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Qualitative Theory
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# Correction to: On Stacked Planar Central Configurations with Five Bodies when One Body is Removed

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## Correction to: Qual. Theory Dyn. Syst. (2013) 12:293–303 https://doi.org/10.1007/s12346-012-0084-y

The final part of the proof of Lemma 2.7 in [3] is not correct as pointed out in [1]. The correct part, after Eq. (12), is the following one.

Suppose that $r_5 \neq (0,0)$. Then, from Eq. (12), we have $d_6^3\lambda + M = 0$, which is equivalent to

$$\frac{\lambda}{M} + R = 0,$$

where $R = d_6^{-3}$, for every mass $m_5$. The last equation can be written as

$$R - \frac{\sum_{1\le i<j\le 5} \frac{m_i m_j}{r_{ij}}}{\sum_{1\le i<j\le 5} m_i m_j r_{ij}^2} = 0,$$

which is equivalent to

$$R \sum_{1\le i<j\le 5} m_i m_j r_{ij}^2 - \sum_{1\le i<j\le 5} \frac{m_i m_j}{r_{ij}} = 0.$$

The original article can be found online at https://doi.org/10.1007/s12346-012-0084-y.

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Separating $m_5$ in the last equation, we have

$$m_5\left(R\sum_{i=1}^{4} m_i d_6^2 - \sum_{i=1}^{4}\frac{m_i}{d_6}\right) + R\sum_{1\le i<j\le 4} m_i m_j r_{ij}^2 - \sum_{1\le i<j\le 4}\frac{m_i m_j}{r_{ij}} = 0.$$

In the last equation, the factor multiplying $m_5$ is null, then we get

$$R\sum_{1\le i<j\le 4} m_i m_j r_{ij}^2 - \sum_{1\le i<j\le 4}\frac{m_i m_j}{r_{ij}} = 0,$$

which is equivalent to

$$R - \frac{\sum_{1\le i<j\le 4}\frac{m_i m_j}{r_{ij}}}{\sum_{1\le i<j\le 4} m_i m_j r_{ij}^2} = 0.$$

Note that the quotient in the last equation is the Lagrange multiplier of the four co-circular bodies divided by $\mathcal{M} = m_1 + m_2 + m_3 + m_4$. Let $\lambda_4$ be this Lagrange multiplier. So the last equation can be written as

$$R + \frac{\lambda_4}{\mathcal{M}} = 0. \tag{13}$$

Since the four co-circular bodies are in a convex central configuration with positions disposed counterclockwise, the following inequalities must hold (see for instance [2] p. 349)

$$R_{13}, R_{24} < -\frac{\lambda_4}{\mathcal{M}} < R_{12}, R_{14}, R_{23}, R_{34}.$$

On the other hand, using the perpendicular bisector theorem, we see that in a co-circular central configuration the center of the circle belongs to the interior of the convex hull of the quadrilateral, see [2]. Thus, from the geometry of a quadrilateral inscribed in the circle of radius $d_6$ with the center of the circle inside the convex hull of the quadrilateral, at least one side is greater than $\sqrt{2}d_6$. Thus, Eq. (13) is never satisfied, because the four co-circular bodies are in a convex central configuration. Therefore, in order to satisfy (12) we must have $r_5 = (0, 0) = \mathcal{C}$ and the proof is complete.

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