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The Impact of Writing Notations and Response Types: A Spatial Representation Study on Fractions

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Abstract
The combined effect of writing notations and response types on the spatial representation of fractions remains unclear. The present study explored this by employing a magnitude comparing task with irreducible and single-digit proper fractions as target stimuli as well as 1/2 ($\frac{1}{2}$) as benchmarks for right-handed undergraduate students (64 males, 76 females; Mean age = 20.65 years, *SD* = 1.75, with normal or corrected-to-normal vision) in E-Prime 2.0 Professional Software platform. We found that: 1) for the horizontal response type, the horizontal writing notation of fractions did not produce a reliable SNARC effect or reversed SNARC effect; 2) for the same response type, the vertical writing notation of fractions elicited a SNARC effect; 3) for the vertical response type, both vertical and horizontal writing notations of fractions led to a reversed SNARC effect. These results indicated a combined effect of writing notations and response types on the spatial representation of fractions. Specifically, writing notations can affect the spatial representation of fractions under the condition of horizontal response types but not the vertical response types.

Keywords Fraction · Response Types · SNARC Effect · Writing Notations

Introduction

The association between numerical and spatial cognition has been confirmed and emphasized for about two decades (Meng et al., 2019; Shaki & Fischer, 2018). Currently, one of the most famous evidences of spatial-numerical associations is the spatial-numerical association of response codes effect (SNARC effect) (Dehaene et al., 1993; Meng et al., 2019). It is a phenomenon wherein participants' left hands have faster responses for small numbers, while their right hands have faster responses for large numbers in magnitude comparing tasks or parity judgment tasks. The effect has been found in various numbers, such as positive numbers (Fischer & Shaki., 2016), negative numbers (Fischer & Shaki., 2017), integers (Dehaene et al., 1993), decimals (Sun et al., 2017), fractions (Toomarian et al., 2019). In addition to the SNARC effect, the phenomenon wherein participants' left hands have faster responses for large numbers, while their right hands have faster responses for small numbers (namely reversed SNARC effects) have also been found in many kinds of numbers (Shaki et al., 2009; Zeng et al., 2022), including fractions (Bonato et al., 2007). At first, the SNARC effect was found by Dehaene et al. (1993) in a study of number representation. From their original findings in France, many researchers have started to pay attention to the field of the associations between numbers and space, trying to find more evidence that numerical cognition connects to spatial perceptions. Exhilaratingly, a great deal of studies about the question appeared after the classical study and proved numbers have an association with the space, whatever cultures (Cipora et al., 2019; Zebian, 2005). Recently, Meng et al. (2019) even found SNARC effect existed in numerosity, specifically, responses were faster on the left for smaller nonsymbolic ratio magnitudes and faster on the right for larger nonsymbolic ratio magnitudes. However, studies about the SNARC effect of fractions are relatively few, especially the study of influencing factors on the fractional SNARC effect. According to Siegler et al. (2020), fraction study is a difficulty in children's mathematics learning. Exploring the relationship between fractions and space can provide guidance

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for fraction learning, and offer a more effective approach to developing fraction teaching plans. The current study intends to illustrate the impacts of writing notations and response types on the spatial representation of fractions, and further clarify the mechanism of fractional SNARC effect.

The simple history of fractions

As one of the most common numbers, '*a fraction (from Latin fractus, "broken") represents a part of a whole or, more generally, any number of equal parts. When spoken in everyday English, a fraction describes how many parts of a certain size there are, for example, one-half, eight-fifths, three-quarters.*' (Fraction, 2022). The fractions are invented to deal with some practical problems, in which the parts contained in a whole need to be examined or calculated. When people were faced with such problems, fractions were invented because this kind of problem cannot be solved by integers. The earliest fractions were the reciprocals of integers, which were used by the Egyptians (Howard, 1990; Math history, 2022). The modern fraction (with fraction bar) is first attested in the work of a Muslim mathematician named al-Hassar ("Earliest Uses of Symbols for Fractions" 2022).

Previous studies on SNARC effect of fractions

So far, a few studies have focused on the SNARC effect of fractions, most of these studies investigated the influence of fractional number types on the fractional SNARC effect. In the modern mathematics, fractions can be categorized into various types, such as unit fractions, proper fractions, improper fractions, reducible fractions, irreducible fractions, single-digit fractions, multidigit fractions, and so on. All types of fractions can be investigated from the perspective of spatial-numerical associations. Bonato et al. (2007) paid attention to single-digit unit fractions (Namely, single-digit proper fractions with 1 as the numerator). They compared the magnitude of 1/5 (the benchmark) and that of single-digit unit fractions (denominators varying from 1 to 9, except 5). They found a reversed SNARC effect in their results, which proving the existence of spatial-numerical associations in fractions. They concluded that participants adopted a componential strategy of only accessing denominators. Specifically, participants '*associated the denominator to the left response when it was smaller than the denominator of the reference and to the right response when it was large*' (p.1413). Because the magnitude of a fraction is inversely proportional to that of its denominator for single-digit unit fractions, the small magnitude of fractions is associated with right response, while the large magnitude of fractions is associated with left response, namely, the reversed SNARC effect emerges. From the perspective of developmental psychology, Liu et al. (2013) replicated the experiments of Bonato et al. (2007) on sixth-grade children and found the same reversed SNARC effect. It also indicated that sixth-grade children adopted a componential strategy of only accessing denominators when they processed single-digit unit fractions. However, when fractional types were changed, the spatial representation of fractions became different. Toomarian and Hubbard (2018) adapted the study of Bonato et al. (2007) by using the irreducible and single-digit proper fractions as stimuli and found that fractional '*stimulus properties dramatically impact spatial representations of fractions*' (p.1761). The components of a fraction (e.g., numerators) especially played a pivotal role in determining the type of fractional SNARC effect. When numerators were same (e.g., all numerators are 1, as Bonato et al. (2007) did in their study), the consequence was the presence of a reversed SNARC effect. However, the SNARC effect was elicited when numerators were different (e.g., $\frac{2}{3}$, $\frac{3}{5}$). The authors concluded that individuals processed fractions by using two different strategies. One of the strategies was a componential strategy, by which individuals only processed the magnitude of a denominator or numerator and led to a reversed SNARC effect of fractions. The other strategy was a holistic strategy, by which individuals processed the magnitude of a fraction and caused a SNARC effect of fractions. Specifically, which strategy is utilized depends on experimental tasks and contexts, especially the type of fractions (DeWolf & Vosniadou, 2015; Schneider & Siegler, 2010; Toomarian & Hubbard, 2018). Individuals tend to adopt a more straightforward and simple strategy rather than a complicated strategy even though the complicated strategy is beneficial to problem solving.

Besides investigating the impact of fractional number types on the fractional SNARC effect, other studies examined whether the performance of the fractional SNARC effect can predict higher-order fractional mathematic grades. Toomarian et al. (2019) '*investigated how individual implicit and explicit spatial representations of fractions relate to fraction knowledge and other formal measurements of math achievement*' (p. 9). They found that the performance of spatial representations of fractions was not a significant predictor of algebra scores.

As we mentioned above, only a few studies have paid attention to the spatial representation of fractions. Currently, there are still lots of questions about the mechanism of fractional SNARC effect. For example, in addition to fractional types, whether other potential factors can determine the SNARC effect of fractions?

