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CLASSICAL AND QUANTUM PLASMAS

On quantum electrodynamic processes in plasmas interacting with strong lasers

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Abstract
In the realm of laser-plasma interactions, if the laser intensity is strong enough, quantum effects play a significant role. Due to the large separation between the length scale at which quantum electrodynamic processes form and both the typical separation between particles in a plasma and the wavelength of optical lasers, it is possible in many situations of interest to take into account quantum electrodynamic processes in a fairly straightforward way. If one considers plasmas of increasingly large densities, the presence of many particles can render potentially unavoidable a full quantum treatment of the dynamics. Here, two kinds of multi-particle effects are discussed within strong-field Quantum Electrodynamics. First, we show how a correct description of coherence effects in the radiation emitted by a two-electron wave packet in a strong laser field requires a quantum treatment also if the quantum nonlinearity parameter of the system is much smaller than unity. Secondly, we indicate that at solid-state densities the presence of several particles within the formation region of radiation by an electron in a strong electromagnetic field may alter the emission probability itself, such that in general it is not possible to disentangle collective effects from quantum effects.

Keywords Quantum electrodynamics · Strong laser fields · Collective quantum effects · Quantum coherent emission

1 Introduction

The rapid development of optical lasers with high intensities made it possible to envisage experiments where plasmas or particle beams interact with electromagnetic fields of strengths which were unseen in a laboratory (Ehlotzky et al. 2009; Di Piazza et al. 2012), except in the proximity of highly-charged atomic nuclei. There have already been such experiments which employed laser peak intensities on the order of $10^{21}\ \mathrm{W/cm^2}$ (Cole et al. 2018; Poder et al. 2018), and optical lasers able to deliver pulses with intensities higher than $10^{23}\ \mathrm{W/cm^2}$ are currently being built [ELI, XCELS]. According to the present knowledge, optical lasers in this intensity regime can induce nonlinear electrodynamical effects, both classical and quantum, which have not been fully tested yet. In general, it is extremely challenging to study the interaction of matter with fields of such a high intensity, and typically there are many approximations that one needs to use depending on the particular system being studied. In this work, we will assess some limits of the current treatment of quantum electrodynamical effects in plasma physics and, in particular, we will estimate the density at which the tight interplay between collective and quantum effects cannot be easily disentangled.

In the context of laser–matter interaction, a laser can be considered "strong" if it can accelerate matter initially at rest up to relativistic speeds in a time which is short when compared with the field's oscillation period; when applying this criterion to electrons,[1] it allows us to define a Lorentz- and gauge-invariant parameter

$$\xi = \frac{|e|\mathcal{E}}{m\omega c}, \tag{1}$$

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[1] Notice that although in this work we will mainly refer to electrons, similar considerations can be made for other particles.

where c is the speed of light in vacuum, $e < 0$ is the electron's charge and m its mass, $\mathcal{E}$ is the field's peak strength, and ω is the central frequency of oscillation of the laser. Since for $\xi \gtrsim 1$ relativistic effects become increasingly important, the trajectories of electrons interacting with a strong laser field depend nonlinearly on the field's amplitude. It is worth noticing that aside of this classical interpretation of ξ, by multiplying and dividing by $\hbar$ the right hand side of Eq. (1) one can write ξ as

$$\xi = |e|\mathcal{E}\frac{\hbar}{mc}\frac{1}{\hbar\omega}, \quad (2)$$

i.e. ξ is an estimate of the work that the field can perform on the electron in a reduced Compton wavelength $\lambda_C = \hbar/mc = 3.9 \times 10^{-11}$ cm in units of the laser photons' energy $\hbar\omega$. Equation (2) suggests that for $\xi \gg 1$ processes where multiple photons are exchanged between the laser field and electrons are non-negligible.

