

On Wittgenstein's Dispensation with "=" in the *Tractatus* and its Philosophical Background. A Critical Study

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Abstract

In this essay, I critically analyze Wittgenstein's dispensation with "=" in a correct concept-script. I argue *inter alia* (a) that in the *Tractatus* the alleged pseudo-character of sentences containing "=" or=-sentences remains largely unexplained and propose how it could be explained; (b) that at least in some cases of replacing=-sentences with equivalent identity-sign free sentences the use of the notion of a translation seems inappropiate; (c) that in the *Tractatus* it remains unclear how identity of the object as that which is expressed by identity of the sign should be understood specifically; (d) that there are=-sentences which have no obvious equivalent in Wittgenstein's novel notation; (e) that Wittgenstein's adherence to (non-relational) identity, although he dispenses with "=", is probably motivated by his desire to ensure that the expressive power of an identity-sign free concept-script of first-order is on a par with standard first-order logic containing "=". In the concluding section, I critically discuss some claims in Lampert and Säbel (*The Review of Symbolic Logic, 14*, 1–21, 2021) and defend Wehmeier's account of pseudo-sentences in the *Tractatus* (2012) against the objections they raise.

Keywords Identity-sign free sentence · Pseudo-sentence · Senseless sentence · Substitutivity · Translation · Equivalence

1 Introduction

In the *Tractatus*, Wittgenstein acknowledges Frege's and Russell's achievements in logic but he also criticizes several features of their concept-scripts. He therefore feels motivated to envision a new and "correct" concept-script, one which avoids the shortcomings to which his fellow logicians allegedly fall prey and at the same time exceed them in logical accuracy. To meet the correctness requirement with respect

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to the treatment of identity, Wittgenstein dispenses with "=" and expresses identity of the object by identity of the sign and difference of objects by difference of the signs (T 5.53). This implies that no object of the domain¹ has more than one name and that, roughly speaking, different object variables take different values. Thus, together with "=" coreferential names which could be substituted one for the other in sentences *salva veritate* disappear from the screen, and the free and bound variables are no longer construed in exactly the same way as in standard first-order logic. This has a number of consequences, for example, regarding the definability of certain object-signs. I shall return to this shortly.²

From the outset, it is clear that owing to Wittgenstein's dispensation with "=" in his envisaged concept-script, identity, if it is to be retained in logic, must be re-interpreted, that is, he must understand it in a way which (a) is independent of the meaning which has been attached to "=" in line with the objectual view according to which identity is a relation in which every object uniquely stands to itself or the metalinguistic view of identity according to which identity is explained in terms of the coreferentiality or mutual substitutivity of two singular terms and (b) fits together with his novel notation. In the *Tractatus*, Wittgenstein takes exactly this route.

Original as Wittgenstein's innovation may appear at first glance, it is not immune from criticism. Generally speaking, it is not enough to present, without providing any explanation, a little more than a handful of identity-sign free counterparts of sentences which in the language of standard first-order logic with identity contain "=" (in brief,=-sentences), and infer from this that "=" is not an essential constituent of a correct concept-script. The soundness of the inference might be challenged if one or the other example of a =-sentence could be presented, which has no obvious equivalent in Wittgenstein's novel notation, and thus could not appropriately replace the former in it, unless Wittgenstein would not insist that the old and the new sentence must be equivalent. Yet the legitimacy of untightening the semantic and logical relations between =-sentences and their identity-sign free counterparts below the threshold of equivalence would require a special argument. There are further objections to and complaints about Wittgenstein's approach some of which I shall discuss in due course. In particular, one would like to know how we should understand more specifically the non-relational character of identity of the object and in what sense it is supposed to be logically sound and systematically fruitful. Imposing a ban on "=" in a newly developed concept-script with the aim of achieving utmost notational parsimony is one thing. The consequence that this has for our understanding of the nature and role of identity in logic is another. This connection is largely passed over in silence in the *Tractatus*. To all appearances, it is the notational aspect of expressing identity of the object via identity of the sign that is in the foreground.

¹ According to Wittgenstein's remarks in T 4.2211 and T 5.535, the first-order domain might be taken to be infinite; see in this respect Rogers and Wehmeier 2012, p. 539 f.

² I translate "Satz" throughout this essay as "sentence" to avoid ambiguity to which the use of the word "proposition" might give rise in certain contexts.

As I indicated above, by a correct concept-script Wittgenstein has a formal language in mind which is more or less modelled on Frege and Russell's formal languages but is supposed to be more prudently designed than theirs. It is said to be governed "by *logical* grammar – by logical syntax" (T 3.325),³ and its projected construction follows the guidance principle of narrowing down the notational repertoire to the absolutely essential. In short, Wittgenstein wishes to impose those constraints on a concept-script which in his view guarantee its logical perfection, austerity and best possible handling. Dispensing with "=" and re-interpreting identity is just a paradigm case for him in pursuit of this objective.⁴

Wittgenstein observes that due to the absence of "=" in a correct concept-script equations and sentences which include equations as components, for example, the law of transitivity of identity, as well as quantified sentences containing "=", for example (in Wittgenstein's notation), "(x): $fx \supset x=a$)" or "($\exists x, y$).f(x, y). x=y)" cannot be written down in it (cf. T 5.531, T 5.532, T 5.5321, T 5.534). The observation is trivial. (I shall say more about this in Section 3.) What is not trivial is Wittgenstein's characterization of =-sentences as pseudo-sentences (Scheinsätze). It is striking that in this context he withholds important information from his readers. In particular, he does not explain (a) in which sense the alleged pseudocharacter of =-sentences should be understood but confines himself to noting in one place (T 6.2) that the sentences of mathematics are equations, and therefore pseudo-sentences; (b) what the logical and semantic relations between = -sentences and their identity-sign free counterparts are supposed to be; (c) whether he regards only the tautologies⁵ (and contradictions) among = -sentences as senseless or would stigmatize = -sentences across the board as senseless, and if so, why and with which consequence for the status of the corresponding identity-sign free sentences which in all likelihood are supposed to be equivalent to the=-sentences in a sense of equivalence left unspecified; (d) why despite the dispensation with "=" he adheres to identity nonetheless; (e) how identity of the object as that which is expressed by identity of the sign should be understood specifically since it is not construed as a relation, in particular, it is not understood as a relation in which every object uniquely stands to itself⁶; (f) whether he thinks that in his envisioned concept-script he could still define a new object-sign "b" without having a coreferential object-sign "a" on hand which is already known and to be replaced by "b" via stipulation, or whether he thinks that he has to get along without proper definitions of object-signs

³ "The concept-script of Frege and Russell is such a language, although it still fails to exclude all mistakes" (T 3.325).

⁴ In his Introduction to the *Tractatus*, Russell (1961, p. xvii) misinterprets Wittgenstein's remarks on "=" and identity. It is clear that in his envisioned concept-script Wittgenstein dispenses only with "=" but not at the same time with identity. What he does reject, are standard conceptions of identity.

⁵ In a letter to Russell of 15 December 1913 (Wittgenstein 1969, p. 273), Wittgenstein writes: "The question concerning the nature of identity cannot be answered before the nature of tautologies is explained. The question concerning the nature of the latter is, however, the fundamental question of all logic."

⁶ Wittgenstein does not claim that "=" does not denote a relation but rather that identity is not a relation. This claim suggests that a novel, non-relational conception of identity is required, if identity is not to be abandoned in logic.

but is willing to pay the price; (g) whether by removing or excluding "=" from his notation he intends to kill two birds with one stone, that is, not only achieve utmost notational parsimony, even at the risk that the innovation may appear idiosyncratic and in need of getting used to, but also ensure (i) that the identity-sign free counterparts are not subject to the classification as pseudo-sentences, (ii) that the expressive power, say, of the envisaged identity-sign free concept-script of firstorder is on a par with standard first-order logic containing "=" but, thanks to the new device to express identity (of the object), is notationally put on the right track in this respect, and (iii) that the new treatment of identity is logically justified on a large scale.