The notations of fractions

Considering the special attribute that fractional numbers consist of a numerator and denominator, fractions can be written with two different notations. One notation is the fraction with a horizontal fraction bar. It was introduced by the Arabs. They improved the Hindu notation of ancient fractions by inserting a horizontal bar between the two numbers and attested this notation in a work around 1200 ("Earliest Uses of Symbols for Fractions" 2022). In modern mathematics, the notation of fractions, which consists of a numerator displayed above a line and a non-zero denominator displayed below that line, is known as the vertical fraction. It often occurs in many printed documents today. The other notation is the fraction with a diagonal fraction bar (also called a solidus bar or virgule bar). It was found in an early handwritten document named "Ledger of 1718". In this document where quantities of tea and coffee transactions were listed, fraction bars were replaced with forward slashes ("Earliest Uses of Symbols for Fractions" 2022). Similarly, the notation of fractions, which consists of a numerator displayed before a slash and a non-zero denominator displayed after that line, also has a nickname (horizontal fractions) in modern mathematics. The horizontal fraction is commonly used in official documents and electronic texts today.

Most previous studies of the mechanism of fractional SNARC effect used vertical writing notations as stimuli (Bonato et al., 2007; Liu et al., 2013; Toomarian & Hubbard, 2018). The horizontal writing notation was ignored. In China, children learn vertical fractions initially in their primary schools (Research and Development Center of Mathematics Curriculum Materials, 2014). At that time, they are very young. However, they start to be acquainted with horizontal fractions not until using some electronic calculating devices or reading relatively formal electronic texts. Therefore, they are more familiar with vertical fractions. Compared to horizontal fractions, Chinese people are habituated to adopt vertical fractions as operands to calculate in mental arithmetic tasks due to their early learning experiences. According to previous studies, individuals' familiarity with numbers can influence the representation of numbers (Ebersbach et al., 2008; Lipton and Spelke, 2005). Given that, it is possible that Chinese people adopt different mental representation modes when process fractions with different writing notations. Moreover, from the perspective of the digital spatial arrangement, the relative position between numerators and denominators is different in the two writing notations of fractions. Thus, the writing notation of fractions may be a potential modulator that determines individuals' spatial representation of fractions. It is necessary to clarify the influences of writing notations on the fractional SNARC effect.

Spatial reference frame and response types

According to the spatial reference frame account, the spatial reference frame of numbers is a means of representing the locations of numbers in space (Freksa et al., 1998). It determines individuals' spatial representation of numbers (Viarouge et al., 2014; Wood & Fischer, 2008). Previous studies have proved that only when the spatial reference frame has the same directional dimension as individuals' response, the spatial-numerical association occurs (Mourad & Leth-Steensen, 2017). Namely, when individuals spatially represent numbers by using spatial reference frames, they have to choose those frames that have the same directional dimension as their responses. Specifically, when horizontally respond to numbers, individuals have horizontal reference frames, such as the left-to-right reference frame or the right-to-left reference frame. Oppositely, when vertically respond to numbers, vertical reference frames will be utilized, such as the bottom-to-up reference frame or the top-to-down reference frame (Viarouge et al., 2014). Therefore, individuals' spatial representation of numbers must be different between horizontal and vertical response types because spatial reference frames of the two response types are approximately orthometric. Currently, the reasonable inference has been proved in the field of integers. Namely, previous studies have demonstrated different response types lead to distinct spatial representation of integers.

For the horizontal response type, it was initially adopted by Dehaene et al. (1993) in a parity judgment task. In this task, participants were instructed to judge whether an integer was odd or even by pressing left- and right-located keys. The results showed that participants' left-hand responses were faster when the stimulus was a small number, while their right-hand responses were faster when the stimulus was large. Namely, the horizontal SNARC effect occurred under the horizontal responsive condition. The theory of the mental number line (MNL) explains this phenomenon by highlighting the mechanism that numbers are represented in a horizontally arranged continuum. It is called MNL. The orientation of the MNL depends on individuals' culture and early experiences. For most people living in western cultures, they have a left-to-right reading habit, and their MNL is consequently oriented from left to right. Smaller numbers are located on the left, while larger numbers are found on the right (Aleotti et al., 2020; Dehaene et al., 1993). In fact, the MNL is often considered as a specific reference frame (Wood et al., 2006). Thus, the study of Dehaene et al. (1993) indicates that individuals' horizontal response type with a horizontal reference frame leads to a horizontal SNARC effect of integers. For the vertical response type, it was initially used by Ito and Hatta (2004) in a parity judgment task. They imitated the task of Dehaene et al. (1993) but changed the instruction of horizontal responses to the vertical responsive one. They found that '*the subjects responded faster to large numbers with the top choice than with the bottom choice, whereas the reverse held true for small numbers*' (p. 670), which indicated a vertical SNARC effect. They inferred that the vertical SNARC effects occurred because individuals vertically represented numbers depending on ordinal information rather than quantitative information of numbers. Specifically, Japanese individuals learn mathematics by using a kind of mathematical diagram, in which larger numbers locate in the upper right, whereas smaller numbers locate in the lower left. This ordinal information of numbers (from lower left to upper right) is internalized to a bottom-to-up reference frame in the vertical dimension. Therefore, in their Japanese participants, the vertical SNARC effects appeared, instead of reverse vertical SNARC effects. Then, Hartmann et al. (2014) found a reversed vertical SNARC effect of integers when individual responded by their feet instead of hands. Cooney et al. (2021) demonstrated that even at an early stage of formal education, children could flexibly assign magnitude of integers to the horizontal and vertical dimensions. In addition to the horizontal SNARC effect, children also had the vertical SNARC effect. What's more, Li et al. (2017) found the vertical SNARC effect was flexible for Chinese people. Namely, when Chinese people use different numerical notations, such as Arabic, simplified Chinese, and complex Chinese numerical notations, their spatial-numerical association is various in the vertical dimension. To sum up, different response types can change spatial-numerical associations of integers.

Given the impact of response types on the SNARC effect of integers, it is reasonable to speculate that response types also play a key role in individuals' spatial representation of fractions. However, previous studies have only paid attention to the spatial-numerical associations of fractions under the horizontal responsive condition. The vertical response type has been neglected. Therefore, it is necessary to investigate the potential influence of vertical response type on individuals' spatial representation of fractions and the difference between two response types.

The current study

By abovementioned literature reviews, writing notations of fractions and response types can be inferred as the possible influencing factors of the spatial-numerical associations of fractions. Thus, we attempt to explore the fractional SNARC effect under various combinations of writing notations and response types. Specifically, vertical fractions are horizontally responded to in Experiment 1; horizontal fractions are horizontally responded to in Experiment 2; vertical fractions are vertically responded to in Experiment 3; and horizontal fractions are vertically responded to in Experiment 4. We hypothesize that 1) Chinese people represent fractions of different writing notations in a distinct way due to dissimilar familiarity with these writing notations; 2) Chinese people represent fractions differently in two distinct response types as a result of using dissimilar spatial reference frames; 3) Writing notations interact with response types because the spatial information is not only included in the writing notation of fractions (vertical or horizontal writing notations), but also included in the response types (horizontal or vertical responses). To sum up, we assume that the spatial representation of fractional numbers is influenced by a combined effect of response types and writing notations of fractions. Due to the flexibility of numerical representation in vertical dimensions (Qiao et al., 2016), the influence of writing notations on the fractional SNARC effect only exists under the condition of horizontal response types but does not exist for vertical response types.

