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The effective pressure law for permeability of clay-rich sandstones

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Abstract: To study the relative sensitivity of permeability to pore pressure P_p and confining pressure P_c for clay-rich rocks, permeability measurements were performed on samples of four clay-rich sandstones. A new method (hereafter denoted the "slide method") was developed and used for analyzing the permeability data obtained. The effective pressure coefficients for permeability n_k were calculated. The values of n_k were found to be greater than 1.0 and insensitive to changes in pressure. These results confirmed observations previously made on clay-rich rocks. Also, the coefficients n_k obtained had different characteristics for different samples because of differences in the types of clay they contained. The effective pressure law ($\sigma_{eff}=P_c-n_kP_p$) determined using the slide method gave better results about $k(\sigma_{eff})$ than classic Terzaghi's law ($\sigma_{eff}=P_c-P_p$).

Key words: Permeability, effective pressure coefficient, slide method, clay, sandstones

1 Introduction

The functional relationship among permeability k, confining pressure P_c, and pore pressure P_p can be expressed as $k=f(\sigma_{eff})f(P_c-n_kP_p)$, where $P_c-n_kP_p$ is the effective pressure σ_{eff} and n_k the effective pressure coefficient for permeability, a measure of the sensitivity of permeability to pore pressure relative to the sensitivity to confining pressure (Bernabé, 1987; Al-Wardy and Zimmerman, 2004). Applying the effective pressure law for permeability helps understand how permeability changes with pore pressure in reservoirs and allows the setting up of an optimal production strategy. So the effective pressure coefficient for permeability is important for oil and gas production activities.

A large amount of experimental work on the permeability effective pressure law has been reported. Three groups of experimental results can be summarized: 1) n_k equals 1.0 (Brace et al, 1968; Coyner, 1984; Bernabé, 1987). 2) n_k ranges from 0 to 1.0 (Nur et al, 1980; Bernabé, 1986; Warpinski and Teufel, 1992; Zheng et al, 2008; Li and Xiao, 2008; Li et al, 2009a; 2009b). Nur et al (1980) studied two artificial rocks and observed that n_k were constant (n_k values of 0.43 and 0.86) while the others found that n_k varied with confining pressure and pore pressure. 3) n_k values are greater than 1.0 (Zoback and Byerlee, 1975; Nur et al, 1980; Al-Wardy and Zimmerman, 2004; Ghabezloo et al, 2009), indicating that pore pressure has more effect on permeability than confining pressure. Ghabezloo et al (2009) found that n_k was variable while the others observed constant n_k.

Theoretical studies of permeability effective pressure coefficients are sparse. For single-mineral rocks, Bernabé (1986) calculated n_k on the basis of a "tunnel" crack with an elliptical cross-section in an infinite body. He found that n_k is a function of the crack aspect ratio ε (the ratio of the short to the long cross-sectional dimensions) and the Poisson' ratio v of the solid phase, changing from $1/2(1-v)$ for $\varepsilon = 1.0$ (cylindrical channel with circular cross section) to 1.0 for $\varepsilon = 0$ (infinitely thin crack). Berryman (1992) obtained n_k expression on the assumption of a homogenous, one-constituent mineral frame. Furthermore he pointed out that the range of n_k was [ϕ 1.0], where ϕ is the porosity. For two-constituent rocks, Zoback and Byerlee (1975) proposed the conceptual "clay-shell" model to explain why n_k was observed greater than 1.0. Al-Wardy and Zimmerman (2004) derived another clay-particle model and developed analytic solutions for n_k for both the clay-shell and clay-particle models. They found that n_k is related to clay content and the elastic moduli of the rock and the clay. Berryman (1992) presented an equivalent two-constituent model of porous rocks and deduced the expression of n_k. His model can be used to interpret why n_k is observed to be significantly greater

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than unity in rocks containing clays.

In this paper, we conducted a laboratory study aimed at determining the value of n_k in clay-rich sandstones. One important goal was to investigate the effect of clay on permeability at different pressures. So permeability measurements were performed on rock samples subjected to cycles of pore pressure. The "slide method" was developed and used for analyzing the experimental data. The results showed that the measured values of n_k were greater than 1.0 and affected by the types of clay present in the various rocks.