2 Wittgenstein's Dispensation with "=" in a Correct Concept-script

In what follows, I shall critically comment on (a) – (g). In doing so, I intend to shed some new light on the philosophical background of Wittgenstein's innovation. I analyze only a few technical or purely logical details that the innovation involves. For further information in this respect, I refer the reader to Fogelin, 1983, Floyd, 2007, Landini, 2007, Rogers & Wehmeier, 2012 and Lampert & Säbel, 2021. These works make valuable contributions to our understanding of Wittgenstein's elimination of "=" in a correct concept-script. However, due to the limitation of space I cannot comment on all of them appropriately in this essay.

With regard to (a): Wittgenstein does not explain in which sense the alleged pseudo-character of=-sentences should be understood. To begin with, I take the connotation of the term "pseudo-sentence" in the *Tractatus* to be pejorative. Clearly, it is meant neither in a positive nor in a neutral sense. Wittgenstein's examples of a pseudo-concept and of pseudo-sentences in T 4.1272 suggest that he associates a deprecatory connotation with the term "pseudo-sentence".⁷ This seems to follow also from the premise that he most likely regards a concept-script which contains pseudo-sentences as incorrect: "And now we see that in a *correct* concept-script [my emphasis] pseudo-sentences like 'a=a', ' $a=b.b=c. \supset a=c$ ', '(x).x=x', ' $(\exists x).x=a$ ', etc. cannot even be written down" (T 5.534).⁸

Why are equations and probably=-sentences in general considered pseudosentences? This remains unexplained in the *Tractatus*. If based on the remarks in T 5.534 and T 6.2 Wittgenstein were to say that it is the sheer occurrence of "=" in sentences which establishes their pseudo-character he would not thereby provide a justification for characterizing=-sentences as pseudo-sentences. Thus, I assume that in

⁷ Wittgenstein gives here examples of nonsensical pseudo-sentences. Yet this does not mean that he regards all pseudo-sentences as nonsensical. He obviously does not. The =-sentences which he lists in T.531, T 5.532. T 5.5321 and T 5.534 are not considered to be nonsensical. However, the tautologies among his examples of =-sentences are considered to be senseless. I shall argue below that Wittgenstein probably regards all =-sentences as senseless, not only the tautologies (and contradictions) among them.

⁸ Wittgenstein hastens to add that this also disposes of all the problems that were connected with such pseudo-sentences. This can be doubted. He mentions in particular all the problems to which Russell's axiom of infinity gives rise and claims that they can be solved at this point.

Wittgenstein's view it is a special feature of "=", namely its (alleged) redundancy or dispensability in a correct concept-script, which is responsible for the pseudo-character of=-sentences. In saying this, I do not wish to rule out that he thought (a) that the pseudo-character of=-sentences is due to the fact that they are only representential devices or (b) that they apparently say what can only be shown or, perhaps, that they are supposed to offend against the dictum "What can be shown, cannot be said" (T 4.1212). Moreover, I do not wish to exclude (c) that Wittgenstein thought that the identity-sign free sentences that he lists in T 5.531, 5.532, 5.5321 shake off the pseudo-character of the corresponding=-sentences not only for one but for two reasons (of possibly different weight). Yet in the *Tractatus* there is direct evidence neither for (a) nor for (b) nor for (c). Moreover, if Wittgenstein in fact held that, for example, "a=b" apparently says what can only be shown – namely that "b" can be substituted for "a" – this would seem to contravene his remarks in T 4.241, 6.23 and 6.24. It is true that in T 6.23 he claims that it must be *shown* by the two expressions themselves whether one can be substituted for the other. Yet this does not imply that he thinks that "a=b" apparently says that "b" can be substituted for "a" nor does it imply that the substitutivity of "b" for "a" cannot be said or expressed at all but only be shown.

In his diary notes of 6 and 20 October 1914 (Wittgenstein, 1969, p. 94, 104), Wittgenstein explains his understanding of the term "pseudo-sentence", while for some reason he does not deem this necessary in the *Tractatus*⁹:

All combinations of signs which apparently say something that can only be shown are pseudo-sentences.

Pseudo-sentences are those which, if analyzed, only show again that which they were supposed to say.

According to these explanations, a combination of signs is a pseudo-sentence if that which it apparently says is exactly that which can only be shown. If a sentence S of a language L apparently says something that cannot be shown in L (or in any other language if we literally rely on the first explanation), then it seems that Wittgenstein would not necessarily classify S as a pseudo-sentence. It seems that he considers *saying something apparently* a necessary but not a sufficient condition for a sentence to count as a pseudo-sentence. According to the second explanation, the sentence which is supposed to say something and the sentence which, if analyzed, only shows again what it was supposed to say, are probably taken to be one and the same sentence. ¹⁰ By contrast, if we take the wording of the first explanation at face value, the sentence which only shows that which a sentence apparently says is not necessarily the same as the latter (although Wittgenstein may tacitly have implied this). To see this, suppose that he thought that in the language of first-order logic

⁹ In his diary note of 6 October 1914 (Wittgenstein 1969, p. 97), Wittgenstein mentions "p" is true" and in his diary note of 29 November 1914 (Wittgenstein 1969, p. 123) "(x)x=a" as a pseudo-sentence. He claims that in view of his proposed identity-sign free notation "(x)x=a" and similar sentences would lose all appearance of justifiability. Finally, in his diary note of 10 June 1915 (Wittgenstein 1969, p. 149), his examples of pseudo-sentences are " $p \lor \neg p$ " and " $p.\neg p$ ". He even goes so far as to say that these are not sentences at all.

¹⁰ It is not clear what Wittgenstein means here precisely by "analyzing".

with "=" $(LFOL^{-})$ the equation "a=b" apparently says (or, in the light of his showsay dictum: cannot say at all) that "b" can be substituted for "a". If on assumption the apparently stated (or possibly ineffable) substitutivity of "b" for "a" in $LFOL^{=}$ can only be *shown* in *LFOL*⁼ or in another suitable language then, following the first explanation, "a=b" is considered a pseudo-sentence in LFOL⁼. Unless a certain restriction is imposed on the clause "can only be shown in LFOL"", it is not necessarily linked to "a=b". The sentence " $f(a) \leftrightarrow f(b)$ " of LPOL⁼, for example, could be interpreted in such a way that it only shows what on assumption the LFOL⁼-sentence "a=b" apparently says, namely that "b" can be substituted for "a" (or that "a" and "b" are intersubstitutable).¹¹ Furthermore, the first explanation of "pseudo-sentence" does not rule out that showing the substitutivity of "b" for "a" may be delivered by a sentence which is a constituent of a language that does not contain "=". The sentence " $f(a) \leftrightarrow f(b)$ " of LFOL would be a potential example. In sum, granted that non-trivial substitutivity can be *shown* in a language, it need not depend or rest on the use of "=" unless it has been stipulated that it does. Yet showing nontrivial substitutivity does presuppose the availability of (distinct) coreferring terms. Showing the substitutivity of "a" for "a" would be just as trivial as saying or showing that a is identical with a.¹² However, expressing or showing the substitutivity of "a" for "a" is, in my view, only a border-line case of substitutivity without any decisive impact on the logic of identity. I shall return to this in Section 3. Dispensing with "=" in a formal language may go hand in hand with the abandonment of both coreferentiality and substitutivity, but it need not. In his envisioned concept-script, Wittgenstein chooses the first option. In it, a sentence cannot show (nor state) that "b" can be substituted for "a". Coreferentiality and substitutivity have fallen off the grid once "=" was left behind and identity was construed as that which is expressed by identity of the sign.

It is perfectly possible that in the *Tractatus* Wittgenstein would have endorsed his diary explanations of "pseudo-sentence" but as I already indicated, there is no evidence for that. In the *Tractatus*, the pseudo-character of=-sentences is sparsely explained by simple appeal to the occurrence of the (dispensable) sign "=" (T 6.2). However, in their excellent essay 'Tractarian First-Order Logic: Identity and the N-Operator' (2012, p. 548), Rogers and Wehmeier claim that

Wittgenstein evidently thought that the formulas in 5.534 were pseudo-propositions *because* they violated the say-show distinction, for the earliest known draft of his proposal to eliminate the identity sign (Wittgenstein 1961, p. 34) is immediately preceded by the remark that became *Tractatus* 4.1212: "What *can* be shown, *cannot* be said."...Identities of the form x = y, where x and y are distinct, attempt to say that the variables x and y are to be assigned the same object – in violation of what is shown, in a correct notation, by the use of distinct variables.

