International Journal of Automotive Technology, Vol. 22, No. 4, pp. 1011−1022 (2021)
DOI 10.1007/s12239−021−0091−6

pISSN 1229−9138/ eISSN 1976−3832

DYNAMIC FINITE ELEMENT ANALYSIS OF ROLLING NON-PNEUMATIC TIRE

Ravivat Rugsaj[1,2] and Chakrit Suvanjumrat[1,2]*

[1]Department of Mechanical Engineering, Mahidol University, Salaya, Nakhon Pathom 73170, Thailand
[2]Laboratory of Computer Mechanics for Design (LCMD), Department of Mechanical Engineering, Faculty of Engineering, Mahidol University, Salaya, Nakhon Pathom 73170, Thailand

(Received 3 July 2020; Revised 15 October 2020; Accepted 16 December 2020)

ABSTRACT−This research aimed to develop a dynamic finite element (FE) model of non-pneumatic tire (NPT) to study the mechanical behaviors of rolling NPT. The generalized Maxwell's viscoelastic material was combined with linear elastic material model. This material model was fitted with results of tensile and compressive testing on actual NPT specimen. The FE model of rolling NPT on curvature and flat surface under supporting load of 14 kN and speed of 11 km/hr were carried out to compare with the actual experiment of rolling NPT on drum testing machine. The high-speed video camera was used to capture NPT's spoke deformation at various angular position corresponding to time. The comparison results shown good agreement between the actual experiment and both FE models, which yielded average error of 3.68 and 3.89% for FE model of rolling NPT on drum and flat surface, respectively. The maximum dynamic values of impact force, displacement, and stress were higher than static values of 1.06, 1.16 and 1.805 times, respectively. Finally, the limitation of rolling NPT's performance on flat plate was analyzed. The shear stress between shear band and spoke under maximum load of 20 kN and velocity of 15 km/hr was found at 1.7277 MPa.

KEY WORDS : Finite element method, Non-pneumatic tire, Viscoelastic material, Dymamic behaviors

1. INTRODUCTION

Airless tire or non-pneumatic tire (NPT) is recently developed to eliminate the drawback of traditional pneumatic tire, which are the risk of getting flat while driving and the requirement of periodic maintenance or proper air pressure. Some of tire main functions were provided vehicles to road interface, transferred the vehicle force to the road, and absorbed impacting from the road and irregularities (Gent and Walter, 2006). Therefore, the knowledge and understanding of dynamic characteristic of NPT are required for improvement and development distinctly.

The general concept of NPT could be defined as a combination of an annular shear band including tread and polymer spokes. The function of spoke structure was replaced air pressure in pneumatic tire and distributed load to shear band (Rhyne and Cron, 2006).

Over the last decade, the interest in design and development of NPT are significantly growth with an observable rate. Many researchers focused on studying the mechanics and development of NPT. There were included load carrying capacity, vertical stiffness, and rolling resistance. At the early stage of development, the numerical models of NPT was developed to study the performance of NPT (Rhyne *et al*., 2006). However, the early simplified model could only capture deformation in shear mode of shear band component. The model was later improved by the authors using modified Timoshenko's curve beam theory to take bending and circumferential extension mode, which were critically required to estimate NPT behavior into account. The developed analytical model could be used to estimate contact pressure based on 2D quasi-static analysis. On the other hand, the rolling resistance could be estimated using steady state rolling analysis, which Fourier's series of shear strain and material complex modulus were used to model viscoelastic properties of the shear band (Gasmi *et al*., 2012). However, the steady state rolling analysis is limit to model with continuous mesh, which isn't suitable for the analysis of NPT structure that normally has separated spokes feature. Another disadvantage of this approach is inability to capture transient dynamic behavior of spoke at each time step during rolling, which is important information and arguably required in designing a tire.

The performance of NPT depended on shear band and spoke structure. Thus, design of NPT requires in-depth study and analysis on geometries and materials of its components. The finite element method (FEM) was a powerful tool to predict the performance of NPT based on

**Corresponding author*. e-mail: chakrit.suv@mahidol.ac.th

different testing conditions, which varied geometries, materials, and another geometric factors (Phromjan and Suvanjumrat, 2018a, 2018b, 2018c). The thermo-viscoelastic model of NPTs was developed to predict energy loss and the heat generation during rolling. The Yeoh hyperelastic model, which was obtained from tension and compression tests, and viscoelastic material model, which the parameters were obtained from dynamic mechanical analysis (DMA), were used to model elastic and inelastic behavior of NPT components, respectively. The steady state rolling analysis was then used to predict cyclic strain energy, which was converted into heat generation and temperature distribution (Yoo *et al.*, 2017). For better understanding of mechanical behavior of NPT and validity of FE models, the material properties obtained from actual NPT components were required. The waterjet cutting technique was used to prepare tension and compression testing specimens from NPT components including tread and spokes. The hyperelastic material constants, which were then derived from the stress-strain relationship, were implemented into FE model of the specimens. The analysis results of tensile and compressive testing were shown to be in a good agreement with the experiment, thus proved the validity of the hyperelastic models (Rugsaj and Suvanjumrat, 2018b). The inverse method involving FEM could also be used to search for accurate material properties of NPT's spoke with complex shape. The 3 points bending specimens were prepared from actual NPT's spokes with curvature. The inverse method was then performed on the force-displacement relationship was obtained from 3 points bending test and compared with the FEM to search for accurate material properties of the spokes (Rugsaj and Suvanjumrat, 2018a). Recently, a 3D FE model of NPT for designing proper radial spokes was developed based on validated hyperelastic material models. The vertical stiffness analysis of NPT was found to be in a good agreement with the experiment. The validated FE model of NPT was then used to search for the proper spoke thickness to provide the same vertical stiffness value which estimated from pneumatic tire at the same size (Rugsaj and Suvanjumrat, 2019).

