Correction to: Infinite Dimensional Holomorphic Homogeneous Regular Domains

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There is a gap in the proof of the last assertion of Lemma 2.3 in the paper [2] which appeared in Volume 30 of this journal. The arguments given there are only valid in finite dimensions since the boundary $\partial\Omega$ need not be weakly closed in infinite dimension. In what follows, we present a complete proof including the infinite dimensional case and amend the statement of the last assertion in this lemma for a bounded *convex* domain Ω .

Lemma 2.3 was only used in the proof of Theorem 5.3, which can be replaced by the amended lemma below without affecting the rest of the original proof of the theorem.

Amended Lemma 2.3 *Let (p_k) be a sequence in a bounded convex domain Ω in an isomorph V of a Hilbert space such that $\lim_{k \rightarrow \infty} \sigma_\Omega(p_k) = 0$. Then there is a subsequence (p_j) of (p_k) such that*

$$\lim_{j \rightarrow \infty} d(p_j, \partial\Omega) = 0.$$

Further, there is a sequence (p'_j) in Ω such that $\lim_{j \rightarrow \infty} \sigma_\Omega(p'_j) = 0$, and for each j , there exists a boundary point $q_j \in \partial\Omega$ with $\|p'_j - q_j\| = d(p'_j, \partial\Omega)$.

The original article can be found online at <https://doi.org/10.1007/s12220-019-00145-y>.

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Proof We only need to prove the last assertion. Let (p_j) satisfy

$$d(p_j, \partial\Omega) \rightarrow 0 \quad \text{as } j \rightarrow \infty.$$

We will make use of a result in [1, Theorem 3.2], which states that in a reflexive Banach space V , if the complement $V \setminus C$ of a non-empty closed set C in V is convex, then C is almost proximal, in other words, there is a dense G_δ set A in $V \setminus C$ such that for each $x \in A$, there is a point $z \in C$ satisfying

$$\|x - z\| = d(x, C).$$

The given Banach space V is reflexive. We apply the above result to the set $C = V \setminus \Omega$, which is almost proximal. By continuity of the squeezing function σ_Ω , there is an open neighbourhood N_j of p_j such that $\sigma_\Omega(x) < 2\sigma_\Omega(p_j)$ for all $x \in N_j$ and for each j . By density of A , one can find $p'_j \in A \cap N_j$ for which there exists $q_j \in C$ satisfying

$$\|p'_j - q_j\| = d(p'_j, C) = d(p'_j, V \setminus \Omega) \leq d(p'_j, \partial\Omega)$$

where the last inequality holds because the boundary $\partial\Omega$ is contained in $V \setminus \Omega$.

If $q_j \notin \partial\Omega$, then $q_j \notin \overline{\Omega}$ since $q_j \notin \Omega$. Since all points in the open convex set Ω are internal points [3, p. 413], the line segment $\{p'_j + \alpha(q_j - p'_j) : 0 \leq \alpha \leq 1\}$ joining p'_j and q_j must intersect the boundary $\partial\Omega$ at some point $\omega = p'_j + \beta(q_j - p'_j) \in \partial\Omega$ with $0 < \beta < 1$. It follows that

$$\|p'_j - q_j\| \leq d(p'_j, \partial\Omega) \leq \|p'_j - \omega\| = \beta\|p'_j - q_j\| < \|p'_j - q_j\|$$

which is impossible. Hence we have $q_j \in \partial\Omega$ and $\|p'_j - q_j\| = d(p'_j, \partial\Omega)$.

Finally, $\sigma_\Omega(p'_j) < 2\sigma_\Omega(p_j)$ for all j implies $\lim_j \sigma_\Omega(p'_j) = 0$. $\square$

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