¹¹ " $f(a) \leftrightarrow f(b)$ " or " $\forall f(f(a) \leftrightarrow f(b))$ " is just the *definiens* in Leibniz's definition of "a = b".

 $^{^{12}}$ Cf. Wittgenstein 1967, p. 190. In a conversation with Waismann on the topic *equation and substitution rule*, Wittgenstein says that even the statement "*a* cannot be substituted for *a*" can be understood if one is aware that one is faced with a new calculus and knows the rules of its grammar.

The use of "attempt to say" is here a slightly infelicitous choice of phrasing. A sentence, taken by itself or even in the mouth of a speaker or the hand of a writer does not attempt to say anything. It is the speaker or writer who may attempt to say something by using a sentence. For Wittgenstein, identities of the form "x = y" do not even apparently say anything since he construes "x = y" as a functional expression.¹³ Regarding the citation of "x = y", Rogers and Wehmeier also refer to "x" and "y" as variables and thus, on the face of it, seem to construe "x = y" as a functional expression. However, they would probably argue as follows: since, for the sake of exposition, we "represent names as free variables under a fixed variable assignment" (p. 543), we may well consider "x = y" a sentence. Now if we construe "a" and "b" as before as names and choose the sentence "a = b" instead of "x = y" and assume, for the sake of argument, that in the *Tractatus* Wittgenstein holds that "a=b" apparently says that the name "b" can be substituted for the name "a", and if we finally adjust the statement by Rogers and Wehmeier accordingly, we obtain: Equations of the form "a=b" apparently say that the name "b" can be substituted for the name "a" – in violation of what is shown, in a correct concept-script, by the use of the distinct names "a" and "b", namely that b is different from a. If my interpretation of Wittgenstein's second diary explanation of "pseudo-sentence" carries conviction, then Rogers and Wehmeier might be missing a point that Wittgenstein makes in it. As I stressed earlier, that which a sentence apparently says and that which can only be shown is here probably taken to be the same by him. Now, independently of whether Rogers and Wehmeier would accept the intended illustrativeness of my example or not, pointing out that in Wittgenstein's correct concept-script the use of "a" and "b" (possibly) shows that a and b are different objects – which is almost the opposite of what "a=b" was supposed to say apparently in $LFOL^{=}$ – and to infer from this that "a = b" is a pseudo-sentence in LFOL⁼ does not accord with Wittgenstein's diary explanations of what he understands by a pseudo-sentence. Strictly speaking, reference to what is (possibly) shown by the use of "a" and "b" in a correct concept-script - the difference of a and b - is irrelevant for an understanding of the supposed pseudo-character of "a = b" in line with Wittgenstein's diary explanations. I said "(possibly) shows" since in the Tractatus Wittgenstein states that difference of objects is *expressed* by difference of the signs.¹⁴

With regard to (b): Wittgenstein does not explain what the logical and semantic relations between = -sentences and their identity-sign free counterparts are supposed to be. Let me focus on the semantic relation between = -sentences and the identity-sign free counterparts which Wittgenstein leaves unspecified. The logical relations between the sentences of these two kinds are discussed in

¹³ Independently of Wittgenstein's position regarding "=" in the *Tractatus*, we can say, quite in the spirit of Frege, that the epistemically relevant use of "x=y" qua sign of a binary object-relation presupposes the availability of coreferential, but non-synonymous singular terms in a quantifier-free language *L*. Otherwise, from an epistemological point of view, "x=y" would be a non-rolling vehicle in *L*.

¹⁴ In fairness to Rogers and Wehmeier, I should mention that in another place (see p. 549) they also draw attention to Wittgenstein's statement in T 6.2 that the pseudo status of pseudo-sentences is due to their containing "=".

detail, for example, by Rogers & Wehmeier, 2012 and Lampert & Säbel, 2021 (with the focus on quantifier-free = sentences in the latter case). When Rogers and Wehmeier argue that Wittgenstein's identity-sign free counterparts of certain quantified = -sentences are *translations* of the latter they invoke the relation of samesaying.¹⁵ However, if the translation of a sentence A into a sentence B rests crucially on the fact that the relation of samesaying holds between A and B (that B means the same as A in the sense of either strict or approximate synonymy), hence, that there is in general no translation of A into B which rests exclusively on syntactic considerations (possible exceptions such as natural language active-passive pairs aside), then it would seem that any tautology - due to its supposed say-vacuum, senselessness and non-possession of truth-conditions - could be indiscriminately "translated" into any other: all tautologies say the same thing, namely nothing at all. In other words, if the samesaying relation qua criterion of a correct translation is applied to sentences which are declared to say nothing, it fails to single out a specific sentence which could be regarded as *the* correct translation of the sentence to be translated.¹⁶ Thus, shall we say that the notion of a sense-preserving translation of a sentence A into a sentence B is perverted if both A and B avowedly say nothing or are senseless? If so, it would perhaps be recommendable to apply the term "translation" at most in a partial sense when Wittgenstein's correlation or replacement of =-sentences with identity-sign free sentences is at issue. For at least in those cases in which the two correlated sentences are both tautologies we are facing senselessness on both sides of the respective correlation, Wittgenstein thinks. In the *Tractatus*, he does not use the term "translation" when he suggests replacing = -sentences with identity-sign free sentences, nor does he apply a term like "equivalent" or "coreferential (coreferring)" or "tantamount" or "have the same meaning" or "synonymous" to pinpoint the semantic relation between them. In fact, he does not specify this relation at all¹⁷ nor does he explain why he (probably) thinks that the counterparts which he lists in T 5.531, T 5.532 and T 5.5321 are equivalent to their "=" containing ancestors, apart from stating generally that

¹⁵ In future work, I plan to comment on their meticulous analysis, based on work by Hintikka 1956, especially of identity-sign free counterparts of quantified=-sentences which Wittgenstein lists in the *Tractatus*. Following Hintikka, Rogers and Wehmeier distinguish between a *weakly exclusive* and a *strongly exclusive* interpretation of Wittgenstein's identity-sign free counterparts. They argue, *inter alia*, that a variant of the strongly exclusive interpretation is compatible with Wittgenstein's programmatic view in the *Tractatus*. Appealing to work by Ramsey, Rogers and Wehmeier further argue in support of Hintikka's identification of the weakly exclusive interpretation with Wittgenstein's intended identity convention. They end their discussion of Wittgenstein's new conception of identity and difference by noting (p. 555) that

one cannot expect all valid schemas of classical first-order logic to hold in Wittgenstein's alternative logic. We can however legitimately insist that if Wittgenstein's notation is to be considered an adequate alternative to standard first-order logic, the translation of every valid sentence in the Russellian notation must be valid in the Wittgensteinian notation.

¹⁶ It is obvious that a sentence which is recognized as expressing a determinate thought cannot be the translation of a senseless sentence and that the latter cannot be the translation of the former.

¹⁷ Stating only that instead of "--" I write "..." does not reveal anything specific about the semantic relation between sentences of the first kind and those of the second.

the former do not contain "=" and that identity of the object is expressed by identity of the sign.¹⁸

With regard to (c): Wittgenstein does not explain whether he regards only the tautologies (and contradictions) among = -sentences as senseless or would characterize all = -sentences as senseless, and if so, why and with which consequence for the status of the corresponding identity-sign free sentences which in all likelihood are supposed to be equivalent to the = -sentences in a sense of equivalence left unspecified. After having commented on expressions of the form "a = b" in T 4.241 and T 4.242, Wittgenstein observes in T 4.243:

Expressions like 'a = a', and those derived from them, are neither elementary sentences nor is there any other way in which they have sense. (This will become evident later.)