The polyurethane (PU), which is normally used as material of NPT spoke, has low viscoelastic energy loss. The thermoviscoelastic model of NPT should be developed to predict rolling energy loss and dynamic characteristic of NPT while rolling. The FEM based on hyperelastic and viscoelastic material model was developed to study the effects of viscoelastic energy loss and rolling resistance on shear band. The porous shear band with the same effective shear modulus to continuous shear band was compared. The decrease in rubber volume showed to have reduce energy loss due to hysteresis (Ju *et al.*, 2013). The parametric studies along with design of experiment (DOE) and sensitivity analysis were performed on developed NPT model to study the effects of geometric and materials. The spokes thickness, shear band thickness, and shear modulus were varied, while the rolling resistance, contact pressure, and vertical stiffness were observed. The analysis showed that shear modulus of PU and shear band thickness had the most effects on NPT, which the NPT with higher shear modulus and shear band thickness had lower rolling resistance corresponding to lower shear deformation while rolling (Veeramurthy *et al.*, 2014). The rolling resistance was then optimized on NPT with hexagonal cellular spoke. The spoke thickness, cell angle, and shear band thickness were selected as design variables, while the contact pressure and vertical stiffness were selected as designed constraints (Kim *et al.*, 2015). The effects of spoke structures on NPT's performance were later discussed, which three different spoke structures were studied using FEM. The contact pressure, vertical stiffness, and stress were observed using quasi static analysis, which spoke structure geometries and shear layers were shown to have most significantly effects on spoke deformation and contact pressure distribution (Aboul-Yazid *et al.*, 2015). Recently, the static and dynamic behaviors of NPT were study using FEM. The three different honeycomb spoke structures with same cell wall thickness and load carrying capacity were selected. The maximum stress in NPT's treads and spokes and load carrying capacity of NPT were found to be lower and higher than pneumatic tire, respectively. The stress under dynamic loading was also found to be higher than the static loading. In addition, the rolling resistance was also predict, which the honeycomb structure with smaller angle was found to yield lower rolling resistance due to lower mass and therefore less deformation (Jin *et al.*, 2018). These previous works provided extensive details and contributions to the design and development of NPTs by FEM. The rolling resistance analysis could be carried out to achieve the dynamic performance of NPT. However, it was still limited to predict viscoelastic energy loss by using certain function in FE software. Consequently, the dynamic behavior of NPT during actual rolling condition still are critically important and need to be observed. Thus, in-depth study of mechanical properties including static and dynamic behaviors are still required.

This research aimed to develop the FE model of NPT based on dynamic approach to study the mechanical behaviors of NPT while rolling on flat road. The algorithm of transient dynamic analysis was proposed in section 2. In section 3, the actual experiment of rolling NPT on drum testing machine was performed and the high speed video camera was used to capture spoke deformation of the rolling NPT. In section 4, the FE model of rolling NPT on curvature and flat surface, which were represented drum and road surface, were developed and the analysis results were compared with the experiment to prove validity of the model. In section 5, the FE model of NPT on flat surface was selected to study the dynamic behaviors of NPT while rolling. The results were then analyzed and discussed as concluded in the last section of the article.

2. TRANSIENT DYNAMIC ANALYSIS

In order to solve highly nonlinear response that occur during interaction between rolling tire and drum/road surface, the dynamic transient analysis was performed. In this research, the direct integration with single step Houbolt operator was used to solve implicit dynamic contact analysis of NPT. The method has advantage over standard Houbolt operator in this case due to its flexible adaptive time step, while the standard Houbolt method was restricted to fixed time step. The single step Houbolt method is also preferred due to its unconditional stable nature, second order accuracy, and due to asymptotically annihilating (Chung and Hulbert, 1994). The method can be started with equilibrium equation of linear dynamics as follows:

$$M\ddot{x} + C\dot{x} + Kx = F \tag{1}$$

where M, C, K are the mass, viscous damping, and stiffness matrix, respectively. F is the applied load vector, while x is the displacement vector. The initial conditions are given as:

$$x(0) = d_0 , \tag{2}$$

$$\dot{x}(0) = v_0 \tag{3}$$

Given the equation of motion, the general form of single step algorithm of equation of motion can be written as follows:

$$\begin{aligned} &\alpha_{m1} M a_{n+1} + \alpha_{c1} C v_{n+1} + \alpha_{k1} K d_{n+1} \\ &+\alpha_m M a_n + \alpha_c C v_n + \alpha_k K d_n \\ &= \alpha_{f1} F_{n+1} + \alpha_f F_n \end{aligned} \tag{4}$$

The expression for velocity and acceleration are given by:

$$d_{n+1} = d_n + \Delta t v_n + \beta \Delta t^2 a_n + \beta_1 \Delta t^2 a_{n+1} , \tag{5}$$

$$v_{n+1} = v_n + \gamma \Delta t a_n + \gamma \Delta t^2 a_{n+1} \tag{6}$$

where d_n, v_n and a_n are used to denote approximation of $x(t_n)$, $\dot{x}(t_n)$, and $\ddot{x}(t_n)$, respectively. The coupled equation of motion can be reduced to a series of uncoupled single degree of freedom system by using standard modal decomposition technique, which the single degree of freedom form of the single step algorithm may be written as follows:

$$A_l X_{n+1} = A_r X_n + L F_n \tag{7}$$

where

$$X_n = \left\{ d_n, \Delta t v_n, \Delta t^2 a_n \right\}^{\mathrm{T}} , \tag{8}$$

$$F_n = \left\{ f(t_n), f(t_{n+1}) \right\}^{\mathrm{T}} , \tag{9}$$

$$A_l = \begin{bmatrix} 1 & 0 & -\beta_1 \\ 0 & 1 & -\gamma_1 \\ \alpha_{k1}(\omega\Delta t)^2 & \alpha_{c1} 2\xi(\omega\Delta t)^2 & \alpha_{m1} \end{bmatrix} , \tag{10}$$

$$A_r = \begin{bmatrix} 1 & 0 & \beta \\ 0 & 1 & \gamma \\ -\alpha_k(\omega\Delta t)^2 & -\alpha_c 2\xi(\omega\Delta t)^2 & -\alpha_m \end{bmatrix} , \tag{11}$$

$$L = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \alpha_f & \alpha_{f1} \end{bmatrix} , \tag{12}$$

The asymptotic annihilation condition was imposed on the algorithm to reduce of independent parameters. The asymptotic annihilation require roots of characteristic polynomial to have zero magnitude. Thus, the requirements of asymptotic annihilation in terms of algorithmic parameters are given as follows:

$$\alpha_k = 0 \tag{13}$$

$$\beta = \gamma \tag{14}$$

$$\beta_1 = \gamma + \gamma_1 \tag{15}$$

In order to stabilize simulation result, this paper selects γ and γ_1 is -1/2 and 3/2, respectively. The second order accuracy can be achieved by impose Equations (13) ~ (15) into local truncation error of Equation (7). In undamped system, equality constraints on the parameter can be achieved as follows:

$$\alpha_m = -\frac{1}{2} \tag{16}$$

$$\alpha_{k1} = \frac{1}{2\beta_1} \tag{17}$$

In damped system, the equality constrains can be achieved as follows:

$$\alpha_c = -\frac{2\beta + \beta_1}{4\beta_1^2} \tag{18}$$

$$\alpha_{c1} = -\frac{2\beta + 3\beta_1}{4\beta_1^2} \tag{19}$$

The algorithm is perform by considered as the un-force system in the first step of the analysis by let $\alpha_{m1} = 1$. The non-zero forcing terms should be add later, which in this case second order accuracy given equality constraints as follows:

$$\alpha_{f1} = \alpha_{k1} \tag{20}$$

$$\alpha_f = \alpha_k \tag{21}$$

With asymptotic annihilation and second order accuracy, the 9 independent algorithm parameters that found in general single step method were reduced to 2 free parameters. The final form of the equation can be achieved by substituting velocity and acceleration into equilibrium equation. The result can be written as follows:

$$\begin{aligned}
&\left\{\frac{1}{\beta_1 \Delta t^2 \alpha_{k1}} M + \frac{\alpha_{c1}\gamma_1}{\beta_1 \Delta t \alpha_{k1}} C + K\right\} \Delta u = \\
&F_{n+1} - Ku_n + \frac{1}{\beta_1 \Delta t^2 \alpha_{k1}} M \left\{\Delta t v_n + \beta \Delta t^2 a_n\right\} \\
&-\frac{\alpha_{c1}}{\alpha_{k1}} C \left\{ v_n + \gamma \Delta t a_n - \frac{\gamma_1}{\beta_1 \Delta t}\left\{\Delta t v_n + \beta \Delta t^2 a_n\right\}\right\} \\
&-\frac{a_m}{a_{k1}} M a_n - \frac{\alpha_c}{\alpha_{k1}} C v_n
\end{aligned} \tag{22}$$