2 Materials and methods

2.1 Rock samples

Four samples were selected for permeability measurements and their properties are listed in Table 1. X-ray experiments showed that the amount of clay present in each sample is 22.5%, 20.3%, 15.0% and 19.0% respectively. Clay minerals mainly consisted of mixed layer illite/smectite (26%-40%) and mixed layer chlorite/smectite (31%-48%). Sample S4, in which kaolinite was not detected, had greater amounts of illite and chlorite than the other samples.

Table 1 The types and content of clay minerals

Sample	Permeability mD	Porosity %	Clay %	Clay mineral, %					
				S	I/S	I	K	C	C/S
S1	0.299	9.89	22.5	20	35	10	2	2	31
S2	0.561	12.31	20.3	–	39	8	–	6	47
S3	0.764	13.02	15.0	–	40	7	2	3	48
S4	0.139	8.53	19.0	–	26	28	–	11	35

Notes: S: Smectite; I: Illite; K: Kaolinite; C: Chlorite; I/S: Mixed layer illite/smectite; C/S: Mixed layer chlorite/smectite

2.2 Experimental procedure

The permeability measurements were conducted with nitrogen gas as the pore fluid so that any chemical and capillary effects that might arise from clay/water reactions and imperfect saturation could be minimized. The measurement apparatus was described in detail in a previous paper (Li et al, 2009a; 2009b). Three cycles of pore pressure with different confining pressures were performed for each sample, and the confining pressure and pore pressure values were set according to reservoir conditions. The measurements in each test were repeated five times and averaged to determine permeability. The viscosities of nitrogen gas at different pressures and temperatures were obtained from the tables of the Beijing Chemical Industrial Company Inc. (1979).

Using nitrogen may cause problems at low pore pressure because of the Klinkenberg effect (Klinkenberg, 1941; Li et al, 2009a). However, the minimum pore pressure in our tests was 6 MPa so that the Klinkenberg corrections can be neglected (Warpinski and Teufel, 1992; Li et al, 2009a; 2009b). Also the study of Bernabé (1986; 1987) and Li et al (2009a; 2009b) revealed that history effects had essentially vanished after two seasoning cycles. So the samples were subjected to at least two seasoning cycles to avoid loading history effects.

2.3 Slide method

From the definition of the effective pressure law, the effective pressure coefficient for permeability, n_k, can be defined as follows (Bernabé, 1986; 1987):

$$n_k = -\frac{(\partial k / \partial P_\mathrm{p})_{P_\mathrm{c}}}{(\partial k / \partial P_\mathrm{c})_{P_\mathrm{p}}} \tag{1}$$

The two partial derivatives in Eq. (1) can be evaluated by calculating the variations of pressure through changing P_c by δP_c and changing P_p by δP_p independently to make the variations δk of permeability the same, so Eq. (1) of n_k is then given by:

$$n_k = \left(\frac{\delta P_\mathrm{c}}{\delta P_\mathrm{p}}\right)_k \tag{2}$$

The calculation procedures of n_k are as follows:

1) Establishing the relationship among permeability k, confining pressure P_c, and pore pressure P_p by the quadratic response-surface method (Li et al, 2009a; 2009b).

$$k^{(\lambda)} = a_1 + a_2 P_\mathrm{c} + a_3 P_\mathrm{p} + a_4 P_\mathrm{c}^2 + a_5 P_\mathrm{c} P_\mathrm{p} + a_6 P_\mathrm{p}^2 \tag{3}$$

Here, we used the Box-Cox transformation, and $k^{(\lambda)} = ((k/k_0)^\lambda - 1)/\lambda$, where $k^{(\lambda)}$ denotes the transformed permeability and k_0 is a normalizing constant. The parameter λ was normally expected to lie between –3 and +3, with $\lambda = 0$ corresponding to log transformation. The coefficients a_i were calculated by least squares regression. Thus, if we combine Eq. (1) with Eq. (3), the values of n_k can be obtained by the response-surface method (Li, et al, 2009a; 2009b).

2) Calculating the coordinates of two points in the confining pressure-pore pressure plane (e.g. M (P_c1, P_p1) and N (P_c2, P_p2)), based on Eq. (3), when δk is equal to zero. If we know the values of any three among P_c1, P_p1, P_c2 and P_p2, the remaining pressure can be solved using Eq. (4).