That "a = a" and the sentences that are supposed to be derivable from "a = a" are not elementary sentences means (i) that they do not assert the existence of a state of affairs (T 4.21), (ii) that they are not a concatenation of names (T 4.22), (iii) that they are not a truth-function of themselves (T 5) and (iv) that they are not a truth-argument of a sentence (T 5.01). Unfortunately, Wittgenstein does not say which sentences he considers to be derivable from "a=a", nor does he explain in which way they could be derived from "a = a". He does not give a single example of such a derivation. Regarding the sentences derivable from "a = a", he presumably has not only "(x).(x=x)", " $(\exists x).x=a$ " and " $(x):fx \supset x=a$ " in mind, but also "a=b. b=c. $\supset a=c^{\prime\prime}$, " $(\exists x,y).(f(x,y). x=y)$ " and perhaps even all other=-sentences. If so, this would confirm my hunch that Wittgenstein regards all =-sentences as senseless pseudo-sentences. In T 4.243 (recall the quotation above), he seemingly alludes to T 5.531, T 5.532, T 5.5321 and T 5.534. However, in these paragraphs it does not become evident that all the =-sentences that he lists there do not have sense. What about the identity-sign free counterparts? Are any of them supposed to be senseless too and if so, why? Wittgenstein does not raise this question. However, it is clear

¹⁸ As far as I can see, it is only in a diary note of 27 October 1914 (Wittgenstein 1969, p. 108) that Wittgenstein makes a remark on the semantic relation between a=-sentence and the identity-sign free counterpart: "For it is clear that 'aRa' would have the same meaning as 'aRb.a=b'. Thus, one can let disappear the pseudo-sentence 'a=b' by means of a completely analyzed notation." It seems clear that Wittgenstein would have to argue for each individual case of a = -sentence and its identity-sign free counterpart that the two sentences have the same meaning. Any generalization in this respect would seem to be ungrounded. Furthermore, we cannot safely infer from the remark in the diary that in the Tractatus Wittgenstein thought that =-sentences and their identity-sign free counterparts have the same meaning, quite apart from the fact that in the Tractatus he would likewise have to argue for sameness of meaning on a case by case basis. It is conceivable that in the Tractatus he tacitly holds that in some cases of pairing =-sentences with identity-sign free sentences he can justifiably assume sameness of meaning in the sense of synonymy while in other such cases the semantic relation must be set weaker and could be more appropriately characterized, say, in terms of equivalence. And in some such cases he might have realized that it is hard to argue for the (requisite) equivalence of the=-sentence and its assumed identity-sign free correlatum. Whatever Wittgenstein thought about the semantic relation at issue, it is noticeable that he does not analyze a single example, let alone justify the supposed equivalence (or even sameness of meaning) of the =-sentences and the identity-sign free counterparts that he lists in T 5.531, T 5.532 and T 5.5321.

that he at least considered the tautologies (and contradictions) among the identitysign free sentences to be devoid of sense.

According to Wittgenstein's lights, every identity-sign free counterpart sentence which is a tautology says nothing. In the case of "f(a,b).a=b", which is not a tautology but presumably considered senseless by him nonetheless,¹⁹ the counterpart "f(a,a)" or "f(b,b)" probably is a tautology from his point of view and if so, is regarded as senseless. It is therefore hard to see how identity of the object could in fact be expressed by a sentence "f(a,a)" or "f(b,b)". Yet if Wittgenstein insisted that "f(a,a)" or "f(b,b)" does express identity of the object, then it is clear that in his concept-script "f(b,b)" does not and cannot express the same thing as "f(a,a)", for "a" and "b" are treated as names of different objects. Consequently, it is not the case that both "f(a,a)" and "f(b,b)" can be the appropriate transcription or appropriate identity-sign free counterpart of "f(a,b).a=b". Rogers and Wehmeier (p. 547) even claim that neither "f(a,a)" nor "f(b,b)" can count as an adequate translation of "f(a,b).a = b", for in either case "f(a,a)" and "f(b,b)" fail to have the same truth-conditions as "f(a,b).a=b".²⁰ Frege would say that "f(a,b).a=b" and "f(a,a)" may have the same truth-conditions but only if "a" and "b" not only corefer but also express the same sense. Now, if Wittgenstein considers "f(a,a)" and "f(b,b)" to be senseless tautologies and "f(a,b).a=b" to be senseless too, although for a different reason, then it seems that from his viewpoint we could not even properly say that "f(a,a)" (or "f(b,b)) and "f(a,b).a=b" fail to have the same truth-conditions. In saying this, I presume that Wittgenstein denies the possession of truth-conditions not only to tautologies and contradictions but to all other sentences that he regards as lacking a sense or even as non-sensical.

Note that nowhere in the *Tractatus* does Wittgenstein say that in his projected concept-script identity of the object is *shown* by identity of the sign.²¹ In his diary note of 29 November 1914 (Wittgenstein, 1969, p. 123), he writes that in his notation identity could always be *indicated* by identity of the sign. Yet we cannot infer from this remark that in the *Tractatus* he tacitly holds that identity of the object is shown and can only be shown by identity of the signs. If Wittgenstein held this, then according to his dictum about the *impossibility* of saying that which *can* be shown we should assume that identity of the object could not be said or expressed at

¹⁹ In a diary note of 11 November 1914, Wittgenstein claims: "Since 'a=b is not a sentence, 'x=y' is not a function, a 'class 'x(x=x)' is an absurdity and likewise the so-called null class" (Wittgenstein 1969, p. 117).

²⁰ Rogers and Wehmeier (2012, p. 547) point out that their translation procedures only pertain to scenarios in which neither language contains distinct but coreferring names so that this example is said to have no bearing on their translatability results. Yet this is clearly not the scenario that Wittgenstein is imagining in the *Tracatus*. Suppose that for some reason he had confined himself to replacing in his projected correct concept-script only=-sentences of the quantifier-free fragment of a standard first-order language with their identity-sign free counterparts. In my view, this would have made sense by his own lights only if the names flanking "=" are considered to corefer and thus to be intersubstitutable. The tautology "a=b.b=c. $\supset a=c$ " is just one telling example.

 $^{^{21}}$ In T 4.1211, he says though that two sentences "*fa*" and "*ga*" show that the same object is talked about in both. I think that Frege would basically agree with that.

all by a sentence. But this would be at variance with his statement that he *expresses* identity of the object by identity of the sign. Since this is a key statement in the *Tractatus* which occurs only once in it, we should take it at face value.

With regard to (d): Wittgenstein does not mention his motive(s) for adhering to identity despite the elimination of "=" in a correct concept-script. It is therefore not easy to figure out what he had in mind in this respect. Did he think that in his envisioned identity-sign-free concept-script identity is ineluctable since, for example, every sentence in which the same name or the same object variable occurs twice or more often *must* be understood as expressing (or showing) identity as he understands it? (b) Or did he think that after having banned "=" from the notational inventory the expressive power of a classical first-order language with identity must be preserved cost what it may, hence, that precisely for this reason one cannot do without identity in logic? Or was he guided by both (a) and (b) or did he perhaps even have another reason? I cannot give a definite answer but I favour option (b).

It seems to me that if Wittgenstein did not intend to express something logically relevant by identity of the sign, he could have spared himself the trouble of dispensing with "=" and of interpreting identity in a new, non-standard fashion. In that case, pursuing the strategy of utmost notational parsimony may not have paid off to the degree he thought it would.

With regard to (e): Wittgenstein does not explain how identity of the object as that which is expressed by identity of the sign should be understood specifically since it is not construed as a relation, in particular, it is not understood as a relation in which every object uniquely stands to itself. In T 5.53, Wittgenstein says that he expresses identity of the object by identity of the sign. He deliberately avoids saying that in this way he expresses the identity of an object with itself. According to T 5.5303 (cf. Wittgenstein, 1969, § 216), this would amount to saying nothing.²² Unfortunately, Wittgenstein leaves the nature of identity of the object as he understands it unspecified.²³ Do we know what identity of the object is supposed to be if we are told that it is that which is expressed by identity of the sign? Hardly. What we do know is only that identity so expressed is not considered a relation. "It is self-evident that identity is not a relation between objects" (T 5.5301). Following his conviction, Wittgenstein might have written instead that identity is not a relation at all. In his view, it is neither a relation between coreferential signs.