Experiment 1

Methods

Participants

We adopted recruitment by releasing advertisements on campuses and a random sampling approach to choose the participants. Thirty-five right-handed undergraduate students (16 males, 19 females; mean age = 19.85 years, $SD = 1.53$) with normal or corrected-to-normal vision voluntarily participated in the experiment. The participants gave their consent to participate all these studies. None of the participants had participated in similar experiments before, nor were they aware of the purpose of the current experiment. All of them were given a small reward for their participation in the studies. All present studies were approved by the relevant ethics committee.

Stimuli and apparatus

The experiment was performed using E-Prime 2.0 Professional Software (Psychology Software Tools, Pittsburgh, PA). The stimuli included 26 irreducible and single-digit proper fractions in a vertical writing notation (e.g.,$\frac{2}{3}$,$\frac{3}{5}$). They were exhibited on a 15-inch screen with a resolution of 1024 × 768 pixels. Because the value of $\frac{1}{2}$ is in the midpoint of values of irreducible and single-digit proper fractions, as Toomarian and Hubbard (2018) did, we adopted $\frac{1}{2}$ as a benchmark, to ensure an equal number of fraction values greater and less than the comparison benchmark. After that, 13 fractions were greater than $\frac{1}{2}$ (including: $\frac{2}{3}$, $\frac{3}{4}$, $\frac{3}{5}$, $\frac{4}{5}$, $\frac{4}{7}$, $\frac{5}{6}$, $\frac{5}{7}$, $\frac{5}{8}$, $\frac{5}{9}$, $\frac{6}{7}$, $\frac{7}{8}$, $\frac{7}{9}$,$\frac{8}{9}$) and the others less than $\frac{1}{2}$ (including: $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$, $\frac{3}{7}$, $\frac{3}{8}$, $\frac{2}{5}$, $\frac{2}{7}$, $\frac{2}{9}$,$\frac{4}{9}$). All stimuli were presented in black text about 1.8 cm wide and 2.7 cm tall (visual angle: 1.5° × 2.8°). Participants respond to stimuli with a standard QWERTY keyboard.

Procedure

Participants were seated before a screen positioned approximately 68 cm away from their eyes. During the experiment, participants were first instructed by the response rule of a magnitude comparing task. Then, a fixation ('+') was displayed at the centre of the screen with a white background for 600 ms. Next, a blank screen appeared for 1000 ms. A fraction (the target stimuli) was then flashed on the screen before disappearing after 3000 ms or after a response was made by the participants. Participants were instructed to make the comparing decision and respond as soon as possible on the premise of ensuring the correct answer. After the trial ended, succeeding trials automatically began.

There were two blocks with 20 practice trials and 416 experimental trials (each block had 10 practice trials in practice stage and 208 experimental trials in formal experiment stage). In the practice stage, fractions emerged randomly but not repeatedly. In the formal experiment stage, every fraction emerged 16 times (in each block, every fraction emerged 8 times). In the first block, participants were instructed to respond to fractions less than $\frac{1}{2}$ with the 'd' key, and to fractions greater than $\frac{1}{2}$ with the 'k' key. In the second block, participants were instructed to respond to fractions greater than $\frac{1}{2}$ with the 'd' key, and to fractions less than $\frac{1}{2}$ with the 'k' key. The order of the blocks was counterbalanced across participants. Thus, participants horizontally responded to fractional stimuli by utilizing horizontally distributed keys. A 5-min break was allotted between the two blocks. Participants were also given a break for every 52 trials completed. Participants had to obtain an accuracy rate of at least 85% in every practice stage before beginning formal experiment stage in each block (see Fig. 1).

Study design

In the experiment, we adopted a 2 (magnitude: large '$>\frac{1}{2}$', small '$<\frac{1}{2}$') × 2 (response orientation: left, right) within-subject study design. Every independent variable contains two levels. For the magnitude, all stimuli of fractions are larger than $\frac{1}{2}$ at the first level. Oppositely, all stimuli of fractions are smaller than $\frac{1}{2}$ at the second level. For the response orientation, participants respond to stimuli with the 'd' key (located in the left position of a standard QWERTY keyboard and corresponding to a left response) at the first level. Reversely, participants respond to stimuli with the 'k' key (located in the right position of a standard QWERTY keyboard and corresponding to a right response) at the second level. Dependent variables include 1) the reaction time (RT) and 2) the reaction time difference (dRT).

Data analysis

According to the standard of data screen in the studies of Toomarian and Hubbard (2018) about fractional SNARC effects, the following types of data will be excluded from data analysis: 1) data from subjects whose accuracy rate in formal experiment stages is less than 85%; 2) data from error response trials; 3) data form potentially random response trials (trials with RT < 300 ms); 4) data form abnormal long-time response trials (trials with RT > 2000 ms). In this experiment, Toomarian's exclusion criteria of data analysis was adopted.

We used repeated measures ANOVA with magnitude (large '$>\frac{1}{2}$', small '$<\frac{1}{2}$'), and response orientation (left, right) as within-subject factors. The dependent variable was the RT. If a SNARC effect or reversed SNARC effect existed, a significant interaction between the magnitude and response orientation would be found. To further confirm

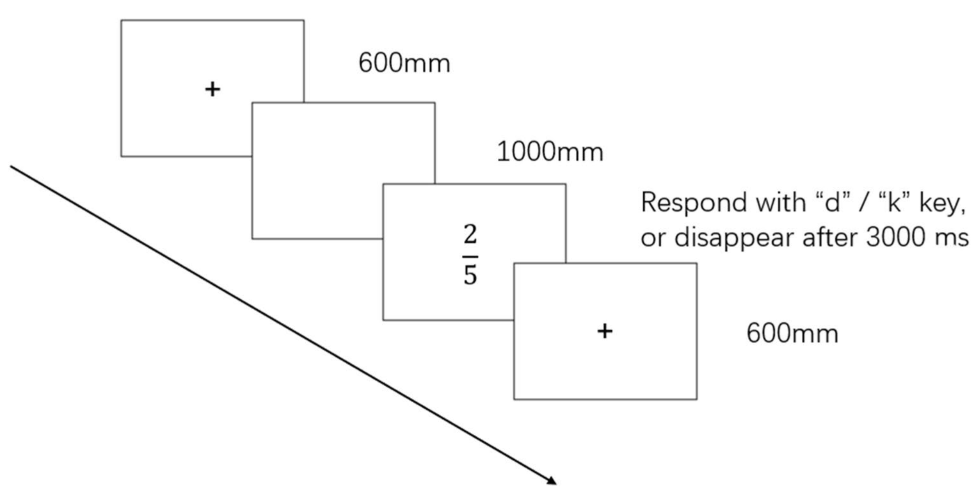

Fig. 1 The processing of Experiment 1

and estimate whether there was a SNARC effect or reversed SNARC effect, the repeated measures regression analysis approach described in previous studies (Ito & Hatta, 2004; Toomarian & Hubbard, 2018) was used. In this approach, the following steps was carried out. Firstly, the dRT was calculated for each fraction across all participants. Specifically, the value of dRT was computed by subtracting the average RT for the left response from the average RT for the right response because most individuals have a left-to-right MNL. Secondly, the regression analysis step was conducted by using the magnitude as a predictor variable. In this step, if the regression weight significantly deviates from zero through the t-test, it means that a SNARC effect or reversed SNARC effect has occurred. In addition, through this step, besides the significance level, a positive or negative regression slope was also obtained. The negative regression slope is equal to a negative correlation between magnitude and dRT, which indicates a SNARC effect. Oppositely, the positive regression slope is equal to a positive correlation between magnitude and dRT, which indicates a reversed SNARC effect.