3. DRUM TESTING EXPERIMENT OF NPT

The commercial NPT, Tweel 12N16.5 SSL ALL TERRAIN, which was developed by Michelin, is tested for dynamic behaviors using drum testing machine, Kayton DTM-350MS by Kayton Industry Co., Ltd., as shown in Figure 1. The drum diameter was 1.706 m according to the testing standard, ASTM 28580; Passenger car, truck and bus tire rolling resistance measurement method. The compression load was measured by load cell with the precision of ±0.5 %. The rotation speed could be defined under the error less than ±2 km/hr. The NPT was pressed against the drum with pressing load of 14 kN while the drum was rotated at speed of 11 km/hr. In addition, the high speed video camera, Photron FASTCAM mini UX50, is used to capture the spoke's deformation at various rotation angle and time during rolling as shown in Figure 2. The frame rate of 3,000 fps and resolution of 1024 × 1024 pixels were used to capture the deformation behavior of NPT spoke. The specification of selected commercial NPT is shown in Table 1.

Figure 1. Drum testing of NPT, Tweel 12N16.5 SSL ALL TERRAIN by Michelin.

Figure 2. Setting up of high speed video camera to record NPT's spoke deformation during rotation.

Table 1. Specification of selected NPT Tweel model.

Parameter	Value
Model	SSL ALL TERRAIN
Size	12N16.5
Max Load at 15 km/hrs (lb)	4400
Weight (lb)	226

4. DYNAMIC FINITE ELEMENT MODELING OF NPT

The finite element (FE) model of NPT was created using FE software, MSC.Patran. The FE model of NPT is shown in Figure 3. The width and diameter of NPT model was 309 mm and 860 mm, respectively. The FE model of NPT consisted of 3 main components including 1) tread, 2) shear band, which was the combination of solid tire-like sidewall and belt layers, and 3) spoke, which normally made of polyurethane. The NPT components are showed in exploded state as shown in Figure 4. The linear elastic and viscoelastic material model were used to model elastic and inelastic behavior of NPT components, respectively. The viscoelastic

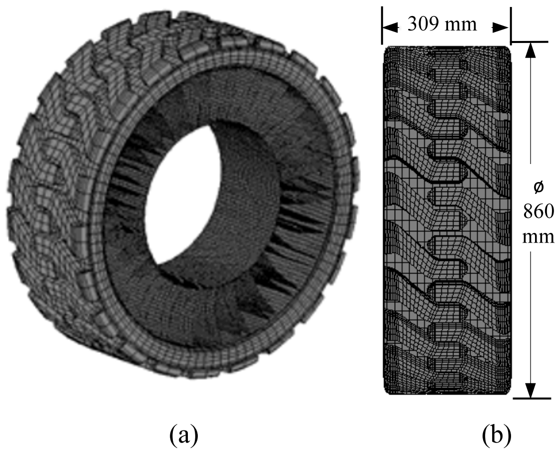


Figure 3. (a) Finite element model of NPT, (b) overall dimension of NPT.

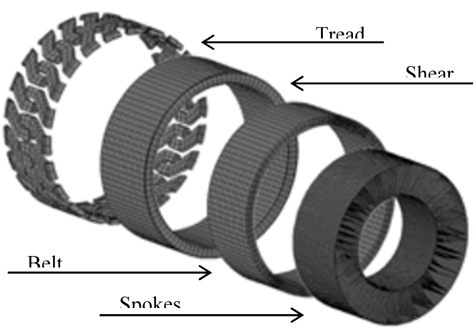


Figure 4. NPT's components.

properties were required to properly estimate inelastic deformation that occur during cyclic deformation by rolling. The inelastic stress and strain responsed exhibit of phase delay, which was the cause of hysteresis energy loss. The stress-strain relationships of NPT components were obtained from previous research, which the NPT components was cut into tensile, compression, and bending test specimens using waterjet cutting technique. The linear elastic material is used to model elastic behavior of NPT's components, in which the simplified modulus of elasticity and details of finite elements use for modeling NPT components are shown in Table 2 (Rugsaj and Suvanjumrat, 2018b, 2018a). In addition, the generalized Maxwell's viscoelastic material model is used to model inelastic behavior of NPT components, can be expressed by the following equations.

$$G(t) = G_0 - \sum_{i=1}^{n} G_i\left(1 - e^{-t/\tau_i}\right) \tag{23}$$

$$\tau_i = \eta_i / E_i \tag{24}$$

Where $G(t)$ is Shear Relaxation Modulus, G_0 is Shear Modulus at time, $t=0$, G_i is i^{th} term of Shear Modulus, τ_i is i^{th} term of Relaxation Time (sec), E_i is Modulus of Elasticity, and η_i is viscoelasticity. The required viscoelastic material constants (Yoo *et al*., 2017) for generalized Maxwell equations are shown in Table 3. The reinforced bar or rebar element is used to model belt layers while the tying equation is used to embed the belt elements into rubber element as shown in Figure 5.

The dynamic FE models of rolling NPT on drum and flat surface were then created to analyze the NPT behavior during rolling state. It should be noted that while the FE model of rolling NPT on drum was identical, thus it had be a lot easier to compared with the experiment. The analysis of rolling NPT on drum was very hard to obtain due to

Table 2. Details of FE models of NPT components and estimated modulus of elasticity.