What about identity of the sign? I suggest that Wittgenstein does not construe it as a relation either. In particular, he does not consider it a relation in which every

²² In my view, Wittgenstein's argument in T 5.5301 that identity is not a relation between objects (a relation in which an object stands to itself) is not conclusive; see AMONYMOUS.

²³ In the notes dictated to G. E. Moore in Norway in April 1914, Wittgenstein claims that the sign of identity expresses the internal relation between a function and its argument: $\varphi a \Box \Box (\exists x)$. $\varphi x.x = a: \equiv: \varphi a$. He characterizes here internal relations "as relations between types, which cannot be expressed in sentences, but are all shown in the symbols themselves, and can be exhibited systematically in tautologies" (Wittgenstein 1969, p. 248; cf. p. 184 and T 5.441, T 5.47). Thus, "one can see that the sentence ($\exists x$). $\varphi x.x = a$ is a tautology, if one expresses the *truth-conditions* of ($\exists x$). $\varphi x.x = a$ successively" (Wittgenstein 1969, p. 248).

sign uniquely stands to itself, regardless of whether he has the sign-token qua concrete spatio-temporal object or the sign-type qua abstract object in mind. If he did consider identity of the sign a relation, how could identity of the sign express identity of the object which is supposed to be non-relational? Wittgenstein must have been aware that both a sign-token and a sign-type are objects. Yet when he states that identity of the object is expressed by identity of the sign he probably thinks of any object that is not a sign but is referred to by a sign.

With regard to (f): Wittgenstein does not say whether he thinks that in his envisioned concept-script he could still define a new object-sign "b" without having a coreferential object-sign "a" on hand which is be replaced by "b" via stipulation, or whether he thinks that he can get along without proper definitions of object-signs. In T 4.241, he writes:

If by means of an equation I introduce a new sign "b" and stipulate that it should replace a certain sign "a" that is already known, then (like Russell) I write the equation – definition – in the form "a = b Def." The definition is a rule dealing with signs.

What Wittgenstein writes here is just water under the bridge when a little later in the *Tractatus* he makes a case for dispensing with "=" in a correct concept-script. As I already noted, one significant consequence of the removal of "=" and the related new way of expressing identity of the object is the absence of coreferential names in the concept-script. This means that Wittgenstein can no longer define a new object-sign "b" in such a way that it shall replace a sign "a" that is already known. For "a" and "b" must be taken to corefer in order to license the definitional stipulation that "b" shall replace "a", that is, in a definition Wittgenstein cannot just replace a sign "a" that is already known by a new sign "b" without any consideration of the reference or meaning of "a" which is already known and is bestowed on "b". Wittgenstein does not address this issue. If he had acknowledged that the sign of an object cannot be defined in his concept-script for the reason just explained, he would perhaps have denied that this has a far-reaching impact on his project by pointing out that definitions construed as stipulations about the replacement of one sign by another are, from a logical point of view, dispensable.²⁴ Be that as it may, towards the end of the Tractatus, he still defines some expressions and he does so in accordance with what he writes in T.4241, namely by using "=" in the definition and, thus in his view by relying on the stipulation that the new sign shall replace the sign that is already known. In T 6.02, he sets up five definitions (etc.) in which "=" occurs. And in T 6.241, after having pointed out that the method by which mathematics arrives at its equations is that of substitution, he proves "2 X 2=4" by starting with the definition: $(\Omega^{\nu})^{\mu'} x = \Omega^{\nu X \mu'} x$ Def. In the proof which proceeds from this definition, he uses "=" eight times. It would have been interesting to see how

²⁴ In T 3.343, Wittgenstein writes: "Definitions are rules for translating from one language into another." "A name cannot be dissected any further by means of a definition: it is a primitive sign" (T 3.26). "The meanings of primitive signs can be explained by means of elucidations" (T 3.263). The last statement tallies with Frege's view in *Grundgesetze der Arithmetik*.

Wittgenstein would have proved the identity-sign free counterpart of "2 X 2=4", that is in his sense, "f(2 X 2, 2 X 2)" or "f(4,4)".

With regard to (g): Wittgenstein fails to explain whether by removing "=" from his notation he intends not only achieve utmost notational parsimony, but also ensure (i) that the identity-sign free counterparts are not subject to the classification as pseudo-sentences, (ii) that the expressive power, say, of the envisaged identity-sign free concept-script of first-order is on a par with standard first-order logic containing "=" but, thanks to the new device to express identity (of the object), is notationally put on the right track in this respect, and (iii) that the new treatment of identity is logically justified on a large scale.

(1) If in Wittgenstein's judgement the pseudo-character of equations and truth-functional connections of equations were exclusively due to the circumstance that they are only (simple or complex) representational devices which govern the mutual substitutivity of coreferential names, then it would be clear that none of the identity-sign free counterparts of those representational devices is a pseudo-sentence for him. For he does not construe the identity-sign free counterparts as representational devices.²⁵ (2) If the pseudo-character of = -sentences in general is seen to be due to the use of the dispensable sign "=", as we may assume in the light of the available evidence in the Tractatus, then it is most likely that Wittgenstein does not construe any identity-sign free counterpart as a pseudo-sentence. It seems rather obvious that he does not consider any of the identity-sign free counterparts that he lists in T 5.531, T 5.532 and T 5.5321 a pseudo-sentence. (3) Assuming with near certainty that Wittgenstein regards a concept-script which contains pseudo-sentences, in particular = -sentences, as incorrect,²⁶ it follows also from this premise that in a correct concept-script none of the identity-sign free counterparts is considered to be a pseudo-sentence. I presume that in Wittgenstein's view replacing pseudosentences containing "=" with identity-sign free pseudo-sentences would be a pointless endeavour. Why should he do this? The sheer saving of a sign, which in standard first-order logic with identity occupies an important place, is presumably not his only motive for breaking radically new ground regarding identity. It seems appropriate to see his approach to identity not in isolation but in the context of his overall view of logic and the constraints that he wishes to impose on a correct concept-script in general.

To conclude this section, let me return to my claim in the Introduction that the soundness of Wittgenstein's inference from the mention of a few identity-sign free counterparts of = -sentences to the dispensability of "=" in a correct concept-script might be disputed if one or the other example of a=-sentence which has no identity-sign free equivalent could be presented. Here are two potential candidates for this.

 $^{^{25}}$ I assume that for Wittgenstein the representential device character of equations follows from the nature and status of "=".

²⁶ Yet much indicates that Wittgenstein does not regard a concept-script which contains senseless sentences as incorrect. I have no illuminating explanation for that.

(i) According to Wittgenstein, the truth-functional compound "a=b. b=c. $\supset a = c$ ", which he quotes in T 5.534 as a pseudo-sentence, connects three representational devices: "a=b", "b=c" and "a=c". Thus, the compound sentence is supposed to read as follows: If "b" can be substituted for "a" and "c" can be substituted for "b", then "c" can be substituted for "a".²⁷ Shall we therefore assume that "a=b. b=c, $\supset a=c$ " is considered a complex representational or substitution device? I am inclined to assume this. Yet whatever Wittgenstein thought about this, I further assume that on the grounds that "a=b", "b=c" and "a=c" are not elementary sentences – I infer this from the assumption that he considers them to be derivable from "a=a", cf. T 4.243 – he does not construe these equations as truth-arguments of "a=b. b=c. $\supset a=c$ " (cf. T 5.01) or in other words: he does not consider "a=b. b=c. $\supset a=c$ " to be a truth-function of elementary sentences.²⁸ In his view, it is a senseless tautology with three distinct representential devices or substitution rules that stand in a certain logical relation to one another. Now, which sentence is supposed to replace "a=b. b=c. $\supset a=c$ " in an identity-sign free concept-script? Again, identity as that what Wittgenstein expresses by identity of the sign is not a relation, in particular, it is not an equivalence relation, nor is it a substitution device. If I am right, then "a=b. b=c. $\supset a=c$ " has no equivalent identity-sign free *correlatum* in a concept-script à la Wittgenstein. Identity in his sense is not transitive nor does it possess a property which in his notation might coherently "replace" the transitivity of "=". It is objectual yet non-relational.

