Correction to: Measuring consistency of interval-valued preference relations: comments and comparison

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Correction to: Operational Research

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In the original publication of this article, it was stated that the number of quasi interval multiplicative reciprocal matrices (IMRMs) with n order is

$$C_{n(n-1)/2}^1 + C_{n(n-1)/2}^2 + \cdots + C_{n(n-1)/2}^{n(n-1)/2} = 2^{n(n-1)/2} - 1. \quad (1)$$

In fact, the case without a quasi-positive interval is not considered for the entries above or below the diagonal of a quasi IMRM in (1). When all quasi IMRMs are computed, the number is given as

$$C_{n(n-1)/2}^0 + C_{n(n-1)/2}^1 + C_{n(n-1)/2}^2 + \cdots + C_{n(n-1)/2}^{n(n-1)/2} = 2^{n(n-1)/2}. \quad (2)$$

For example, consider the following matrix:

$$\bar{A}_1 = \left[\begin{array}{c|ccc} & x_1 & x_2 & x_3 \\ \hline x_1 & [1, 1] & [2, 3] & [1/2, 1] \\ x_2 & [1/3, 1/2] & [1, 1] & [1/6, 1/2] \\ x_3 & [1, 2] & [2, 6] & [1, 1] \end{array} \right],$$

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Eight quasi IMRMs are determined from $\bar{A}_1$ by considering the locations of quasi intervals such as $\bar{A}'_1$:

$$\bar{A}'_1 = \left[\begin{array}{c|ccc} & x_1 & x_2 & x_3 \\ \hline x_1 & [1, 1] & [2, 3] & [1/2, 1] \\ x_2 & [1/2, 1/3] & [1, 1] & [1/6, 1/2] \\ x_3 & [2, 1] & [6, 2] & [1, 1] \end{array} \right].$$

Hereafter the intervals in *italic* stand for the quasi ones.

Moreover, according to Theorem 1 in Liu et al. (2020), there is a permutation of objectives such that any quasi IMRM can be changed to a matrix where all quasi-positive intervals are above the diagonal and the standard ones locate below the diagonal. However, it is found that Theorem 1 in Liu et al. (2020) does not hold for $\bar{A}'_2$ and $\bar{A}'_3$ where

$$\bar{A}'_2 = \left[\begin{array}{c|ccc} & x_1 & x_2 & x_3 \\ \hline x_1 & [1, 1] & [2, 3] & [1, 1/2] \\ x_2 & [1/2, 1/3] & [1, 1] & [1/6, 1/2] \\ x_3 & [1, 2] & [6, 2] & [1, 1] \end{array} \right],$$

and

$$\bar{A}'_3 = \left[\begin{array}{c|ccc} & x_1 & x_2 & x_3 \\ \hline x_1 & [1, 1] & [3, 2] & [1/2, 1] \\ x_2 & [1/3, 1/2] & [1, 1] & [1/2, 1/6] \\ x_3 & [2, 1] & [2, 6] & [1, 1] \end{array} \right].$$

That is, the result of Theorem 1 in Liu et al. (2020) is wrong. An obvious reason is that the number of quasi IMRMs is bigger than that of the permutations due to the following inequality:

$$2^{n(n-1)/2} \geq n!, \quad n \geq 3. \quad (3)$$

A correct statement of the theorem is provided below:

Theorem 1 Assume that a quasi IMRM $\bar{A}' = (\bar{a}'_{ij})_{n \times n}$ is constructed from a standard IMRM $\bar{A} = (\bar{a}_{ij})_{n \times n}$. $\bar{a}_{ij}^\circ$ stand for the quasi-positive intervals in $\bar{A}'$ for $i \in V_i$ and $j \in V_j$ where V_{ij} are the subsets of $\{1, 2, \dots, n\}$. If there is a permutation of objectives σ such that

$$\sigma(i) < \sigma(j), \quad \text{or} \quad \sigma(i) > \sigma(j), \quad (4)$$

for $\forall i \in V_i$ and $\forall j \in V_j$, then $\bar{A}'$ can be changed to the matrix where all the entries above the diagonal are quasi-positive intervals and all the entries below the diagonal are standard intervals.

Proof Suppose that there is a permutation of objectives σ such that $\sigma(i) < \sigma(j)$ for $\forall i \in V_i$ and $\forall j \in V_j$. This means that under the permutation σ , the quasi-positive entries

are above the diagonal. Since the number of quasi-positive intervals is $n(n-1)/2$, the entries above the diagonal are all quasi-positive and the result is satisfied.

On the other hand, let us assume that there is a permutation of objectives σ such that $\sigma(i) > \sigma(j)$ for $\forall i \in V_i$ and $\forall j \in V_j$. Without loss of generality, we only consider the following matrix:

$$\bar{A}' = (\bar{a}'_{ij})_{n \times n} = \begin{bmatrix} & x_1 & x_2 & \cdots & x_n \\ x_1 & [1, 1] & [a_{12}^-, a_{12}^+] & \cdots & [a_{1n}^-, a_{1n}^+] \\ x_2 & [a_{21}^+, a_{21}^-] & [1, 1] & \cdots & [a_{2n}^-, a_{2n}^+] \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_n & [a_{n1}^+, a_{n1}^-] & [a_{n2}^+, a_{n2}^-] & \cdots & [1, 1] \end{bmatrix}.$$

In order to move the quasi-positive entries below the diagonal to the positions above the diagonal, one only needs to change the permutation of objectives as $(x_n, x_{n-1}, \dots, x_1)$. Then one can obtain the changed matrix as

$$\bar{A}'_c = \begin{bmatrix} & x_n & x_{n-1} & \cdots & x_1 \\ x_n & [1, 1] & [a_{n(n-1)}^+, a_{n(n-1)}^-] & \cdots & [a_{n1}^+, a_{n1}^-] \\ x_{n-1} & [a_{(n-1)n}^-, a_{(n-1)n}^+] & [1, 1] & \cdots & [a_{(n-1)1}^+, a_{(n-1)1}^-] \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_1 & [a_{1n}^-, a_{1n}^+] & [a_{1(n-1)}^-, a_{1(n-1)}^+] & \cdots & [1, 1] \end{bmatrix}.$$

It is seen from $\bar{A}'_c$ that all the entries above the diagonal are quasi-positive entries. The proof is completed. $\square$

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

References

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