Results

In the experiment, data from a total of 35 subjects were collected, and data of five subjects were completely excluded. The final data exclusion rate was 6.10%. The average RT in each condition is shown in Fig. 2.

The repeated measures ANOVA revealed that there was a significant main effect for both the response orientation [F (1, 29) = 5.36, $p < 0.05$, $\eta^2 = 0.16$] and magnitude [F (1, 29) = 18.36, $p < 0.001$, $\eta^2 = 0.39$]. In addition, a significant interaction was observed between the magnitude and response orientation [F (1, 29) = 5.48, $p < 0.05$, $\eta^2 = 0.16$]. A simple effect analysis found that for small magnitudes, participants' left response was faster than their right response ($p < 0.001$). However, there was no significant difference between participants' left response and right response ($p > 0.05$) for large magnitudes. The above results demonstrate the existence of a SNARC effect (see Fig. 2). Moreover, the dRT regression analysis showed a significant negative regression slope (B = -88.78; t (29) = -3.61; $p < 0.001$), confirming the existence of a SNARC effect.

Discussion

To explore the spatial representation of fractional numbers, we used the magnitude comparing task, in which fractional numbers were exhibited with a vertical writing notation and were horizontally responded to by the participants, in Experiment 1. The results indicated the presence of a SNARC effect and replicated the results of Toomarian and Hubbard (2018), who also used vertical fractional numbers and a horizontally responsive manner. Thus, we can ensure that under a horizontally responsive condition, vertical fractions indeed elicit a SNARC effect. It indicates that the association between vertical fractions and space is a stable psychological effect, and horizontally response to irreducible and single-digit proper vertical fractions indeed induces a spatial representation of a left to right mental number line.

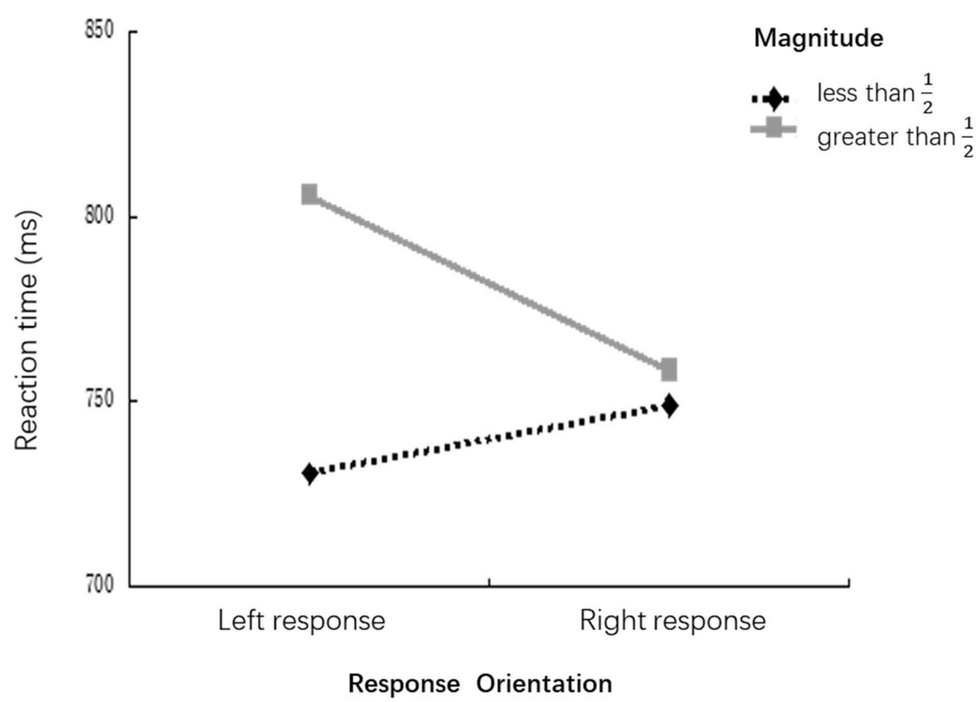

Fig. 2 Average reaction times and the interaction between the magnitude and response orientation in Experiment 1

According to this result, developing new fractional teaching approaches for operating horizontal mental number lines is a key to improving individuals' fractional learning. The study of Barbieri et al. (2020) has assisted struggling learners to make durable gains in their conceptual understanding of fractions by this kind of teaching approach.

Horizontal writing notation of fractions is a common notation in official documents and electronic texts. However, Chinese people are more accustomed to reading horizontal fractions rather than calculating horizontal fractions mentally. Because of their early learning experiences, Chinese people are better at operating vertical fractions in the mind. Therefore, it is rational that the fractional SNARC effect is different between horizontal and vertical writing notations. On these grounds, we aimed to examine the spatial representation of fractions under the condition of horizontally respond to horizontal fractions, and to elucidate the potential mechanism.

Experiment 2

Methods

Participants

We adopted recruitment by releasing advertisements on campuses and a random sampling approach to choose the participants. Thirty-five right-handed undergraduate students (14 males, 21 females; mean age = 21.57 years, $SD = 2.15$) with normal or corrected-to-normal vision voluntarily participated in the experiment. None of the participants had participated in similar experiments before, nor were they aware of the purpose of the current experiment. All of them were given a small reward for their participation in the studies. All present studies were approved by the relevant ethics committee.

Stimuli and apparatus.

The experiment was performed using E-Prime 2.0 Professional Software as in Experiment 1. The stimuli included 26 irreducible and single-digit proper fractions in a horizontal writing notation (e.g., 2/5, 3/7). As the first experiment, we adopted $\frac{1}{2}$ as the benchmark but used its horizontal writing notation. After choosing 1/2 as a benchmark, there were 13 fractions greater than 1/2 (including 2/3, 3/4, 3/5, 4/5, 4/7, 5/6, 5/7, 5/8, 5/9, 6/7, 7/8, 7/9, 8/9) and the remaining fractions were less than 1/2 (including 1/3, 1/4, 1/5, 1/6, 1/7, 1/8, 1/9, 3/7, 3/8, 2/5, 2/7, 2/9, 4/9). All stimuli were presented in black text using size 48 Arial font. Other characters of the stimuli in this experiment were similar to those in the first experiment.

Procedure

Except for adopting the horizontal writing notation of stimuli, the same procedures used in the first experiment were retained.

Study design

In the experiment, we adopted a 2 (magnitude: large '> 1/2', small '< 1/2') × 2 (response orientation: left, right) within-subject study design. Every independent variable contains two levels. For the magnitude, all stimuli of fractions are larger than 1/2 at the first level. Oppositely, all stimuli of fractions are smaller than 1/2 at the second level. The levels of the response orientation are the same as that in Experiment 1. Dependent variables include 1) the RT and 2) the dRT.

Data analysis

Apart from analysing the magnitude of horizontal fractions, the same approaches of data analysis used in the first experiment were also applied to the second experiment.

Results

In the experiment, data from a total of 35 subjects were collected, and data of two subjects were completely excluded. The final data exclusion rate was 6.30%. The average RT in each condition is shown in Fig. 3.