(ii) By which identity-sign free sentence does Wittgenstein intend to replace the tautology "a=a"? The sentence "f(a,a)" or "f(b,b)" is already taken to replace "f(a,b).a=b". It cannot serve as an identity sign-free equivalent of both "a=b" and "a=a". Furthermore, there is no evidence that in the *Tractatus* Wittgenstein would have chosen, say, " $F(a) \lor \neg F(a)$ " as the identity-sign free counterpart of "a=a". Examples like these and others may show that the belief in the correct "translatability" of *all*=-sentences into identity-sign free counterparts is probably an illusion.²⁹

²⁷ So construed, the sentence "a=b. b=c. $\supset a=c$ " is, in my view, not senseless: it states an eminently learnable and forgettable fact.

²⁸ Wittgenstein contends in T 5.32 that all truth-functions are results of successive applications to elementary sentences of truth-operations. (T 5.41).

²⁹ Long before I read Lampert and Säbel 2021, I had the suspicion that "a=a" and "a=b. b=c. $\supset a=c$ " may be two examples of =-sentences that defy adequate "translatability" into identity-sign free counterparts. It is striking that in T 5.331, T 5.532 and T 5.5321 Wittgenstein provides an identity-sign free transcription neither of "a=a" nor of "a=b. $\supset b=a$ " nor of "a=b. b=c. $\supset a=c$ ". If he had worked out a concept-script in accordance with his novel conception of identity and his dispensation with "=" and had extended the concept-script to include second-order logic, he could not have formulated a secondorder abstraction principle like Hume's Principle " $N_xF(x)=N_xG(x) \leftrightarrow Eq_x(F(x),G(x))$ " or an instance of it. An identity-sign free formulation such as " $f(N_xF(x), N_xF(x)) \leftrightarrow Eq_x(F(x),F(x))$ " or " $f(N_xG(x), N_xG(x))$ $\leftrightarrow Eq_x(G(x),G(x))$ " along the lines of Wittgenstein's replacement of "f(a,b).a=b" with "f(a,a)" or "f(b,b)" could never take the place of Hume's Principle. Yet from his viewpoint in his middle period, Wittgenstein may have denied that this would be a severe handicap. In the early 1930s, he regarded the significance and utility of abstraction principles like Hume's Principle in a foundation of arithmetic with increasing suspicion. And Frege's Basic Law V had been thoroughly discredited by Russell three decades prior to that.

Finally, just a point of clarification and a brief look at Wittgenstein's abandonment of his *Tractatus* conception of identity in his middle period. Regardless of my preceding criticism, Wittgenstein's position in the *Tractatus* vis-à-vis "=" and identity is not without merit. "[It] gives rise to challenging questions which are not altogether easy to answer" (Frege, 1967, p. 143).

In his so-called middle period, Wittgenstein's view regarding the nature of identity and the meaningfulness of equations has changed, perhaps as a result of a Damascus Road experience in his logical outlook. (One important example of a conversion of his views, besides identity, is his critique of the "theory" of elementary sentences in the Tractatus; cf. Wittgenstein, 1967, p. 73 f.; 1969, p. 210 ff.) First, in the middle period the dispensation with "=" in the Tractatus is not mentioned, let alone endorsed. Second, identity (of the object) is no longer characterized as that which is expressed by identity of the sign. Third, equations of the form "a=b" are not labelled representential devices. In particular, "non-degenerate" mathematical equations are not disesteemed as not expressing a thought, and =-sentences in general, including mathematical equations, are no longer downgraded as senseless pseudo-sentences. However, Wittgenstein still regards "degenerate" equations of the form "a = a" as senseless, but he no longer calls them pseudo-sentences.³⁰ I presume that in the early 1930s he had abandoned the idea of developing a new conceptscript, one which was thought to be superior to the concept-scripts that Frege and Russell had worked out, at least with respect to the first-order portion they embrace.

3 More on Pseudo-sentences in the Tractatus: Critical Comments on Lampert and S\u00e4bel's Critique of Wehmeier's Account

In this concluding section, I return to Wittgenstein's view of pseudo-sentences in connection with his elimination of "=" in a correct concept-script. In their interesting essay 'Wittgenstein's Elimination of Identity in Quantifier-Free Logic' (2021), Lampert and Säbel deal to a large extent with Wehmeier's analysis of Wittgenstein's treatment of identity and "=" in the *Tractatus*. At the ouset of Section 5.1, they claim that "there is a second problem with Wehmeier's account of Wittgenstein's elimination of identity". (Recall my critical observation that in the *Tractatus* Wittgenstein dispenses with "=" but not with identity.) The first problem is said to concern the treatment of individual constants in sentences of the form "a=b" which, for reasons of space, I shall not address in this section. I confine myself to making just one remark.

In Section 4 of their article, Lampert and Säbel (2021) analyze in great detail the "translatability" of "a=b" into an equivalent identity-sign free formula. While, as we have seen, Rogers and Wehmeier (2012) argue that Wittgenstein's "translation " of "a=b" into "Faa" or "Fbb" is inadequate – "a=b" and "Faa" (or "Fbb")

³⁰ In his critical discussion of Ramsey's theory of identity, Wittgenstein (1964 p. 141 ff., 1967, p. 189 ff., 1969, p. 315 ff.) argues that equations play a dual role in the arithmetical calculus. They are concrete, individual configurations and at the same time supposed to function as substitution rules.

are considered to have different truth-conditions – Lampert and Säbel (2021, 4.1) develop an alternative account that is intended to reveal the viability of the identity-sign free counterpart of "a=b" suggested by Wittgenstein.

The second problem indicated above concerns Wehmeier's interpretation of Wittgenstein's understanding and treatment of pseudo-sentences in the *Tractatus*. Special emphasis is placed on the account in Rogers & Wehmeier, 2012. However, I fail to see that Lampert and Säbel adduce a compelling argument against Wehmeier's interpretation of Wittgensten's use of the term "pseudo-sentence".

Since in Section 5.1 of their essay Lampert and Säbel deal only with quantifierfree logic, they confine themselves to considering two of the four sentences that in T 5.534 Wittgenstein characterizes as pseudo-sentences: (a) "a=a" and (b) "a=b. $b=c. \supset a=c$ ". In my view, Lampert and Säbel do not elucidate as completely as possible Wittgenstein's specific interpretation of a pseudo-sentence in the *Tractatus*. Without attempting to clarify this as far as it goes, in particular with respect to=-sentences, the question why Wittgenstein would consider formulas (a) and (b) to be pseudo-sentences can hardly be given a clear-cut answer.³¹ Lampert and Säbel write:

For one, it seems clear that neither of the two is interpretable as a substitution license or prohibition fixing the use of names in any way. Second, these formulas lack matrices, consisting purely of identity statements.

For the sake of convenience. I abbreviate the two claims in this quotation as (A) and (B). To begin with, I fail to see what impact the "lack of matrices" is supposed to have on the pseudo-character that Wittgenstein assigns to "a=a" and " $a=b.b=c. \supset a=c$ ". Regarding the characterization of =-sentences as pseudo-sentences, there is no textual evidence that Wittgenstein is concerned with a "lack of matrices". Thus, I consider claim (B) largely irrelevant when we try to figure out what it is in the Tractatus that Wittgenstein understands by the pseudo-character of =-sentences. In contrast to the practice in his diaries, in the Tractatus Wittgenstein does without a general explanation of the term "pseudo-sentence". I presume that this is due not to carelessness but rather to the insight that especially in the light of his (likely) interpretation of the whole class of =-sentences as pseudo-sentences in the last third of the Tractatus - after having dealt with non-sensical pseudosentences at an earlier stage, namely in T 4.1272 – any attempt to furnish a uniform and generally applicable interpretation of "pseudo-sentence" might be bound to fail. Non-sensical pseudo-sentences – for example, whenever in a sentence the word "object" is not "expressed" by the variable name but as a concept-word we are

³¹ In their Section 5.1, Lampert and Säbel frame the following definition: "Two classical formulas φ and ψ have equivalent exclusive translations iff either (1) both are classified as pseudo-propositions from the exclusive standpoint, or (2) neither φ nor ψ are classified as pseudo-propositions *and* their translations are equivalent." The definition presupposes that the meaning of the term "pseudo-proposition" has been explained prior to the definition. However, it is not quite clear what Lampert and Säbel understand by a pseudo-sentence in Wittgenstein's sense of this term.