Parameters	NPT's components		
	Tread	Shear band	Spoke
Element type	Hexagonal	Hexagonal	Quadrilateral
Number of element	2,288	11,904	35,500
Average element edge length (mm)	19.36	16.15	8.74
Average element thickness (mm)	-	-	5.8
Modulus of Elasticity	8 GPa	32 GPa	61.8 MPa

Table 3. Generalized Maxwell's viscoelastic material constants of NPT's components.

i^{th} term	τ_i	G_i	
		Spoke	Shear Band
1	0.2	0.125	0.2
2	0.02	0.125	0.2
3	0.002	0.125	0.2

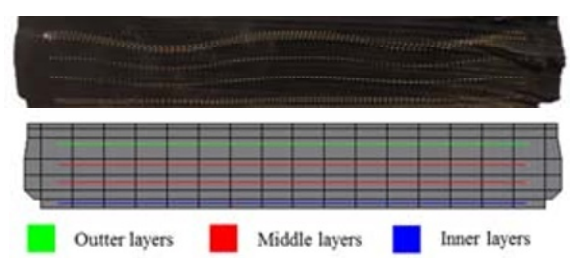
(a)
(b)


Figure 5. Cross sectional of shear band and its belt layer details and positions of (a) NPT and (b) FEM.

complexity of contact calculation between the tire and rotating curvature surface resulting in high computational resource and analysis time. Moreover, the analysis of NPT on drum at high rolling speed was impractical and very hard to analyze due to several reasons. Firstly, the time step became very small in acceleration process during start of rolling step, which the tire was starting to rotate from zero velocity by driven drum under applied load. In this case, the time step was going to be cut-off or decrease constantly in every incremental step resulting in very long analysis time before the tire could even got to desirable velocity and became stable. Another problem was that the time step value was suffered from constantly update due to vibrating behavior of the rolling tire. In addition, there were constantly changed in contact area calculation of curve surface, which also resulted in time step adaptation. The fluctuation in contact force constantly changed contact surface resulting in difficult calculation of frictional force, which used to drive the rolling tire by driven drum. In this research, the FE model of rolling NPT on flat surface was developed as an alternative approach for the requirement of dynamic characteristic and simulation of rolling NPT. The dynamic finite element analysis (FEA) was performed by FE software, MSC. Marc 2010, and using the personal computer with Intel CPU Core i7-7700 3.60 GHz and DDR RAM 8 GB.

4.1. FE Modeling of Rolling Tire on Drum Testing Machine

The FE model of rolling tire on drum was created. The drum was modeled as rigid surface with diameter of 1.706 m, which was identical to the actual experiment. The tire was pressed against the drum with load of 14 kN. After the load was fully applied, the drum was assigned to rotate at angular velocity of 4.56 rad/sec, which could be converted to linear velocity of 11 km/hr. The Coulomb's friction model was used to model friction between the tire and drum with the friction coefficient value of 0.8. The FE model of rolling tire on drum testing machine is shown in Figure 6. The drum was rotated to about 1 sec to ensure the analysis to be stable. The spoke deformation at specific rotating angles and various time steps were then collected.

4.2. FE modeling of Rolling Tire on Flat Surface

The FE model of rolling tire on flat surface was created. The flat surface used to represent the road surface and was modeled to be rigid. The boundary conditions was set to be identical with the analysis on rotating drum, which the NPT was pressed against the flat surface with load of 14 kN. After the load was fully applied, the NPT was assigned to rotate at angular velocity of 7.11 rad/sec, which could be converted to linear velocity of 11 km/hr. The Coulomb's friction model was used to model friction between the NPT and the flat surface with the friction coefficient value of 0.8. The FE model of rolling tire on flat surface is shown in

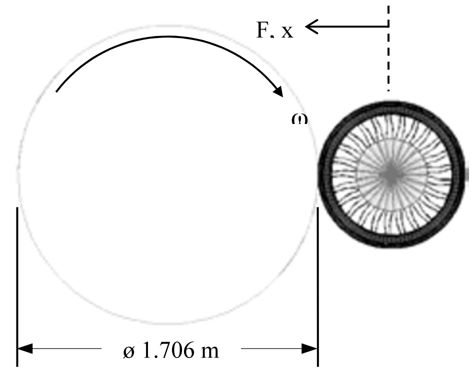


Figure 6. FE model of rolling tire on drum testing machine.

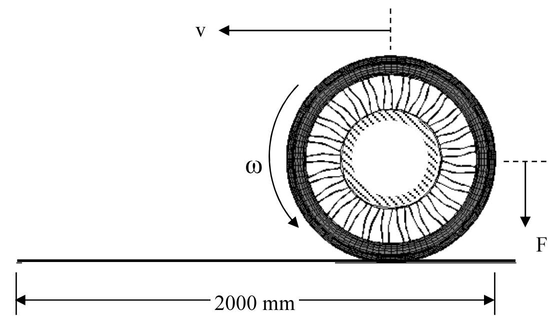


Figure 7. FE model of rolling tire on flat surface.

Figure 7. The NPT was assigned to be rotated for 1 sec to ensure the analysis to be stable. The spoke deformation at specific rotating angles and various time steps were then collected.

5. RESULTS AND DISCUSSION

5.1. The FEA Results of Rolling NPT

The FEA of rolling NPT on drum testing machine and flat surface had been performed. The spoke deformation behaviors were used to validate the FEA of rolling NPT at high speed on the drum. This was to ensure the same force-deformation behavior under dynamic loading between the experiment and FEA. In the previous work, the force-deformation relationship of the FEA under static loading was validated, which was found the average error less than 6.81 % (Rugsaj and Suvanjumrat, 2019). The overall NPT's displacement in loading direction and deformation were also found to directly relate to the vertical force. Consequently, the positions of spoke deformation would be captured at high speed with the high speed video camera and should be extracted to be a useful tool for comparing with the FEA. The deformed spokes of rolling NPT by FEA are compared with the recorded spoke deformation by the high speed video camera are shown in Figure 8. It was explicitly illustrated the same deformed spokes of both FEA and experiment.

The displacements of nodes on NPT's spoke were collected at specific angular positions, i.e. 30 °, 60 °, 90 °, 120 °, 150 °, and 180 °, during rolling. As for the experiment, the spoke deformation was recorded using high speed video camera. The spoke deformations at each angular location corresponding to the nodal position of FEA were then captured and converted into Cartesian coordinates by means of image processing technique. The nodal displacements of FEA of NPT on rolling drum and flat surface are then compared with the spoke deformation obtained from the experiment to validate the model as shown in Figure 9. The spoke deformation was observed to be bending due to compression at lower portion of the spoke which was the contacting side of NPT. On the other hand, the upper portion of the spoke was found to be in tension from supporting the load that distributed along the tire. The maximum spoke bending was found when it rotated to be normal with surface.

The average error of FE model of rolling NPT on drum and flat surface compared to the experiment was found to be 3.68 and 3.89 %, respectively. Thus, the FE model was shown to be in a good agreement with the experiment and should be reliable in analysis of rolling NPT's dynamic behaviors. The processing time requires for analysis rolling NPT of both methods is shown in Table 4. The analysis time of rolling NPT on flat surface was found to be faster due to less complexity in contact and time step calculation. The analysis results also shown that FE model of rolling NPT on drum and flat surface yielded nearly identical result when compared to the experiment. In addition, both FE model of rolling NPT were found to be in a good agreement with the experiment and can be used in design and analysis of rolling NPT. However, the rolling NPT on flat surface model is preferable due to less complexity in modeling and analysis. Thus, in this research, the FE model of rolling NPT on flat surface was selected for the observation of dynamic behaviors.

