International Review of Economics (2022) 69:339–349
https://doi.org/10.1007/s12232-022-00394-4

RESEARCH ARTICLE

Abilities and the structure of the firm

Tobias Hiller[1]

Received: 14 July 2021 / Accepted: 15 April 2022 / Published online: 19 May 2022

Abstract

In this paper, we enhance the production games approach introduced by Hiller (Manag Decis Econ 40(5):520–525, 2019) with aspects from a recent article by Morelli and Park (Games Econ Behav 96(1):90–96, 2016), to analyze how abilities of employees influence the structure and the wage scheme of the firm. The analysis is done within the framework of cooperative game theory. Concretely, we apply the coalition structure approach and the χ value (Casajus in Games Econ Behav 65(1):49–61, 2009).

Keywords Production games · Abilities · Structure of the firm · χ reward function · Stability

JEL Classification C71 · D21 · J24 · M52

1 Introduction

In a recent paper, Hiller (2019) introduced production games within the framework of cooperative game theory. He was inspired by the team games approach introduced by Hernández-Lamoneda and Sánchez-Sánchez (2010). The main idea of Hiller (2019) is illustrated by an example. Imagine a firm has a task that requires a certain number of employees t. For example, we have $t = 3$. By adding a fourth employee, the firm will place the best three of the four employees in a team to achieve the task. If the firm has six employees, maybe two teams of three will be formed. In Hiller (2019) the worth of a group of employees is determined from the maximum worth of possible coalitions with size t. The ability of a single employee is modeled only indirectly by the worths of the coalitions he or she belongs to. Another recent paper also deals with teams and their structure within firms using cooperative game theory (Morelli and Park 2016) In their model, every employee has certain ability. For achieving a task, a certain number of employees t and a certain sum of abilities

✉ Tobias Hiller
hiller@wifa.uni-leipzig.de

[1] Department of Microeconomics, University of Leipzig, Leipzig, Germany

z are necessary. After the relative threshold is crossed, the worth of a team increases linearly with the aggregate ability of its members. Hence, there is no graduality of worths as modeled by t in Hiller (2019).

The objective of our paper has two thrusts. First, the two theoretical approaches mentioned above are combined into one. Then, we determine the stable assignment of employees to teams for this approach. For the first step, we assume that every employee i has an ability a_i. These abilities of employees determine the worth of coalitions. Additionally, the graduality exists. Only the abilities of the best t (or any multiple of t) members of a team are considered for determining the worth of a team. With reference to our example from the beginning, the worth of a group with four employees and $t = 3$ is the aggregate ability of the three employees with highest ability.

The main question is how to assign employees to teams and how this decision does depend on the heterogeneity of the employee's abilities. We assume that the employees are rewarded for the worth created by their team. Our aim is to find stable structures of teams, i.e., the number of teams and their internal structure. A structure of teams in the firm is stable if there is no other structure that raises the wages of all employees in at least one new team. One could interpret these structures as enforceable from the point of view of the employer. A higher range of stable structures is associated with a higher degree of freedom for the employer in deciding on the firm structure. We analyze which structures of teams are stable in three steps.[1] First, we look at symmetric employees, meaning that all employees have the same ability. In a second step, we allow for two types of ability and finally, we analyze a firm in which every employee has a different ability. One result of all cases is that assigning more employees than necessary in a team lowers the stability of the team. In addition, a higher degree of heterogeneity, meaning that more levels of ability exist, leads to smaller opportunities for employers to structure teams; the size and composition of stable teams are more prescribed.

For the purpose of the article, we use the framework of cooperative game theory. To model structures of teams in cooperative game theory, firm structures (FS, also called coalition structures) are used. These structures divide employees into disjointed teams (or components). To answer the question of how to reward the employees, a reward function for games with a firm structure is used. The most popular function is the χ function (Casajus 2009). This function is team-efficient, meaning that the worth of the team is divided among the team members,[2] and reflects the outside options of the employees. The better an employee's outside options (possibilities of cooperating with other employees in the firm), the higher the employee's share within the team. Other reward functions for FS games being team-efficient and reflecting the outside options of employees are introduced by Wiese (2007) and

[1] This analysis is based on similar questions in the article by Morelli and Park (2016). Whereas these authors analyze inequality with respect to the payoffs, we focus on inequalities in the abilities of the employees.

