The influence of vaccination on the control of JE with a standard incidence rate of mosquitoes, pigs and humans

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Abstract

In this article, a nonlinear mathematical model used for the impact of vaccination on the control of infectious disease, Japanese encephalitis with a standard incidence rate of mosquitoes, pigs and humans has been planned and analyzed. During the modeling process, it is expected that the disease spreads only due to get in touch with the susceptible and infected class only. It is also assumed that due to the effect of vaccination, the total human population forms a separate class and avoids contact with the infection. The dynamical behaviors of the system have been explored by using the stability theory of differential equations and numerical simulations. The local and global stability of the system for both equilibrium states under certain conditions has been studied. We have set up a threshold condition in the language of the vaccine-induced reproduction number $R(\alpha_1)$, which is fewer than unity, the disease dies in the absence of the infected population, otherwise, the infection remains in the population. Furthermore, it is found that vaccine coverage has a substantial effect on the basic reproduction number. Also, by continuous efforts and effectiveness of vaccine coverage, the disease can be eradicated. It is also found a more sensitive parameter for the transmission of Japanese encephalitis virus by using sensitivity analysis. In addition, numerical results are used to investigate the effect of some parameters happening the control of JE infection, for justification of analytical results.

Keywords JEV · Vaccine-induced reproduction number · Sensitivity analysis · Standard incidence · Global stability

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1 Introduction

Japanese encephalitis (JE) is an illness of the brain caused through the Japanese encephalitis virus (JEV) and is transmitted from the pig population to the human population through bloodsucking insects *Culex* mosquitoes, Centers for Disease Control, (CDC). The JEV is maintained in a natural enzootic cycle between reservoir population (especially pigs), mosquitoes and humans. The transmission of JEV, for the most part, happens in the country in horticultural zones, where floodwater system and rice development are related [1–4]. In the areas where JEV is endemic the annual incident range from 1 to 10 cases per 10,000 person per year affected children are under 16 years of age [5]. The time duration of infection to illness in infected humans is about 5–15 days and initial symptoms are fever, headache and vomiting [3,6–8]. Most humans who are infected with only a few symptoms or have no symptoms and JE can be more deadly in advanced stages, National Vector Borne Disease Control Program, (NVBDCP).

The first human case of JEV was registered in Japan in 1935 and in India in 1955 [3,9]. Unfortunately, there is no specific treatment available that can cure Japanese encephalitis, including the management of some basic supportive medical care and complications in medicine. Those who experience a more severe form of the disease will have to undergo treatment at the hospital. The finding of JE disease ought to be performed by utilizing a JEV-explicit immunoglobulin M (IgM) counteracting agent catch ELISA for serum and cerebrospinal liquid (CSF). JEV-explicit IgM can be estimated in the CSF of most patients by four days after the commencement of side effects and in serum by seven days after inception. The prevention of JE is mainly based on the control of mosquitoes and the vaccination of the human population. To control the mosquito population, some methods like using insecticides, reducing the number of breeding sites on your property, by killing the mosquito larvae where the insect's breed and bed nets are used in endemic regions. Mathematical modeling of infectious disease has a long history for understanding the effects of vaccination. Through the speedy rate of vaccine development, medical science has saved millions of lives. Smallpox, influenza, polio and lots of supplementary diseases are conquering examples in this regard [10]. In the present life, it is bizarre and uncommon for a youngster not to get an antibody. An individual can get accessible immunizations for a malady common in his general vicinity. Some sickness immunizations are as of now created and anybody can take the antibody if the specific malady is in danger for it. Currently, several groups of vaccines available in the world like inactivated Vero cell culture-derived JE vaccine, SA 14-14-2 live attenuated JE vaccine, Ixiaro JE vaccine, etc. But Ixiaro is the only licensed JE vaccine available in the United States [11]. It was avowed in march 2009 for the use of the human population aged more than 17 years. Later on, May 2013, avowed for the use of children whose aged lies between 2 months to 16 years [1,6,12]. The essential inoculation plans for Ixiaro is two portions intramuscularly administrated on 0–28 days. For youngsters from 2 months to 2 years, each dose is 0.25 ml, for grownups and kids for more than 3 years, each dose is 0.5 ml. Prior, In India, the mouse brain-derived JE antibody was utilized, yet the productivity was insufficient to meet the national prerequisite [13,14]. The SA 14-14-2 live attenuated JE vaccine has been utilized in China for over 20 years with

an effectiveness of 80% for single-dose and 97.5% for a two-doses routine with 1 year interim among youngsters under 15 years old [11,14]. Thus, the effectiveness of the JE vaccine is more than 90%, but the time duration of protection with the JE vaccine is not clear, but its effectiveness appears to decrease over time [1,6].

Japanese encephalitis spreading fast in all states of India except Meghalaya, Himachal Pradesh, Jammu and Kashmir, Dadra, Daman, Punjab, Diu, Nagar Haveli, Gujarat, Lakshadweep and Sikkim [1]. This disease is seasonal, most human cases reported in north India in May–October. The highest rate of this disease reported from the states of Uttar Pradesh, Bihar, Andhra Pradesh, Assam, Kerala, Goa, Haryana, Karnataka, Tamil Nadu and West Bengal [5,9]. In modeling of such infectious disease, many authors have already worked on epidemic models to minimize the infection of JE and maximize the rate of recovered population [15]. In recent years, mathematical models have been used to understand the interplay between various traditional measures such as the mobility of epidemic outbreaks [16,17] and vaccination [5,18–20] quarantine, behavior change [21].

When the immunization is accessible, a general inquiry emerges; How to allocate and implement this immunization. Unquestionably, we can't inoculate all people to control the sickness from the human population on account of social and moral issues, significant expenses, and so forth. In this manner, the legitimate conveyance and usage systems are critical to controlling JEV using vaccines. The adequacy of such a vaccination assignment procedure can be resolved through the investigation of a mathematical model. Right now, we need to illuminate this significant issue and would like to give a valuable rule to the arrangement producer.

To appropriately actualize vaccination battles in the human population, the recognized and insightful idea may be to inoculate individuals having a spot with explicit social affairs or territories vulnerable to contamination. On account of mosquito dependent infectious diseases, the vaccination can be an important tool to help control the spread of infection in such infectious disease and encephalitis is one of them. It is pointed out that in the above model, vaccination has been studied without considering the effective role of the mosquito population which depends on the human population-related factor. In this work, we construct and analyze a modified mathematical model of De et al. [9], by incorporating the effect of vaccination on the spread of JE, which is the most effective control strategy against JE. In addition, we use a realistic standard incidence rate for direct contact between susceptible and infectious. We might want to express that we don't consider any structure in the model analyzed down right now, these references can be utilized to improve the present work.

Kar et al. [19] have been studied the SIR epidemic mathematical model to minimize the infection rate and to maximize the recovered rate of the human population by using vaccination program and optimal control strategy to SIR model. Singh et al. [22] have found that the development rate of the mosquito population relies upon the increasing density of ecological discharge by the human population. Also, the growth rate of the mosquito populations can be controlled by controlling the increasing density of ecological discharge. Iannelli et al. [23] build an SIS mathematical model to contemplate the effect of inoculation by expecting the immunization age, super-infection, immaculate immunization and furthermore talked about the asymptotic behavior of the equilibriums state. Raul Prealta et al. [24], developed an SVIR mathematical model

to study the effect of the vaccine by considering the immune loss rate, which depends on the vaccine-age under the condition that during vaccination individuals have the same immunity.

In the present investigation, modifications to the SVIR model for JEV-affected humans and SI model for JEV-affected pigs [9] are considered by reducing the vaccinated compartment of a pig. The main objective is to analyze the effect of vaccination on the human population for the prevention of Japanese encephalitis with a standard incidence rate of mosquitoes and pigs. In the case of mosquitoes there is nothing to explain because it is obvious that vaccination, medicine is not possible for that compartment. Hence we consider susceptible, vaccinated, infected and recovered (SVIR) compartment for humans and susceptible, infected (SI) compartment of mosquitoes and pigs (cf. the Fig. 1). Disease transmission of the susceptible human population depends on the standard incidence rate of infected mosquitoes. Similarly, JEV spreads among the susceptible mosquito population when it comes in contact with infected human and pig populations. In this way, JE transmission for the susceptible mosquito population should be dependent on the rate of standard incidence on infected pig and human populations.