5.2. Dynamic Behaviors Analysis

The displacement and impact force of rolling NPT at various time from 0 to 2 sec was analyzed and collected. In this part of the article, the velocity of NPT was varied to study the effect of dynamic parameters to NPT's performance. The variable velocity of NPT included 11, 13, and 15 km/hr, which corresponded to angular velocity of 7.10593, 8.39793, and 9.68992 rad/sec, respectively. The maximum value of varied velocity was selected to be 15 km/hr due to velocity limit in actual NPT's specification.

The displacement was collected at NPT's center of rotation, while the impact force was collected at interface between the NPT's tread and flat surface. The NPT was pressed against flat surface with fixed load of 14 kN, then the NPT was assigned a start to rotate from zero to final

(a)

(b)

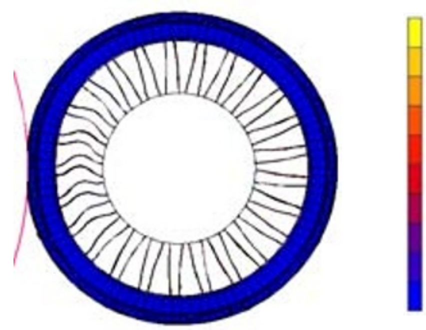


Figure 8. (a) Captured spoke deformation of rolling NPT on drum using high speed video camera (b) Stress and deformation of rolling NPT's spoke under dynamic loading.

Table 4. Analysis time for each analysis type.

Analysis type	Analysis time (hrs)
Static	0.32
Dynamic on drum	329.71
Dynamic on flat surface	32.56

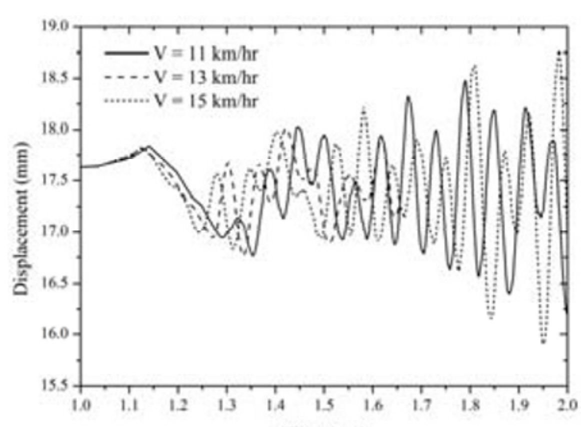


Figure 10. The displacement at NPT's center of rotation at various time.

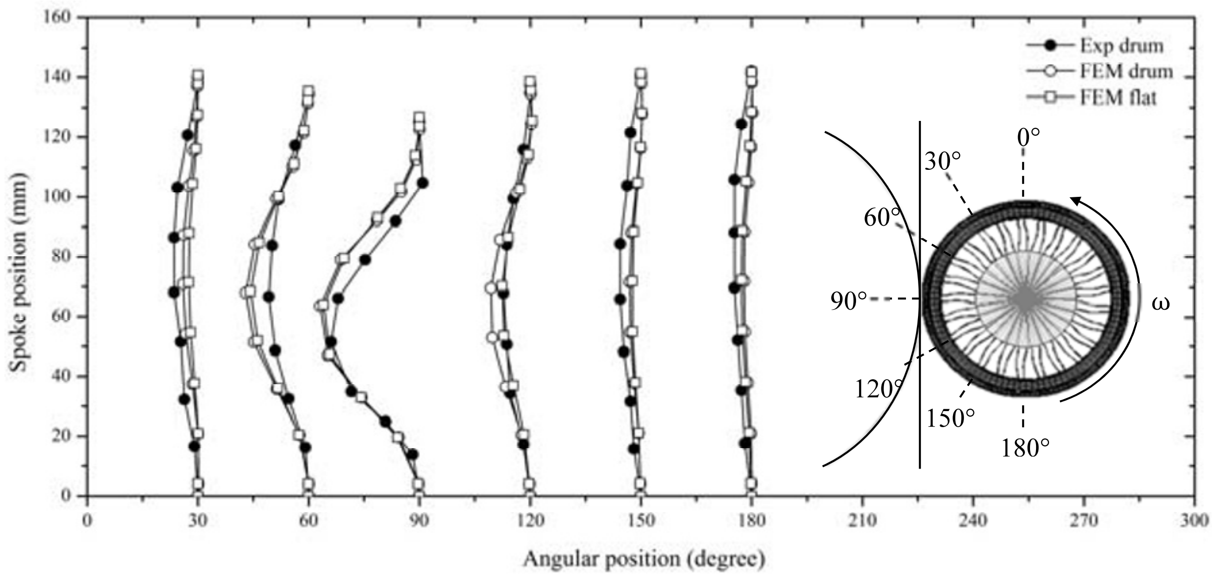


Figure 9. The spoke deformation at various angular positions of rolling NPT and schematic diagram of angular position with corresponding drum and flat surface.

velocity in 1 sec. The acceleration stage was required in this comparation to prevent diverge that might occur when the NPT was rotated at high speed. The displacements of a center of rolling NPT at velocity of 11, 13, 15 km/hr are collected as shown in Figure 10. The impact force between NPT's tread and flat surface at the velocity of 11, 13, 15 km/hr is collected as shown in Figure 11. The displacement was found to be fluctuate in sine wave-like manner with average value of 17.42 mm, which was nearly identical to initial displacement of 17.64 mm after load was fully applied statically at 14 kN.

The maximum and minimum values of displacement and impact force were collected from the analysis results. The total displacement was then computed from difference between maximum and minimum displacement, while vibrating frequency at NPT's center of rotation could be computed using the average period between peak to peak of displacement graph. The dynamic parameters obtain from processing the analysis data are shown in Table 5. It was found that total displacement, maximum impact force, and vibrating frequency showed increasing trend when NPT's velocity was raised. Thus, high rolling velocity of NPT may affect to ride comfort or cause damage to vehicle's suspension system.

The stress at various time was collected from the model. The maximum stress was found to be occured at upper part of spokes due to tension during loading and rolling. In the same manner, the maximum shear stress was found at interface between shear band and outer ring of spoke, which

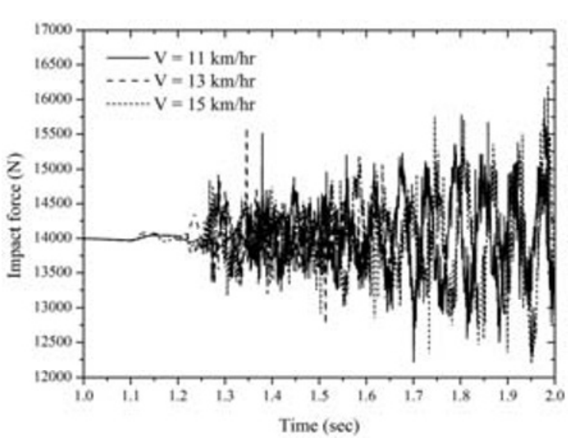


Figure 11. The impact force at various time.