[2] According to Aumann and Drèze (1974), components are active groups as in our understanding. In contrast, the Owen value (Owen ,1977) interprets components as bargaining unions.

Alonso-Meijide et al. (2015), for example. The problem of determining the optimal number and quality of employees in the firm is beyond the scope of his article. Some other missing aspects are mentioned in Sect. 5 by suggesting further research options.

One line of related literature on how to structure teams are hedonic games.[3] These games model the formation of coalitions (groups) of players when players have preferences for one group over others (Banerjee et al. 2001; Bogomolnaia and Jackson 2002). One article using hedonic games and employees with different level of ability was developed by Barberà et al. (2015). They identify conditions under which stable structures consists of non-segregated teams. The result is similar to our result in the case of two level of ability. In this case, we also identify situations that result in one mixed team. Another approach was developed by Piccione and Razin (2009). Their article uses exogenous power relations over the set of coalitions of agents to determine a stable order (groups). This approach does not consider explicit how (heterogeneity of) abilities influences the set of stable teams. One model taking this aspect into account was introduced by Damiano et al. (2010). The model considers two effects: peer effect and pecking order. The peer effect means that individuals prefer to cooperate with more capable peers in the same organization. The pecking order effect models the opposite; the wish of individuals to be in a good rank according to their ability within the organization. In equilibrium, segregation occurs for the more capable and less capable individuals whereas the intermediate able employees are in mixed teams. Also, one result of our model is a segregation of employees of different abilities. In the case of only two level of ability, we have one mixed team. In the case of n abilities, teams are minimal heterogeneous.

The remainder of this paper is structured as follows. Basic notations of cooperative game theory are given in Sect. 2. The next section introduces production games. Section 4 presents the results. Section 5 concludes with a summary of the results and some questions for further research.

2 Preliminaries

A TU (transferable utility) game is a pair (N, v). $N = \{1, 2, \dots, n\}$ is the set of players. The coalition function v specifies for every subset $K \subseteq N$ a certain worth $v(K)$ reflecting the economic abilities of K, i.e., $v : 2^N \to \mathbb{R}$ with $v(\emptyset) = 0$. If v is symmetric, there exists a function $f : N \to \mathbb{R}$ such that $v(K) = f(|K|)$ for all non-empty sets $K \subseteq N$. A coalition function is called monotone if $v(K) > v(S)$ for all $S \subseteq K \subseteq N$.

A value is an operator ϕ that assigns (unique) payoff vectors to all games (N, v) (i.e., uniquely determines a payoff for every player in every TU game). One important value is the Shapley value. For calculating a player's payoff, rank orders ρ on N are used. They are written as $(\rho_1, \dots, \rho_n)$ where ρ_1 is the first player in the order, etc. The set of these orders is denoted by $RO(N)$; $n!$ rank orders

[3] The relation of hedonic games to the transferable utility approach of cooperative game theory which our article belongs to is outlined in Casajus (2008).

exist. The set of players before i in rank order ρ including i is called $K_i(\rho)$. For player i, the Shapley payoff is (Shapley 1953):

$$\mathrm{Sh}_i(N,v) = \frac{1}{n!} \sum_{\rho \in RO(N)} v\big(K_i(\rho)\big) - v\big(K_i(\rho)\backslash\{i\}\big). \tag{1}$$

In our paper, we interpret the Shapley value as a reward function and the payoff vector represents the wage for each employee.

The Shapley reward function assumes that all employees work together and the worth $v(N)$ is distributed to all employees. Since we assume a firm structure with teams producing a worth, the reward function should take this structure into account. Hence, the members of each team of the firm should be rewarded for the worth produced by this team.

A firm structure is a partition $\mathcal{P}$ of N into non-empty teams $G_1, \ldots, G_m$, $\mathcal{P} = \{G_1, \ldots, G_m\}$, with $G_i \cap G_j = \emptyset$, $i \neq j$and$N = \bigcup G_j$. The team containing employee i is denoted by $\mathcal{P}(i)$. The set of partitions of N is $\mathfrak{P}(N)$. A FS game is a game with a firm structure, $(N, v, \mathcal{P})$. A FS reward function is an operator φ that assigns wage vectors to all FS games $(N, v, \mathcal{P})$. The χ reward function is a FS reward function. It divides the worth of a team, $v(\mathcal{P}(i))$, among its members, $j \in \mathcal{P}(i)$. In contrast to the Aumann-Drèze function (Aumann and Drèze 1974), the χ function accounts for outside options — the possibilities of an employee cooperating with employees outside his team. The formula for computing the χ wage of employee $i \in N$ is Casajus (2009) :

$$\chi_i(N, v, \mathcal{P}) = \mathrm{Sh}_i(N, v) + \frac{v(\mathcal{P}(i)) - \sum_{j \in \mathcal{P}(i)} \mathrm{Sh}_j(N, v)}{|\mathcal{P}(i)|}. \tag{2}$$