The organization of the manuscript is as follows: In the next Sect. 2, notations and assumptions along with full explanation are provided. In Sect. 3, the mathematical formulation of the JE endemic model with a schematic diagram of the transmission of JEV puts in place and derive the model equations. We provide the region of biologically feasible solutions in Sect. 4, together with their positivity and boundedness. Dynamical behavior (existence of equilibrium states, basic reproduction number and stability) of the framework has been discussed in Sect. 5. In Sect. 6, vaccine-induced reproduction number and model parameters sensitivity investigation has been discussed to study the effects of controllable parameters of the JE model on the basic reproduction number and infected human population in Sect. 7 while the simulations and discussions are presented in Sect. 8 and conclusions in Sect. 9.

2 Notations and assumptions

In our proposed model, it is assumed that the JEV starts its life cycle from the pig population and then spreads into the human population through *Culex* mosquitoes. It is also assumed that all the newborn's pigs and mosquitoes are susceptible, but newborn humans are not susceptible. In addition, JEV transmitted only in susceptible individuals. Many variables and constants(parameters) symbols have utilized in our proposed mathematical epidemiological model, including four compartments for humans, two compartments for pigs and mosquitoes, which are given in Tables 1 and 2.

3 Model formulation

In our present investigation, we consider susceptible, vaccinated, infected and recovered (SVIR) model for human population, susceptible, infected (SI) models for mosquito and pig population. The model sub-divides the total human population

Table 1 Model variables and their descriptions

Symbol	Descriptions
$S_h(t)$	Susceptible human population (people who are in danger of getting tainted by disease) at any time t
$I_h(t)$	Infected human population (the people who can transmit disease) at any time t
$V_h(t)$	Vaccinated human population (the human who was vaccinated and are currently immune, however not totally) at any time t
$R_h(t)$	Recovered human population (the human who is now immune but not completely) at any time t
$S_m(t)$	Susceptible mosquito population (able to compress the disease) at any time t
$I_m(t)$	Infected mosquito population (equipped for transmitting the disease to the human population) at any time t
$S_p(t)$	Susceptible pig population (pigs who are able to contract the JE disease) at any time t
$I_p(t)$	Infected pig population (pigs who are equipped for transmitting the JE illness to susceptible mosquitoes) at any time t

N_h into sub-population of susceptible humans ($S_h(t)$), vaccinated humans ($V_h(t)$), infected humans ($I_h(t)$) and recovered humans ($R_h(t)$), so that $N_h = S_h(t) + V_h(t) + I_h(t) + R_h(t)$. The total mosquito population N_m is divided into sub-population of susceptible mosquitoes ($S_m(t)$) and infected mosquitoes ($I_m(t)$), so that $N_m = S_m(t) + I_m(t)$. Also, the total pig population N_p is sub-divided into susceptible pigs $S_p(t)$ and infected pigs ($I_p(t)$) such that $N_p = S_p(t) + I_p(t)$. In SVIR model, humans are recruited at vaccinated birth rate $\alpha_1 \lambda_h$ and unvaccinated birth rate $(1 - \alpha_1) \lambda_h$. It is assumed that the JE spreads among the $S_h(t)$ due to contribution of the $I_m(t)$ [5]. Hence, the transmission of JE disease to susceptible humans should be a standard incidence rate of an infected mosquito population and it is in the form of $B \beta_{mh} \frac{I_m}{N_h} S_h$. Similarly, the JE transmitted among susceptible mosquitoes ($S_m(t)$) due to contact with infected pigs and human populations. Hence, the transmission of JE disease to susceptible mosquitoes should be standardized incidence rate of infected pig and human population and it is in the form of $\left(C \beta_{pm} \frac{I_p}{N_p} + B \beta_{hm} \frac{I_h}{N_p} \right) S_m$. The interaction between $I_m(t)$ and $S_h(t)$, $I_p(t)$ and $S_m(t)$ are assumed to be rated B , C respectively. Ultimately, the JE spreads among $S_p(t)$ due to contact with $I_m(t)$. Hence, the JE transmission to susceptible pigs should be a standard incidence rate of infected mosquitoes and it is in the form of $C \beta_{mp} \frac{I_m}{N_p} S_p$. Also, we consider the proportion of the infected human population become recovered naturally and with medicine control at a rate of τ_1 and τ_2 respectively. After that, some of the recovered individuals go to susceptible class at rate f_1 . Humans in each compartment are assumed to die with a natural mortality μ_h and die with the disease at a rate of λ . In the SI model, the recruitment rate of mosquitoes and pigs is λ_m and λ_p respectively. The natural mortality of the mosquito population in each compartment is μ_m . Also, the pig populations in each compartment are assumed to die with natural mortality of μ_p . Further, we consider that in the whole modeling process the disease transmission is possible only in susceptible individual class, for other classes JE spreads among them after they become susceptible class only. So, we do not consider direct JE transmission from vaccinated

Table 2 Model parameters, its description and their values used for numerical calculation

Symbol	Description	Value	Source
α_1	The rate at which humans are vaccinated	0.50	[9]
λ_h	The rate at which humans are recruited	1.14	Assumed
λ_m	The rate at which susceptible mosquitoes are generated	1.51	Assumed
λ_p	The rate at which susceptible pigs are recruited	0.405	Assumed
μ_p	Natural mortality of pigs	0.01	Assumed
μ_h	Natural mortality of humans	0.001538	Assumed
μ_m	Natural mortality of mosquitoes	0.03	Assumed
β_1	The rate at which vaccinated human goes to susceptible class of human	0.001	[9]
B	Average biting rate on susceptible human by infected mosquitoes per day	0.7	[9]
C	Average biting rate on susceptible pig by infected mosquitoes per day	0.9	[9]
λ	Disease-related death rate of infected human population	0.0222	[9]
τ_1, τ_2	The rate at which infected human becomes susceptible without and with medicine respectively	0.5	[9]
$1/\eta_h$	Mean viremic period of JEV (in days)	4	[1]
δ	Effectiveness of medicine on the human population	0.10	[9]
β_{mh}	JEV transmission potential rate from infected mosquitoes to susceptible humans	0.2	[9]
β_{pm}	JEV transmission potential rate from an infected pigs to susceptible mosquitoes	0.3	[9]
β_{mp}	JEV transmission potential rate from an infected mosquitoes to the susceptible pigs	0.3	[9]
α_2	The rate at which infected pigs go to the susceptible class of pigs by medicine	0.5	[9]
u_1	Drug control of pig population	0.5	[9]
N_h	The absolute human population at time t	2500	Assumed
N_m	The absolute mosquito population at time t	12,500	Assumed
N_p	The absolute pig population at time t	50	Assumed
f_1	Rate at which infected human goes to susceptible class at time t	0.02	Assumed

Fig. 1 Schematic diagram representing dynamics for the transmission of JE disease in humans, pigs and mosquitoes

humans class to infected human class. The schematic diagram for the dynamics of transmission of JE has appeared in Fig. 1.

The mathematical model is described by following system of ODEs:

$$\frac{dS_h}{dt} = (1 - \alpha_1)\lambda_h + f_1 R_h + \beta_1 V_h - \left(B\beta_{mh} \frac{I_m}{N_h} + \mu_h \right) S_h \quad (1)$$

$$\frac{dV_h}{dt} = \alpha_1 \lambda_h - (\beta_1 + \mu_h) V_h \quad (2)$$

$$\frac{dI_h}{dt} = B\beta_{mh} \frac{I_m}{N_h} S_h - (\tau_1 \eta_h + \tau_2 \delta + \mu_h + \lambda) I_h \quad (3)$$

$$\frac{dR_h}{dt} = (\tau_1 \eta_h + \tau_2 \delta) I_h - (f_1 + \mu_h) R_h \quad (4)$$

$$\frac{dS_m}{dt} = \lambda_m - \left(C\beta_{pm} \frac{I_p}{N_p} + B\beta_{hm} \frac{I_h}{N_h} + \mu_m \right) S_m \quad (5)$$

$$\frac{dI_m}{dt} = \left(C\beta_{pm} \frac{I_p}{N_p} + B\beta_{hm} \frac{I_h}{N_h} \right) S_m - \mu_m I_m \quad (6)$$

$$\frac{dS_p}{dt} = \lambda_p + \alpha_2 u_1 I_p - C\beta_{mp} \frac{I_m}{N_p} S_p - \mu_p S_p \quad (7)$$

$$\frac{dI_p}{dt} = C\beta_{mp} \frac{I_m}{N_p} S_p - \alpha_2 u_1 I_p - \mu_p I_p \quad (8)$$

