

Hydraulic fracture initiation theory for a horizontal well in a coal seam

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Abstract: A series of experiments were performed to determine rock mechanical parameters related to hydraulic fracturing of coal. The effect of confining pressure and pore pressure on the strength of coal was studied. Experimental results show that the coal seam in the study areas has a relatively low elastic modulus, high Poisson's ratio, high fragility and is easily broken and compressed. The coal seam is considered as a transversely isotropic medium, since the physical properties in the direction of bedding plane and orthogonal to the bedding plane vary markedly. Based on the generalized plane strain model, stress distribution for an arbitrarily orientated wellbore in the coal seam was determined. In a horizontal well, hydraulic fracturing was found to initiate in the coal seam mass due to tensile failure, or from cleats due to shear or tensile failure. For those coal seams with abundant natural cleats, hydraulic fracture initiation can be induced by any of these mechanisms. In this study, hydraulic fracture initiation criteria for a horizontal well in a coal seam were established.

Key words: Hydraulic fracturing, coal seam, transversely isotropic, fracture initiation, cleat, natural fracture

1 Introduction

Hydraulic fractures usually initiate in the direction of maximum crustal stress. The fracture initiation pressure can be determined from the crustal stress condition on the wellbore wall (Rice, 1968; Nordgren, 1972; Hossain et al, 1999; Bruno and Nakagawa, 1991; Whittaker et al, 1992; Zhang and Chen, 2010a; 2010b; 2010c). Most coal is naturally fractured, with closely spaced, regular, planar fractures that are collectively known as cleats. For coal seams with natural cleats (plane cleat & tip cleat), fracture initiation becomes complicated since cleats have certain occurrence and geometric space distribution. Different alignments and trends of cleats, various crustal stresses and wellbore geometric configurations all contribute to differences in wellbore stress conditions, thus leading to various fracture initiation scenarios (Thomson, 1998; Zhou et al, 2008; 2010; Thiercelin, 2009; Jeffrey et al, 2009; Chen et al, 2010; Zhang and Chen, 2010a; 2010b; 2010c).

Generally coal seam gas reservoirs consist of sedimentary rocks with smooth laminar structures. Under normal conditions, the elastic modulus perpendicular to the bedding plane is slightly greater than that in the direction parallel to the bedding plane, and Poisson's ratio, as well as other

mechanical parameters, also vary between these two orientations. This lamellar rock has mechanical and physical properties that are quite similar in the direction parallel to the bedding plane and indeed they are often considered the same in practical calculations; nevertheless, these properties vary greatly between the transverse direction (parallel to the bedding plane) and the longitudinal direction (perpendicular to the bedding plane). Therefore, sedimentary rock, especially coal seams (with abundance plane cleats, tip cleats & laminations), is very often considered as a transversely isotropic medium (Zhao and Chen, 2006; Akulich, 2008; Olson, 2008; Zhang and Chen, 2009; Jin et al, 2009; 2011).

So far this factor has not been thoroughly taken into consideration when designing coal seam fracturing in China. Based on experimental results, several researchers summarized the transverse isotropic properties of coal seam (Li, 1997; Zhang, 2009; Li et al, 2000; Zhang and Chen, 2010a). However, they did not take stress condition of adjacent rock into consideration nor propose a comprehensive theory of the initiation of hydraulic fracturing. Based on a theoretical approach and rock mechanical tests on coal seam material, this paper determines the stress distribution of adjacent rock in arbitrary directions. Hydraulic fracture initiation for a horizontal well in the coal seam mass due to tensile failure and, from cleats due to shear or tensile failure is discussed. For those coal seams with an abundance of natural cleats, hydraulic fracture initiation can be caused by

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Received August 22, 2012

any of these mechanisms. Hydraulic fracture initiation criteria for horizontal wells in coal seams are also established.

2 Analysis of coal seam mechanical properties

Determination of basic physical parameters of rocks is very useful in terms of analyzing hydraulic fracture propagation. The physical properties of rocks are obtained by laboratory or field tests. There is no doubt that mechanical parameters determined in the field are more accurate and realistic. But considering the high cost and uncertainties, some researchers prefer laboratory tests when determining these parameters. The original stress state of coal seam must be taken into account in laboratory tests. Researchers usually retrieve data such as well depth, confining pressure, pore pressure etc. from geological information before laboratory tests (Li, 1994).