The repeated measures ANOVA revealed that there was a significant main effect for both the response orientation [F (1, 32) = 7.01, $p < 0.05$, $\eta^2 = 0.18$] and magnitude [F (1, 32) = 15.05, $p < 0.05$, $\eta^2 = 0.32$]. However, there was no significant interaction between the magnitude and response orientation [F (1, 32) = 0.07, $p > 0.05$], which demonstrates the disappearance of a reliable SNARC effect and reliable reversed SNARC effect. Moreover, the dRT regression analysis showed a nonsignificant positive regression slope ($B = 9.05$; t (32) = 0.47; $p > 0.05$), confirming the disappearance of a reliable SNARC effect and reliable reversed SNARC effect.

Discussion

In Experiment 2, we displayed horizontal fractional numbers to participants and instructed them to make a horizontal response to explore the spatial representation of fractions. The results did not illustrate a reliable SNARC effect or a reliable reversed SNARC effect, unlike previous

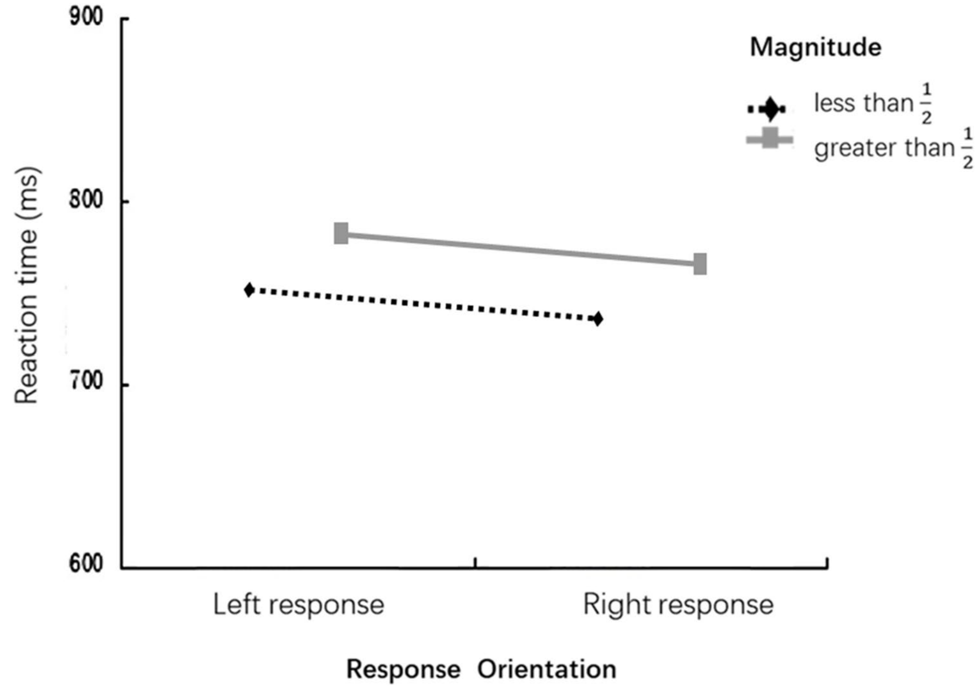

Fig. 3 Average reaction times and the interaction between the magnitude and response orientation in Experiment 2

studies (Bonato et al., 2007; Liu et al., 2013; Toomarian & Hubbard, 2018). The potential reason is that processing of horizontal fractions is susceptible to individuals' familiarity of the horizontal writing notation. For Chinese participants, they are more familiar with vertical fractions compared to horizontal fractions. They are habituated to mentally operate vertical fractions rather than horizontal fractions due to their early learning experiences. When calculating horizontal fractions, they usually depend on some electronic devices, such as a calculator or a computer. Thus, processing horizontal fractions is more complicated in this task, which makes participants adopt different processing strategies. Some participants with relatively higher mathematical levels tend to adopt a holistic strategy and focus on the actual value of a fraction. However, for participants with relatively lower mathematical levels, they may adopt a componential strategy, and take notice of more on the numerator or the denominator. Although we used irreducible and single-digit proper fractions trying to avoid a componential strategy, the above-mentioned speculations of different processing strategies are still very possible because the best-documented error in fraction reasoning and fraction arithmetic is treating numerators or denominators as independent whole numbers (Fitzsimmons et al., 2020; Ni & Zhou, 2005; Siegler et al., 2020). The independent-whole-number errors are common in people with relatively lower mathematical levels, such as community college students (Siegler et al., 2013). The error is also common in relatively hard fractional tasks in which individuals estimate fractions under a time constraint, or estimate fractions with large components, or compare fractions close in decimal distance (Fitzsimmons et al., 2020). No matter people with relatively lower mathematical levels or situations with relatively hard tasks, the possibility of strategy errors is greatly enhanced. In our Experiment 2, the relatively harder task may lead to the utilization of different strategies, and further triggers the disappearance of the SNARC effect and of reversed SNARC effect.

In previous studies, the response type was found to have an impact on the SNARC effect of integers. Hung et al. (2008) used parity judgement tasks to explore the SNARC effect between horizontal and vertical responses. They found that the SNARC effect only existed in horizontal response types when stimuli were Arabic numbers. However, when stimuli were numeral notations in Simplified Chinese, the SNARC effect only existed under the condition of vertical response types.

Therefore, response type is a latent impact factor for the spatial representation of fractions. In Experiment 3, we instructed participants to process vertical fractions in a vertical responsive manner to explore whether this assignment triggers a dissimilar fractional SNARC effect.

Experiment 3

Methods

Participants

We adopted recruitment by releasing advertisements on campuses and a random sampling approach to choose the participants. Thirty-five right-handed undergraduate students (16 males, 19 females; mean age = 20.54 years, $SD = 1.19$) with normal or corrected-to-normal vision voluntarily participated in the experiment. None of the participants had participated in similar experiments before, nor were they aware of the purpose of the current experiment. All of them were given a small reward for their participation in the studies. All present studies were approved by the relevant ethics committee.

Stimuli and apparatus

The stimuli and apparatus utilized in this experiment were the same as in the first experiment.

Procedure

In the first block, participants were instructed to respond to fractions less than $\frac{1}{2}$ with the '6' key and to fractions greater than $\frac{1}{2}$ with the 'b' key. In the second block, participants were instructed to respond to fractions greater than $\frac{1}{2}$ with the '6' key and to fractions less than $\frac{1}{2}$ with the 'b' key. The order of the blocks was counterbalanced across participants. Thus, participants vertically responded to fractional stimuli. Except for changes in response keys (replaces horizontally distributed response keys to vertically distributed response keys), other procedures used in the first experiment were retained (see Fig. 4).

Study design

In the experiment, we adopted a 2 (magnitude: large '$>\frac{1}{2}$', small '$<\frac{1}{2}$') × 2 (response orientation: lower, upper) within-subject study design. Every independent variable contains two levels. For the response orientation, participants respond to stimuli with the '6' key (located in the upper position of a standard QWERTY keyboard and corresponding to an upper response) at the first level. Reversely, participants respond to stimuli with the 'b' key (located in the lower position of a standard QWERTY keyboard and corresponding to a lower response) at the second level. The levels of the magnitude are the same as that in Experiment 1. Dependent variables include 1) the RT and 2) the dRT.

Data analysis

We adopted the same approach of data analysis used in the first experiment. However, in this experiment, the vertical response was adopted. Thus, the computational formula of dRT changed. According to previous studies (Ito & Hatta, 2004; Schwarz & Keus, 2004), most individuals associate large numbers with the upper side and small numbers with the lower side. If individuals represent numbers in a vertical spatial dimension, most of them have a lower-to-upper MNL (in other words, a bottom-to-top MNL). Moreover, it is a consensus in Chinese cultures that the upper position means the large, while the lower position means the small (Tang et al., 2015; Wu et al., 2016). Participants in this study were asked to put numbers (1–9) on vertical lines according to their intuition, and they all put small numbers below big ones. To sum up, in the current experiment, the value of dRT was calculated by subtracting the average RT for the lower response from the average RT for the upper response.