For our simplicity we can re-write Eqs. (1)–(8) in notations form as-

$$\frac{dS_h}{dt} = a_{11} + f_1 R_h + \beta_1 V_h - (a_{13} I_m + \mu_h) S_h \quad (9)$$

$$\frac{dV_h}{dt} = a_{21} - a_{22} V_h \quad (10)$$

$$\frac{dI_h}{dt} = a_{13} I_m S_h - A I_h \quad (11)$$

$$\frac{dR_h}{dt} = a_{12} I_h - (f_1 + \mu_h) R_h \quad (12)$$

$$\frac{dS_m}{dt} = \lambda_m - (a_{31} I_p + a_{32} I_h + \mu_m) S_m \quad (13)$$

$$\frac{dI_m}{dt} = (a_{31} I_p + a_{32} I_h) S_m - \mu_m I_m \quad (14)$$

$$\frac{dS_p}{dt} = \lambda_p + a_{41} I_p - a_{42} I_m S_p - \mu_p S_p \quad (15)$$

$$\frac{dI_p}{dt} = a_{42} I_m S_p - d I_p \quad (16)$$

with initial conditions $S_h(0) = S_{h0} > 0$, $V_h(0) = V_{h0} \geq 0$, $I_h(0) = I_{h0} \geq 0$, $R_h(0) = R_{h0} > 0$, $S_m(0) = S_{m0} > 0$, $I_m(0) = I_{m0} \geq 0$, $S_p(0) = S_{p0} > 0$, $I_p(0) = I_{p0} \geq 0$.

where,

$$a_{11} = (1 - \alpha_1) \lambda_h, \quad a_{12} = \tau_1 \eta_h + \tau_2 \delta, \quad a_{13} = \frac{B \beta_{mh}}{N_h}, \quad a_{22} = \beta_1 + \mu_h, \quad a_{31} = \frac{C \beta_{pm}}{N_p}$$

$$a_{21} = \alpha_1 \lambda_h, \quad a_{32} = \frac{B \beta_{hm}}{N_h}, \quad a_{41} = \alpha_2 u_1, \quad a_{42} = \frac{C \beta_{mp}}{N_p}, \quad d = a_{41} + \mu_p,$$

$$A = a_{12} + \mu_h + \lambda.$$

4 Positivity and boundedness of solutions

Now, we present the positivity and boundedness of solutions of the system of Eqs. (1)–(8). For the system of Eqs. (1)–(8) to be biologically significant and well-proposed, essentially we have to show that all the solutions with non-negative initial conditions will remain positive for all time $t > 0$. In the theoretical study of disease transmission, the boundedness of the system measures that the system is biologically legitimate and well-behaved [16]. Biologically, validation of the model will be appeared by the accompanying theorem.

Theorem 4.1 *If $S_h(0)$, $V_h(0)$, $I_h(0)$, $R_h(0)$, $S_m(0)$, $I_m(0)$, $S_p(0)$ and $I_p(0)$ are non negative, then $S_h(t)$, $V_h(t)$, $I_h(t)$, $R_h(t)$, $S_m(t)$, $I_m(t)$, $S_p(t)$ and $I_p(t)$ are also non-negative for all time $t > 0$. Moreover*

$$\lim_{t \rightarrow \infty} \sup N_h(t) \leq \frac{\lambda_h}{\mu_c}, \quad \lim_{t \rightarrow \infty} \sup N_m(t) \leq \frac{\lambda_m}{\mu_m} \text{ and } \lim_{t \rightarrow \infty} \sup N_p(t) \leq \frac{\lambda_p}{\mu_p}$$

where $\mu_c = \min\{\mu_h, \lambda\}$, $N_h(t) = S_h(t) + V_h(t) + I_h(t) + R_h(t)$, $N_m(t) = S_m(t) + I_m(t)$, $N_p(t) = S_p(t) + I_p(t)$

Furthermore,

$$\text{If } N_h(0) \leq \frac{\lambda_h}{\mu_c} \text{ then } N_h(t) \leq \frac{\lambda_h}{\mu_c}. \text{ Similarly } N_m(0) \leq \frac{\lambda_m}{\mu_m} \text{ then } N_m(t) \leq \frac{\lambda_m}{\mu_m}$$

$$\text{and } N_p(0) \leq \frac{\lambda_p}{\mu_p} \text{ then } N_p(t) \leq \frac{\lambda_p}{\mu_p}.$$

The proof is excluded. The feasible region for the system of Eqs. (1)–(8) is given by

$$D = D_h \times D_m \times D_p \subset \mathbb{R}_+^4 \times \mathbb{R}_+^2 \times \mathbb{R}_+^2$$

where,

$$D_h = \left\{ (S_h, V_h, I_h, R_h) \in \mathbb{R}_+^4 : S_h + V_h + I_h + R_h \leq \frac{\lambda_h}{\mu_c} \right\}$$

$$D_m = \left\{ (S_m, I_m) \in \mathbb{R}_+^2 : S_m + I_m \leq \frac{\lambda_m}{\mu_m} \right\}$$

and

$$D_p = \left\{ (S_p, I_p) \in \mathbb{R}_+^2 : S_p + I_p \leq \frac{\lambda_p}{\mu_p} \right\}$$

is positive invariant.

5 Dynamical behavior of the system

5.1 Equilibrium states and basic reproductive number

Clearly, the system (9)–(16) has following two equilibrium states, namely
(1) *Disease or infection-free steady state* $E_0 = (\bar{S}_h, \bar{V}_h, \bar{I}_h, \bar{R}_h, \bar{S}_m, \bar{I}_m, \bar{S}_p, \bar{I}_p)$.
where,

$$\bar{S}_h = \frac{a_{11}a_{22} + a_{21}\beta_1}{a_{22}\mu_h}, \quad \bar{V}_h = \frac{a_{21}}{a_{22}}, \quad \bar{I}_h = 0, \quad \bar{R}_h = 0, \quad \bar{S}_m = \frac{\lambda_m}{\mu_m}, \quad \bar{I}_m = 0,$$

$$\bar{S}_p = \frac{\lambda_p}{\mu_p}, \quad \bar{I}_p = 0.$$

The existence of equilibrium E_0 is obvious. It demonstrates that if the model has an infected population, then the equilibrium magnitude of susceptible human, mosquito

and pig population will reach the value $\bar{S}_h = \frac{a_{11}a_{22}+a_{21}\beta_1}{a_{22}\mu_h}$, $\bar{S}_m = \frac{\lambda_m}{\mu_m}$, $\bar{S}_p = \frac{\lambda_p}{\mu_p}$ respectively and vaccinated human population will remain at its equilibrium $\bar{V}_h = \frac{a_{21}}{a_{22}}$.

Let us define threshold parameter R_{01} , as

$$R_{01} = \frac{C^2 \beta_{pm} \beta_{mp} \lambda_m \lambda_p}{N_p^2 \mu_m^2 \mu_p (\alpha_2 u_1 + \mu_p)}$$

Biologically, R_{01} is the average number of secondary Culex mosquitoes produced by single Culex mosquitoes [25]. To maintain the Culex mosquitoes in the environment, R_{01} must be greater than one. To measure the disease transmission potential in such a population, the basic reproduction number R_0 can be established by using the concept of the next-generation matrix approach method. Since the system of Eqs. (9)–(16) are consistent, therefore the conditions for new infection and transition terms are given by the matrices

$$F = \begin{pmatrix} \frac{\partial F_1}{\partial I_h} & \frac{\partial F_1}{\partial I_m} & \frac{\partial F_1}{\partial I_p} \\ \frac{\partial F_2}{\partial I_h} & \frac{\partial F_2}{\partial I_m} & \frac{\partial F_2}{\partial I_p} \\ \frac{\partial F_3}{\partial I_h} & \frac{\partial F_3}{\partial I_m} & \frac{\partial F_3}{\partial I_p} \end{pmatrix}; \quad V = \begin{pmatrix} \frac{\partial V_1}{\partial I_h} & \frac{\partial V_1}{\partial I_m} & \frac{\partial V_1}{\partial I_p} \\ \frac{\partial V_2}{\partial I_h} & \frac{\partial V_2}{\partial I_m} & \frac{\partial V_2}{\partial I_p} \\ \frac{\partial V_3}{\partial I_h} & \frac{\partial V_3}{\partial I_m} & \frac{\partial V_3}{\partial I_p} \end{pmatrix}$$

where,

$$F_1 = a_{13} I_m S_h, \quad F_2 = (a_{31} I_p + a_{32} I_h) S_m, \quad F_3 = a_{42} I_m S_p, \quad V_1 = A I_h, \quad V_2 = \mu_m I_m \\ V_3 = d I_p$$

then,

$$F = \begin{pmatrix} 0 & a_{13} \bar{S}_h & 0 \\ a_{32} \bar{S}_m & 0 & a_{31} \bar{S}_m \\ 0 & a_{42} \bar{S}_p & 0 \end{pmatrix}; \quad V = \begin{pmatrix} A & 0 & 0 \\ 0 & \mu_m & 0 \\ 0 & 0 & d \end{pmatrix}$$