Table 5. Dynamic parameters of rolling NPT on flat surface at varied velocity and a constant load of 14 kN.

Parameters	Velocity		
	11 km/hr	13 km/hr	15 km/hr
Maximum displacement (mm)	18.48	18.57	18.77
Minimum displacement (mm)	16.21	16.12	15.91
Total maximum displacement (mm)	2.27	2.45	2.86
Maximum impact force (kN)	15.51	15.59	16.21
Vibrating frequency (Hz)	18.47	19.12	18.47

which position was beside node in contact with flat surface at that time. The deformation of spoke while rolling at various time is shown in Figure 12. The details of maximum stress at spoke and shear stress at spoke-shear band interface of NPT with velocity of 11 km/hr are shown in Figures 13 (a) and 13 (b), respectively. The maximum stress at spoke and interface at various rolling velocity is shown by time history in Figures 14 and 15, respectively. It should be note that both maximum stress and shear stress position at given time were found to constantly change because spoke and interface between spoke and shear band were constantly change during rolling. Thus, the maximum stress and shear stress would always occur at opposite side of spoke and same side of interface that touching the flat surface, respectively. In addition, as it was found that the spoke's maximum deformation was increased when the velocity was higher. This also results in higher stress corresponding to increase velocity. The presence of maximum shear stress at spoke-shear band interface might occur due to highly difference in modulus of elasticity of both interface's materials. Thus, this difference caused NPT separation happened at the interface, which resulted in shear stress concentration during rolling.

In order to study the effects of load to maximum stress and shear stress, the load was varied. The viable loads were 14, 16, and 18 kN, while the velocity was fixed at 11 km/hr. The maximum stress and shear stress with varied load are plotted as shown in Figures 16 and 17, respectively. The dynamic parameters including maximum, minimum

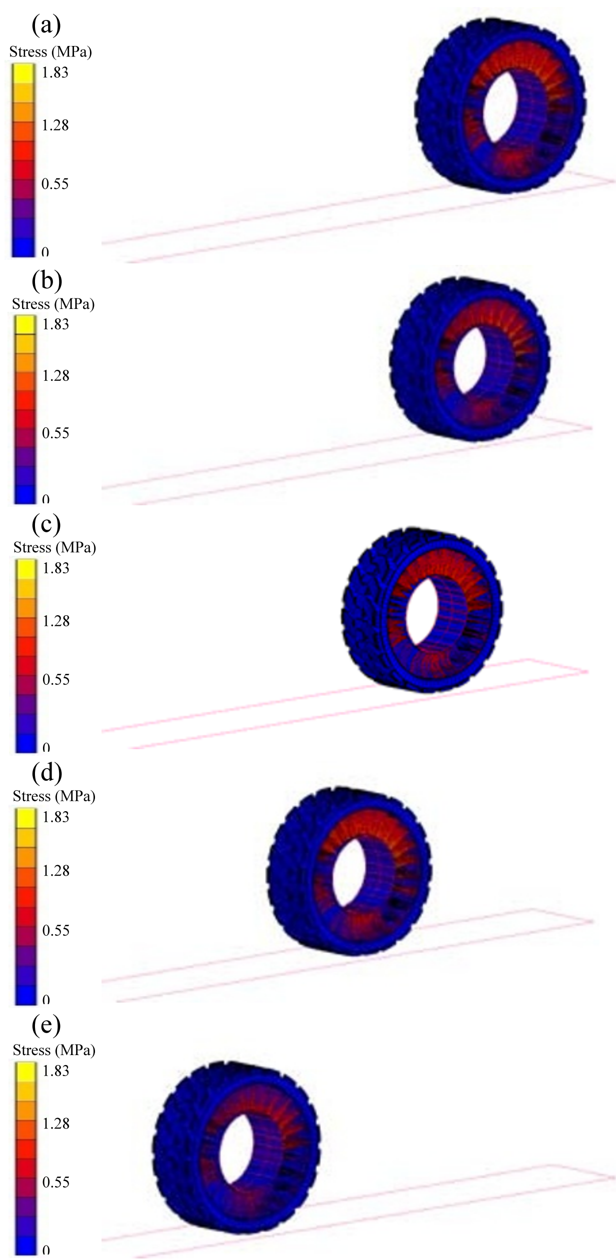


Figure 12 Stress and deformation of rolling NPT with load of 14 kN and velocity of 11 km/hr at various time (a) 1 sec, (b) 1.25 sec, (c) 1.5 sec, (d) 1.75 sec, and (e) 2 sec.

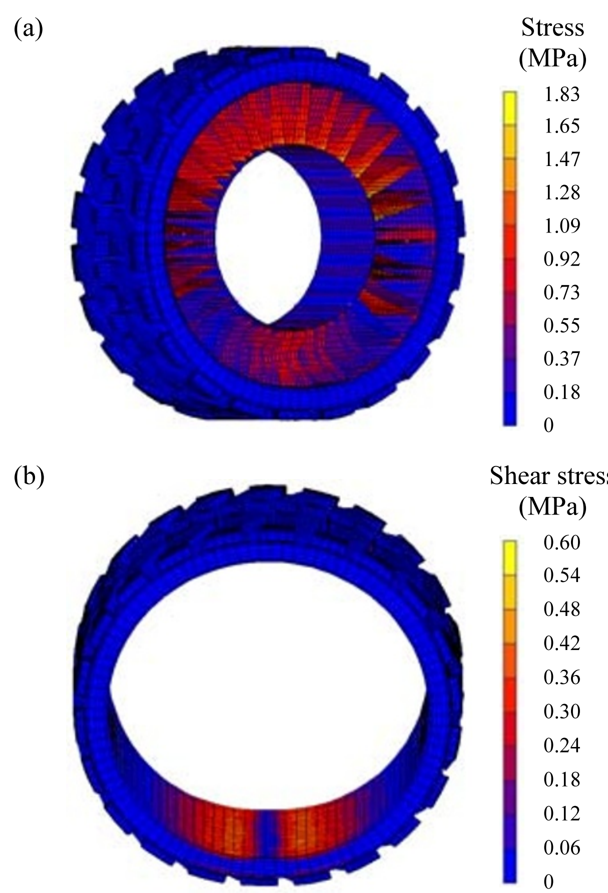


Figure 13. (a) Stress and deformation analysis results and (b) Shear stress analysis results at spoke-shear band interface of rolling NPT at velocity of 11 km/hr and load of 14 kN.