To estimate the influence of quantum effects in the dynamics of an electron moving through a laser field, it can be shown (Ritus 1985; Berestetskii et al. 1982; Baier et al. 1998) that the quantity that governs them is the ratio between the electric field of the laser in the rest frame of the electron and $\mathcal{E}_{cr} = m^2c^3/\hbar|e| = 1.3 \times 10^{16}$ V/cm, the critical field of Quantum Electrodynamics (QED); this ratio is given by the Lorentz- and gauge-invariant parameter (Ritus 1985; Di Piazza et al. 2012)

$$\chi = \frac{(pk)}{m\omega}\frac{\mathcal{E}}{\mathcal{E}_{cr}}, \quad (3)$$

where $k^\mu = (\omega/c, \boldsymbol{k})$ is the central laser's four-wave-vector, p^μ is the electron's momentum, and we write the product between any pair of four-vectors a^μ and b^μ as (ab), i.e., $(ab) = g_{\mu\nu}a^\mu b^\nu$ (the metric tensor is assumed to be $g_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$).

In the presence of extremely strong electromagnetic fields ($\xi \gg 1$), it is impractical (or even fundamentally impossible) to study QED at the required perturbative order; thus, non-perturbative approaches are needed; this regime of QED is called Strong-Field QED (SF-QED). If the electromagnetic field can be considered as a classical background, and if the Dirac equation can be solved exactly in the presence of such background, it is convenient to perform calculations in the Furry picture (Furry 1951), where the total electromagnetic field is split into a classical background plus a quantized part, and the states over which perturbation theory is performed are solutions of the Dirac equation in the classical background. For electromagnetic plane waves, the states which are adopted in the Furry picture are the so-called Volkov states (Volkov 1935; Berestetskii et al. 1982).

Studying SF-QED effects in complicated systems, such as plasmas, is a nontrivial effort; in fact, whereas for SF-QED calculations it is desirable to have highly symmetric electromagnetic field backgrounds where the Dirac equation can be solved, plasmas are typically composed of a large number of charged particles whose dynamics are in general not tractable analytically, not even classically. Because of their complexity, plasmas are mostly studied with the aid of computer simulations; of the various numerical schemes which have been devised, arguably the most widely used ones are the so-called "Particle-in-Cell" (PIC) codes (Dawson 1983). In a PIC simulation, the matter content of the plasma is discretized in an appropriate number of (electrically charged) superparticles which move according to the Lorentz equation at discrete time steps, whereas a mean-field approximation of the electromagnetic field is calculated at each time step on a grid by summing the contributions given by neighboring superparticles (Birdsall et al. 1991). Notice that in such classical simulations there is not any upper bound on the maximum frequency at which radiation can be emitted, although the fact that the electromagnetic field is discretized on a grid implies that its oscillations modes with wavelength smaller than the grid size cannot be accounted for (see Shannon 1949), and the process by which radiation is emitted is continuous; this is in striking contrast with the assumptions of QED, where energy between charged particles is exchanged in the form of photons, i.e. discrete energy quanta, and four-momentum conservation implies that there is a cutoff for the highest frequency at which a charged particle can emit a photon. Also predicted by QED there are purely quantum processes, such as pair production (Oppenheimer and Plesset 1933; Breit and Wheeler 1934; Bethe and Heitler 1934), QED cascades (see Di Piazza et al. 2012 and references therein), and photon–photon scattering (Euler and Kockel 1935; Ritus 1985; Di Piazza et al. 2012), which have no classical analogue. If the quantum nonlinearity parameter χ associated with the electrons in the plasma is not much smaller than unity, it is expected that at least some quantum effects are indeed important in the dynamics of plasma.

It is reasonable, at least in some regimes, to extend PIC simulations of plasmas interacting with intense laser fields to include some SF-QED effects (Gonoskov et al. 2015); in order to do so it is crucial to note that for $\xi \gg 1$ and for ultra-relativistic particles the typical length scale over which SF-QED phenomena are formed is much smaller than the laser wavelength (Baier et al. 1998; Baier and Katkov 2005), and this makes it possible to consider the quantum processes as taking place instantaneously, with rates which depend only on the local value of the electromagnetic field and which can be calculated analytically in SF-QED within the local constant field approximation (LCFA). Notice that this approximation can break down in the low-frequency portion of the radiation (Di Piazza et al. 2018), even when $\xi \gg 1$.