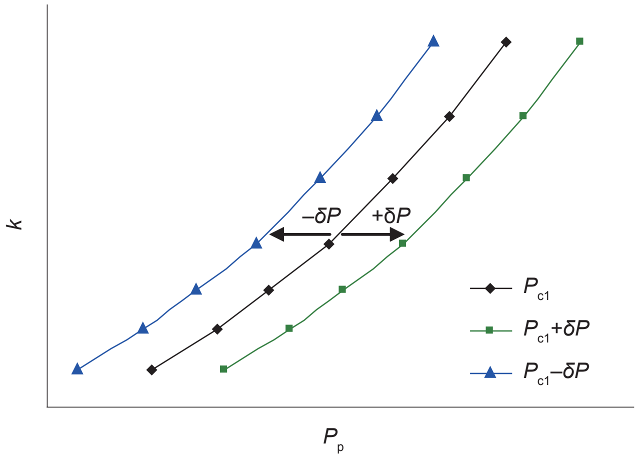
$$\begin{aligned} & a_2 P_\mathrm{c1} + a_3 P_\mathrm{p1} + a_4 P_\mathrm{c1}^2 + a_5 P_\mathrm{c1} P_\mathrm{p1} + a_6 P_\mathrm{p1}^2 \\ & = a_2 P_\mathrm{c2} + a_3 P_\mathrm{p2} + a_4 P_\mathrm{c2}^2 + a_5 P_\mathrm{c2} P_\mathrm{p2} + a_6 P_\mathrm{p2}^2 \end{aligned} \tag{4}$$

3) Calculating the effective pressure coefficient n_k. Assuming the point M (P_c1, P_p1] is given, if we slide P_c1 by δP (Fig. 1(a)) and $P_\mathrm{c2} = P_\mathrm{c1} \pm \delta P$, P_p2 can be solved (solP_p2); if we slide P_p1 by δP (Fig. (1b)) and $P_\mathrm{c2} = P_\mathrm{c1} \pm \delta P$, P_c2 can be

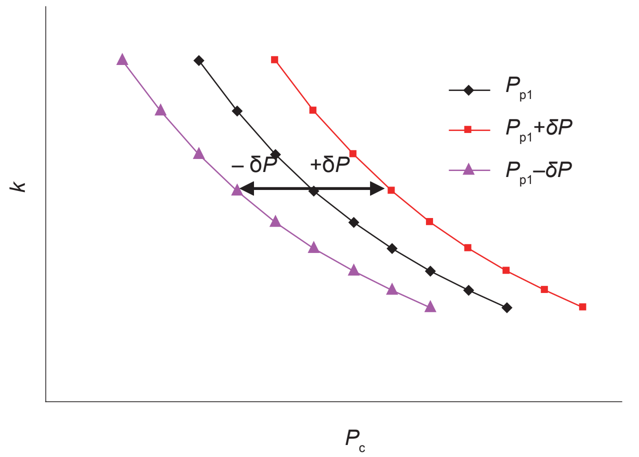
solved (solP_{c2}). Thus, there are four solutions satisfying Eq. (4). So n_k can be expressed as follows:

$$n_k = \left(\frac{\delta P_c}{\delta P_p}\right)_k = \left(\frac{P_{c1} - P_{c2}}{P_{p1} - P_{p2}}\right)_k = \left(\frac{\delta P}{P_{p1} - \mathrm{sol}P_{p2}}\right)_k$$
$$or \left(\frac{\delta P}{P_{p1} - \mathrm{sol}P_{p2}}\right)_k \ or \left(\frac{P_{c1} - \mathrm{sol}P_{c2}}{-\delta P}\right)_k \ or \left(\frac{P_{c1} - \mathrm{sol}P_{c2}}{-\delta P}\right)_k \quad (5)$$

The sliding value δP is commonly the same and, furthermore, is chosen to be small (e.g. 1 MPa). All values calculated are averaged for the determination of effective pressure coefficient n_k. It should be noted that we can get two roots for either P_{c2} or P_{p2} when solving Eq. (4). However, the appropriate root can be identified based on Eq. (5).