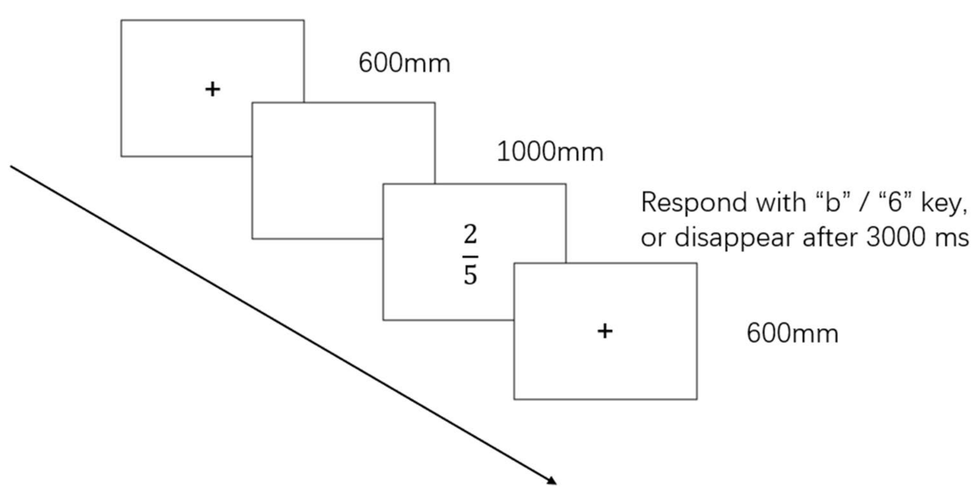

Fig. 4 The processing of Experiment 3

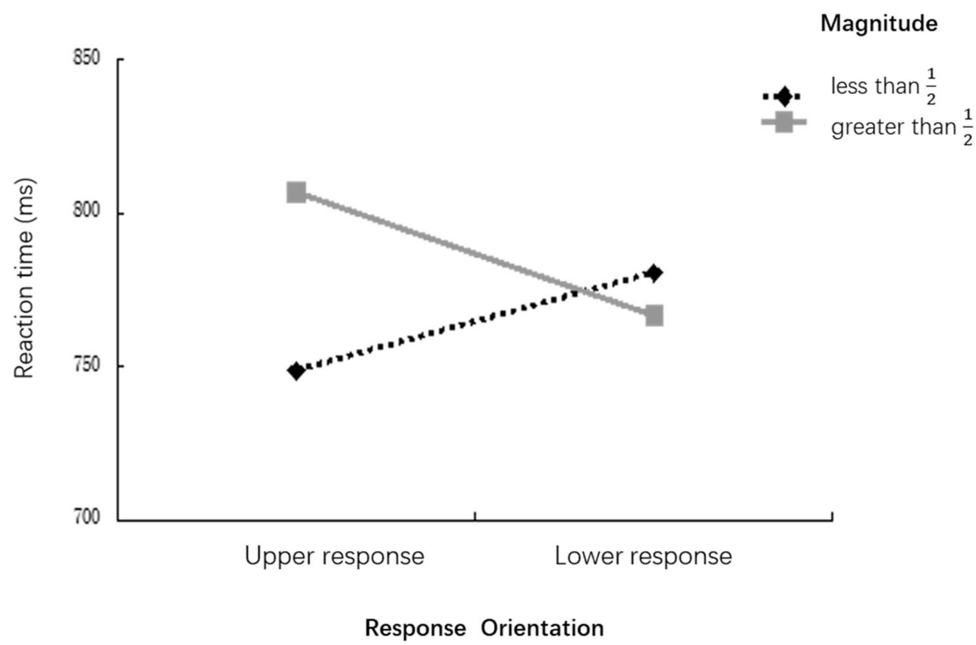

Fig. 5 Average reaction times and the interaction between the magnitude and response orientation in Experiment 3

Results

In the experiment, data from a total of 35 subjects were collected, and data of four subjects were completely excluded. The final data exclusion rate was 6.30%. The average RT in each condition is shown in Fig. 5.

The repeated measures ANOVA revealed that there was no significant main effect for response orientations [F (1, 30) = 0.48, $p > 0.05$]. However, the magnitude had a significant main effect [F (1, 30) = 5.72, $p < 0.05$, $\eta^2 = 0.16$]. A significant interaction was observed between the magnitude and response orientation [F (1, 30) = 9.09, $p < 0.05$, $\eta^2 = 0.23$]. A simple effect analysis found that, for small magnitudes, the upper response ('6' key) was faster than the lower response ('b' key) ($p < 0.05$). However, the lower response ('b' key) was faster than the upper response ('6' key) ($p < 0.05$) for large magnitudes. The above results demonstrate the existence of a reversed SNARC effect (see Fig. 5). Moreover, the dRT regression analysis showed a significant positive regression slope ($B = 116.41$; t (30) = 6.25; $p < 0.001$), confirming the existence of a reversed SNARC effect.

Discussion

In Experiment 3, we displayed vertical fractional numbers to participants and instructed them to make a vertical response to explore the spatial representation of fractions. The results illustrated a reversed SNARC effect, which was inconsistent with those of Toomarian and Hubbard (2018). Although vertical fractional numbers are used in both the study of Toomarian and Hubbard (2018) and our current study, the response type is different. In both studies, vertical proper fractions can generate an analogous numerical sequence in vertical dimensions. Specifically, the analogous numerical sequence consists of two numerical elements – a numerator and denominator. The magnitude of elements in the sequence increased from the top to bottom because a numerator is always smaller than a denominator for proper fractions. Consequently, participants perceived the monotonously ordinal information in the analogous numerical sequence, namely ordinal information from top to bottom. Due to the vertical response type in our study, participants needed a vertical reference frame (Mourad & Leth-Steensen, 2017). Coincidentally, the abovementioned analogous numerical sequence elicited a top-to-down reference frame. Previous study (Toomarian & Hubbard, 2018) and our first experiment have proved that individuals adopt a holistic strategy to process irreducible and single-digit proper fractions in a vertical writing notation. Namely, individuals process the magnitude of a vertical fraction rather than the magnitude of fractional components in magnitude comparing tasks. Therefore, with the influence of the holistic strategy and the top-to-down reference frame, our participants' upper responses were faster when the stimulus was a small number (less than $\frac{1}{2}$), while their lower responses were faster when the stimulus was large (greater than $\frac{1}{2}$). Given the abovementioned consensus in Chinese cultures (the upper indicates the large; the lower indicates the small), the phenomenon here was a vertical reversed SNARC effect. In the study of Toomarian and Hubbard (2018), their instructions of horizontal response types led their participants to have a horizontal reference frame. It was not influenced by the vertical writing notation of fractions. Therefore, the difference of spatial representation of fractions appears between the study of Toomarian and Hubbard (2018) and our Experiment 3.

If vertical notation of fractions interfered the spatial representation of fractions in vertical responsive condition. Then, what spatial representation of fractions is in horizontal writing notations under the condition of vertical response types. In Experiment 4, we examined participants' spatial representation of horizontal fractions under the condition of a vertical response type.