For infection-free equilibrium state, the next generation matrix (FV^{-1}) is calculated as:

$$FV^{-1} = \begin{pmatrix} 0 & \frac{a_{13} \bar{S}_h}{\mu_m} & 0 \\ \frac{a_{32} \bar{S}_m}{A} & 0 & \frac{a_{31} \bar{S}_m}{d} \\ 0 & \frac{a_{42} \bar{S}_p}{\mu_m} & 0 \end{pmatrix}$$

The potent characteristic root of matrix FV^{-1} is the basic reproduction number R_0 [26]. It is calculated as

Fig. 2 This figure shows the variation of R_0 with respect to μ_m

$$\begin{aligned}
 R_0^2 &= \frac{a_{31}a_{42}\bar{S}_p\bar{S}_m}{\mu_m d} + \frac{a_{13}a_{32}\bar{S}_h\bar{S}_m}{\mu_m A} \\
 &= \frac{B^2\beta_{mh}\beta_{hm}\lambda_m\lambda_h}{N_p N_h \mu_m^2 A \mu_h} \left[1 - \frac{\alpha_1 \mu_h}{\lambda_h(\beta_1 + \mu_h)} \right] + \frac{C^2\beta_{pm}\beta_{mp}\lambda_m\lambda_p}{N_p^2 \mu_m^2 \mu_p (\alpha_2 u_1 + \mu_p)} \quad (17)
 \end{aligned}$$

Biologically, R_0 is the average number of secondary cases generated by one primary JEV infected person, which has been introduced in the susceptible population, in which some individuals have been vaccinated. From Eq. (17), it is found that as the increase with the parameters β_1 , μ_h , μ_m , μ_p and α_2 , the value of R_0 decreases. Also the parameters B , C , β_{mh} and β_{pm} affects the value of R_0 significantly. From the expression of R_0 , it is also found that vaccination rate α_1 affects the value of R_0 . It is imperative to take note that, the above parameters assume a significant job in the investigation of global threshold dynamics with regards to R_0 . Pronouncedly, if $R_0 < 1$, the disease will stop after some time and $R_0 > 1$ disease will stay endemic and irresistible population will arrive at positive continued levels [2,4,27,28]. Clearly, from the perspective of controlling the JE infection, one would normally need to decrease the R_0 . In this way, it is advantageous to research how we can successfully diminish R_0 by sensible usage of vaccines and different parameters. The variation of R_0 w.r.t. some μ_m , B , μ_h , μ_p are shown in the Figs. 2, 3, 4 and 5 (Table 3).

5.2 Local stability around E_0

The local stability of infection-free equilibrium state (E_0) is determined after the characteristic roots of the Jacobian matrix $J(E_0)$ of system (9)–(16) by following theorem:

Theorem 5.1 *The infection-free equilibrium state (E_0) is locally asymptotically stable (LAS) if $R_0 < R_{01} < 1$ and unstable if $R > R_{01} > 1$.*

See “Appendix A” for proof of this theorem.

Fig. 3 The figure shows the changes in R_0 with respect to B

Fig. 4 This figure shows the variation in R_0 with respect to μ_h

Fig. 5 This figure shows the variation in R_0 with respect to μ_p

Table 3 The impact of the basic reproduction number (R_0) as for parameters μ_m , B , μ_h and μ_p where all the parameter esteem continue as before as in Table 2

Figures	Parameter	R_0	Explanation
2	$\mu_m > 0.05$	$R_0 < 1$	This signifies that if $\mu_m > 0.05$, $R_0 < 1$ as R_0 is a decreasing function for μ_m . Thus, if the decay rate of mosquitoes exceeds $\mu_m = 0.05$, disease can be eradicated from the population
3	$B > 1.4$	$R_0 > 1$	This shows that if $B > 1.4$, $R_0 > 1$ as R_0 is a increasing function for B . But if $B < 1.4$, $R_0 < 1$ population can be make disease-free
4	$\mu_h > 0.0005$	$R_0 < 1$	From this, we found that μ_h also help to eradicate the infection from environment when $\mu_h > 0.0005$
5	$\mu_p > 0.02$	$R_0 < 1$	This figure signifies that disease can be control from the population when $\mu_p > 0.02$, because if μ_p increases, R_0 decreases

Remark 5.1 The local stability of E_0 will depend as the sign of $\phi_6 (= -\mu_m(1 - R_{01}))$ and $\phi_8 (= \frac{-d(1-R_0^2)}{1-R_{01}})$. The infection-free equilibrium is locally asymptotically stable if $R_0 < 1$ and $R_{01} < 1$. Biologically, $R_{01} < 1$ implies the absence of Culex mosquito population. In this manner, the infection-free equilibrium must be steady without the Culex mosquito population, in any case, disease despite everything perseveres, regardless of whether the susceptible have no immediate contact with the infected population.

5.3 Global stability around E_0

In this section, we have studied the global behavior of infection-free equilibrium point for the system of Eqs. (9)–(16).

Theorem 5.2 *Assumed that*

$$\mu_p = a_{31}\bar{S}_m \quad (18)$$

If $R_0 \leq 1$, the the infection-free equilibrium state E_0 is globally asymptotically stable (GAS) on D .

See “Appendix B” for proof of this theorem.

(2) *Endemic or positive equilibrium $E_1 = (S_h^*, V_h^*, I_h^*, R_h^*, S_m^*, I_m^*, S_p^*, I_p^*)$.*
where,

$$S_h^* = \frac{a_{11} + f_1 R_h^* + \beta_1 V_h^*}{a_{13} I_m^* + \mu_h}, \quad V_h^* = \bar{V}_h, \quad I_h^* = \frac{a_{13} I_m^* S_h^*}{A}, \quad R_h^* = \frac{a_{12} I_h^*}{f_1 + \mu_h} \quad (19)$$

$$S_m^* = \frac{\lambda_m}{a_{31} I_p^* + a_{32} I_h^* + \mu_m}, \quad I_m^* = \frac{(a_{31} I_p^* + a_{32} I_h^*) S_m^*}{\mu_m} \quad (20)$$

$$S_p^* = \frac{\lambda_p + a_{41}I_p^*}{a_{42}I_m^* + \mu_p}, \quad I_p^* = \frac{a_{42}I_m^*S_p^*}{d} \quad (21)$$

Using the value of Eqs. (19) and (21) in the expression for I_m^* in the Eq. (20). This shows that the endemic equilibrium of this model satisfy the Eq.

$$I_m^*(K_1I_m^{*2} + K_2I_m^* + K_3) = 0 \quad (22)$$

where,

$$\begin{aligned} K_1 &= a_{13}a_{42}[a_{31}\lambda_m\lambda_p\{f_1(\lambda + \mu_h) + \mu_hA\} + \mu_m\mu_p\{a_{11}a_{32}(f_1 + \mu_h) + f_1(\lambda\mu_m \\ &\quad + \mu_h\mu_m + a_{32}\beta_1V_h^*) + \mu_h(A\mu_m + a_{32}\beta_1V_h^*)\}] \\ K_2 &= A\mu_h(f_1 + \mu_h)(T_0d + a_{42})\mu_m^2\mu_p + a_{13}[\mu_m\mu_p d\{(f_1 + \mu_h)(a_{11}a_{13} + a_{32}\beta_1V_h^*) \\ &\quad + f_1\mu_m(\lambda + \mu_h) + A\mu_m\mu_h\} - a_{31}a_{42}\lambda_m\{(a_{11}\mu_p + \beta_1\mu_pV_h^*)(1 + f_1) + \lambda_p(A\mu_h \\ &\quad + f_1(\lambda + \mu_h))\}] \\ K_3 &= -(f_1 + \mu_h)\mu_m\mu_h d[A\mu_h\mu_m\mu_p(R_{01} - 1) + a_{22}a_{31}\lambda_p(R_0^2 - R_{01})] \end{aligned}$$

The root $I_m^* = 0$ of Eq. (22) correspond to the infection-free equilibrium (E_0) and the positive roots of the equation

$$K_1I_m^{*2} + K_2I_m^* + K_3 = 0 \quad (23)$$

correspond to endemic equilibria point. From Eq. (23) we have

$$I_m^* = \frac{-K_2 \pm \sqrt{K_2^2 - 4K_1K_3}}{2K_1}$$

If $R_0 > R_{01} > 1$, $K_3 < 0$ and hence $(K_2^2 - 4K_1K_3) > 0$. Thus, the positive equilibrium point is as

$$I_m^* = \frac{-K_2 + \sqrt{K_2^2 - 4K_1K_3}}{2K_1} \quad (24)$$

But if $R_0 < R_{01} < 1$, $K_3 > 0$ and hence $(K_2^2 - 4K_1K_3) < 0$, this gives imaginary value of I_m^* . To verify above numerically, we use the set of parametric values from Table 2. Thus, the components of the endemic equilibrium point (E_1) are found as

$$\begin{aligned} S_h^* &= 300.251, \quad V_h^* = 206.667, \quad I_h^* = 0.329196, \quad R_h^* = 2.5561, \quad S_m^* = 40.9769 \\ I_m^* &= 3.9106, \quad S_p^* = 16.3444, \quad I_p^* = 1.2642 \end{aligned}$$