(a) Sliding confining pressure P_c



(b) Sliding pore pressure P_p

Fig. 1 Schematic diagrams of the slide method

3 Results

3.1 Permeability measurements

Fig. 2 shows the permeability results for four samples at different confining pressures while cycling pore pressure ("L" and "U" stand for decreasing pore pressure (loading process) and increasing pore pressure (unloading process) respectively). The results show that permeability decreased with increasing confining pressure, and increased with increasing pore pressure. We observed differences in permeability during decreasing and increasing cycles of P_p, which showed that hysteresis effects were still present, although history effects were eliminated by seasoning the samples.

3.2 Effective pressure coefficients

The values of n_k can be calculated by two methods: the response-surface method and the slide method. The results were listed in Table 2. The values of n_k in the decreasing P_p cycle were different from the ones in the increasing P_p cycle at the equivalent pressure for the same P_c, due to stress hysteresis.

Table 2 The values of effective pressure

Sample	The effective pressure coefficient n_k			
	The response-surface method		The slide method	
	Decreasing pore pressure	Increasing pore pressure	Decreasing pore pressure	Increasing pore pressure
S1	0.75-1.65	0.34-1.72	1.16	0.93
S2	0.14-5.59	0-8.31	4.04	2.47
S3	3.74-16.59	2.19-5.88	6.16	4.05
S4	0.78-8.15	0.54-13.19	2.61	1.89

We needed a way to assess the validity of n_k, because the two methods produced different values. The most natural method was to compare the two effective pressure laws determined here: $\sigma_{eff}= P_c - n_{ks}P_p$ (n_{ks} was computed by the slide method) and $\sigma_{eff}= P_c - n_{ks}P_p$ (n_{kr} was computed by the response-surface method). This can be done graphically by comparing plots of k versus σ_{eff} for both effective pressure laws and observing which one gave the lowest dispersion of the data points. The comparison can be quantified by calculating the correlation coefficient R^2 (the range of R^2 was [0, 1], which characterizes the correlation among the variables — the bigger R^2 is, the more closely the variables are related to each other, and the smaller is the root mean square of residuals s. Exponential and polynomial functions were used respectively for k–σ_{eff} curve-fitting. The results of this analysis were in Table 3.

From Table 3, it was concluded that: 1) Whatever the curve-fitting function was, for every sample the correlation coefficient R^2 based on the slide method (R_s^2) was always bigger than the one based on the response-surface method (R_r^2), and the root mean square of residuals s_s was smaller than s_r. 2) The coefficient R^2 from the polynomial-fitting function was bigger than the one from the exponential-fitting function, but s from the polynomial-fitting function was lower than the one from the exponential-fitting function. Therefore,