The preparation of rock specimens is an important part

of these experiments. During the sampling process, we cored in both transverse and longitudinal directions, since the anisotropy of physical properties would be considered. The specific sampling process is shown in Fig. 1. Taking confining pressure and pore pressure into account, seven specimens were tested according to the above requirements.

A comparison of compression and tensile tests shows the anisotropy of mechanical properties (Table 1). On average, the elastic modulus of coal seams ranges from 4 to 5 GPa, which is smaller than that of normal sandstones. Poisson's ratio stays around 0.33-0.35, which is significantly greater than that of other normal rock. These properties are determined by its special structure and evolution. The previous determination proved that the coal seam in the study areas has relatively low elastic modulus, high Poisson's ratio, high fragility and is easily broken and compressed. Moreover, the elastic modulus and Poisson's ratio in the longitudinal direction are greater than that in the transverse direction. A clear anisotropy also occurs in compression strength and tensile strength.

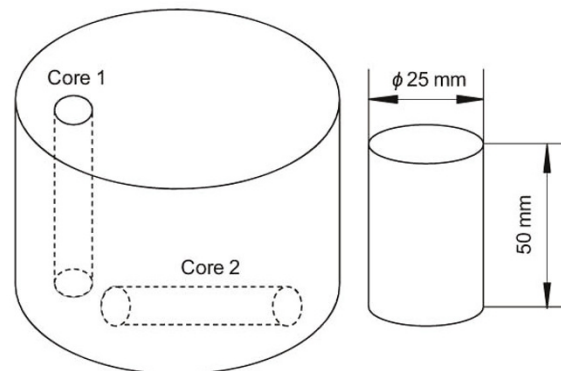


Fig. 1 Sampling process

Table 1 Comparison of anisotropy of mechanical properties of coal seam

Sample	Elastic modulus E , GPa		Poisson's ratio ν		Compression strength σ , MPa		Tensile strength σ , MPa	
	Vertical	Horizontal	Vertical	Horizontal	Vertical	Horizontal	Vertical	Horizontal
1'	6.500	0.300	0.50	0.10	19.6	13.2	0.25	0.80
2'	2.797	0.391	0.43	0.14	13.6	6.80	0.27	0.75
3'	1.718	0.696	0.37	0.21	12.4	4.70	0.12	0.27
4'	3.192	0.416	0.35	0.11	9.80	4.40	0.25	0.54
5'	1.437	0.579	0.41	0.27	10.7	6.40	0.26	0.46

A coal gas reservoir is a sedimentary formation with a laminar structure. From a macroscopic point of view, its physical properties are anisotropic. A coal seam is generally considered as orthogonally anisotropic in terms of its mechanical response. Taking practical tests of mechanical properties into consideration, it can be further simplified as transversely isotropic. Consequently, considering a coal seam as transversely isotropic ($k>1$, $n>1$) is relatively close to the actual in-situ situation.

3 Stress distribution in a horizontal wellbore in a coal seam

The stress-strain relations of linear elastic materials is

described by Hooke's Law. Thus the matrix form of stress distribution at any point can be expressed as:

$$[\boldsymbol{\varepsilon}] = [\boldsymbol{D}][\boldsymbol{\sigma}] \quad (1)$$

where $[\boldsymbol{\sigma}] = [\sigma_x \sigma_y \sigma_z \tau_{xy} \tau_{yz} \tau_{zx}]^T$ is the stress array;

$[\boldsymbol{\varepsilon}] = [\varepsilon_x \varepsilon_y \varepsilon_z \gamma_{xy} \gamma_{yz} \gamma_{zx}]^T$ is the strain array; and $[\boldsymbol{D}]$ is a 6×6 elastic matrix. Any element in these matrices is constant, and can be determined from the elastic properties of the material. Du et al (1986) and Liu and Lu (1979) proved that the principal strain directions of transversely isotropic materials are the same as that of the principal stress. Suppose the y axis

is the principal elastic direction that is normal to the bedding plane and the x axis and z axis are two other principal elastic directions, respectively. Lekhnitskii (1981) proved that for transversely isotropic materials, the independent elastic constants can be reduced from 21 to 5. These constants are $a_{11}, a_{12}, a_{13}, a_{33}, a_{44}$. By representing E, ν, G as elastic modulus, Poisson's ratio and shear modulus in the xz plane