Since quantum processes in a PIC simulation are then calculated on top of each individual classical trajectory of each superparticle, they cannot take into account quantum processes involving multiple particles in the initial state; this will be discussed in Sect. 2, with a focus on the simplest case of a two-electron wave packet interacting with an electromagnetic plane wave, and in Sect. 3 we will derive an estimate for the densities at which we expect multiparticle effects to influence the probabilities of quantum processes like the emission of a photon.

Below, natural units with $\hbar = c = 1$ and $\alpha = e^2 \approx 1/137$ are employed.

2 Multi-particle effects in nonlinear single compton scattering

In Classical Electrodynamics, the energy emitted by a system of identical charges in accelerated motion can scale with the square of their number (Jackson 1999); this phenomenon is called coherent emission and, generally speaking, it takes place if during their motion the distance among the charges is smaller than the wavelength of the emitted radiation. In this section, we will show how quantum effects can introduce a frequency cutoff (without any classical analogue) above which coherent emission does not occur. We will see that the SF-QED predictions for the emission of radiation can depend non-trivially on the number of particles which participate in the process and this leads to qualitative differences with respect to the classical predictions (Angioi and Di Piazza 2018).

In SF-QED, the lowest order process by which a charged particle in a strong field can emit radiation is called Nonlinear Single Compton Scattering (NSCS). In NSCS, an electron which is dressed by a background field emits a single photon and recoils (see Ritus 1985; Krajewska et al. 2017; Di Piazza et al. 2012 and references therein). Since in order to find the above-mentioned dressed states it is necessary to be able to solve the Dirac equation in the background field, laser fields are usually approximated with plane waves, i.e. they are given by a four-vector potential which depends only on $\phi = (nx)$, where $n^\mu = k^\mu/\omega$. Thus, we will approximate the four-vector potential of a laser pulse linearly polarized along the x-axis and propagating along the positive z-direction with $\mathcal{A}^\mu_L(\phi) = (0, \boldsymbol{\mathcal{A}}^\mu_L(\phi)) = \mathcal{A}^\mu \psi_L(\phi)$, where $\mathcal{A}^\mu = (0, -\mathcal{E}/\omega, 0, 0)$, $n^\mu = (1, 0, 0, 1)$, and $\psi_L(\phi)$ is a smooth function with compact support; this approximation is valid as long as the electron collides nearly head-on with the laser field and almost at its focus, provided that the transverse excursion of the electron is much smaller than the laser waist size, which occurs if ξmc^2 is much smaller than the initial energy of the electron (Landau and Lifshitz 1975; Berestetskii et al. 1982).

Although NSCS rates are typically calculated for electrons having initially a well-defined momentum, one can consider a more general initial state by computing NSCS rates for electron wave packets. In general, one would expect to find in the spectrum of the emitted photon some quantum interference effects between the different components of the wave packet; this would be analogous to the classical interference which is present in the radiation emitted by a charge distribution. As it was shown in Angioi et al. (2016), however, NSCS rates for single-electron wave packets in a plane wave can be obtained by averaging emission spectra of electrons having initially a well-defined momentum over the initial momentum distribution of the wave packet. This has to do with the fact that, due to some conservation relations (namely, the on-shell conditions and the conservation of three light-cone components of the four-momentum between the initial and final state), by measuring the final state of the process it is possible in principle to know exactly the initial state. This is equivalent to a which-path information in the double slit experiment and it prevents quantum interference from taking place. Note that were the background field not a plane wave but a focused field the above-mentioned conservation relations would not be valid anymore, and in general interference can take place (for more information about NSCS in focused fields, see Di Piazza 2014, 2015, 2017; Heinzl and Ilderton 2017).