facing a non-sensical pseudo-sentence (cf. T 4.1272)³² – as such differ from *sense*less pseudo-sentences which in Wittgenstein's view (pointed out earlier) probably is a class which comprises all=-sentences, not only=-tautologies (and=-contradictions). Note that quantified = -sentences prevail among the = -sentences that in T 5.531 - T 5.534 Wittgenstein lists and with an eye to his envisaged correct conceptscript replaces with their identity-sign free counterparts. In the context of the Tractatus, the feature of senselessness is neither a necessary nor a sufficient condition for a sentence (p) to be classified as a pseudo-sentence. We know that all tautologies and contradictions are said to be senseless, but not all tautologies and contradictions are considered to be pseudo-sentences by Wittgenstein.³³ In fact, he characterizes only those tautologies and contradictions as (senseless) pseudo-sentences which contain the sign "=". Lampert and Säbel's claim (2021, 5.1) that in the Tractatus "[t]autologies are never marked as Scheinsätze" is therefore false. Needless to say, non-sensicalness is not a necessary condition for a sentence (p) to be classified as a pseudo-sentence either. However, under the assumption that Wittgenstein considers every non-sensical sentence a pseudo-sentence - the assumption seems reasonable to me - non-sensicalness is a sufficient condition for a sentence (p) to belong to the class of pseudo-sentences.

In the Tractatus, Wittgenstein downgrades (a) and (b) as pseudo-sentences. He does so on the grounds that they contain the sign "=" which, to repeat, he takes to be dispensable in a correct concept-script. This seems to be all that we can reliably say by appeal to textual evidence in the Tractatus about the pseudo-character of (a) and (b). (Recall the statement in T 6.2: "The sentences of mathematics are equations, and therefore pseudo-sentences.") Wittgenstein understands "=" as expressing substitutivity of one name "b" for another name "a" flanking "=", in fact as mutual substitutivity of "a" and "b". With regard to claim (A) in the quotation above, I observe that Wittgenstein's interpretation of "=" as the key constituent of a substitution device or substitution rule (see also T 6.24) is presumably a feature that he also considers relevant for his characterization of =-sentences as pseudosentences. I used the word "presumably" since in the Tractatus Wittgenstein does not expressly say this. We have seen that in his judgement a representational device like "a=b" is neither an elementary sentence nor an argument of a truth-function. Construed as a rule that governs the mutual substitutivity of "a" and "b", "a=b" is, unlike an elementary sentence or a truth-function of elementary sentences, most likely considered not to be bivalent by Wittgenstein, that is, he does not construe

 $^{^{32}}$ "So one cannot say, for example, 'There are objects', as one might say, 'There are books'...And it is nonsense to speak of the total number of objects... '1 is a number', 'There is only one zero', and all similar expressions are nonsensical. (It is just as nonsensical to say, 'There is only 1', as it would be to say, '2+2 at 3 o'clock equals 4'.)" Wittgenstein does not expressly say that these sentences are pseudosentences but I assume that he does regard them as nonsensical pseudo-sentences.

³³ Recall Wittgenstein's remark in T 4.243: "Expressions like "a=a", and those derived from them, are neither elementary sentences nor is there any other way in which they have sense." In Wittgenstein's view, all=-tautologies are senseless on the ground that they are tautologies. According to T 4.243, nontautologous=-sentences that he considers to be derivable from "a=a" such as "a=b" and others as well, fall likewise under the verdict of senselessness although for a different reason.

"a=b" as a sentence that is either true or false. If this is correct, it would make no sense for him to say that "a=b" expresses that "b" can be substituted for "a" salva veritate. Nevertheless, I presume that Wittgenstein (tacitly) distinguishes between correct and incorrect substitution devices. Plainly, "a=b" governs the substitutivity of "b" for "a" and of "a" for "b" correctly only if "a" and "b" corefer.

Now suppose that Wittgenstein does indeed consider the metalinguistic substitutivity interpretation of "=" relevant for his characterization of=-sentences as pseudo-sentences. Suppose further that Lampert and Säbel assent to this, although they do not say this expressly. Under these assumptions, I tentatively propose that we take the conclusion we are supposed to draw from (A) and (B) to be this: that according to Wittgenstein's understanding of the term "pseudo-sentence" regarding = -sentences he would hardly be justified in declaring (a) and (b) as pseudosentences. Granted this would seem awkward, and I am not sure whether Lampert and Säbel would accept or reject the proposal. However, in Section 5.1 they do not provide a conclusive argument for their thesis that neither (a) nor (b) can be interpreted as a substitution license in Wittgenstein's sense. Claiming - as they do in Section 5.2, and in my view correctly – that "a=a" is a redundant substitution rule does not deliver a compelling argument for the correctness of (A) regarding "a=a". One may insist that a vacuous substitution rule is still a substitution rule, although, as Wittgenstein might say, a degenerate one. (Recall that in his middle period, he calls "a=a" a degenerate equation.) Admittedly, there is no direct evidence that in the *Tractatus* Wittgenstein considered "a = a" a borderline case of a substitution license. Yet I am inclined to assume that he would not have denied "a=a" the substitutivity character. To be sure, Wittgenstein is not committed to any attempt to make the impossible possible, namely to save "a=a" from triviality, in order to possibly legitimize the substitutivity-interpretation of "a=a". The substitutivity-interpretation paradigmatically applies of course to the standard case of an equation, to one of the form "a=b". Furthermore, from a logical point of view, we see that if "a=a" is construed as a trivial or redundant substitution license, it is causing no harm. So understood, "a=a" does not lead to any result that may have an adverse impact on our reasoning about identity. And the pseudo-character that Wittgenstein assigns to "a=a" is, from a logical point of view, likewise nothing threatening. Even if due to the (epistemic) triviality of the sentence " $a = a^{34}$ it is never used in a calculus of classical first-order logic with identity,³⁵ this need not undermine its substitutivity character which in the present scenario is supposed to rest soley on what I take to be Wittgenstein's unrestricted metalinguistic reading of "=".³⁶

³⁴ "a = a" is trivial no matter how it is interpreted, objectually or metalinguistically.

³⁵ Note that in *Grundgesetze* I, §50 Frege proves "a = a", objectually construed, as a theorem: IIIe. He says that although "a = a" is by his elucidation of the equality-sign (in Frege 1893, §7) obvious [*selb-stverständlich*], it is worth seeing how it can be developed from Basic Law III (in modern notation): $|-g(a=b) \rightarrow g(\forall f(f(a) \rightarrow f(b))))$, in Frege's words: the truth-value $\forall f(f(a) \rightarrow f(b))$ falls under every concept under which the truth-value a = b falls.

 $^{^{36}}$ I do not quite understand why Lampert and Säbel claim (in 5.2) that "*a*=*b*" "by itself should similarly be considered ill-formed and therefore denied translation." I assume that they mean: translation in an equivalent identity-sign free sentence.

As far as the truth-functional sentence " $a=b.b=c. \supset a=c$ " is concerned, I have already suggested that Wittgenstein construes it as a complex representential device. It governs the substitution of "c" for "a", if "b" can be substituted for "a " and "c" for "b". In contrast to "a=a", Wittgenstein's probably considers " $a=b.b=c. \supset a=c$ " a non-trivial substitution device that consists of logically interconnected non-trivial substitution devices. So understood, " $a=b.b=c. \supset a=c$ " seems acceptable for someone who endorses classical logic and follows Wittgenstein in interpreting "=" as expressing, in the standard case of an equation, the (mutual) substitutivity of one sign for another (coreferential) sign.³⁷

Lampert and Säbel observe that most interpreters have taken 5.534 to exclude formulas like a=a from the correct concept-script. They go on to write: "Wehmeier, as we have seen, takes a different approach." In relation to what? Wehmeier does of course not choose a different path regarding Wittgenstein's actual exclusion of "a=a" from a correct concept-script. Only the exclusion of =-formulas such as "a=a" from a correct concept-script was at issue in the statement that precedes Lampert and Säbel's statement on Wehmeier's approach. However, the authors unduly shift the focus and comment instead on Wehmeier's suggestion to "translate" sentences of the form "a=a" into tautologies and sentences of the form "a=b" into contradictions by. In any event, there is or at least should be *unanimous* agreement among scholars dealing with the *Tractatus* that Wittgenstein banishes all=-sentences from a correct concept-script. There is no other way to understand him.