(parallel to the bedding plane) respectively, and E', ν', G' as elastic constants in the direction of y axis (normal to the bedding plane), the stress distribution in a horizontal wellbore in an actual coal gas formation can be sketched as shown in Fig. 2. Its wellbore axis is parallel to the coal seam bedding plane, which is the isotropic xz plane. The stress-strain relations along the wellbore wall are shown in Eq. (2).

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{1}{E} & -\frac{\nu'}{E'} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu'}{E'} & \frac{1}{E'} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & -\frac{\nu'}{E'} & \frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{E} + \frac{1}{E'} + 2\frac{\nu'}{E'} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2(1+\nu)}{E} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{E} + \frac{1}{E'} + 2\frac{\nu'}{E'} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{bmatrix} \tag{2}$$

where E and E' denote the modulus of elasticity in the plane of isotropy and in the direction normal to the plane of isotropy, respectively, MPa; ν and ν' denote Poisson's ratio in the plane of isotropy and Poisson's ratio in the direction normal to the plane of isotropy respectively. $\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{xz}, \tau_{yz}$ are far field stresses in the Cartesian coordinate system, MPa; $\varepsilon_x, \varepsilon_y, \varepsilon_z, \gamma_{yz}, \gamma_{xz}, \gamma_{xy}$ are far field strains in Cartesian coordinates; MPa.

The degree of anisotropy k and n are defined to represent the difference of elastic parameters in the vertical and horizontal directions.

$$\begin{cases} k = \frac{E'}{E} \\ n = \frac{\nu'}{\nu} \end{cases} \tag{3}$$

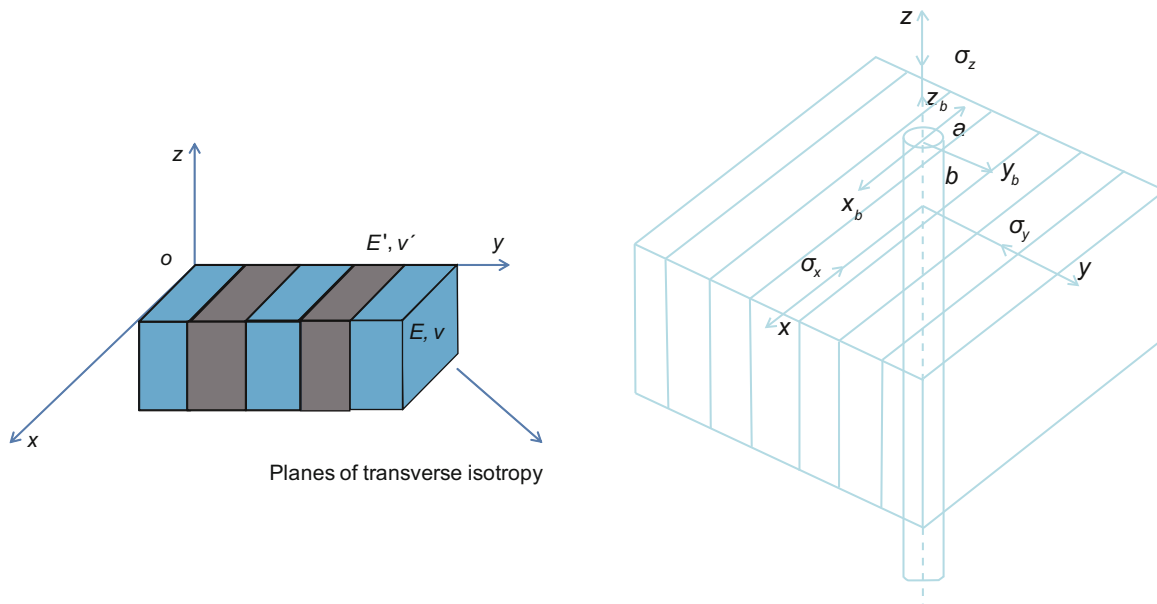


Fig. 2 Elastic parameters and stress analysis of borehole in a coal formation transversely isotropic in a plane striking parallel to the hole axis