Experiment 4

Methods

Participants

We adopted recruitment by releasing advertisements on campuses and a random sampling approach to choose the participants. Thirty-five right-handed undergraduate students (18 males, 17 females; mean age = 20.65 years, $SD = 1.53$) with normal or corrected-to-normal vision voluntarily participated in the experiment. None of the participants had participated in similar experiments before, nor were they aware of the purpose of the current experiment. All of them were given a small reward for their participation in the studies. All present studies were approved by the relevant ethics committee.

Stimuli and apparatus

The stimuli and apparatus used in the second experiment were retained in this experiment.

Procedure

Except for adopting the horizontal writing notation of stimuli, other procedures used in the third experiment were retained.

Study design

In the experiment, we adopted a 2 (magnitude: large '> 1/2', small '< 1/2') × 2 (response orientation: lower, upper) within-subject study design. Every independent variable contains two levels. The levels of the magnitude are the same as that in Experiment 2. The levels of the response orientation are the same as that in Experiment 3. Dependent variables include 1) the RT and 2) the dRT.

Data analysis

Apart from analysing the magnitude of horizontal fractions, other approaches used for data analysis in Experiment 3 were retained in this experiment.

Results

In the experiment, data from a total of 35 subjects were collected, and data of two subjects were completely excluded.

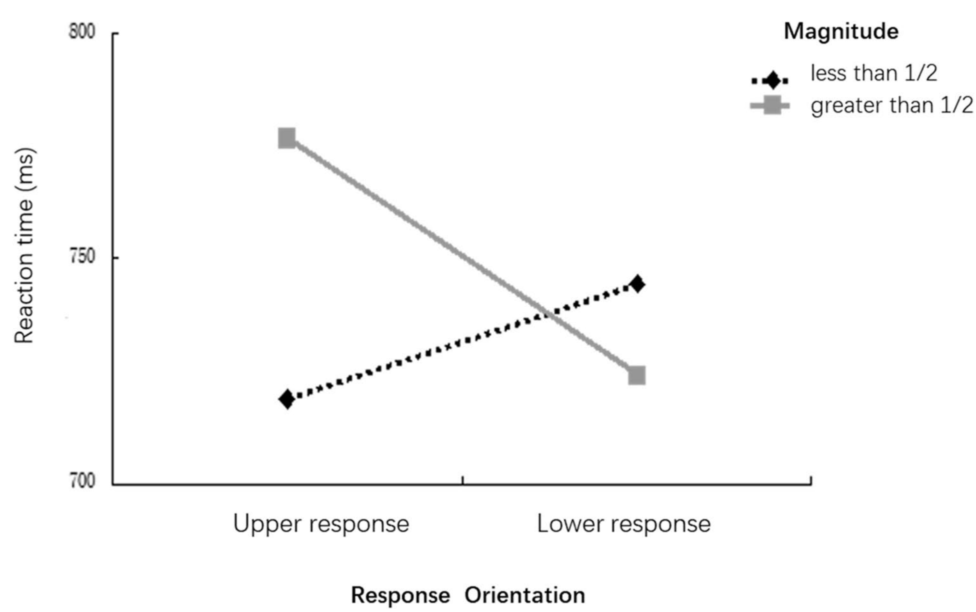

Fig. 6 Average reaction times and the interaction between the magnitude and response orientation in Experiment 4

The final data exclusion rate was 6.40%. The average RT in each condition is shown in Fig. 6.

The repeated measures ANOVA revealed that there was a significant main effect for both the response orientation [F (1, 32) = 5.45, $p < 0.05$, $\eta^2 = 0.15$] and magnitude [F (1, 32) = 7.44, $p < 0.05$, $\eta^2 = 0.19$]. In addition, a significant interaction was observed between the magnitude and response orientation [F (1, 32) = 5.48, $p < 0.01$, $\eta^2 = 0.21$]. A simple effect analysis found that participants reacted faster for small fractions in the upper response condition ($p < 0.01$). However, in the lower response condition, participants' RT had no significant difference between small and large fractions ($p > 0.05$). The above results demonstrate the existence of a reversed SNARC effect (see Fig. 6). Moreover, the dRT regression analysis showed a significant positive regression slope ($B = 127.17$; t (32) = 6.66; $p < 0.001$), confirming the existence of a reversed SNARC effect.

Discussion

To explore the spatial representation of fractional numbers in Experiment 4, we used the magnitude comparing task, in which fractional stimuli were presented using a horizontal notation and were vertically responded to by the participants. The results indicated a reversed SNARC effect. This was inconsistent with the study of Ito and Hatta (2004), which processed integers in a vertically responsive manner and found a SNARC effect. As Li et al. (2017) found in previous study, the spatial-numerical association of Chinese people is various in the vertical dimension when they process different numerical notations. The difference between the result of Ito and Hatta (2004) and ours could be attributed to adopting distinct stimuli. Stimuli are integers in their study, while those are horizontally irreducible and single-digit proper fractions in our Experiment 4.

In vertical responsive conditions, Chinese people have a vertical reference frame. For using the vertical reference frame more effectively, Chinese people may adopt an inner transform replacing a horizontal writing notation of fractions with a vertical one in the minds. Thus, they can easily map fractional numbers on vertical reference frames. This speculation of an inner transform is fit for the fact that Chinese people are habituated to mentally operate vertical fractions rather than horizontal fractions (Research and Development Center of Mathematics Curriculum Materials, 2014). Through the inner transform, the spatial representation of fractions in Experiment 4 is similar to those in Experiment 3. Namely, the reversed SNARC effect is triggered in Experiment 4, just as it happens in Experiment 3.

General Discussion

The current study explored the influence of writing notations and response types on the spatial representation of fractions. The existence of the fractional SNARC effect in the first experiment is congruent with the study of Toomarian and Hubbard (2018). In their study, participants used irreducible and single-digit proper fractions (e.g., $\frac{1}{8}$,$\frac{2}{5}$) and $\frac{1}{2}$(a benchmark) to perform magnitude comparing tasks, finding a fractional SNARC effect. They attributed the SNARC effect to a holistic strategy adopted by participants. Through comparing with the study of Bonato et al. (2007), they proved that individuals choose different processing strategies according to types of fractions. Meert et al. (2010) instructed participants to perform magnitude comparing tasks by using stimuli of fractional pairs and found a significant numerical distance effect. They proposed that participants used a holistic strategy to process the magnitude of fractions because fractions in any fractional pair did not have a common component in their task. Namely, the same numerator or denominator did not exist between two fractions in a fractional pair. Under the condition of the first experiment in our study, participants may adopt a holistic strategy when horizontally process irreducible and single-digit proper fractions in a vertical writing notation. It reconfirmed that the holistic strategy is one of the most common strategies in processing vertical fractions.

Previous studies have demonstrated that in addition to the holistic strategy, people also use a componential strategy to process fractions. For example, Liu et al. (2013) instructed Chinese sixth-grade students to perform magnitude comparing tasks with fractional and decimal stimuli. The numerator and denominator of fractions were 1 and any digit from 1 to 9 (except 5), respectively. The benchmark was $\frac{1}{5}$ or 0.2. The authors found a reversed SNARC effect under the condition of fractional numbers, and a SNARC effect under that of decimals. For this finding, it is reasonable that participants adopt a componential strategy to process fractions when a common component (e.g., numerator 1) exists. Similarly, Rivera and Soylu (2018) investigated semantic processing in fractional magnitude comparing tasks. They found that shared fractional components interfered with the comparison of fractions because these shared components indirectly evoked an advantage of componential strategy. Individuals tended to compare fractions with these same numerical components.