Since we interpret the χ computation as wage, the employees are rewarded solely on the basis of performance. In addition, the χ reward function distributes the worth generated by the members of a team to the employees. This means that the total output produced by the employees is divided among them. Of course, this is not true in real firms. There, only a fraction of the produced worth is distributed among the employees. Our results in the paper are unchanged if the χ wages of all employees are multiplied by a constant $0 < c < 1$.

Later on, we will use the concept of stability. A firm structure $\mathcal{P}$ for (N, v) is χ stable iff for all $\emptyset = K \subseteq N$ there is some $i \in K$ such that (Hart and Kurz 1983; Wiese 2007; Casajus 2009)

$$\chi_i(N, v, \mathcal{P}) \geq \chi_i(N, v, \{K, N\backslash K\}). \tag{3}$$

Hence, starting from $\mathcal{P}$ it is not possible to raise the wages of all $i \in K$, if K is separated in one team. In other words, the structure of teams in the firm is stable, if no other structure raises the wage of all employees in at least one new team. For enforcing a structure in the firm, stability in the firm is a major element, meaning that a team of employees cannot espouse a new structure of the firm.

3 Production games

In our model, the worth of a coalition is determined by four variables / factors. These factors model the production conditions of the firm. Besides the set K, the abilitiy of employees influence the worth. The ability of an employee i is denoted by a_i, $a_i \in \mathbb{R}^+$. The vector of abilities for all employees is a. The number of employees that is necessary to achieve the task is $t \in \mathbb{N}^+$. Additionally, a coalition must exceed a certain aggregated ability (or power) z to be productive, $z \in \mathbb{R}^+$. Using this information, we introduce the production function p :

$$p(K,a,t,z) = \begin{cases} \max \sum_{l=1}^{d \cdot t} a_i \text{ with } i \in K, & k \geq t \text{ and } \max \sum_{l=1}^{d \cdot t} a_i \geq z \\ 0, & else \end{cases} \tag{4}$$

with $d = \max \left\{ d \in \mathbb{N}^+ | d \leq \frac{k}{t}, k \geq t \right\}$. Considering K, the task is done d times. To determine the worth of K in the first line, the best $d \cdot t$ abilities of employees in K are summarized. A production game is denoted by (N, p).

The Shapley reward function could also be applied to production games. Obviously from Eqs. 1 and 4, $\text{Sh}_i(N, p)$ is increasing in a_i. In addition, we know from Eq. 4 that the Shapley wage for i is increasing more slowly then a_i since the marginal contribution of i is lower then a_i in rank orders in which the employee substitutes only an employee with lower ability.

If a production function p is the basis for a FS game, it is called a FS production game $(N, p, \mathcal{P})$. For these games, the χ reward function determines wages for employees.

4 Results

In this section, we present results for different level of heterogeneity with respect to ability. The first one is a firm with symmetric employees meaning that all employees have the same ability $\overline{a}$. The production function of a symmetric production game is given by:

$$p(K,a,t,z) = \begin{cases} d \cdot t \cdot \overline{a}, & k \geq t \text{ and } d \cdot t \cdot \overline{a} \geq z \\ 0, & \text{else.} \end{cases} \tag{5}$$

where $d = \max \left\{ d \in \mathbb{N}^+ | d \leq \frac{k}{t}, k \geq t \right\}$. The task is done d times and $d \cdot t$ employees use their ability for this.

In symmetric production games, the employees obtain equal Shapley wages — the outside options of employees are the same. Hence, the employees χ wage is the average worth of their team G. For χ stable firm structures we have:

Theorem 1 *In monotone symmetric FS production games* $(N, p, \mathcal{P})$ *with* $p(N) > 0$, *only partitions* $\mathcal{P} = \{G_1, \ldots, G_s, G_{s+1}, \ldots, G_m\}$ *with*

- $\sum_{j\in\{G_1,\dots,G_s\}} a_j < z$ and
- $|G_{s+1}|,\dots,|G_m| \in \{l|l = b\cdot t\}$ with $b\in\mathbb{N}^+$ and $\sum_{j\in G_h} a_j \geq z$ for all $G_h \in \{G_{s+1},\dots,G_m\}$ are χ stable.