The situation is different when one considers initial states containing multiple electrons (Angioi and Di Piazza 2018), such as:

$$|\Psi\rangle = \frac{1}{\sqrt{\mathcal{N}}} \int \frac{d^3p_1}{(2\pi)^3\sqrt{2\varepsilon_1}} \frac{d^3p_2}{(2\pi)^3\sqrt{2\varepsilon_2}} \times \rho_1(\boldsymbol{p}_1)\rho_2(\boldsymbol{p}_2) a^\dagger_{s_1}(\boldsymbol{p}_1) a^\dagger_{s_2}(\boldsymbol{p}_2)|0\rangle. \tag{4}$$

Here, $\mathcal{N}$ is a normalization factor such that $\langle\Psi|\Psi\rangle = 1$, $\varepsilon_i = \sqrt{m^2 + \boldsymbol{p}_i^2}$, $\rho_1(\boldsymbol{p}_1)$ and $\rho_2(\boldsymbol{p}_2)$ are scalar functions, and $a^\dagger_s(\boldsymbol{p})$ is an operator which creates an electron with spin s and momentum $\boldsymbol{p}$ when acting on the vacuum of the Dirac field $|0\rangle$.

Although when computing NSCS with the initial state given by Eq. (4), the plane-wave conservation relations still apply, depending on the characteristics of the initial wave packet it can be fundamentally impossible to know which electron actually emitted the photon, and this prevents the reconstruction of the initial state from the final state; because of this, it is possible to observe coherent emission in the quantum spectra (Angioi and Di Piazza 2018). This "kinematical indistinguishability" condition for coherent emission in the quantum case is more restrictive than the one in the classical case, so that quantum effects can forbid coherent emission even at frequencies where classical mechanics predicts it to occur.

As an example of multi-particle effects in NSCS rates, we show in Fig. 1 the energy emission spectrum from a

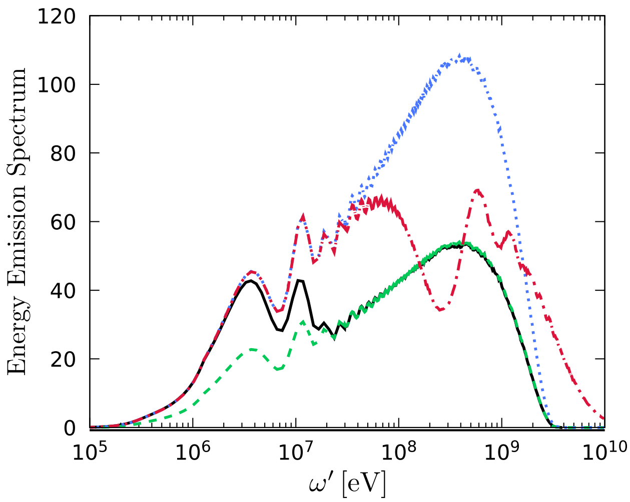

Fig. 1 Dimensionless energy emission spectrum for the two-electron wave packet described in the text (solid black line), compared with the classical prediction for an analogous ensemble (dash-dotted red line). As a reference, we show also the single-electron spectrum multiplied by two (dashed green line) and four (dotted blue line)

two-electron wave packet where each electron has initially the same momentum distribution ($|\rho_1(\boldsymbol{p})| = |\rho_2(\boldsymbol{p})|$), which we chose to be Gaussian in the laboratory frame, with average value of the momentum $\bar{\boldsymbol{p}} = (0, 0, -4)$ GeV, transverse standard deviation $\sigma_x = \sigma_y = 10$ keV, and longitudinal standard deviation $\sigma_z = 10$ MeV. Between the two single-particle wave packets, we must[2] introduce a small displacement (i.e., a phase such that $\rho_1(\boldsymbol{p}) = \rho_2(\boldsymbol{p})e^{i\boldsymbol{p}\cdot\boldsymbol{D}}$), which we chose as $\boldsymbol{D} = (50, 50, 1)\,10^{-8}\,\text{eV}^{-1} = (98.7, 98.7, 1.97)$ fm.