5.4 Local stability around E_1

Theorem 5.3 *The endemic equilibrium state (E_1) is LAS if and only if $b_1 > 0$, $b_3 > 0$, $b_4 > 0$ and $b_1 b_2 b_3 > b_3^2 + b_1^2 b_4$ where,*

$$\begin{aligned} b_1 &= A + f_1 + a_{13} I_m^* + a_{22} + \mu_h + \mu_m \\ b_2 &= \frac{1}{a_{13} I_m^* + \mu_m} [-a_{12} a_{13} f_1 I_m^* + A(a_{13} I_m^* + \mu_m)(f_1 + a_{22} + a_{13} I_m^* + \mu_h + \mu_m) \\ &\quad + (a_{13} I_m^* + \mu_m)\{(f_1 + \mu_h)(a_{13} I_m^* + \mu_m) + a_{22}(f_1 + a_{13} I_m^* + \mu_h + \mu_m)\}] \\ b_3 &= \frac{1}{a_{13} I_m^* + \mu_m} [(a_{22}(f_1 + \mu_h)(a_{13} I_m^* + \mu_m) + b_4 + a_{22})(a_{13} I_m^* + \mu_m) + a_{22}(f_1 \\ &\quad + \mu_h - a_{12} a_{22} a_{13} f_1 I_m^*)] \\ b_4 &= a_{22}\{A(f_1 + \mu_h)(a_{13} I_m^* + \mu_m) - 2a_{12} a_{13} f_1 I_m^*\} \end{aligned}$$

See “Appendix C” for proof of this theorem.

Remark 5.2 To verify Theorem 5.3 numerically, we use the set of parametric values from Table 2. Thus, the conditions are as

$$\begin{aligned} b_1 &= 0.285495, \quad b_2 = 1.87217 \times 10^{-2}, \quad b_3 = 3.23177 \times 10^{-4}, \\ b_4 &= 8.08664 \times 10^{-7} \text{ and} \\ b_1 b_2 b_3 &= 1.7273 \times 10^{-6}, \quad b_3^2 + b_1^2 b_4 = 1.70355 \times 10^{-7}. \end{aligned}$$

5.5 Global stability around E_1

Theorem 5.4 *The endemic steady state (E_1) is GAS in the region D , if the following inequalities are satisfied:*

$$\begin{aligned} \max \left\{ \frac{2f_1^2}{f_1 + \mu_h}, \frac{\beta_1^2}{a_{22}}, \frac{5a_{13}^2 I_m^{*2}}{A} \right\} &< (\mu_h + a_{13} I_m^*) \\ a_{13}^2 S_h^{*2} &< \min \left\{ \frac{\mu_m(a_{13} I_m^* + \mu_h)}{6}, \frac{2\mu_m A}{15} \right\} \\ a_{12}^2 &< \frac{2A(f_1 + \mu_h)}{5} \\ \frac{1}{2} \max \left\{ \frac{9a_{31}^2 S_m^{*2}}{d}, \frac{9(a_{31}^2 I_p^{*2} + a_{32} I_h^{*2})}{\mu_m + a_{31} I_p^* + a_{32} I_h^*}, \frac{15a_{32}^2 S_m^{*2}}{2A} \right\} &< \mu_m \\ 3 \max \left\{ \frac{a_{41}^2}{2d}, \frac{a_{42}^2 S_p^{*2}}{\mu_m} \right\} &< (\mu_p + a_{42} I_m^*) \end{aligned}$$

See “Appendix D” for proof of this theorem.

Remark 5.3 To verify Theorem 5.4 numerically, we use the set of parametric values from Table 2. The simulations are done in the following seven cases for different initial starts and graphically displayed in Figs. 6 and 7. In this figure, the variation of

Table 4 The components of endemic equilibrium points for non-linear global asymptotically stable

$S_h^*(0)$	$V_h^*(0)$	$I_h^*(0)$	$R_h^*(0)$	$S_m^*(0)$	$I_m^*(0)$	$S_p^*(0)$	$I_p^*(0)$
275	207	20	30	35	40	17	20
250	207	20	30	25	40	17	20
301	207	20	30	30	40	17	20
301	195	23	30	35	40	17	20
301	200	15	30	35	40	17	20
301	203	17	30	35	40	17	20
301	207	20	30	35	40	17	20

Fig. 6 The plot represents the non-linear stability of (S_h^*, S_m^*)

susceptible humans with susceptible mosquitoes and vaccinated humans with infected humans have been shown respectively. The trajectories start with different initial points reach at the equilibrium point (S_h^*, S_m^*) and (V_h^*, I_h^*) . Therefore, we infer that the system of Eqs. (1)–(8) may be non-linear global asymptotically stable about equilibrium points (S_h^*, S_m^*) and (V_h^*, I_h^*) for the following set of parameters (Table 4).

Also for following set of parameter values, the conditions for the global asymptotical stability of the endemic equilibrium E_1 are satisfied.

Fig. 7 The plot represents the non-linear stability of (V_h^*, I_h^*)

6 Vaccine-induced reproduction number

From Eq. (17), we define the vaccine-induced reproduction number $(R(\alpha_1))$ as

$$R(\alpha_1) = \sqrt{R_{01} + \frac{B^2 \beta_{mh} \beta_{hm} \lambda_m \lambda_h}{A \mu_m^2 N_h \mu_h N_p} \left[1 - \frac{\alpha_1 \mu_h}{\lambda_h (\beta_1 + \mu_h)} \right]} \quad (25)$$

Now,

$$R'(\alpha_1) = -\frac{B^2 \beta_{mh} \beta_{hm} \lambda_m}{2R(\alpha_1) A N_p N_h \mu_m^2 (\beta_1 + \mu_h)} < 0$$

From above it is clear that $R(\alpha_1)$ is a decreasing function of α_1 if $\alpha_1 \geq 0$. This signalize that the effect of vaccination in reducing vaccine-induced reproduction number. Also, when vaccination is zero i.e. $(\alpha_1 = 0)$ then Eq. (25) can be written as

$$\sqrt{R_{01} + \frac{B^2 \beta_{mh} \beta_{hm} \lambda_m \lambda_h}{A \mu_m^2 N_h \mu_h N_p}} = R_c^* \quad (26)$$

From Eqs. (25) and (26), we can write the vaccination as

$$R(\alpha_1) \leq R_c^* \text{ for } 0 \leq \alpha_1 \leq 1$$

Fig. 8 The graph represents the variation of vaccine-induced reproduction number $R(\alpha_1)$ with vaccination rate α_1

If $R_c^* < 1$ then $R(\alpha_1) < 1$, therefore E_0 is locally asymptotically stable. Also if $R_c^* = 1$, then $R(\alpha_1) \leq 1$ this is also the case in which E_0 locally asymptotically stable. From Eqs. (25) and (26) we can write,

$$\sigma R_c^* \leq R(\alpha_1) \leq R_c^* \text{ where } \sigma = 1 - \frac{\alpha_1 \mu_h}{\lambda_h (\beta_1 + \mu_h)} \text{ and } 0 \leq \sigma \leq 1$$

If $\sigma R_c^* > 1$ then $\sigma > \sigma_c = \frac{1}{R_c^*}$. From this we find $R(\alpha_1) > 1$, therefore no amount of vaccination can bring $R(\alpha_1)$ less than one.

Hence, σ_c defines the stationary values for the vaccine related reduction rate of infection. Therefore, if the vaccination level $\alpha_1 > \sigma_c$, then possibility of disease eradication from population can be maximized (Fig. 8).

Solving Eq. (17) for α_1 , we get a critical vaccination level α_{1c1} , is given by

$$\alpha_{1c1} = \frac{(R_c^{*2} - R_0^2) N_p A \lambda_h \mu_m^2}{a_{13} a_{32} V_h^*}$$

Here, α_{1c1} is always positive, since $R_c^{*2} > R_0^2$. If $\alpha_1 > \alpha_{1c1}$, $R(\alpha_1) < 1$ as $R(\alpha_1)$ is a decreasing function for $\alpha_1 \geq 0$. In this manner, in the event that the vaccination coverage α_1 surpasses the condition α_{1c2} , at that point disease can be eradicated. From Eq. (19), we also find that

$$\alpha_{1c2} = \frac{V_h^* (\beta_1 + \mu_h)}{\lambda_h}$$

This critical vaccination level α_{1c2} guarantees that the disease annihilation when inoculated people stay at its harmony. Along these lines, if the vaccination rate α_1 is with the end goal that $\alpha_1 > \max\{\alpha_{1c1}, \alpha_{1c2}\}$, at that point disease annihilation is conceivable in the human population.