Based on the theory of elasticity, Ong (1994), Aadnoy (1987, 1989) and Amadei (1983, 1984, 1996) presented stress distribution equations for a wellbore in an anisotropic formation.

$$\begin{cases} \sigma_x = \sigma_{x,0} + 2Re[\mu_1^2 \phi_1'(z_1) + \mu_2^2 \phi_2'(z_2) + \lambda_3 \mu_3^2 \phi_3'(z_3)] \\ \sigma_y = \sigma_{y,0} + 2Re[\phi_1'(z_1) + \phi_2'(z_2) + \lambda_3 \phi_3'(z_3)] \\ \tau_{xy} = \tau_{xy,0} - 2Re[\mu_1 \phi_1'(z_1) + \mu_2 \phi_2'(z_2) + \lambda_3 \mu_3 \phi_3'(z_3)] \\ \tau_{xz} = \tau_{xz,0} + 2Re[\lambda_1 \mu_1 \phi_1'(z_1) + \lambda_2 \mu_2 \phi_2'(z_2) + \mu_3 \phi_3'(z_3)] \\ \tau_{yz} = \tau_{yz,0} - 2Re[\lambda_1 \phi_1'(z_1) + \lambda_2 \phi_2'(z_2) + \phi_3'(z_3)] \\ \sigma_z = \sigma_{z,0} - \frac{1}{a_{33}}(a_{31} \sigma_{x,h} + a_{32} \sigma_{y,h} + a_{34} \tau_{yz,h} + a_{35} \tau_{xz,h} + a_{36} \tau_{xy,h}) \end{cases} \quad (4)$$

where *Re* denotes the real part of a complex variable; $a_{31}, a_{32}, a_{33}, a_{34}, a_{35}, a_{36}$ denote elastic constants.

For complex variables $z_k = x + \mu_k y$ ($k = 1, 2, 3$), analytic functions $\phi_k(z_k)$ can be obtained from boundary conditions. Lekhnitskii (1981) summarized the analytic functions at any given point. This paper mainly discusses the stress distribution of a horizontal wellbore in a coal seam, therefore, in the adjacent wellbore wall, we can get $x = a \cos \theta, y = a \sin \theta$. For a coal seam formation with apparent transverse isotropy, these functions can be further simplified as:

$$\begin{cases} \sigma_x = \sigma_{x,0} + 2Re[\mu_1^2 \phi_1'(z_1) + \mu_2^2 \phi_2'(z_2)] \\ \sigma_y = \sigma_{y,0} + 2Re[\phi_1'(z_1) + \phi_2'(z_2)] \\ \tau_{xy} = \tau_{xy,0} - 2Re[\mu_1 \phi_1'(z_1) + \mu_2 \phi_2'(z_2)] \\ \tau_{xz} = \tau_{xz,0} + 2Re[\mu_3 \phi_3'(z_3)] \\ \tau_{yz} = \tau_{yz,0} - 2Re[\phi_3'(z_3)] \\ \sigma_z = \sigma_{z,0} - \frac{1}{a_{33}}(a_{31} \sigma_{x,h} + a_{32} \sigma_{y,h}) \end{cases} \quad (5)$$

with

$$\begin{cases} \phi_1'(z_1) = \frac{1}{2\Delta(\mu_1 \cos \theta - \sin \theta)}(E' \mu_2 - D') \\ \phi_2'(z_2) = \frac{1}{2\Delta(\mu_2 \cos \theta - \sin \theta)}(D' - E' \mu_1) \\ \phi_3'(z_3) = \frac{1}{2\Delta(\mu_3 \cos \theta - \sin \theta)}(F' \mu_2 - \mu_1) \end{cases} \quad (6)$$

For a horizontal well, its in-situ crustal stresses are as follows:

$$\sigma_{x,0} = \sigma_H; \sigma_{y,0} = \sigma_h; \sigma_{z,0} = \sigma_v; \tau_{yz,0} = 0; \tau_{xz,0} = 0; \tau_{xy,0} = 0 \quad (7)$$

where σ_H and σ_h denote the maximum horizontal stress and the minimum horizontal stress respectively; σ_v is the overburden stress.