The proof is given in the "Appendix". All employees receive the same Shapley payoffs due to their symmetry. From this, all employees in teams of the second bullet item receive the same χ wage $\overline{a}$. The (productive) components have size $b\cdot t$, whereby factor b, $b\geq 1$, ensures that the teams achieve the required ability level z. Structuring more employees than necessary in teams lowers the stability of the team since the χ wage of the employees in such a team then falls below $\overline{a}$. The employer has a great deal of freedom in deciding how to structure teams. On the one hand, team size is freely variable with a multiple of t. On the other hand, the employer could exchange the employees arbitrarily due to their symmetry. The remaining employees in components $G_1,\dots,G_s$ obtain χ wage zero. Their exact structure (individually or in smaller teams) does not affect the χ wages of the other employees in $G_{s+1},\dots,G_m$.

Now, we analyze firms with employees with different abilities. In a first step, we assume only two types of employees — high productive employees H and low productive employees L with$N = H\cup L$, $H\cap L = \emptyset$. We have $a_i = \overline{a_h}$ for all $i\in H$ and $a_j = \overline{a_l}$ for all $j\in L$. We assume $t < |H|$, $t < |L|$ and $(t-1)\cdot\overline{a_l} + \overline{a_h} > z > t\cdot\overline{a_l}$. In this interval, the lower bound of ability z influences the size of stable teams with employees with low ability:

Theorem 2 *In monotone FS production games* $(N,p,\mathcal{P})$ *with* $N = H\cup L$ *and* $(t-1)\cdot\overline{a_l} + \overline{a_h} > z > t\cdot\overline{a_l}$ *only partitions* $\mathcal{P} = \{G_1,\dots,G_q,G_{q+1},\dots,G_s,G_{s+1},G_{s+2},\dots,G_m\}$ *with*

- $\left|\bigcup_{1,\dots,q} G_i\right| = \left|L\backslash\{G_{q+1},\dots,G_{s+1}\}\right|$ with $\left|\bigcup_{1,\dots,q} G_i\right|\cdot\overline{a_l} < z$
- $|G_{q+1}|,\dots,|G_s| = b\cdot t$ with $b\in\mathbb{N}^+$, $\{G_{q+1},\dots,G_s\}\cap H = \emptyset$, and $b\cdot t\cdot\overline{a_l}\geq z$
- $|G_{s+1}| = \begin{cases} 0, & |H| = b\cdot t \\ t & else\end{cases}$ with $b\in\mathbb{N}^+$ and $G_{s+1}\cap H = H\backslash\{G_{s+2}\dots,G_m\}$
- $|G_{s+2}|,\dots,|G_m| = b\cdot t$ with $b\in\mathbb{N}^+$ and $\{G_{s+2},\dots,G_m\}\cap L = \emptyset$ are χ stable.

The proof is given in the "Appendix". Employees with high abilities are structured in homogenous teams $G_{s+2},\dots,G_m$. These teams have size t or any multiple of t; the ability bound z does not influence the possibilities of the employer to structure these teams. The employees in these teams obtain χ wage $\overline{a_h}$. At most one mixed team with both types of employees exist. This team has size t. In this team, the players from H receive a lower χ wage than $\overline{a_h}$. However, they cannot create a worthwhile divergent structure on their own, since they are less than t. Therefore, it is also not worthwhile for them to join the other teams with H employees, since these teams would then be larger than necessary, and thus the χ wages for those employees previously in these teams are lower. Employees with low ability are also structured in homogenous teams whose size ensures the achievement of aggregate ability z. In these teams, the employees obtain χ wage $\overline{a_l}$. Finally, some remaining employees from L are structured in teams $G_1, \dots, G_q$ whose worth is zero. Hence, the employees in these teams also get χ wage zero. With respect to the first scenario (symmetric employees), the freedom of the employer is reduced. On the one hand, they have to consider the different level of ability (forming homogenous teams). In addition, they must ensure that only one mixed team exists. On the other hand, the possible sizes of teams with low-ability employees are restricted by the lower aggregated ability bound.

