Correction to: On a Problem of Pichorides

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It is claimed in [1] that the proof of [1, Lemma 21] follows from an adaptation of the corresponding argument of [2], as presented in [4]. We recently realised that this is not the case and at this point we do not know whether [1, Lemma 21] holds as stated or not for $d > 1$. Therefore, [1, Lemma 21] should be disregarded for $d > 1$. Note that the one-dimensional case corresponds to [2, Theorem 3.1].

We remark that although [1, Lemma 21] was used in the proof of [1, Theorem 19], [1, Theorem 19] is correct via an alternative argument that we describe below. We also note that [1, Lemma 21] was not used in any other proof or argument in [1].

Correction of the proof of [1, Theorem 19]

We use the notation of [1].

The one-dimensional case is a consequence of [1, (7.5)] (for $d = 1$) combined with [2, Theorem 3.1].

To establish the two-dimensional case, we shall argue as in [3]. Note that it follows from [1, (7.5)] for $d = 1$ that there exists an absolute constant $C > 0$ such that for all $w \in A_2(\mathbb{R})$, $M, N \in \mathbb{N}$, and $\Lambda \subset \mathbb{N}$ with $\sigma_\Lambda < \infty$ one has

$$\left(\int_{\mathbb{R}} \sum_{j=1}^N [S_M^{(\Lambda)}(g_j)(x)]^2 w(x) dx \right)^{1/2} \leq C \sigma_\Lambda^{1/2} [w]_{A_2}^{3/2} \left(\int_{\mathbb{R}} \sum_{j=1}^N |g_j(x)|^2 w(x) dx \right)^{1/2} \quad (1)$$

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for all Schwartz functions $g_1, \dots, g_N$. It thus follows from (1) and [2, Theorem 3.1] that

$$\left(\int_{\mathbb{R}} \left(\sum_{j=1}^N [S_M^{(\Lambda)}(h_j)(x)]^2 \right)^{p/2} dx \right)^{1/p} \leq A \sigma_{\Lambda}^{1/2} (p-1)^{-3/2} \left(\int_{\mathbb{R}} \left(\sum_{j=1}^N |h_j(x)|^2 \right)^{p/2} dx \right)^{1/p} \quad (2)$$

for all $p \in (1, 2)$ and $h_1, \dots, h_N \in L^p(\mathbb{R})$, where $A > 0$ is an absolute constant.

Fix $p \in (1, 2)$, $N_1, N_2 \in \mathbb{N}$, and $\Lambda_1, \Lambda_2 \subset \mathbb{N}$ with $\sigma_{\Lambda_1}, \sigma_{\Lambda_2} < \infty$ and let f be a Schwartz function on $\mathbb{R}^2$. Let I denote the one-dimensional identity operator. For $y \in \mathbb{R}$, using (2) for $\Lambda = \Lambda_1$, $M = N_1$, $N = N_2$, and $h_j(x) = (I \otimes \Delta_j^{(\Lambda_2)})(f)(x, y)$, $j \in \{1, \dots, N_2\}$, one gets

$$\left(\int_{\mathbb{R}} [S_{N_1, N_2}^{(\Lambda_1, \Lambda_2)}(f)(x, y)]^p dx \right)^{1/p} \leq A \sigma_{\Lambda_1}^{1/2} (p-1)^{-3/2} \left(\int_{\mathbb{R}} [S_{N_2}^{(\Lambda_2)}(f)(x, y)]^p dx \right)^{1/p}, \quad (3)$$

where $S_{N_2}^{(\Lambda_2)}$ acts on the second variable. Hence, if we raise (3) to the p -th power, integrate in the second variable, use Fubini's theorem, and then employ (2) for $\Lambda = \Lambda_2$, $M = N_2$, $N = 1$, we deduce that

$$\|S_{N_1, N_2}^{(\Lambda_1, \Lambda_2)}(f)\|_{L^p(\mathbb{R}^2)} \leq A^2 \sigma_{\Lambda_1}^{1/2} \sigma_{\Lambda_2}^{1/2} (p-1)^{-3} \|f\|_{L^p(\mathbb{R}^2)}. \quad (4)$$

The desired estimate follows from (4) and a standard limiting argument. The case $d \geq 3$ is obtained in a completely analogous way.

Remark In view of the correction presented above, to establish [1, Theorem 19] one only needs [1, (7.5)] for $d = 1$.

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