The electrons interact with a two-cycle laser pulse of invariant strength $\xi = 13$, linearly polarized along the x-axis, propagating along the positive z direction, with a $\sin^4$ temporal envelope, null carrier-envelope phase, and carrier frequency $\omega = 1.55$ eV, i.e., $\psi_L(\phi) = \sin^4(\omega\phi/4)\sin(\omega\phi)$ for $0 \leq \omega\phi \leq 4\pi$ and $\psi_L(\phi) = 0$ elsewhere.

The main features to be understood in Fig. 1 are the discrepancies between the NSCS rates calculated using a two-electron initial state of the form given by Eq. 4, shown as a solid black line, with an analogous classical ensemble (for more details, see Angioi and Di Piazza 2018), shown as a dash-dotted red line.

As it can be seen in Fig. 1, which shows the NSCS rate of emission of photons with frequency ω', the classical and quantum spectra are quite different; this is expected in the high-energy tail of the spectra because, whereas in SF-QED an electron can convert at most all of its kinetic energy into radiation, which in turn implies a cutoff on the maximum frequency of the emitted photon in NSCS, in classical mechanics there is nothing of this sort. Notice that in this regime the quantum nonlinearity parameter has the value $\chi \approx 0.6$, which by itself suggests that indeed quantum effects should be important.

Moreover, if we were to calculate the emission spectrum of each electron individually and then sum the two spectra we would obtain the dashed green line of Fig. 1, that is, the total emission spectrum would be given by the emission spectrum of a single electron multiplied by two, the number of electrons participating in the process. As it is clear from Fig. 1, treating the emission process as if coming from each electron independently for frequencies smaller than $\omega'_Q \approx 1$ MeV would underestimate the correct quantum prediction by a factor of two, because it would neglect the contributions coming from quantum coherence in the process. Similarly, Classical Electrodynamics predicts that the emission should be coherent over a certain frequency range (for $\omega' \lesssim 4 \times 10^7$ eV), but overestimates the cutoff frequency until which coherent emission takes place by about an order of magnitude. This discrepancy is due to the kinematical indistinguishability condition introduced in this section; notice that it can be realized even for $\chi \ll 1$ (see Angioi and Di Piazza 2018).

In summary, there are physical scenarios where both Classical Electrodynamics and SF-QED give inaccurate predictions if the emission by several particles is treated as stemming from each particle independently. In these cases, it is necessary to perform calculations with SF-QED on wave packets. This poses a great challenge because there is no simple way to include multi-particle wave packets in PIC simulations and, even if there was, there would still remain the obstacle that the full quantum state of particles in a plasma is hardly known even in principle.

3 Quantum statistical effects on basic SF-QED processes

In the previous section, we have seen how quantum effects can alter the coherence properties of radiation emitted by two electrons.

In the present section, we would like to estimate the impact of the presence of several particles on the basic QED processes occurring in an intense laser field: the emission of a photon by an electron/positron, indicated as NSCS in the previous section and the transformation of a photon into an electron–positron pair (nonlinear Breit–Wheeler pair production). It is useful to introduce the concept of "formation region" of a QED process (see, e.g., Ritus 1985) by considering the representative example of NSCS. It is convenient to start by considering the emission of radiation by a classical electron in a background

[2] This is because, being electrons indistinguishable fermions, they cannot occupy the same single-particle state.