7 Model parameters sensitivity investigation

In this section, we want to examine the relationship between input parameters and the results of the proposed model using the reproduction number R_0 . To assist us with discovering the parameters that highly affect R_0 , we dissected the R_0 to decide if immunization, treatment of infectives and mortality of mosquitoes can prompt the successful disposal or control of the JE in the population. The parameters chosen to perform the sensitivity analysis were B , μ_m , δ and α_1 . We evaluate the sensitivity indices for $B = 0.7$, $\mu_m = 0.06$, $\delta = 0.1$, $\alpha_1 = 0.5$ and all other parameters are same as in Table 2. For sensitivity analysis of the model, we define

$$Z_x^s = \frac{\partial x}{\partial s} \cdot \frac{s}{x}$$

called normalized forward sensitivity index for a variable x and it depends on a parameter s [29]. The sensitivity indices of R_0 w. r. t. the parameters B , μ_m , δ and α_1 are calculated as

$$Z_{R_0}^B = 0.01743, \quad Z_{R_0}^{\mu_m} = -0.01587, \quad Z_{R_0}^{\delta} = -0.001987, \quad Z_{R_0}^{\alpha_1} = -0.00495$$

The fact is that $Z_{R_0}^B = 0.01743$ signify that 1% increase in B , will produce 0.01743% increase in R_0 , keeping all other parameters fixed. When the parameters μ_m , δ and α_1 increase by 1%, while keeping all others fixed then the values of R_0 decrease by 0.01587%, 0.001987% and 0.00495% respectively. Overall a lower value of R_0 is better because it increases the chance of disease eradication in the areas. Therefore, all the above prevention practices should be considered to reduce the parameter B , while increases the parameters μ_m , u_1 and α_1 , instead it should be favored. It is seen that the impact of vaccination on people assumes an overwhelming job in the control of the disease. On the off chance that the contact rate (B) between susceptible people and infected mosquitoes can be managed then Japanese encephalitis can be killed totally from the population. It is additionally seen that the infected human population consistently expanding as for B and it is progressively delicate for the spread of JE disease, (cf. Figs. 9, 10). That's why we have chosen the parameter B only for the model sensitivity investigation.

Figure 9 shows the sensitivity plot of the basic reproduction number R_0 for α_1 and β_1 . From this, we found that a higher vaccination rate α_1 and a low contact rate β_1 will keep $R_0 < 1$. Also, in Fig. 10, the sensitivity plot of the R_0 regarding B and α_1 has appeared. This implies a higher estimation of α_1 and a low contact pace of B will keep $R_0 < 1$.

8 Simulations and discussions

In this paragraph, we provide some simulations on the dynamics of the system (1)–(8). Moreover, to check how the human, mosquito and pig populations are affected by various parameters, we perform simulations by using the parameter values as shown

Fig. 9 The surface plot shows the variation of R_0 with respect to β_1 and α_1

Fig. 10 The surface plot shows the variation of R_0 with respect to B and α_1

in Table 2, and numerical results are manifested graphically in Figs. 11, 12, 13, 14, 15, 16, 17 and 18 by using MATLAB(R2014a) software. In Fig. 11, the variation of the human population with vaccination is shown and in Fig. 12, the variation of the human population without vaccination is shown. From these two figures, we can study the effect of vaccination on the human population. In the case of vaccination, the susceptible population is below 50 and the vaccinated population is above 50, whereas in case of without vaccination, the vaccinated level goes to zero and susceptible level increases. We also see that vaccination does not affect the infected and recovered human population. This signifies that vaccination only affects susceptible individuals

Fig. 11 The graph shows the variation of human population with vaccination at endemic equilibrium

Fig. 12 The graph shows the variation of human population without vaccination at endemic equilibrium

Fig. 13 The figure shows the variation of mosquito population at endemic equilibrium

Fig. 14 The figure shows the variation of pig population at endemic equilibrium

Fig. 15 The figure shows the effect of susceptible human with no control, all control, vaccine control and medicine control

Fig. 16 The figure shows the effect of vaccinated human with no control, all control, vaccine control and medicine control

Fig. 17 The plot shows the effect of infected human with no control, all control, vaccine control and medicine control

Fig. 18 The plot shows the effect of recovered human with no control, all control, vaccine control and medicine control

if individuals infected, vaccination does not affect the infected population. Figures 13 and 14, show the variation of mosquito and pig population at an endemic equilibrium point. These two figures (Figs. 13, 14) show the stability of the endemic equilibrium when R_0 is larger than one and the threshold condition T_0 of the JE virus larger than one. This reflects that the virus can be dominant when the T_0 and R_0 are larger than one. We observed in Fig. 15, that due to the control strategy, the number of susceptible human decreases in society. This signifies that the spread of JE infection in society can also be controlled through the proper implementation of vaccines, medicine and some effective prevention. This strategy also shows that the effect of vaccine and medicine control on susceptible humans is almost the same. This reflects that some effective medical treatment and medicine can also control the spread of JE infection. Whereas in Fig. 16, the number of vaccinated human increases in the community. This shows that the community can make infection-free through the effective treatment and vaccination control of a human. From Fig. 17, we observed that due to the control strategy, initially decreases the infected human but after some time it stabilized. This shows that the vaccination strategy has no significant impact on the infected human. Finally, in Fig. 18 we observed that due to controlled strategy, the number of recovered human increases in the community but after some time it stabilized. This shows that infected humans can be immune (but not completely) through some basic supportive medical care and treatment.

9 Conclusions

In this article, perspective on human, mosquito and pig population, a nonlinear scientific model for JEV has been proposed and examined to contemplate the impact of vaccination on control of infectious disease, Japanese encephalitis. The model displays two equilibria, to be a specific disease-free and endemic equilibrium. The first equilibrium exists without any conditions but the second equilibrium exists if the threshold parameter $R_{01} > 1$. We have discovered another threshold parameter in terms of vaccine, vaccine-induced reproduction number $R(\alpha_1)$ which is, if not as much as unity, the disease ceases to exist, without infected population otherwise the infection is kept up in the population and disease turns out to be progressively endemic. The system of the differential equations has a unique infection-free steady state which is locally asymptotically stable if $R_{01} < 1$ and unstable if $R_{01} > 1$. Furthermore if R_{01} approaches to 1, E_0 becomes unstable but E_1 exist. If $R_{01} > 1$ then E_1 exists and it is locally asymptotically stable under certain condition. Also for both the equilibrium point global stability have to show by using Liapunov's function in the region of attraction Ω under certain conditions. Model analysis shows, if the vaccination program has adequate capacity to mobilize a high degree of population, there is an abundance of mosquitoes in the environment, then the disease can be controlled. Moreover, it is found that vaccine coverage has a substantial effect on the basic reproduction number, R_0 . Also, by continuous efforts and effectiveness of vaccine coverage, the disease can be eradicated. Also, found by sensitivity investigation, if 1% increase in the parameters μ_m , δ and α_1 keeping all others parameters fixed, then the value of R_0 decreases by 0.01587%, 0.001987% and 0.00495% respectively. Also, if we increase 1% in B then

R_0 increases by 0.01743%. This signifies that the parameter B is more sensitive to the transmission of JEV. Then again, in the event that the vaccination rate is over a critical level α_{1c_1} , at that point the JE doesn't take a type of pestilence gave the density of mosquito population stays at its equilibrium level. During the entire investigation of the JEV model, it is discovered that vaccination is the main long haul control measure against JE in the population. The mosquitoes reduction strategies may incorporate the end of mosquito breeding sites, keeping environmental factors perfect and clean. On the other hand, vaccinating the susceptible individuals in the human population with the end goal that vaccination level is kept up over the critical vaccination level α_{1c_2} and the vaccine efficacy is sufficiently high so that vaccinated individuals are not infected by infected mosquitoes. Along these lines, our present paper has been investigated various sorts of control procedures and their adequacy in controlling the JE disease by utilizing the steadiness hypothesis of differential conditions and the basic reproduction number. From the clinical perspective, we accept that our model examination is gainful for persons and control of the JE disease.