In the next step of our analysis, we allow again for more ability classes. Concretely, we assume for simplification $a_i \neq a_j$ for all $i,j \in N$ with $i \neq j$. Additionally, we assume $\sum_{i \in K} a_i > z$ for at least one $K \subseteq N$, $k \geq t$. Again, we state what team structures are χ stable:

Theorem 3 *In monotone FS production games* $(N, p, \mathcal{P})$ *with* $a_i \neq a_j$ *for all* $i,j \in N$, $i \neq j$, *only partitions* $\mathcal{P} = \{G_1, \dots, G_q, G_{q+1}, \dots, G_m\}$ *with*

- $\sum_{i \in \{G_1, \dots, G_q\}} a_i < z$, $a_i < a_j$ for all $i \in \{G_1, \dots, G_q\}$ and $j \in \{G_{q+1}, \dots, G_m\}$
- $\left|G_{q+1}\right|, \dots, \left|G_s\right| = b \cdot t$, with $b \in \mathbb{N}^+$, $\sum_{i \in G_h} a_i > z$, $\sum_{i \in G_h} a_i - a_j < z$ for all $j \in G_h$, $G_h \in \{G_{q+1}, \dots, G_s\}$ and $a_i > a_j$ for $i \in G_l$, $j \in G_{l-1}$, $l = m, \dots, q+1$.

The proof is in the "Appendix". The last indent of the theorem implies that stable structures for the most productive employees are teams with size t. Analogous to the case with only two level of ability, stable groups are as homogenous as possible, meaning that the difference between the ability of the most able employee and the most unable employee are minimized. At some level of ability, it is necessary to form groups with size $b \cdot t > t$ to ensure the achievement of the aggregate ability z. Intuitively, the results reflect the following example. If the t most able employees in a firm are distributed over several components, they receive a lower χ wage than if these t employees are in one component. Teams that are too large have a similar effect. For example, if a team has a size of $b = 2$ and the t most productive employees can create a new team that is productive by splitting off, their χ wages increase. The range of group sizes that are stable, as in the case of only two levels of ability, does not exist. Thus, the employer has fewer opportunities to structure the teams in stable structures.

Our results fit in with results by Morelli and Park (2016). In their article, a higher inequality of employees in a firm leads to more teams (Proposition 2 in Morelli and Park (2016)). In our model, larger stable teams are conceivable if the workers are homogeneous (Theorem 1). With the introduction of heterogeneity, these possibilities decrease (Theorem 2). Finally, the introduction of completely different abilities leads to teams that are just as large as necessary (Theorem 3), and thus tends to lead to more teams for a fixed set of N.

5 Conclusion

In this paper, we enhance the production games approach introduced by Hiller (2019) with aspects from a recent article by Morelli and Park (2016). In addition to this technical development, we have obtained initial results for this new modeling of production conditions. We analyze how heterogeneity of abilities of employees influences the set of stable team structures in firms and, hence, determine the freedom of employers to structure the employees in teams.

First, we look at symmetric employees. For these employees, we prove that firm structures are stable with size t or any multiple of t. In the case of only two classes of abilities for employees, we state that usually employees are structured in homogenous teams. In addition, the groups of low productive employees tend to have a greater size or, in other words, the degree of freedom for the employer to decide on the size of groups is lower for less able employees since the lower bound of ability z is effective. In the case of n levels of ability, stable teams are as homogenous as possible. For the most productive employees, the size of stable teams is t. The employees with lower ability are stable structured in larger teams. The freedom of the employer to decide on the size of stable teams is reduced with respect to the scenario with only two abilities.

However, our analysis of production games is only a first step and many questions remain unanswered. For example, we do not consider different ts for the productive sectors in the firm. Another starting point for future research is the integration of interrelationships among the teams. With this integration, the existence of the firm as coordinating element between different teams could be modeled.

This paper could also inspire empirical research. One question could be whether one reason for firms to offer vocational training is to align the abilities of employees (reduce the number of levels of ability) and, hence, to increase the freedom for structuring the employees in teams.

