Correction to: On the Relationship Between D'Angelo q -Type and Catlin q -Type

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John D'Angelo brought to the authors' attention the following counterexample to the claim in Corollary 2.11 of [1] that for any ideal $\mathcal{I}$ of holomorphic germs the infimum in the definition of the D'Angelo q -type is achieved and equal to the generic value: $\mathcal{I} = (z_1^3 - z_2 z_3, z_2^2) \subset \mathcal{O}_0$ [2]. This example is due to John D'Angelo and Martino Fassina; see also [3]. Here the D'Angelo 1-type $\Delta_1(\mathcal{I}, 0) = +\infty$, the D'Angelo 2-type

$$\Delta_2(\mathcal{I}, 0) = \inf_{\{a_1, a_2, a_3\}} \Delta_1 \left((z_1^3 - z_2 z_3, z_2^2, a_1 z_1 + a_2 z_2 + a_3 z_3), 0 \right) = 3,$$

whereas

$$\text{gen.val}_{\{a_1, a_2, a_3\}} \Delta_1 \left((z_1^3 - z_2 z_3, z_2^2, a_1 z_1 + a_2 z_2 + a_3 z_3), 0 \right) = 4.$$

The corresponding claim for real hypersurfaces found in Proposition 2.10 of [1] is likewise invalidated by writing the sum of squares hypersurface corresponding to $\mathcal{I}$ in $\mathbb{C}^4$ given by $r = \text{Re}\{z_4\} + \|z_1^3 - z_2 z_3\|^2 + \|z_2^2\|^2$.

As the results in [1] are proven with respect to the generic value, we introduce the following definition:

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Definition 0.1 Let $2 \leq q \leq n$. If $\mathcal{I}$ is an ideal in $\mathcal{O}_{x_0}$,

$$\tilde{\Delta}_q(\mathcal{I}, x_0) = \text{gen.val}_{\{w_1, \dots, w_{q-1}\}} \Delta_1\left((\mathcal{I}, w_1, \dots, w_{q-1}), x_0\right),$$

where the generic value is taken over all non-degenerate sets $\{w_1, \dots, w_{q-1}\}$ of linear forms in $\mathcal{O}_{x_0}$, $(\mathcal{I}, w_1, \dots, w_{q-1})$ is the ideal in $\mathcal{O}_{x_0}$ generated by $\mathcal{I}, w_1, \dots, w_{q-1}$, and Δ_1 is the D'Angelo 1-type. Likewise, if M is a real hypersurface in $\mathbb{C}^n$ and $x_0 \in M$,

$$\tilde{\Delta}_q(M, x_0) = \text{gen.val}_{\{w_1, \dots, w_{q-1}\}} \Delta_1\left((\mathcal{I}(M), w_1, \dots, w_{q-1}), x_0\right),$$

where $(\mathcal{I}(M), w_1, \dots, w_{q-1})$ is the ideal in $C_{x_0}^\infty$ generated by all smooth functions $\mathcal{I}(M)$ vanishing on M along with $w_1, \dots, w_{q-1}$.

By substituting the D'Angelo q -type Δ_q by $\tilde{\Delta}_q$, all the results and proofs [1] remain true.

References

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