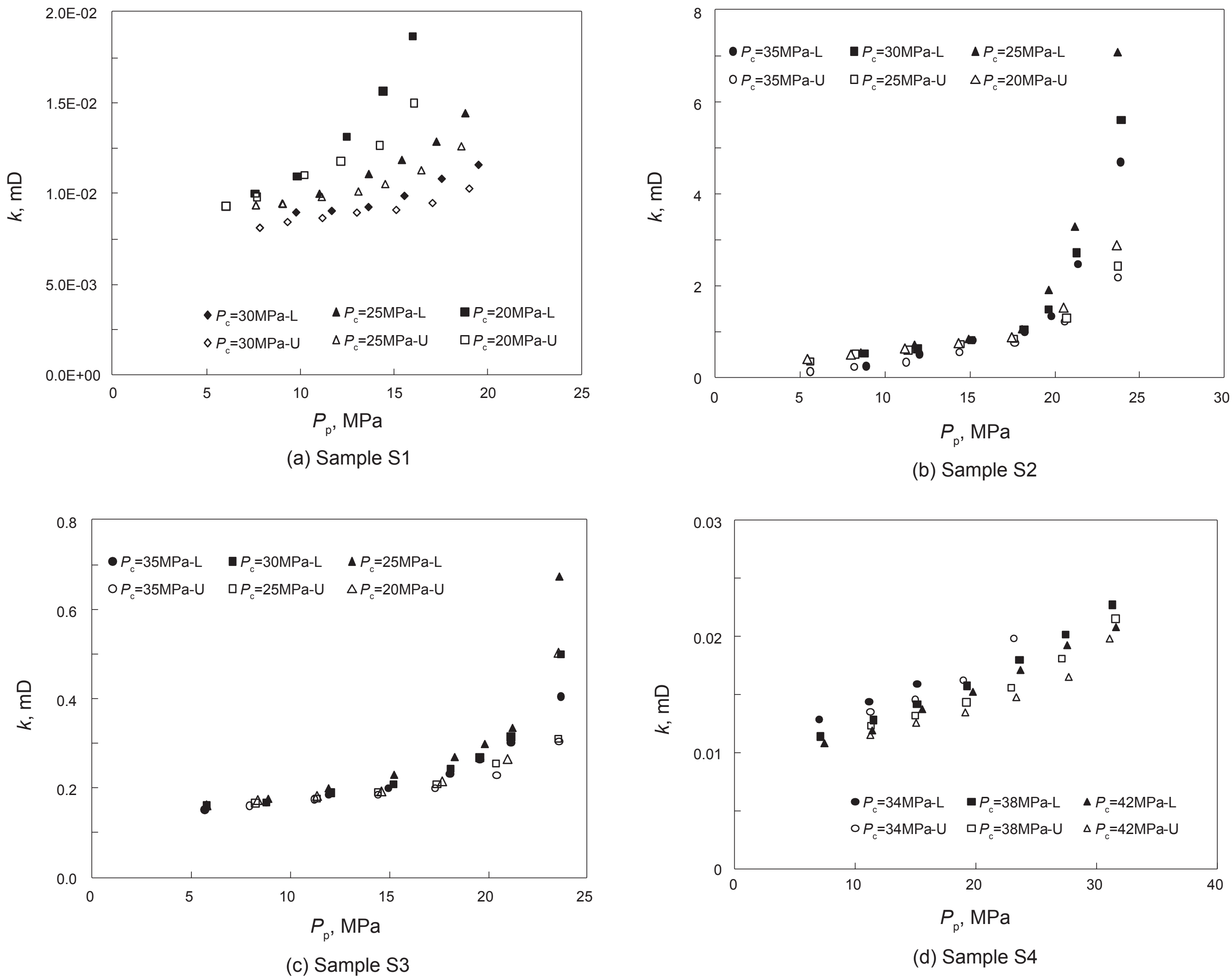


Fig. 2 The relation of k–P_c–P_p

Table 3 Correlation coefficients R^2 and root mean square of residuals s

Sample	Cycle	Exponential function				Polynomial function			
		R_r^2	s_r, mD	R_s^2	s_s, mD	R_r^2	s_r, mD	R_s^2	s_s, mD
S1	Decreasing P_p	0.6440	3.55E-04	0.8830	2.22E-04	0.7192	3.12E-04	0.9466	1.36E-04
	Increasing P_p	0.7628	1.93E-04	0.9079	1.21E-04	0.7772	1.78E-04	0.9642	8.10E-05
S2	Decreasing P_p	0.9231	4.21E+00	0.9052	1.55E-01	0.9352	1.31E+00	0.9800	7.03E-02
	Increasing P_p	0.8096	7.87E-02	0.9130	4.47E-02	0.8619	6.05E-02	0.9449	3.83E-02
S3	Decreasing P_p	0.3712	2.28E-02	0.8391	1.32E-02	0.3362	4.66E-02	0.9960	9.53E-03
	Increasing P_p	0.8626	8.26E-03	0.7987	9.87E-03	0.8882	5.86E-03	0.9790	2.50E-03
S4	Decreasing P_p	0.1097	8.10E-04	0.9741	1.24E-04	0.1145	8.06E-04	0.9900	9.63E-05
	Increasing P_p	0.2152	7.15E-04	0.9278	1.90E-04	0.3041	7.16E-04	0.9650	1.00E-04

Notes: Subscripts r and s stand for the response-surface method and the slide method respectively

the k–σ_{eff} curves were fitted better by the polynomial function and the slide method gave better results than the response-surface method (Fig. 3). So we conclude that the slide method should be used to determine the effective pressure law. Moreover, we observed that the values of n_k obtained with the slide method were constant and greater than 1.0 (Table

2), demonstrating that pore pressure had more effect on permeability than confining pressure.