electromagnetic field. The (longitudinal) formation region is the region of the electron trajectory where the radiation along a given direction originates or is formed (see also Jackson 1999). This concept can be also defined quantum mechanically within a semiclassical approach, where the motion of the electrons can still be described classically (Baier et al. 1998). Interestingly, the semiclassical approach is particularly suitable for problems regarding the interaction of ultrarelativistic electrons and strong optical laser fields because the De Broglie wavelength of the electrons ($\sim \lambda_C/\gamma$, with $\gamma \gg 1$ the Lorentz factor of the electron) is much smaller than the wavelength of the laser field (~ 1 μm) (Baier et al. 1998). Now, in the ultrarelativistic case, one can write the Lorentz equation for the electron velocity $\boldsymbol{\beta}$, with $|\boldsymbol{\beta}| \approx 1$ in an external electromagnetic field $(\boldsymbol{E}, \boldsymbol{B})$ in the form

$$m\gamma \frac{d\boldsymbol{\beta}}{dt} = e\boldsymbol{\beta} \times (\boldsymbol{B} - \boldsymbol{\beta} \times \boldsymbol{E}) \tag{5}$$

where terms proportional to $1/\gamma^2 \ll 1$ are neglected. This equation indicates that the motion of an ultrarelativistic electron instantaneously resembles a circular motion with an "effective" angular frequency $\omega_0 = |e(\boldsymbol{B} - \boldsymbol{\beta} \times \boldsymbol{E})|/m\gamma$ and a curvature radius $\rho_0 = 1/\omega_0$. Now, relativistic considerations indicate that the radiation is instantaneously emitted by an ultrarelativistic charge along the forward direction and within a cone of angular aperture of the order of $1/\gamma$ (Jackson 1999). In this way, the radiation along a fixed direction receives contributions from all the parts of the trajectory where the electron's velocity points along that direction, within an angular aperture of the order of $1/\gamma$. Each part, i.e. each formation length will be of the order of $l_\parallel \sim \rho_0/\gamma$. By estimating $\rho_0 \sim m\gamma/|e|\mathcal{E}$ for a laser field of amplitude $\mathcal{E}$, we obtain $l_\parallel \sim \lambda/\xi$, where $\lambda = 2\pi/\omega$ is the central laser wavelength. Now, since the electron moves in the same direction of the emitted radiation, the duration of the radiation pulse along a specific direction is of the order of $\tau_\gamma \sim (1-\beta)l_\parallel \approx \rho_0/2\gamma^3$. This result allows one to estimate the typical emitted photon energy $\omega_\gamma \sim 1/\tau_\gamma \sim 2\gamma^2|e|\mathcal{E}/m \sim \chi\varepsilon$, with ε being the electron energy, and the typical transverse momentum transfer $p_\perp \sim \omega_\gamma/\gamma \sim m\chi$ ("transverse" refers to the direction of the instantaneous velocity of the electron). It should be pointed out that quantum mechanically the estimate $\omega_\gamma \sim \chi\varepsilon$ is found to be valid for $\chi \lesssim 1$ because the energy of the emitted photon cannot exceed the electron energy (Baier et al. 1998). Applying Heisenberg uncertainty principle, one can then conclude that the transverse formation length is $l_\perp \sim \lambda_C/\chi$ (Di Piazza 2017). It can be shown by exploiting the crossing symmetry of QED that the longitudinal and the transverse formation lengths of nonlinear Breit–Wheeler pair production have the same expressions $l_\parallel \sim \lambda/\xi$ and $l_\perp \sim \lambda_C/\chi_\gamma$, where χ_γ reads as in Eq. (3) but with the four-momentum of the electron being replaced by the four-momentum of the incoming photon (Di Piazza 2017).

