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CORRECTION

Correction: Logarithmic Bergman Kernel and Conditional Expectation of Gaussian Holomorphic Fields

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In this article, the statement and proof of Theorem 3.3 in Logarithmic Bergman kernel and Conditional expectation of Gaussian holomorphic fields has been corrected. Please find below the corrected Theorem 3.3.

The set-up for Theorem 3.3 is summarized below:

Let (M, L) be a polarized Kähler manifold of dimension m. We endow L with a Hermitian metric h with positive curvature. And we use $\omega = \frac{i}{2}\Theta_h$ as the Kähler form. By abuse of notation, we still use h to denote the induced metric on the kth power L^k. Then we have a Hermitian inner product on $H^0(M, L^k)$, defined by

$$\langle s_1, s_2 \rangle = \int_M h(s_1, s_2) \frac{\omega^m}{m!}$$

Let V be a smooth subvariety of M, and we denote by $\mathcal{H}_{k,V}$ the subspace of $H^0(M, L^k)$ consisting of sections that vanish along V. And $\mathcal{H}_{k,V}^{\perp}$ is the orthogonal complement. Let $n = \dim V$. The original statement of Theorem 3.3 is

Theorem 1.1 *The restriction map* $R : \mathcal{H}_{k,V}^{\perp} \to H^0(V, L^k)$ *has norm satisfying*

$$\| R \|^2 = O\left(\frac{1}{k^{m-n}}\right)$$

However both the proof and the application of Theorem 3.3 is for the following statement

The original article can be found online at https://doi.org/10.1007/s12220-020-00602-z.

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Theorem 1.2 *The inverse of the restriction map* $R : \mathcal{H}_{k,V}^{\perp} \to H^0(V, L^k)$ *has operator norm satisfying*

$$\| R^{-1} \|^2 = O\left(\frac{1}{k^{m-n}}\right)$$

And it was pointed out by Finski in [1] that there is a gap in the proof of Theorem 3.3. In the proof, we tried to use the Ohsawa–Takegoshi–Manivel extension theorem in the following way. We first endow the line bundle $L^k - K_M$ with the metric $\mathrm{e}^{-k\varphi} \otimes \mathrm{d}_M^V = \mathrm{e}^{-k\varphi+\kappa}$, where $\mathrm{d}V_M = \frac{\omega^m}{m!}$ is the volume form. Let $r(p) = \mathrm{dist}(x, V)$. Then we choose a nonnegative smooth function χ on $[0, \infty)$, which is concave and satisfies the following conditions:

(1) $\chi(x) = x$ for $x \leq \frac{(\log k)^2}{k}$;
(2) $\chi(x)$ is constant for $x \geq \frac{(10 \log k)^2}{k}$.

So $\chi(r^2)$ can be seen as a smooth function on M, which is constant away from V. Then we twist the metric on $L^k - K_M$ by $\mathrm{e}^{\beta_k \chi(r^2)}$ for β_k to be determined. We then claimed that we can choose $\beta_k = k - O\left(\frac{\log k}{\sqrt{k}}\right)$, which is a mistake, since in order to apply the Ohsawa–Takegoshi–Manivel theorem, we need to make sure that $k\varphi - \kappa - \beta_k \chi(r^2)$ is plurisubharmonic, which cannot be guaranteed if $\beta_k = k - O\left(\frac{\log k}{\sqrt{k}}\right)$. We can only have $\beta_k = k - O(\sqrt{k} \log k)$, which is not enough to prove Theorem 3.3. And this mistake makes the proof of Theorem 3.3 invalid.

In Theorem 4.4 of [1], Finski proved that there exist $C > 0$ (independent of k) such that for each $g \in H^0(V, L^k)$, there is $f \in H^0(M, L^k)$ such that for $f|_V = g$

$$\| f \|_{L^2(M)}^2 \leq C k^{n-m} \| g \|_{L^2(V)}^2$$

for k large enough. Since $R^{-1} g$ is the extension of g with minimal L^2-norm, this theorem clearly implies the corrected statement of our Theorem 3.3. So Theorem 3.3 is still valid and the rest of the article is not affected.

References

1. Finski, S.: Semiclassical Ohsawa–Takegoshi extension theorem and asymptotics of the orthogonal Bergman kernel. arXiv:2109.06851