Appendix

Proof of Theorem 1

The proof is analogous to Theorem 1 in Hiller (2019). The employees i in components $G_{s+1}, \dots, G_m$ obtain the χ wage

$$\begin{aligned}\chi_i(N,p,\mathcal{P}) &= \mathrm{Sh}_i(N,p) + \frac{b \cdot p(T) - b \cdot t \cdot \mathrm{Sh}_i(N,p)}{b \cdot t} \\ &= \frac{p(N)}{n} + \frac{t \cdot \bar{a} - t \cdot \frac{p(N)}{n}}{t} = \bar{a}.\end{aligned} \tag{6}$$

Adding one more employee to a team leads to:

$$\chi_i(N,p,\mathcal{P}) = \mathrm{Sh}_i(N,p) + \frac{b \cdot p(T) - b \cdot t \cdot \mathrm{Sh}_i(N,p) - \mathrm{Sh}_i(N,p)}{b \cdot t + 1}, \tag{7}$$

hence, the wage is reduced for the initial employees. Analogously one could argue in the case of reducing the number of employees in a team.

The χ wage for employees in groups $G_1, \ldots, G_s$ is zero. It is not possible to form an alternative partition $\mathcal{P}' = \{K, N \backslash K\}$ with $p(K,a,t,z) > 0$ and $\{G_1, \ldots, G_s\} \cap K \neq \varnothing$ without at least one employee from $G_{s+1}, \ldots, G_m$. The χ wage of this employee is unchanged, hence, $\mathcal{P}$ is χ stable.

Proof of Theorem 2

We start our proof with employees in teams $G_{s+2}, \ldots, G_m$. The employees $i \in H$ in these teams obtain

$$\chi_i(N,p,\mathcal{P}) = \mathrm{Sh}_i(N,p) + \frac{b \cdot t \cdot \overline{a_h} - b \cdot t \cdot \mathrm{Sh}_i(N,p)}{b \cdot t} = \overline{a_h}. \tag{8}$$

Exemplary, forming an alternative partition $\mathcal{P}'$ with $\left|G'_m \cap L\right| = 1$ changes the χ wage of $i \in G'_m$ to

$$\chi_i\left(N,p,\mathcal{P}'\right) = \mathrm{Sh}_i(N,p) + \frac{(b \cdot t - 1) \cdot \overline{a_h} + \overline{a_l} - (b \cdot t - 1) \cdot \mathrm{Sh}_i(N,p) - \mathrm{Sh}_j(N,p)}{b \cdot t}. \tag{9}$$

$$\begin{aligned} &\chi_i(N,p,\mathcal{P}) - \chi_i\left(N,p,\mathcal{P}'\right) \\ &\quad= \frac{\overline{a_h} \cdot b \cdot t - \mathrm{Sh}_i(N,p) \cdot b \cdot t - (b \cdot t - 1) \cdot \overline{a_h} - \overline{a_l} + (b \cdot t - 1) \cdot \mathrm{Sh}_i(N,p) + \mathrm{Sh}_j(N,p)}{b \cdot t} \\ &\quad= \frac{\overbrace{\overline{a_h} - \mathrm{Sh}_i(N,p)}^{>0} + \overbrace{\mathrm{Sh}_j(N,p) - \overline{a_l}}^{<0}}{b \cdot t} > 0 \end{aligned} \tag{10}$$

As described in Sect. 3, the absolute value of the first term is higher than the absolute value of the second term. Hence, the χ wage of i is reduced. Also, adding (removing) one employee to (from) a team from $G_{s+2}, \ldots, G_m$ reduces the χ wage of the existing employees (see proof to Theorem 1).

The employees $i \in H$ in G_{s+1} obtain a lower χ wage then employees in $G_{s+2}, \ldots, G_m$. It is not possible to form an alternative partition $\mathcal{P}' = \{G'_{s+1}, N \backslash G'_{s+1}\}$ with $\left|G'_{s+1} \cap H\right| > \left|G_{s+1} \cap H\right|$ without at least one employee from $G_{s+2}, \ldots, G_m$. The χ wage of this employee is unchanged or reduced. Again, adding one more employee to G_{s+1} reduces the χ wage of the existing employees.

The employees $j \in G_{q+1}, \ldots, G_s$ obtain $\chi_j(N,p,\mathcal{P}) = \overline{a_l}$. Forming an alternative partition $\mathcal{P}'$ with $\left|G'_s \cap H\right| = 1$ increases the χ wage of employees $j \in G'_s$. The χ wage of $i \in G'_s$ is reduced or constant with respect to $\mathcal{P}$.

The χ wage for employees in groups $G_1, \ldots, G_q$ is zero. It is not possible to form an alternative partition $\mathcal{P}' = \{K, N \backslash K\}$ with $p(K,a,t,z) > 0$ and

$\{G_1, \dots, G_q\} \cap K \neq \emptyset$ without at least one employee from $G_{q+1}, \dots, G_m$. The χ wage of this employee is unchanged, hence, $\mathcal{P}$ is χ stable.

