Correction to: A Generalized Fejér's Theorem for Locally Compact Groups

Huichi Huang¹

Published online: 29 January 2019
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Correction to: J Geom Anal (2018) 28:909–920
<https://doi.org/10.1007/s12220-017-9847-7>

As pointed out by Hanfeng Li, we fix a gap in Theorem 1.2.

There are three issues in the paper.

- (1) Let G be a locally compact group with the unit e_G and a fixed left Haar measure μ . Let ν be the right Haar measure given by $\nu(A) = \mu(A^{-1})$ for every Borel subset A of G .

Theorem 1.2 is stated as follows.

Theorem (A generalized Fejér's theorem)

Consider a locally compact group G with a fixed left Haar measure μ and the corresponding right Haar measure ν . Let $\{F_\theta\}_{\theta \in \Theta}$ be an approximate identity of $L^1(G)$. Assume that there exists a local partition $\{A_1, A_2, \dots, A_k\}$ of G such that

$$\lim_{\theta} \int_{A_j} F_\theta(y) d\mu(y) = \lambda_j \text{ for every } 1 \leq j \leq k.$$

For an f in $L^\infty(G)$, if there exists x in G such that $\lim_{\substack{y \rightarrow e_G \\ y \in A_j}} f(y^{-1}x)$ (denoted by $f(x, A_j)$) exists for every $1 \leq j \leq k$, then

$$\lim_{\theta} F_\theta * f(x) = \sum_{j=1}^k \lambda_j f(x, A_j).$$

The original article can be found online at <https://doi.org/10.1007/s12220-017-9847-7>.

✉ Huichi Huang
huanghuichi@cqu.edu.cn

¹ College of Mathematics and Statistics, Chongqing University, Chongqing 401331, People's Republic of China

Moreover if $\lim_{\theta} \sup_{y \in \mathcal{N}^c} |F_{\theta}(y)| = 0$ for any neighborhood $\mathcal{N}$ of e_G , then for every f in $L^1(G, \nu)$ (or $L^\infty(G)$) such that each $f(x, A_j)$ exists for some x in G , we have

$$\lim_{\theta} F_{\theta} * f(x) = \sum_{j=1}^k \lambda_j f(x, A_j).$$

In the proof of Theorem 1.2, there are two corrections:

- (a) The sentence “Now assume that $\lim_{\theta} \sup_{y \in \mathcal{N}^c} |F_{\theta}(y)| = 0$ for any neighborhood $\mathcal{N}$ of e_G and f is in $L^1(G)$ such that each $f(x, A_j)$ exists for some x in G ” in the line -5 on page 5 is changed to “Now assume that $\lim_{\theta} \sup_{y \in \mathcal{N}^c} |F_{\theta}(y)| = 0$ for any neighborhood $\mathcal{N}$ of e_G and f is in $L^1(G, \nu)$ such that each $f(x, A_j)$ exists for some x in G ”.
- (b) the last identities are changed to:

$$\limsup_{\theta} \left| \int_{\mathcal{N}^c} F_{\theta}(y) f(y^{-1}x) d\mu(y) \right| \leq \left[\limsup_{\theta} \sup_{y \in \mathcal{N}^c} |F_{\theta}(y)| \right] \|f\|_{L^1(G, \nu)} = 0.$$

- (2) Statement of Corollary 2.2 is changed to the following:

Corollary Given any approximate identity $\{F_{\theta}\}_{\theta \in \Theta}$ of $L^1(G)$ and local partition $\{A_1, \dots, A_k\}$ of G , if for an f in $L^\infty(G)$, every $f(x, A_j)$ exists, then there exists a subnet Θ_1 of Θ such that every $\lim_{\theta \in \Theta_1} \int_{A_j} F_{\theta}(y) d\mu(y)$ exists (denoted by $\lambda_j(\Theta_1)$) and

$$\lim_{\theta \in \Theta_1} F_{\theta} * f(x) = \sum_{j=1}^k \lambda_j(\Theta_1) f(x, A_j).$$

Moreover if $\lim_{\theta \in \Theta_1} \sup_{y \in \mathcal{N}^c} |F_{\theta}(y)| = 0$ for every neighborhood $\mathcal{N}$ of e_G , then for every f in $L^1(G, \nu)$ (or $L^\infty(G)$) such that every $f(x, A_j)$ exists for some x in G , we have

$$\lim_{\theta \in \Theta_1} F_{\theta} * f(x) = \sum_{j=1}^k \lambda_j(\Theta_1) f(x, A_j).$$

- (3) Before Corollary 3.5, add a definition.

For $k = (k_1, \dots, k_d)$ in $\{0, 1\}^d$, define

$$J_k = \prod_{l=1}^d J_{k_l}$$

with $J_0 = (-\infty, 0)$ and $J_1 = [0, \infty)$.

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