displacement, maximum impact force and vibrating frequency are collected and summarized in Table 6. The maximum stress at spoke and shear stress at spoke-shear band interface at varied velocity and varied load are summarized as shown in Tables 7 and 8, respectively. The maximum stress was increased by the increase of velocity and load. It was observed that increasing load had more effect on shear stress than increasing in velocity. This all happened because the load had directly impacted on maximum vertical deformation of NPT (Rugsaj and Suvanjumrat, 2019). Thus, the stress was significantly increased with higher load than velocity as a result. The relationship between load and shear stress was estimated using linear regression technique. The NPT shear stress equation is given by:

$$\tau(f) = 0.177f - 1.8123 \tag{25}$$

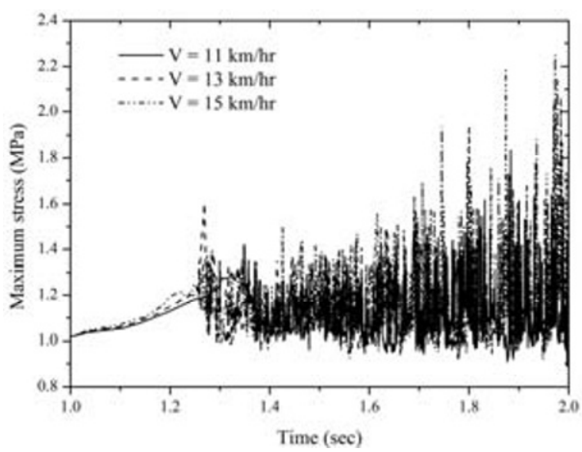


Figure 14. The maximum stress at spoke at varied velocity of 11, 13, and 15 km/hr.

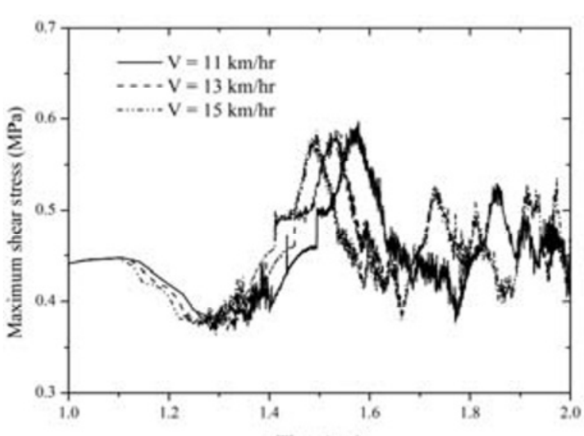


Figure 15. The maximum shear stress at spoke-shear band interface at varied velocity of 11, 13, and 15 km/hr.

where τ and f are shear stress (MPa) and load (kN), respectively. The average error was found to be 9.74 %, while coefficient of determination (R^2) was found to be 0.8959. The equation can used to predict shear stress that should occur at the interface of shear band and outer portion of spoke at any given load. The shear stress value at maximum designed load of 20 kN referring to the NPT's specification was found to be 1.7277 MPa. The maximum allowable shear stress at interface can be roughly estimated at 3 MPa according to the spoke's yield stress of 6 MPa. The load of 20 kN caused the shear stress was 0.5759 times of the allowable shear stress. Thus, the shear stress at interface may cause the spoke and shear band damaged due to fatigue by repeating load.

The separation occurs as shown in Figure 18. It should be noted that the maximum load of NPT was specified at

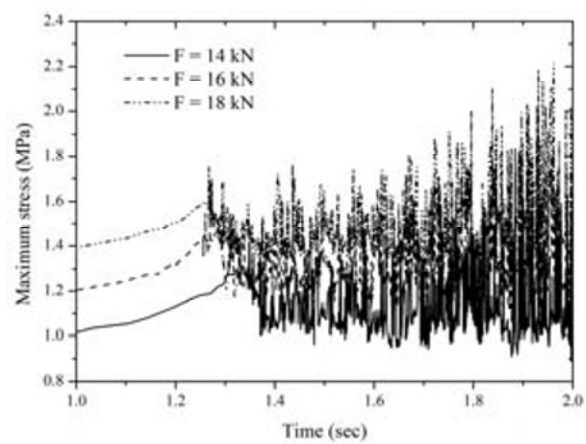


Figure 16. The maximum stress at spoke at a constant velocity of 11 km/hr and varied load of 14, 16, and 18 kN.

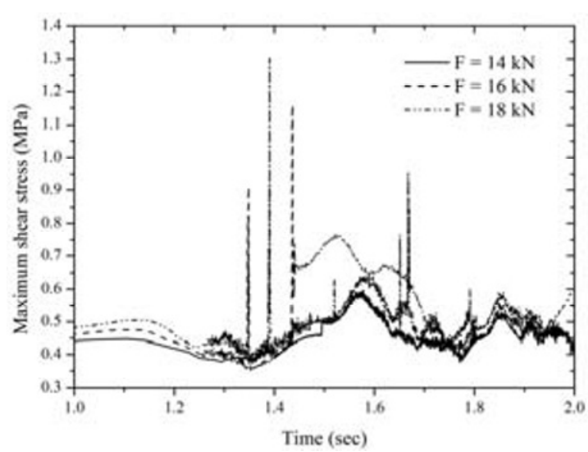


Figure 17. The maximum shear stress at spoke-shear band interface at a constant velocity of 11 km/hr and varied load of 14, 16, and 18 kN.

Table 6. Dynamic parameters of rolling NPT on flat surface at varied load and a constant velocity of 11 km/hr.

Parameters	Load		
	14 kN	16 kN	18 kN
Maximum displacement (mm)	18.48	21.06	23.41
Minimum displacement (mm)	16.20	17.90	19.86
Total maximum displacement (mm)	2.27	3.16	3.54
Maximum impact force (kN)	15.51	18.29	20.68
Vibrating frequency (Hz)	18.47	18.83	18.94

Table 7. Maximum stress and shear stress of rolling NPT on flat surface at varied velocity and a constant load of 14 kN.

Parameters	Velocity		
	11 km/hr	13 km/hr	15 km/hr
Maximum dynamic stress (MPa)	1.832	1.946	2.252
Maximum dynamic shear stress (MPa)	0.596	0.589	0.588

Table 8. Maximum stress and shear stress of rolling NPT on flat surface at varied load and a constant velocity of 11 km/hr.

Parameters	Load		
	14 kN	16 kN	18 kN
Maximum dynamic stress (MPa)	1.832	1.905	2.222
Maximum dynamic shear stress (MPa)	0.596	1.159	1.304

Figure 18. Separation at spoke-shear band interface.

44,000 lb or 20 kN at the speed of 15 km/hr. In addition, the peak of maximum stress and shear stress upon time history was found to be occured at the same time at the peak of the impact force. Moreover, the peak of the impact force is observed to be occured when node at the outer ring of the spoke are contact with the flat surface without presence of supporting spoke at that time (Figure 12 (b)). As previous discussions, the dynamic parameters that obtained from the dynamic FEA of rolling NPT e.g. maximum and total displacement, impact force, vibrating frequency, stress and shear stress, can be used to predict important parameters regarding tire performance including riding comfort, fatigue resistance, damage to vehicle suspension systems, and NPT's durability.

