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CORRECTION

Correction to: Uniformly Compressing Mean Curvature Flow

Wenhui Shi[1] · Dmitry Vorotnikov[1]

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In the sketch of the proof of Theorem 4 we claimed that a weak solution (ξ, κ, σ) can be obtained by following the same lines as in the proof of [1, Proposition 3.4 and Theorem 3]. However, an appropriate argument should be slightly different due to different boundary conditions. The aim of this addendum is to fill that gap.

More precisely, in [1] at the free end one has $\sigma^\epsilon(t,0) = \kappa^\epsilon(t,0) = 0$ for all $t \in [0,T]$. Therefore, the uniform (w.r.t. to ϵ) L^2 bound for $(\sigma^\epsilon, \kappa^\epsilon)$ follows directly from the Poincaré inequality and the uniform L^2 bound for $(\partial_s\sigma^\epsilon, \partial_s\kappa^\epsilon)$. In our case, the vanishing boundary conditions for σ^ϵ and κ^ϵ are no longer satisfied. We thus need some additional estimates to obtain the uniform L^2 bound.

We first estimate the spatial average $\overline{\sigma^\epsilon}(t) := \int_{\mathbb{S}^1} \sigma^\epsilon(s,t)\,\mathrm{d}s$. Indeed, an integration by parts and Cauchy–Schwarz yield

$$|\overline{\sigma^\epsilon}(t)| = \left|\int_{\mathbb{S}^1} \kappa^\epsilon \cdot \partial_s\xi^\epsilon\,\mathrm{d}s\right| = \left|-\int_{\mathbb{S}^1} \partial_s\kappa^\epsilon \cdot \xi^\epsilon\,\mathrm{d}s\right| \le \|\partial_s\kappa^\epsilon(t,\cdot)\|_{L^2(\mathbb{S}^1)}\|\xi^\epsilon\|_{L^2(\mathbb{S}^1)},\ \forall t \in [0,T].$$

Thus

$$\|\overline{\sigma^\epsilon}\|_{L^2([0,T])} \le \sup_{t\in[0,T]} \|\xi^\epsilon(t,\cdot)\|_{L^2(\mathbb{S}^1)}\|\partial_s\kappa^\epsilon\|_{L^2(Q_T)},$$

where the right-hand side is uniformly bounded (cf. [1, Proposition 3.1]). Thus from Poincaré inequality $\|\sigma^\epsilon - \overline{\sigma^\epsilon}\|_{L^2(Q_T)} \le C(T)\|\partial_s\sigma^\epsilon\|_{L^2(Q_T)}$ we obtain $\|\sigma^\epsilon\|_{L^2(Q_T)} \le C$ with C independent of ϵ.

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✉ Dmitry Vorotnikov
mitvorot@mat.uc.pt

Wenhui Shi
wshi@mat.uc.pt

1 CMUC, Department of Mathematics, University of Coimbra, 3001-501 Coimbra, Portugal

It remains to show that κ^ϵ is uniformly bounded in $L^2(Q_T)$. To this end, we note that from the definition of κ^ϵ one has $\partial_s \xi^\epsilon = \epsilon \kappa^\epsilon + \frac{\kappa^\epsilon}{\sqrt{\epsilon + |\kappa^\epsilon|^2}}$, whence

$$\sigma^\epsilon = \kappa^\epsilon \cdot \partial_s \xi^\epsilon = |\kappa^\epsilon| \left(\epsilon |\kappa^\epsilon| + \frac{|\kappa^\epsilon|}{\sqrt{\epsilon + |\kappa^\epsilon|^2}} \right) \geq |\kappa^\epsilon| |\partial_s \xi^\epsilon|.$$

Observing that

$$|\partial_s \xi^\epsilon| = \epsilon |\kappa^\epsilon| + \frac{|\kappa^\epsilon|}{\sqrt{\epsilon + |\kappa^\epsilon|^2}} \geq \epsilon + \frac{1}{\sqrt{1+\epsilon}} > 1$$

provided $|\kappa^\epsilon| \geq 1$, we infer that $|\kappa^\epsilon| \leq \sigma^\epsilon$ when $|\kappa^\epsilon| \geq 1$. Thus,

$$\int_{Q_T} |\kappa^\epsilon|^2 \, \mathrm{d}s\mathrm{d}t = \int_{\{(s,t)\in Q_T : |\kappa^\epsilon| < 1\}} |\kappa^\epsilon|^2 \, \mathrm{d}s\mathrm{d}t + \int_{\{(s,t)\in Q_T : |\kappa^\epsilon| \geq 1\}} |\kappa^\epsilon|^2 \, \mathrm{d}s\mathrm{d}t$$
$$\leq |Q_T| + \int_{Q_T} |\sigma^\epsilon|^2 \, \mathrm{d}s\mathrm{d}t \leq C,$$

where C does not depend on ϵ.

Reference

1. Shi, W., Vorotnikov, D.: The gradient flow of the potential energy on the space of arcs. Calc. Var. **58**, 59 (2019)

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