Now, it is useful to recall here the so-called Landau–Pomeranchuk–Migdal effect (Landau 1953; Migdal 1956), where interference effects between adjacent scattering sites within the longitudinal formation length lead to a suppression of the radiation emitted by an ultrarelativistic electron when it crosses a solid medium. The physical origin of the Landau–Pomeranchuk–Migdal effect is essentially the fact that interference effects impede the complete formation of the radiation process, inducing a suppression of the radiation itself. In the present case, we would expect that the presence of many particles within the formation length may in principle alter the probability of emission with respect to its value, calculated within the "single-particle" approach as usually assumed within SF-QED (Di Piazza et al. 2012). By estimating the radiation formation "volume" as $V \sim l_\perp^2 l_\parallel \sim \lambda\lambda_C^2/\xi\chi^2$ and by using the typical values $\xi \sim 10$ and $\chi \sim 1$, one would require a quite large density of the order of 10^{26} cm^{-3}. This value can be, however, reduced already by two orders of magnitudes by considering a more moderate value of the quantum parameter $\chi \sim 0.1$, recalling that quantum effects are already sizable for such values of χ (Neitz and Di Piazza 2013). Another observation is in order, which can decrease the required density at which multiparticle effects may affect in particular the emission of radiation. In fact, as it is also expected physically, it is known that the formation of radiation also depends on the frequency of the radiation itself and it is longer for smaller frequencies (Jackson 1999; Baier et al. 1998). Indeed, one can show that for frequencies much smaller than $\omega_c = 2\gamma^3/\rho_0 \sim \chi\varepsilon$, the radiation at a frequency ω' is emitted up to angles $(\omega_c/\omega')^{1/3}$ times larger than $1/\gamma$, with respect to the instantaneous forward direction (Jackson 1999). Since the longitudinal formation length is proportional to the radiation aperture angle, we can conclude that for the emission of radiation of frequency $\omega' \ll \omega_c$, multiparticle effects may become important at densities $(\omega'/\omega_c)^{1/3}$ smaller than $\xi\chi^2/\lambda\lambda_C^2$. Now, at a laser intensity of about 5×10^{20} W/cm^2 and for $\omega = 1.55$ eV, we have that $\xi \approx 10$ and, for an electron with energy 1 GeV, it is $\chi \approx 0.1$. With such numerical parameters one obtains $\omega_c \approx 100$ MeV and the emission of ultraviolet photons with $\omega' \sim 100$ eV may be altered by multiparticle effects at densities of the order of 10^{22} cm^{-3}.

4 Conclusion

In conclusion, in the first part of the paper we have shown that under certain experimentally relevant physical circumstances, both single-particle classical and quantum electrodynamics give inaccurate predictions if the coherence properties of the radiation by two- or many-electrons bunches are studied. A full quantum approach, where the probability of emission is evaluated by employing the complete wave packet of the incoming electrons, is required. Our results also indicate that a quantum approach might be required to correctly describe interference effects in the emission by many particles, even if the quantum parameter is much smaller than unity.

In the second part of the paper, we have instead estimated the background density of particles required to potentially alter the probability of photon emission (and pair production) calculated via single-particle SF-QED methods. For typical parameters $\xi \sim 10$ and $\chi \sim 1$ exceedingly large densities of the order of 10^{26} cm^{-3} are obtained. However, in the case of soft-photon emission (frequencies of the order of 100 eV) and for $\chi \sim 0.1$ the above value can be reduced by about four orders of magnitude becoming smaller than solid-state densities.

The processes we considered in Sects. 2 and 3 involve multiple particles; thus, one would no longer be justified in using the single-particle quantum transition rates currently employed in PIC codes. Furthermore, the coherent enhancement of the spontaneous emission rates discussed in Sect. 2 is highly dependent on the quantum state of the particles involved in the process; therefore, a full quantum treatment of the plasma seems unavoidable to take this phenomenon into account in numerical computations. In the case of the effect discussed in Sect. 3, however, even the calculation of the transition rates at the fundamental QED level should be reformulated within an approach of non-equilibrium quantum field theory (Berges 2004). Nevertheless, the results presented in Sect. 3 suggest that at typical plasma densities smaller than 10^{22} particles/cm^3 the multi-particle QED effects we described can be ignored, and the present PIC implementation should be adequate in this respect.

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