A fraction has two writing notations: horizontal and vertical notations. In our second experiment, we explored the impact of horizontal writing notations on the representation of fractions under the condition of a horizontal response. The results did not yield a reliable SNARC effect or reversed SNARC effect. Because of early learning experiences of fractions (Research and Development Center of

Mathematics Curriculum Materials, 2014), it is relatively difficult for Chinese people to mentally calculate horizontal fractions compared to vertical fractions, which may lead different participants to adopt distinct processing strategies to process fractions. Some participants use a holistic strategy, others utilize a componential strategy. Consequently, different processing strategies result in the disappearance of a SNARC effect or reversed SNARC effect. The phenomenon of different strategies using is possible because different teachers may prefer adopting distinct teaching strategies to teach knowledge of fractions (Doğan & Tertemiz, 2020; Wilkie & Roche, 2022). Moreover, different students have different mathematical abilities and may adopt distinct strategies. The results of the first and second experiments indicate that, under the condition of a horizontal response type, the fractional SNARC effect is affected by writing notations of fractions. Writing notations and response types indeed have an interaction.

In the field of integers, previous studies have proved that the spatial representation of numbers is different when individuals adopt distinct response types (Dehaene et al., 1993; Ito & Hatta, 2004). Under the condition of horizontal response types, individuals have a horizontal reference frame and their spatial representation of integers is impacted mainly by cultural and embodied factors such as reading habits (Dehaene et al., 1993) and counting habits (Fischer, 2008). Conversely, under the condition of vertical response types, individuals have a vertical reference frame and their spatial representation of integers is frequently modified by effector instructions (Müller & Schwarz, 2007), different number notations (Hung et al., 2008), and contexts (Qiao et al., 2016). Thus, compared to horizontal response types, individuals' spatial representation of numbers is more flexible when they vertically respond to numbers. Namely, vertical representation of numbers is less influenced by cultural environments. Therefore, in the third experiment, we detected the spatial representation of vertical fractions under the condition of vertical response types and found a reversed SNARC effect. This result could be caused by the combined utilization of a vertical writing notation of proper fractions and a vertical response type. In vertical proper fractions, the numerator is always less than the denominator (e.g., $\frac{2}{5}$), which provides participants with ordinal information of numbers in the vertical dimension. In addition, with the help of ordinal information and a vertical response type, a top-to-down reference frame is elicited. In the end, the combined utilization leads to a reversed SNARC effect. It implies that if a type of fractional number has ordinal information in the vertical dimension, the ordinal information will determine the spatial representation of fractions when individuals respond in the same dimension. The result is supported by the study of He et al. (2020). In their study, researchers proved that numerical ordinal sequences can induce attentional SNARC effects.

In the fourth experiment, fractional stimuli were presented using a horizontal notation and were vertically responded to by our participants. Like the third experiment, the fourth experiment exhibited a reversed SNARC effect in the vertical dimension. The potential reason for the reversed SNARC effect is that individuals use an inner transform replacing a horizontal writing notation of fractions with a vertical one. The purpose of the inner transform can be attributed to the facilitation of mapping fractional numbers on vertical reference frames. Through the inner transform, individuals can easily process fractions in the mind. Although the inner transform may increase participants' RTs, the more easily mental comparison reduces RTs. Thus, under the vertical response condition, processing horizontal fractions also causes a reversed SNARC effect with relatively stable RTs. The results of the third and fourth experiments indicate that the spatial representation of fractions was unaffected by notations of fractions under the condition of vertical response types.

It is worth noting why the inner transformed happens in Experiment 4 but not in Experiment 2. The reasons are as follows. According to the study of Mourad and Leth-Steensen (2017), the spatial-numerical association needs a spatial reference frame that has the same directional dimension as individuals' response. In Experiment 4, the vertical response type requires a vertical reference frame. The inner transform from horizontal notations to vertical notations can make it easier to map the transformed vertical fraction to the vertical reference frame. Moreover, through the inner transform, the unfamiliar notation is changed to a familiar one. Thus, the inner transform in Experiment 4 can decrease difficulty of magnitude comparing tasks. It is reasonable to adopt an inner transform in Experiment 4. In contrast, in Experiment 2, the horizontal response type requires a horizontal reference frame. If individuals transform unfamiliar horizontal fractions to familiar vertical ones, the difficulty of magnitude comparing tasks will decrease. However, through the inner transform, individuals need to map vertical notations to a horizontal reference frame. Compared to mapping horizontal notations on a horizontal reference frame, mapping vertical notations on a horizontal reference frame is relatively harder. Therefore, the inner transform in Experiment 2 can also increase the difficulty of magnitude comparing tasks. Individuals have no reason to utilize an unnecessary inner transform in Experiment 2. Given that, Experiment 4 showed a similar reversed SNARC effect to that of Experiment 3, while Experiment 2 did not show a similar spatial-numerical association to that of Experiment 1. Essentially, the difference between Experiment 2 and Experiment 4 is derived from the fact that Chinese individuals' abilities of processing vertical fractions are better than that of processing horizontal fractions.

Conclusion

To sum up, the current study explores two key factors of fractional spatial representations (writing notations and response types) to clarify the mechanism of the fractional SNARC effect. For horizontal response types, the vertical fraction shows a SNARC effect, while the horizontal fraction does not. This indicates that the writing notation can affect the spatial representation of fractions under a horizontal response type condition. For vertical response types, the reversed SNARC effect emerges in both vertical and horizontal fractions. It indicates that the spatial representation of fractions is unaffected by the writing notation under a vertical response type condition. These results demonstrate a combined effect of writing notations and response types on the spatial representation of fractions, and further indicate the following conclusions: when Chinese individuals process fractions in a familiar writing notation, they can represent the magnitude of fractions on the space that has the same dimension as their responses. Oppositely, the spatial-numerical association will disappear when they process fractions in an unfamiliar writing notation, unless they adopt an inner transform replacing unfamiliar fractional notations with familiar ones. In addition, this study proves a priority of the holistic strategy in the processing of irreducible and single-digit proper fractions. Except horizontally respond to horizontal fractions, Chinese individuals adopt a holistic strategy to process irreducible and single-digit proper fractions in magnitude comparing tasks.

Limitations and implications

There are three limitations to this study. First, the experimental materials only involved irreducible and single-digit proper fractions. Follow-up researches can investigate improper and multidigit fractions using the method. Second, in our second experiment, the spatial-numerical association disappears because the task is relatively hard for our participants. This disappearance may be elicited by some confounding factors such as the level of individuals' mathematical abilities, or different strategies for processing fractions. Further studies need to take potential confounding factors into consideration and become finer grained. Third, our results are acquired from Chinese participants. Cross-cultural studies in this field are required to be conducted in the future.

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Authors' contribution statements All authors contributed to the study conception and design. The first draft of the manuscript was written by Yun Pan and Jian Zhang, all authors commented on previous versions of the manuscript. All authors read and approved the final manuscript.

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Data availability The datasets generated during and/or analysed during the current study are available from the corresponding author on reasonable request.

Declarations

Conflict of Interest The authors have no competing interests to declare that are relevant to the content of this article.

Ethical approval Approval was obtained from the School of Psychology Ethics Committee at Guizhou Normal University. The procedures used in this study adhere to the tenets of the Declaration of Helsinki.

Consent to participate and publish Informed consent was obtained from all individual participants included in the study.

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