Proof of Theorem 3

First we analyze employees in teams $G_{q+1}, \dots, G_m$. From proof of Theorem 2 we know that substituting an employee from G_h, $G_h \in \{G_{q+1}, \dots, G_m\}$, by an employee from team G_{h-1} lowers the χ wages of the remaining employees in G_h. Additionally, we know from Theorem 1 that adding/removing an employee to/from a team from $G_{q+1}, \dots, G_m$ reduces the χ wage of the existing employees. Next, we assume for simplification $|G_m| = t, |G_{m-1}| = t$ (we have $b = 1$ for these teams) with $\sum_{G_m} a_i > z$ and $\sum_{G_{m-1}} a_i > z$. An employee $i \in G_m$ obtains:

$$\chi_i(N, p, \mathcal{P}) = \text{Sh}_i(N, p) + \frac{\sum_{G_m} a_i - \sum_{G_m} \text{Sh}_i(N, p)}{t}. \tag{11}$$

Amalgamating both teams in an alternative partition $\mathcal{P}'$ with $G'_m = G_m \cup G_{m-1}$ gives:

$$\chi_i\left(N, p, \mathcal{P}'\right) = \text{Sh}_i(N, p) + \frac{\sum_{G_m} a_i + \sum_{G_{m-1}} a_j - \sum_{G_m} \text{Sh}_i(N, p) + \sum_{G_{m-1}} \text{Sh}_j(N, p)}{2 \cdot t} \tag{12}$$

As described after Eq. 4 we have

$$\sum_{G_m} a_i - \sum_{G_m} \text{Sh}_i(N, p) > \sum_{G_{m-1}} a_j - \sum_{G_{m-1}} \text{Sh}_j(N, p). \tag{13}$$

The χ wage of employee i is reduced in $\mathcal{P}'$ with respect to $\mathcal{P}$.The remarks on the employees in $G_1, \dots, G_q$ are analogous to Theorems 1 and 2, hence, $\mathcal{P}$ is χ stable.

Acknowledgements I am grateful to an anonymous referee for helpful comments on this paper.

Funding Open Access funding enabled and organized by Projekt DEAL.

References

Alonso-Meijide JM, Carreras F, Costa J, Garcia-Jurado I (2015) The proportional partitional Shapley value. Discrete Appl Math 187(1):1–11

Aumann RJ, Drèze JH (1974) Cooperative games with coalition structures. Int J Game Theory 3(4):217–237

Banerjee S, Konishi H, Sonmez T (2001) Core in a simple coalition formation game. Soc Choice Welfare 18(1):135–153

Barberà S, Beviá C, Ponsatí C (2015) Meritocracy, egalitarianism and the stability of majoritarian organizations. Games Econ Behav 91(1):237–257

Bogomolnaia A, Jackson MO (2002) The stability of hedonic coalition structures. Games Econ Behav 38(2):201–230

Casajus A (2008) On the stability of coalition structures. Econ Lett 100(2):271–274

Casajus A (2009) Outside options, component efficiency, and stability. Games Econ Behav 65(1):49–61

Damiano E, Li H, Suen W (2010) First in village or second in Rome. Int Econ Rev 51(1):263–288

Hart S, Kurz M (1983) Endogenous formation of coalitions. Econometrica 51(4):1047–1064

Hernández-Lamoneda L, Sánchez-Sánchez F (2010) Rankings and values for team games. Int J Game Theory 39(3):319–350

Hiller T (2019) Structure of teams—a cooperative game theory approach. Manag Decis Econ 40(5):520–525

Morelli M, Park I-U (2016) Internal hierarchy and stable coalition structures. Games Econ Behav 96(1):90–96

Owen G (1977) Values of games with a priori unions. In: Henn R, Moeschlin O (eds) Essays in mathematical economics & game theory. Springer, Berlin, pp 76–88

Piccione M, Razin R (2009) Coalition formation under power relations. Theor Econ 4(1):1–15

Shapley LS (1953) A value for n-person games. In: Kuhn HW, Tucker AW (eds) Contributions to the theory of games, vol 2. Princeton University Press, Princeton, pp 307–317

Wiese H (2007) Measuring the power of parties within government coalitions. Int Game Theory Rev 9(2):307–322