All these stress distribution equations are based on

Cartesian coordinates (x, y, z). When $\theta=0$, the stress vectors are the same for both Cartesian and cylindrical coordinates. However, with an increase in θ , the difference increases gradually. Therefore, it is necessary to convert these equations into cylindrical form (r, θ, z). Here is the transformation equation:

$$\{\sigma\}_{r\theta z} = [T_\sigma] \{\sigma\}_{xyz} \quad (8)$$

where $[T_\sigma]$ is the transformation matrix.

Substituting this equation into stress distribution equations in Cartesian coordinates gives the stress distribution in cylindrical coordinates for a horizontal wellbore.

$$\begin{cases} \sigma_r = \cos^2 \theta \sigma_x + \sin^2 \theta \sigma_y + \sin 2\theta \tau_{xy} \\ \sigma_\theta = \sin^2 \theta \sigma_x + \cos^2 \theta \sigma_y - \sin 2\theta \tau_{xy} \\ \sigma_z = \sigma_z \\ \tau_{r\theta} = \cos \theta \tau_{yz} - \sin \theta \tau_{xz} \\ \tau_{rz} = \sin \theta \tau_{yz} + \cos \theta \tau_{xz} \\ \tau_{r\theta} = -0.5 \sin 2\theta \sigma_x + 0.5 \sin 2\theta \sigma_y + (\cos^2 \theta - \sin^2 \theta) \tau_{xy} \end{cases} \quad (9)$$

where σ_r denotes the radial stress on the wellbore wall, MPa; σ_θ denotes the tangential stress on the wellbore wall at an angular position θ , MPa; σ_z denotes the axial stress on the wellbore wall at an angular position θ , MPa; $\tau_{r\theta}, \tau_{rz}, \tau_{\theta z}$ are the tangential stresses on the wellbore wall in cylindrical coordinates, MPa.

Substituting experimental data (Table 2) into this transversely isotropic model, the change of shear (with the difference of k and n) along the wall can be easily analyzed. The results are shown in Figs. 3 and 4. Fig. 3 shows that when the angle changes, the degree of variation of shear stress with k & n is different. When $\theta=0$, the shear stress reaches its minimum value, and the tensile stress is greater than the tensile strength of coal seams, the borehole formation begins to fracture. On the contrary, the shear stress decreases with the increase in k , that is to say, a larger degree of surface anisotropy indicates easier collapse. However, Fig. 4 indicates that when $\theta=0$, the shear stress of anisotropy ($n>1$) is less than that in isotropic formations ($n=1$), nevertheless, the degree of Poisson's ratio anisotropy has little effect on shear stress. Therefore, the degree of anisotropy must be reasonably considered when fracturing coal formation.

Table 2 Input data

Wellbore configuration	Liquid head, MPa	0
	Well depth, m	2000
Elastic parameters	E , MPa	23443 (variable)
	E' , MPa	5000
	ν	0.15 (variable)
	ν'	0.35
Crustal stress, MPa	σ_v	15
	σ_{Hmax}	11
	σ_{hmin}	8