Lampert and Säbel (2021) go on to write:

The evidence that Wittgenstein didn't intend the pseudo-propositions of 5.534 to be represented as tautologies or contradictions is very strong, however. In 5.531, the phrase Wittgenstein uses to give his translation is "Ich schreibe also nicht [...], sondern [...]." So it seems obvious that there is an intended contrast, when in 5.534 he says, dass "Scheinsätze wie [...] sich in einer richtigen Begriffsschrift gar nicht hinschreiben lassen (our emphasis), Wehmeier thinks we should interpret this phrase *literally*: Since the new notation doesn't contain the identity sign, a = a literally cannot be written down ... But given the context of the remark this seems artificial. If Wittgenstein had wanted to use a tautology as the exclusive transcription of a = a, it would have been natural for him to write: "Ich schreibe also nicht a = a, sondern $Fa \lor \neg Fa$ " (or any other tautologous formula). Also the expression "gar nicht" ("not at all") seems to indicate that there is nothing to be written down. Finally, if Wittgenstein only meant to say that these pseudo-propositions cannot literally be written down, this would also exclude the sentences in 5.5301 to 5.5321 from being written down since they also include the identity sign. But for these sentences Wittgenstein explicitly offers instructions on how to write them down.

³⁷ According to Wittgenstein, tautological=-sentences are not pseudo-sentences on the grounds that they are senseless. Otherwise he would have to characterize all tautologies as pseudo-sentences which he does not.

I find the line of argument in this passage hard to follow. This is partly due to the fact that some of the statements that Lampert and Säbel make are, in my view, not related to one another in a clearly recognizable way. Moreover, some of their claims I find either vague or misleading or both. The standard first-order language containing "=" and Wittgenstein's (envisaged) identity-sign free concept-script must of course always be clearly distinguished to avoid unnecessary confusion.

First, contrary to what Lampert and Säbel claim at the beginning of the quoted passage, there is no strong evidence that Wittgenstein did not intend to replace the =-sentences that he mentions in T 5.534 with identity-sign free tautologies or contradictions. In my judgement, the authors do not provide it.

Second, I fail to see that Wittgenstein is concerned with the contrast that Lampert and Säbel bring into play by considering two closely related remarks by Wittgenstein. As a matter of fact, the second remark quoted from T 5.534 tallies seamlessly with the first remark quoted from T 5.531. Wittgenstein concludes from T 5.531 – T 5.533 that "=" is not an essential constituent of a concept-script and then concludes from this that certain=-sentences which he downgrades as pseudo-sentences cannot be written down in a correct concept-script. He apparently believes that he is justified in drawing the conclusions (which in a sense are two sides of the same coin), despite the relatively small number of identity-sign free counterparts of=-sentences that he presents and despite the fact that he does not offer identity-sign free equivalents for the pseudosentences "a=a", " $a=b.b=c. \supset a=c$ ", "(x).x=x" and "($\exists x$).x=a" that he mentions in T 5.335. In short, there are no ifs and buts about Wehmeier's interpretation of T 5.534. It is natural, straightforward and unfailingly correct.

However, should any doubt remain on the part of Lampert and Säbel as far as my attempted clarification of the point at issue is concerned, let me make an additional comment. Imagine, for the sake of argument, that in the early twenties Wittgenstein was about to write down a concept-script to his taste following strictly the plan that he had adumbrated in the *Tractatus*, including the dispensation with "=". Disregarding possibly explanatory footnotes or parenthetical remarks, even in the case in which "=" is officially excluded from the primitive vocabulary of the new concept-script Wittgenstein could nonetheless (physically) have written down some=-formulas in the main body of the manuscript in progress, perhaps by juxtaposing them with some identity-sign-free counterparts of=-sentences, guided by the interest, say, of comparing the length of the former with that of the latter. But in doing so Wittgenstein would flagrantly have offended against his notational key principle: the exclusive use of logically indispensable signs in a correct concept-script. In sum, using any=-formulas in his correct concept-script itself, would have been "illegal" by Wittgenstein's own lights and outright preposterous.

Third, the fact that "a=a" cannot be written down in an identity-sign free concept-script is one thing. Suggesting what Wittgenstein's natural choice of the identity-sign counterpart of "a=a" would have been, if he had intended to use a tautology as the identity-sign free equivalent of "a=a", is another. It is unclear how the putative artificiality of Wehmeier's interpretation of T 5.534 is supposed to become apparent by pointing out that if "Wittgenstein had wanted to use a tautology as the exclusive transcription of a=a, it would have been natural for him to write: Ich schreibe also nicht a=a, sondern $Fa \lor \neg Fa$ (or any other tautologous formula)."

Fourth, as I just made clear, there is no denying that the =-sentences that Wittgenstein actually writes down in T 5.5301 - T 5.5321 cannot (sensibly) be written down in his projected identity-sign free concept-script. Wittgenstein must of course write them down in the *Tractatus* in order to be able to suggest identity-sign free sentences that are designed to replace the =-sentences in his projected concept-script. But it is unambiguously understood that he treats these =-sentences as constituents of a standard first-order language with "=" that he seeks to renew regarding identity offers instructions on how to write them down" Lampert and Säbel intend to convey that Wittgenstein writes these sentences down and is entitled to write them down, since he is (tacitly) appealing to the context of a first-order language with "=" in which they figure as wellformed formulas, then their claim is correct, although almost trivial. However, if they wish to express instead that Wittgenstein explicitly offers instructions on how to write down the =-sentences in his new concept-script, then their claim is patently false.

Fifth, Lampert and Säbel go on to write that "even a stronger piece of evidence comes from an interesting letter by Wittgenstein addressed to Ramsey in 1927." Evidence for what? I presume that the authors have their earlier claim in mind that, contra Wehmeier, Wittgenstein did not intend to replace the pseudo-sentences that he mentions in T 5.534 with identity-sign free tautologies or contradictions. (Lampert and Säbel use the words "to be represented by tautologies or contradictions" which to my mind is perhaps an infelicitous choice of phrasing. Representing pseudo-sentences by identity-sign free tautologies or contradictions sounds as if the latter would thereby inherit the pseudo-character of the former, which in Wittgenstein's opinion is not the case.) Lampert and Säbel quote from the letter in which Wittgenstein criticises Ramsey's definition of identity (cf. Section 3 of my essay): "I will try to show that this definition won't do nor any other that tries to make x=y a tautology or a contradiction." The authors add that "from an exegetical point of view this seems to settle the issue." I assume that with "the issue" they still have Wehmeier's view in mind that statements of the form "x=x" are "translatable" into an identitysign free tautology and statements of the form "x = y" are translatable into an identity-sign free contradiction. In my view, we cannot safely infer from Wittgenstein's remark in his letter to Ramsey that Wehmeier's view does not carry conviction or is untenable. First, Wittgenstein's critical remark on Ramsey's definition should not be lumped together with his suggested replacement of =-sentences with (equivalent) identity-sign free sentences in a correct concept-script. In the critical comment on Ramsey's definition of identity, Wittgenstein's view on identity and "=" in the Tractatus plays no role. It is perfectly possible, if not likely, that in 1927 Wittgenstein had already abandoned the idea of dispensing with "=" in a correct concept-script by replacing=-sentences with (equivalent) identity-sign free sentences such that the question of whether, for example, the appropriate identity-sign free counterpart of "a=a" would have to be a tautology or not was obsolete. (I assume that if in the Tractatus Wittgenstein thought that there is an identity-sign free equivalent of "a=a", it would have been a tautology.)

An objection could be raised that my assessment of Lambert and Säbel 2021 in the preceding discussion was rather critical. While this assertion holds some validity, given the instances of ambiguity and lack of coherence in certain aspects of their argumentation, I found myself compelled to adopt this stance. The moral of the story might be this: One should not read into Wittgenstein's text what is not there.

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