Appendix A. Proof of Theorem. 5.1

Proof The Jacobian matrix ($J(E_0)$) of the system (9)–(16) corresponding to equilibrium state E_0 is given by

$$J(E_0) = \begin{pmatrix} -\mu_h & \beta_1 & 0 & f_1 & 0 & -a_{13}\bar{S}_h & 0 & 0 \\ 0 & -a_{22} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -A & 0 & a_{13}\bar{S}_h & 0 & 0 \\ 0 & 0 & 0 & -(f_1 + \mu_h) & 0 & 0 & 0 & 0 \\ 0 & 0 & -a_{32}\bar{S}_m & 0 & -\mu_m & 0 & 0 & -a_{31}\bar{S}_m \\ 0 & 0 & a_{32}\bar{S}_m & 0 & 0 & -\mu_m & 0 & a_{31}\bar{S}_m \\ 0 & 0 & 0 & 0 & 0 & -a_{42}\bar{S}_p & -\mu_p & a_{42} \\ 0 & 0 & 0 & 0 & 0 & a_{42}\bar{S}_p & 0 & -d \end{pmatrix}$$

The characteristic roots of Jacobian matrix ($J(E_0)$) are

$$\begin{aligned} \phi_1 &= -\mu_h, \quad \phi_2 = -a_{22}, \quad \phi_3 = -A, \quad \phi_4 = -(f_1 + \mu_h), \quad \phi_5 = -\mu_m, \\ \phi_6 &= -\mu_m(1 - R_{01}) \\ \phi_7 &= -\mu_p, \quad \phi_8 = \frac{-d(1 - R_0^2)}{1 - R_{01}} \end{aligned}$$

In this article, all parameters are considered positive, therefore all $\phi_i (i = 1, 2, 3, 4, 5, 6, 7) < 0$. Then, Routh–Hurwitz criterion all the characteristic roots of Jacobian matrix ($J(E_0)$) have negative real part if $\phi_8 < 0$ [27,30]. Thus, the infection-free steady state (E_0) is LAS if $R_0 < R_{01} < 1$, and unstable if $R_0 > R_{01} > 1$. $\square$

Appendix B. Proof of Theorem 5.2

Proof To establish the global stability of the infection-free steady state (E_0), consider the following Lyapunov function,

$$V(t) = (S_h - \bar{S}_h \ln S_h) + V_h + I_h + R_h + (S_m - \bar{S}_m \ln S_m) + I_m \\ + (S_p - \bar{S}_p \ln S_p) + I_p$$

Differentiating $V(t)$ w.r.t. ' t ' along the solutions of Eqs. (9)–(16), we get

$$\begin{aligned} V'(t) &= \dot{S}_h \left(1 - \frac{\bar{S}_h}{S_h}\right) + \dot{V}_h + \dot{I}_h + \dot{R}_h + \dot{S}_m \left(1 - \frac{\bar{S}_m}{S_m}\right) + \dot{I}_m + \dot{S}_p \left(1 - \frac{\bar{S}_p}{S_p}\right) + \dot{I}_p \\ &= (a_{11} + f_1 R_h + \beta_1 V_h - (a_{13} I_m + \mu_h) S_h) \left(1 - \frac{\bar{S}_h}{S_h}\right) \\ &\quad + a_{21} - a_{22} V_h + a_{13} I_m S_h - A I_h \\ &\quad + a_{12} I_h - (f_1 + \mu_h) R_h + (\lambda_m - (a_{31} I_p + a_{32} I_h + \mu_m) S_m) \left(1 - \frac{\bar{S}_m}{S_m}\right) \\ &\quad + (a_{31} I_p + a_{32} I_h) S_m \\ &\quad - \mu_m I_m + (\lambda_p + a_{41} I_p - a_{42} I_m S_p - \mu_p S_p) \left(1 - \frac{\bar{S}_p}{S_p}\right) + a_{42} I_m S_p - d I_p \\ &= -V_h \left(\mu_h + \beta_1 \frac{\bar{S}_h}{S_h}\right) - R_h \left(\mu_h + f_1 \frac{\bar{S}_h}{S_h}\right) \\ &\quad - I_h \left(\lambda + \mu_h + a_{32} \bar{S}_m + a_{12} \frac{\bar{S}_h}{S_h}\right) - a_{41} I_p \frac{\bar{S}_p}{S_p} \\ &\quad + I_m (a_{13} \bar{S}_h + a_{42} \bar{S}_p - \mu_m) + I_p (a_{31} \bar{S}_m - \mu_p) \\ &\quad + a_{11} \left(1 - \frac{\bar{S}_h}{S_h}\right) - \mu_h S_h \left(1 - \frac{\bar{S}_h}{S_h}\right) \\ &\quad + \lambda_m \left(1 - \frac{\bar{S}_m}{S_m}\right) - \mu_m S_m \left(1 - \frac{\bar{S}_m}{S_m}\right) + \lambda_p \left(1 - \frac{\bar{S}_p}{S_p}\right) - \mu_p S_p \left(1 - \frac{\bar{S}_p}{S_p}\right) \end{aligned} \quad (27)$$

Using the condition (18) and feasible region D (as explain in Sect. 4), Eq. (27) can be rewritten as

$$\begin{aligned} &\leq -V_h \left(\mu_h + \beta_1 \frac{\bar{S}_h}{S_h}\right) - R_h \left(\mu_h + f_1 \frac{\bar{S}_h}{S_h}\right) \\ &\quad - I_h \left(\lambda + \mu_h + a_{32} \bar{S}_m + a_{12} \frac{\bar{S}_h}{S_h}\right) - a_{41} I_p \frac{\bar{S}_p}{S_p} \end{aligned}$$

$$\begin{aligned}
& + I_m \mu_m \left(\frac{a_{13} \bar{S}_h + a_{42} \bar{S}_p}{\mu_m} - 1 \right) + a_{11} \left(2 - \frac{\bar{S}_h}{S_h} - \frac{S_h}{\bar{S}_h} \right) + \lambda_m \left(2 - \frac{\bar{S}_m}{S_m} - \frac{S_m}{\bar{S}_m} \right) \\
& + \lambda_p \left(2 - \frac{\bar{S}_p}{S_p} - \frac{S_p}{\bar{S}_p} \right) \\
& = -V_h \left(\mu_h + \beta_1 \frac{\bar{S}_h}{S_h} \right) - I_h \left(\lambda + \mu_h + a_{12} \frac{\bar{S}_h}{S_h} \right) \\
& - a_{41} I_p \frac{\bar{S}_p}{S_p} + I_m \mu_m \left(\frac{a_{13} \bar{S}_h + a_{42} \bar{S}_p}{\mu_m} - 1 \right) \\
& - \frac{a_{11} (S_h - \bar{S}_h)^2}{S_h \bar{S}_h} - \frac{\lambda_m (S_m - \bar{S}_m)^2}{S_m \bar{S}_m} - \frac{\lambda_p (S_p - \bar{S}_p)^2}{S_p \bar{S}_p}
\end{aligned}$$

Thus $V(t) \leq 0$ whenever $\left(\frac{a_{13} \bar{S}_h + a_{42} \bar{S}_p}{\mu_m} \right) \leq 1$ i.e. $R_0 \leq 1$. Therefore, LaSalle's invariance Principle, [18,31] implies that E_0 is GAS in D. The proof is complete. $\square$

Appendix C. Proof of Theorem 5.3

Proof The Jacobian matrix for the model system (9)–(16) corresponding to equilibrium E_1 is given by

$$J(E_1) = \begin{pmatrix} -(a_{13} I_m^* + \mu_h) & \beta_1 & 0 & a_{12} & 0 & -a_{13} S_h^* & 0 & 0 \\ 0 & -a_{22} & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{13} I_m^* & 0 & -A & 0 & 0 & a_{13} S_h^* & 0 & 0 \\ 0 & 0 & a_{12} & -(f_1 + \mu_h) & 0 & 0 & 0 & 0 \\ 0 & 0 & -a_{32} S_m^* & 0 & 0 & 0 & 0 & -a_{31} S_m^* \\ 0 & 0 & a_{32} S_m^* & 0 & 0 & -\mu_m & 0 & a_{31} S_m^* \\ 0 & 0 & 0 & 0 & 0 & -a_{42} S_p^* & a_{42} I_m^* & a_{42} \\ 0 & 0 & 0 & 0 & 0 & a_{42} S_p^* & a_{42} I_m^* & -d \end{pmatrix}$$