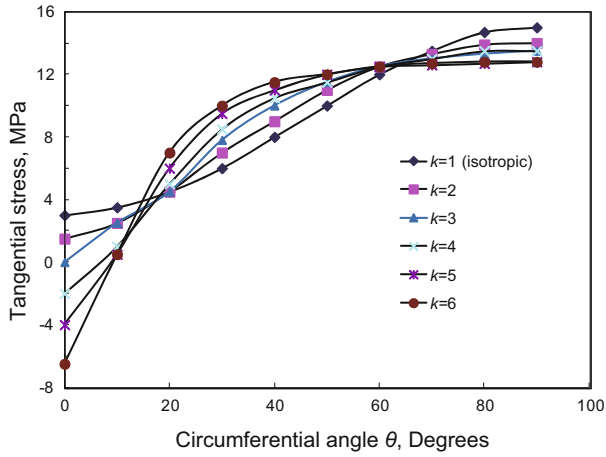


Fig. 3 Shear stress-*k* relations

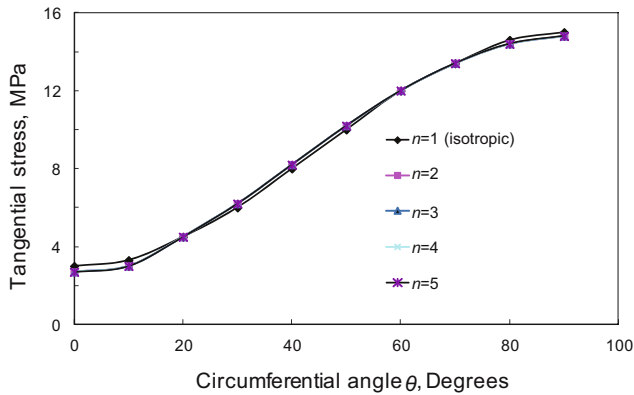


Fig. 4 Shear stress-*n* relations

4 Hydraulic fracture initiation criterion for a horizontal well in a coal seam

4.1 Tensile failure model of hydraulic fractures in coal seams

The fracture pressure of coal formations is closely related to crustal stresses. High liquid head pressure leads to circumferential stress that exceeds the tensile strength of the coal seam, and thus results in fracture initiation and propagation in the formation.

$$\sigma_{\theta} = -S_t \tag{10}$$

where S_t is the tensile strength of rock.

The circumferential stress acting on the wellbore wall σ_{θ} decreases with an increase in bottom hole pressure (BHP). When the BHP reaches a certain value, σ_{θ} becomes negative, which means that the tangential stress transfers from compression to tensile. The wellbore wall experiences fracturing as soon as the tangential stress is greater than the tensile stress of the coal seam, and fracturing occurs where σ_{θ} reaches its minimum value.

The position angle where the tensile failure occurs can be obtained with the following equation:

$$\frac{d\sigma_{\min}(\theta)}{d\theta} = 0 \tag{11}$$

By substituting this position angle θ into Eq. (10), the formation fracture pressure P_f^b when the coal seam fails in tension can be obtained.

Horizontal fractures may occur when the coal formation is shallow enough. Under this circumstance, when the coal seam fractures horizontally, the vertical stress generated by fracturing liquid must be equal to or greater than the overburden pressure σ_v and its vertical tensile strength S_t^v .

Horizontal fracture occurs where:

$$\sigma_z = -S_t^v \tag{12}$$

4.2 Shear failure model of hydraulic fractures along the coal seam cleat

The principal stresses can be calculated using stress components acting on the wellbore wall which are obtained from the above methods. In cylindrical coordinates, the maximum, intermediate and minimum principle stresses can be expressed as follows.

$$\sigma_1 = \sigma_r = p_w \tag{13}$$

$$\sigma_2 = \frac{\sigma_z + \sigma_{\theta}}{2} + \sqrt{\frac{(\sigma_z - \sigma_{\theta})^2}{4} + \sigma_{\theta z}^2} \tag{14}$$

$$\sigma_3 = \frac{\sigma_z + \sigma_{\theta}}{2} - \sqrt{\frac{(\sigma_z - \sigma_{\theta})^2}{4} + \sigma_{\theta z}^2} \tag{15}$$

where p_w denotes the mud weight acting on the borehole wall; $\sigma_1, \sigma_2, \sigma_3$ are the maximum, intermediate, and minimum principle stresses in cylindrical coordinates, MPa.

When existing natural fractures intersect each other, a weak plane model was once used to describe the shear failure of hydraulic fracture along natural fractures. Suppose these natural fractures exist in the main development zone, their alignments and trends are almost the same, thus using the weak plane model to describe the shear failure of hydraulic fractures along natural fractures is reasonable (Zhou et al, 2008; Valkó and Economides, 1994).

The weak plane failure criterion is:

$$\sigma_1 - \sigma_3 = \frac{2(S_w + \mu_w \sigma_3)}{(1 - \mu_w \cot \beta_2) \sin 2\beta_2} \tag{16}$$

where S_w denotes the cohesive stress of the weak plane; μ_w denotes weak plane coefficient of the internal friction; β_2 is the angle between the direction normal to the weak plane and the maximum principal stress.