6. CONCLUSIONS

In this research, the FE model of NPT based on dynamic approach had been developed to study the mechanical behaviors of NPT while rolling. The hyperviscoelastic constitutive equation was used to model elastic and inelastic behavior of NPT. the direct integration with single step Houbolt operator was used to solve implicit dynamic contact analysis of NPT. The FE model of rolling NPT on drum and flat surface were compared to the experiments. The velocity and load varied on flat surface model were 11, 13, 15 km/hr and 14, 16, 18 kN, respectively, to study the dynamic effects on NPT. The conclusion can be summarized as follows:

(1) The FE model of NPT on drum and flat surface had given nearly identical results. The error could be estimated from the spoke deformation. They were 3.68 and 3.89 % when compared to the experiments, respectively.

(2) The analysis time of model on flat surface was found to have shorter analysis time due to less complexity in contact and time step calculation. The FEA times of rolling NPT on drum and flat surface at 0.4 sec were found to be 329.71 and 32.56 hrs, respectively. The analysis time of rolling NPT on drum could be roughly estimated to be 10.13 times when compared to flat surface model.

(3) The dynamic properties including maximum displacement and impact force of rolling NPT with load of 14 kN and velocity of 15 km/hr were found to be 18.77 mm and 16.21 kN, which was 1.06 and 1.16 times when compared to the static case, respectively. In the same manner, the maximum displacement and impact force of rolling NPT with load of 18 kN and velocity of 11 km/hr were found to be 23.41 mm and 20.68 kN, which is 1.08 and 1.15 times when compared to the static case, respectively. This indicated the possibility that the damage would occurred to the suspension system and the NPT itself and riding comfort might become lower when NPT was used with higher velocity and load.

(4) The maximum dynamic stress was found to be occurred at upper portion of spoke at opposite side of flat

surface due to tension. Under the constant velocity and the load of 14, 16, 18 kN, the dynamic maximum stress was happened to be 1.805, 1.583, and 1.597 times of static case, which had the values of 1.015, 1.204, and 1.391 MPa, respectively. This all happened because the increase of load had significantly impacted on maximum spoke deformation and the stress had higher as a result.

(5) The maximum shear stress of rolling NPT was found at interface between shear band and outer ring portion of the spoke. It happened at the same time which the stress and the peak of the impact force were happened. It was found that the shear stress was increased by the increase of carrying load. This indicated that the possibility of separation of interface between shear band and spoke if the NPT is rolling at high speed or load.

(6) The interface shear stress value under maximum load of 20 kN was found to be 1.7277 MPa using the linear regression technique. The maximum allowable shear stress at interface can be roughly estimated to the value of 3 MPa. The load of 20 kN should cause the shear stress increased of 0.5759 times of the allowable shear stress. Certainly, the shear stress under repeated load caused the damage of NPT at interface between shear band and spoke.

The dynamic FE model of rolling NPT on flat surface can be used to predict important dynamic properties of rolling NPT. Thus, the model will prove to be useful in design and development of NPT regarding dynamic effects in the near future.

ACKNOWLEDGEMENT–This work was financially supported by Rubber Technology Research Center (RTEC), Mahidol University and the Thailand Research Fund (TRF) under the TRF Research Grant No. RDG60T0140.

REFERENCES

Aboul-Yazid, A. M., Emam, M. A. A., Shaaban, S. and El-Nashar, M. A. (2015). Effect of spokes structures on characteristics performance of non-pneumatic tires. *Int. J. Automotive & Mechanical Engineering* **11**, **1**, 2212–2223.

Chung, J. and Hulbert, G. M. (1994). A family of single-step houbolt time integration algorithms for structural dynamics. *Computer methods in applied mechanics and engineering* **118**, **1-2**, 1–11.

Gasmi, A., Joseph, P. F., Rhyne, T. B. and Cron, S. M. (2012). Development of a two-dimensional model of a compliant non-pneumatic tire. *Int. J. Solids and Structures* **49**, **13**, 1723–1740.

Gent, A. N. and Watler, J. D. (2006). *The Pneumatic Tire*. National Highway Traffic Safety Administration. Washington DC. WA. USA.

Jin, X., Hou, C., Fan, X., Sun, Y., Lv, J. and Lu, C. (2018). Investigation on the static and dynamic behaviors of non-pneumatic tires with honeycomb spokes, *Composite Structures*, **187**, 27–35.

Ju, J., Veeramurthy, M., Summers, J. D. and Thompson, L. (2013). Rolling resistance of a nonpneumatic tire having a porous elastomer composite shear band. *Tire Science and Technology* **41**, **3**, 154–173.

Kim, K., Heo, H., Uddin, M. S., Ju, J. and Kim, D.-M. (2015). Optimization of nonpneumatic tire with hexagonal lattice spokes for reducing rolling resistance. *SAE Paper No*. 2015-01-1515.

Phromjan, J. and Suvanjumrat, C. (2018a). A suitable constitutive model for solid tire analysis under quasi-static loads using finite element method. *Engineering J.* **22**, **2**, 141–155.

Phromjan, J. and Suvanjumrat, C. (2018b). Material characteristic for capability analysis of solid tire by finite element method. *Key Engineering Materials*, **777**, 416–420.

Phromjan, J. and Suvanjumrat, C. (2018c). Material properties of natural rubber solid tires for finite element analysis. *Key Engineering Materials*, **775**, 560–564.

Rhyne, T. B. and Cron, S. M. (2006). Development of a non-pneumatic wheel. *Tire Science and Technology* **34**, **3**, 150–169.

Rhyne, T. B., Cron, S. M. and Pompier J. P. (2006). *Compliant wheel*. US7013939.

Rugsaj, R. and Suvanjumrat, C. (2018a). Determination of material property for non-pneumatic tire spokes by inverse method. *Key Engineering Materials*, **777**, 411–415.

Rugsaj, R. and Suvanjumrat, C. (2018b). Finite element analysis of hyperelastic material model for non-pneumatic tire. *Key Engineering Materials*, **775**, 554–559.

Rugsaj, R. and Suvanjumrat, C. (2019). Proper radial spokes of non-pneumatic tire for vertical load supporting by finite element analysis. *Int. J. Automotive Technology* **20**, **4**, 801–812.

Veeramurthy, M., Ju, J., Thompson, L. L. and Summers, J. D. (2014). Optimisation of geometry and material properties of a non-pneumatic tyre for reducing rolling resistance. *Int. J. Vehicle Design* **66**, **2**, 193–216.

Yoo, S., Uddin, M. S., Heo, H., Ju, J. and Choi, S. J. (2017). Thermoviscoelastic modeling of a nonpneumatic tire with a lattice spoke. *Proc. Institution of Mechanical Engineers, Part D: J. Automobile Engineering* **231**, **2**, 241–252.