 and 8–10), the method proposed by Kendall (1970), as described in Eq. (38), was used for both the RE and CWR methods.

Next, the original SVI for 2015 and 2016 and the SVI calculated with the ordinal methods with an additional piece of information, i.e. RE, RG and CWR, were compared. The results are presented in Table 4.

5.4 Discussion of the case studies

Considering the results of the case studies, the following observations can be made regarding the weights and SVI:

1. The MAE between the original weights obtained with direct assignment and those obtained with the proposed method is very low and lower than that for the RE method for both case studies.
2. For both case studies, Option 3 of the proposed method was the method with the lowest MAE of the weights (0.60% for Palau de la Música Catalana and 0.22% for CaixaForum Barcelona), followed by Option 2, Option 1 or RG, and RE.
3. The smallest MAE for the weights was found in Option 2 of the proposed method (1.97%) for Palau de la Música Catalana and Option 3 of the proposed method (1.04%) for CaixaForum Barcelona.
4. The SVI obtained with the proposed method is very similar to the original one; in fact, the maximum relative error was 1.74% (for Option 1 or RG in Palau de la Música Catalana) and the minimum was 0.05% (for Option 3 in CaixaForum Barcelona).
5. For Options 2 and 3 of the proposed method the relative error of the SVI was lower than with RE.
6. According to the SVI, the ranking of the seasons of CaixaForum Barcelona with the three options of the proposed method was the same as the original ranking, i.e. 2015 was slightly better than 2016.

It can thus be concluded for the cases studies that the weights obtained by the method proposed in this paper are a very good approximation of the weights obtained with a more demanding method such as direct assignment and yield a better approximation than other ordinal methods such as RE or RG. The advantage of the proposed method is that it requires a smaller amount of data and fewer judgments from the decision maker to produce a very similar set of weights. The final evaluations of the alternatives (in this case, seasons) are very similar to those obtained with the original weights and the ranking obtained is the same.

6 Conclusions

This paper presents a new method for determining cardinal criteria weights on the basis of an ordinal ranking of the criteria and an additional piece of information to be chosen from among several options. The method, called constant weight ratio (CWR), can be considered to fall between purely ordinal ranking methods and cardinal ranking methods and has advantages of both methods, i.e. ease of elicitation and numerical quality.

- Ease of elicitation as only an ordinal ranking and one additional piece of information are required.
- Higher numerical quality than purely ordinal methods as the relative importance of criteria is determined by the information given by the decision maker and is not pre-established by the method as in purely ordinal methods.
- If the decision maker provides the weight of the most important criterion or the sum of the weights of the m most important criteria, they are supposedly more accurate than if they are determined by the method used, assuming that the information provided by the decision maker is accurate. Since the result largely depends on them, it is also more accurate.

This method is suitable for avoiding the excessive decrease in the weight of the most important criteria (or the most important criterion) when the total number of criteria is high and the weight of the most important criteria (or criterion) has to be high.

The advantage of the new method over the other two methods based on a ranking of the criteria and an additional piece of information is the flexibility it offers with regard to the information requested from the decision maker. The decision maker can choose the additional piece of information to be provided from the relative importance of the criteria, the total weight of the most important set of criteria, or the weight of the most important criterion.

The CWR method can be adapted to most situations: many or few criteria and the most important criteria with a high or not so high weight (low values of k or k values near 1). The only case in which it does not adjust so accurately is when the importance ratio of the weight of one criterion to the weight of the preceding one in importance varies greatly from one criterion to another. This is due to the fact that, in this method, the value of k is constant for all weights.

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Declarations

Conflict of interest The authors have no competing interests to declare that are relevant to the content of this article.

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