When $\beta_2 = \phi_w$ or $\beta_2 = \frac{\pi}{2}$, the weak plane would not slide. The weak plane slides only when:

$$\phi_w < \beta_2 < \frac{\pi}{2} \quad (17)$$

Besides, $\sigma_1 - \sigma_3$ must satisfy the weak plane failure criterion Eq. (16). ϕ_w is the friction angle of the weak plane.

In fractured formations, $S_w = 0$, thus the shear failure criterion of hydraulic fractures along the natural fractures is:

$$\sigma_1 - \sigma_3 = \frac{2\mu_w \sigma_3}{(1 - \mu_w \cot \beta_2) \sin 2\beta_2} \quad (18)$$

Consequently, the fracture pressure P_f^z of the shear failure can be calculated.

4.3 Tensile failure model of hydraulic fracture along coal seam cleat

In earth coordinates, the alignment of natural fracture planes is north by east TR degrees. β_2 is equal to the strata dip θ_{DIP} . θ_H is the azimuth of the maximum horizontal stress.

Suppose the natural fracture and wellbore wall intersect at θ , then the normal stress of the fracture plane is:

$$\sigma_n = \sigma_z l_1^2 + \sigma_\theta l_2^2 + \sigma_r l_3^2 \quad (19)$$

$$\begin{cases} l_1 = \sin \theta_{DIP} \\ l_2 = -\sin \theta \cos \theta_{DIP} \cos(TR - \theta_H) \\ \quad + \cos \theta \cos \theta_{DIP} \sin(TR - \theta_H) \\ l_3 = \cos \theta \cos \theta_{DIP} \cos(TR - \theta_H) \\ \quad + \sin \theta \cos \theta_{DIP} \sin(TR - \theta_H) \end{cases} \quad (20)$$

where l_1 , l_2 , and l_3 denote direction cosines of angles between the fracture plane normal direction and three principal stresses.

A tensile failure model of hydraulic fractures along the coal seam cleat can be expressed as:

$$P_f^t = p_w \geq \sigma_n \quad (21)$$

where P_f^t is the tensile failure pressure from cleats, MPa.

For those coal seams with abundant natural cleats, hydraulic fractures may initiate due to any of the following reasons: tensile failure, or from cleats due to shear or tensile failure. The mode of hydraulic fracture initiation and fracture pressure p_f are determined by:

$$p_f = \min \{ p_f^b, p_f^\tau, p_f^t \} \quad (22)$$

where p_f^b is the tensile failure pressure, MPa; p_f^τ is the shear failure pressure from cleats, MPa

If hydraulic fractures initiate in the coal seam mass, the initiation angle can be easily calculated.

5 Conclusions

Based on a series of rock mechanical experiments on samples from coal seams, the following conclusions can be drawn:

1) Confining pressure and pore pressure have a direct effect on the strength of a coal seam; the coal seam in the test area has a relatively low elastic modulus, high Poisson's ratio, high fragility and are easily broken and compressed. Its mechanical & physical properties are markedly different in the directions parallel and normal to the bedding plane, therefore coal seams are often considered as transversely isotropic materials.

2) We consider a coal seam as transversely isotropic based on its mechanical characteristics. Based on the generalized plane strain model, the stress distribution for an arbitrarily orientated wellbore in the coal seam can be determined. The effect of the degree of anisotropy on the stress acting on the wellbore wall is further studied. The results show that when the angle varies, the degree of change in shear stress with the degree of anisotropy is different.

3) Hydraulic fracture initiation in the coal seam mass due to tensile failure, or from cleat due to shear or tensile failure for horizontal wells are also studied in this paper. For those coal seams with an abundance of natural cleats, hydraulic fracture may initiate due to any of above reasons. Hydraulic fracture initiation criteria for horizontal wells in the coal seam are established.

Acknowledgements

The authors are grateful for the financial support from the National Natural Science Foundation of China (No. 51204195, No. 51074171 and No. 51274216).

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(Edited by Sun Yanhua)