There is less scatter for $k(\sigma_{eff})$ with the slide method (solid circles) than both for $k(\sigma_{eff})$ with the response-surface method (open circles) and for $k(\sigma_{eff})$ with Terzaghi's law $\sigma_{eff} = P_c - P_p$(start).

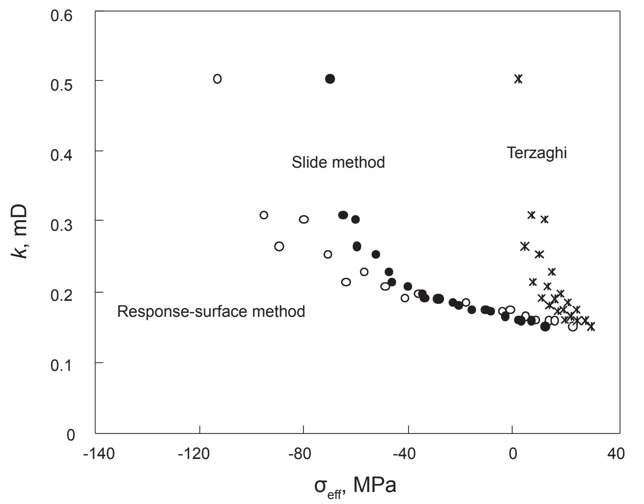


Fig. 3 Assessment of the quality of the effective pressure law for Sample S3

4 Discussion

In the past, Terzaghi's law ($\sigma_{eff} = P_c - P_p$) has usually been referred to as the classic effective pressure law and often applied to analyze rock behavior. Therefore, we compared Terzaghi's law ($\sigma_{eff} = P_c - P_p$) to the effective pressure law obtained using the slide method. We denote R_t^2 and s_t the corresponding correlation coefficients and root mean square of residuals. It was found that 1) R_t^2 values for Samples S1, S2, S3, and S4 are 0.9535, 0.7743, 0.6931, and 0.9851 in decreasing P_p cycles, and 0.9643, 0.8442, 0.6873, and 0.9713 in increasing P_p cycles. 2) The corresponding s_t (mD) values for Samples S1, S2, S3, and S4 are 1.27E-04, 0.231, 1.56E-02, and 1.05E-04 in decreasing P_p cycles, and 7.14E-05, 8.35E-02, 1.09E-02, and 1.14E-04 in increasing P_p cycles. Therefore, R_t^2 was less than R_s^2 and s_t was larger than s_s for Samples S2, S3, and S4, but for Sample S1 they were almost identical because n_k was close to 1.0. These results suggest that the effective pressure law obtained with the slide method gave better results than Terzaghi's law (Fig. 3).

In addition, the results obtained in this paper confirm that n_k is constant and higher than 1.0 in sandstones with a substantial amount of clays (Zoback and Byerlee, 1975; Nur et al, 1980; Berryman, 1992; Al-Wardy and Zimmerman, 2004). From X-ray analysis data, the clay content in the sandstone samples was more than 10%.

However, in Sample S4 the clay content was as high as 19% but n_k was relatively small. From the X-ray analysis, it was found that Sample S4 had more illite and chlorite, and less mixed layers than the other samples, but no smectite and kaolinite. As it is known that smectite, kaolinite, and greater mixed layers are easily compressed, our results suggest that greater value of n_k may be correlated with greater compressibility. On the contrary, illite and chlorite are much harder to compress because the crystallographic interplanar spacing of illite and chlorite is smaller (Ren, 1988; Yang and Ye, 2003; Zhao and Zhang, 1990). In other words, their compressibility is smaller, which makes the value of n_k smaller. In summary, we found that the type of clay also had an influence on the value of n_k.

5 Conclusions

1) The effective pressure coefficients n_k are constant and greater than 1.0 for clay-rich sandstones, which confirms that permeability is more sensitive to change in pore pressure than to change in confining pressure.

2) The slide method was developed and found to be adequate for calculating the coefficient n_k.

3) The effective pressure law with the slide method (n_k>1.0) gave a better relationship between permeability and effective pressure than Terzaghi's law (n_k=1.0).

4) The effective pressure coefficient n_k was affected by clay mineralogy.

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