Clearly, three characteristic roots of $J(E_1)$ are $-\mu_m$, $-(a_{31} I_p^* + a_{32} I_h^* + \mu_m)$, $-a_{42} I_m^*$ and $-d + a_{42}$. The other four characteristic roots of $J(E_1)$ are given by the equation

$$x^4 + b_1 x^3 + b_2 x^2 + b_3 x + b_4 = 0 \quad (28)$$

where,

$$\begin{aligned}
 b_1 &= A + f_1 + a_{13}I_m^* + a_{22} + \mu_h + \mu_m \\
 b_2 &= \frac{1}{a_{13}I_m^* + \mu_m} [-a_{12}a_{13}f_1I_m^* + A(a_{13}I_m^* + \mu_m)(f_1 + a_{22} + a_{13}I_m^* + \mu_h + \mu_m) \\
 &\quad + (a_{13}I_m^* + \mu_m)\{(f_1 + \mu_h)(a_{13}I_m^* + \mu_m) + a_{22}(f_1 + a_{13}I_m^* + \mu_h + \mu_m)\}] \\
 b_3 &= \frac{1}{a_{13}I_m^* + \mu_m} [a_{22}(f_1 + \mu_h)(a_{13}I_m^* + \mu_m) + b_4 + a_{22}\}(a_{13}I_m^* + \mu_m) + a_{22}(f_1 + \mu_h \\
 &\quad - a_{12}a_{22}a_{13}f_1I_m^*)] \\
 b_4 &= a_{22}\{A(f_1 + \mu_h)(a_{13}I_m^* + \mu_m) - 2a_{12}a_{13}f_1I_m^*\}
 \end{aligned}$$

Eq.(28) has negative solutions if and only if $b_1 > 0$, $b_3 > 0$, $b_4 > 0$ and $b_1b_2b_3 > b_3^2 + b_1^2b_4$ i.e. satisfies Routh–Hurwitz criteria [9,32]. Therefore, positive equilibrium (E_1) is LAS. $\square$

Appendix D. Proof of Theorem 5.4

Proof To establish the global stability of the endemic equilibrium state (E_1) , consider the following positive definite function [29].

$$\begin{aligned}
 V &= \frac{1}{2}(S_h - S_h^*)^2 + \frac{1}{2}(V_h - V_h^*)^2 + \frac{1}{2}(I_h - I_h^*)^2 + \frac{1}{2}(R_h - R_h^*)^2 \\
 &\quad + \frac{1}{2}(S_m - S_m^*)^2 + \frac{1}{2}(I_m - I_m^*)^2 \\
 &\quad + \frac{1}{2}(S_p - S_p^*)^2 + \frac{1}{2}(I_p - I_p^*)^2
 \end{aligned}$$

Diff. V w.r.t. time t along the solution of system of Eqs. (9)–(16), we get

$$\begin{aligned}
 \dot{V} &= (S_h - S_h^*)\{f_1R_h + \beta_1V_h - (a_{13}I_m + \mu_h)S_h - \beta_1V_h^* - f_1R_h^* + (a_{13}I_m^* + \mu_h)S_h^*\} \\
 &\quad + (V_h - V_h^*)\{-a_{22}(V_h - V_h^*)\} + (I_h - I_h^*)\{a_{13}I_mS_h - AI_h - a_{13}I_m^*S_h^* + AI_h^*\} \\
 &\quad + (R_h - R_h^*)\{a_{12}I_h - (f_1 + \mu_h)R_h - a_{12}I_h^* + (f_1 + \mu_h)R_h^*\} + (S_m - S_m^*)\{(-a_{31}I_p \\
 &\quad + a_{32}I_p)S_m - \mu_mS_m + (a_{31}I_p^* + a_{32}I_h^*)S_m^* + \mu_mS_m^*\} + (I_m - I_m^*)\{(a_{31}I_p + a_{32}I_h)S_m \\
 &\quad - \mu_mI_m - (a_{31}I_p^* + a_{32}I_h^*)S_m^* + \mu_mI_m^*\} + (S_p - S_p^*)\{a_{41}I_p - a_{42}I_mS_p - \mu_pS_p - a_{41}I_p^* \\
 &\quad + a_{42}I_m^*S_p^* + \mu_pS_p^*\} + (I_p - I_p^*)\{a_{42}I_mS_p - dI_p - a_{42}I_m^*S_p^* + dI_p^*\}
 \end{aligned}$$

On rearranging the terms, $\dot{V}$ reduced to,

$$\begin{aligned}
 \dot{V} &= -(\mu_h + a_{13}I_m^*)(S_h - S_h^*)^2 + \beta_1(V_h - V_h^*)(S_h - S_h^*) - a_{22}(V_h - V_h^*)^2 \\
 &\quad + (a_{12} + a_{13}I_m)(S_h - S_h^*)(I_h - I_h^*) - A(I_h - I_h^*)^2 - a_{31}S_h(S_h - S_h^*)(I_m - I_m^*) \\
 &\quad - \mu_m(I_m - I_m^*)^2 + a_{13}S_h^*(I_m - I_m^*)(I_h - I_h^*) + (a_{31}I_p^* - \mu_m)(I_m - I_m^*)(S_m - S_m^*) \\
 &\quad - a_{31}I_p^*(S_m - S_m^*)^2 + (a_{31}S_m + a_{42}S_p)(I_m - I_m^*)(I_p - I_p^*) - d(I_p - I_p^*)^2
 \end{aligned}$$

$$-a_{42}S_p(I_m - I_m^*)(S_p - S_p^*) - (\mu_p + a_{42}I_m^*)(S_p - S_p^*)^2 - a_{31}S_m(I_p - I_p^*)(S_m - S_m^*) + (a_{41} + a_{42}I_m^*)(I_p - I_p^*)(S_p - S_p^*)$$

Using the region of attraction D (as explain in Sect. 4), we conclude that $\dot{V}$ will be negative definite if it satisfy the following inequalities:

$$f_1^2 < \frac{1}{2}(f_1 + \mu_h)(\mu_h + a_{13}I_m^*) \quad (29)$$

$$\beta_1^2 < a_{22}(\mu_h + a_{13}I_m^*) \quad (30)$$

$$a_{13}^2S_h^{*2} < \min \left\{ \frac{\mu_m(a_{13}I_m^* + \mu_h)}{6}, \frac{2\mu_m A}{15} \right\} \quad (31)$$

$$a_{13}^2I_m^{*2} < \frac{A(\mu_h + a_{13}I_m^*)}{5} \quad (32)$$

$$a_{12}^2 < \frac{2A(f_1 + \mu_h)}{5} \quad (33)$$

$$\left(\frac{9a_{31}^2S_m^{*2}}{2d} \right) < \min\{2(\mu_m + a_{31}I_p^* + a_{32}I_h^*), \mu_m\} = \mu_m \quad (34)$$

$$(a_{31}I_p^* + a_{32}I_h^*)^2 < \frac{2\mu_m(\mu_m + a_{31}I_p^* + a_{32}I_h^*)}{9} \quad (35)$$

$$\frac{15a_{32}^2S_m^{*2}}{2A} < \min\{\mu_m, \mu_m + a_{31}I_p^* + a_{32}I_h^*\} = \mu_m \quad (36)$$

$$a_{41}^2 < \frac{2d(\mu_p + a_{42}I_m^*)}{3} \quad (37)$$

$$a_{42}^2S_p^{*2} < \frac{\mu_m(\mu_p + a_{42}I_m^*)}{3} \quad (38)$$

From Eqs. (29), (30), (31), (32) and (33) we get

$$\max \left\{ \frac{2f_1^2}{f_1 + \mu_h}, \frac{\beta_1^2}{a_{22}}, \frac{5a_{13}^2I_m^{*2}}{A} \right\} < (\mu_h + a_{13}I_m^*) \quad (39)$$

$$a_{13}^2S_h^{*2} < \min \left\{ \frac{\mu_m(a_{13}I_m^* + \mu_h)}{6}, \frac{2\mu_m A}{15} \right\} \quad (40)$$

$$a_{12}^2 < \frac{2A(f_1 + \mu_h)}{5} \quad (41)$$

Similarly from Eqs. (34), (35) and (36), we get

$$\frac{1}{2} \max \left\{ \frac{9a_{31}^2S_m^{*2}}{d}, \frac{9(a_{31}^2I_p^{*2} + a_{32}I_h^{*2})}{\mu_m + a_{31}I_p^* + a_{32}I_h^*}, \frac{15a_{32}^2S_m^{*2}}{2A} \right\} < \mu_m \quad (42)$$

Also, from Eqs. (37) and (38), we get

$$3\max \left\{ \frac{a_{41}^2}{2d}, \frac{a_{42}^2 S_p^{*2}}{\mu_m} \right\} < (\mu_p + a_{42} I_m^*) \quad (43)$$

Hence, we shows that V is a Liapunov's function for the system of equations (9)–(16), provided the conditions (39)–(43) hold. $\square$

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