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CORRECTION

# Correction to: The total intrinsic curvature of curves in Riemannian surfaces

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In the paper [2], in the statements of the main results, Theorems 1–9 and Propositon 3, one has to assume in addition that the curve $\mathbf{c}$ is rectifiable.

The main point is that the equivalence in formula (2.7), namely:

$$\mathrm{TC}_{\mathcal{M}}(\mathbf{c}) < \infty \iff \mathrm{TC}(\mathbf{c}) < \infty$$

holds true for rectifiable curves $\mathbf{c}$, whereas it is false in general that if $\mathrm{TC}_{\mathcal{M}}(\mathbf{c}) < \infty$, then $\mathrm{TC}(\mathbf{c}) < \infty$. If one e.g. takes a curve in $\mathcal{S}^2$, the unit sphere in $\mathbb{R}^3$, that winds around an equator infinitely many times, its total intrinsic curvature is zero but its length and total curvature are both infinite.

Our mistake goes back to a flaw that we recently found in [1, Thm. 6.3.1], where Alexandrov-Reshetnyak erroneously stated that if the geodesic turn of a spherical curve is finite, then its spatial turn is also finite. This is true if the spherical diameter of the curve is smaller than a dimensional constant $\delta_0$. In this case, in fact, for polygonal curves in $\mathcal{S}^2$ they obtain the inequality $\mathbf{k}^*(P) \leq \pi + 2\mathbf{k}_{\mathcal{S}^2}(P)$.

Therefore, their statement holds true provided that the curve can be divided in a finite number of arcs each one with spherical diameter smaller than $\delta_0$. However, the latter property is false, in general, if the curve fails to be rectifiable, as the previous example shows.

Dealing with rectifiable curves $\mathbf{c}$ in $\mathcal{M}$, in fact, by the smoothness and compactness of $\mathcal{M}$, the normal curvature of the geodesic arcs of $\mathcal{M}$ is uniformly bounded, and hence we recover the nontrivial implication $\Rightarrow$ in the previous equivalence by arguing as in the model case $\mathcal{M} = \mathcal{S}^2$ considered in [1].

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For that reason, all the main results in [2] hold true for rectifiable curves with finite total intrinsic curvature.

## References

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