



# Multifractal Characteristics Analysis of Spatial State of Prefecture-Level Cities in China

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Received: 28 October 2020 / Accepted: 20 November 2022 / Published online: 28 January 2023  
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## Abstract

In the last 40 years, Chinese cities have seen extensive development across a wide array of spheres. In this study, we applied a power-law and multifractal analysis to characterise **22** indices for 288 prefecture-level cities in mainland China. The data used to characterise the indices is representative of the year 2012. The results show that the Gross domestic product (GDP) is regular and exhibits multifractal spatial characteristics. Specifically, most developmental fields exhibit coupling in conjunction with being chaotic. **Furthermore, eight indices** with multifractal characteristics clearly reflect the spatial complexity of the corresponding fields and the volatility between prefecture-level cities. Overall, the tertiary industry has undergone nongeneralised development. On the other hand, the construction industry resembles a bubble economy, and the spatial layout of traditional industries has a homogenising effect in Middle and East China. This study examines China's overall spatial characteristics and states based on the above-mentioned analyses; additionally, the study focuses on the early 2010s. The contribution of this study provides a quantitative analysis paradigm based on a multifractal approach.

**Keywords** Multifractal Analysis · Power Law · Spatial State · Hierarchical Evolution · Prefecture-Level City · China

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## Introduction

China implemented the reform and opening-up policy in 1978. This policy largely entailed urbanisation, industrialisation, and informatisation (referred to as the ‘three processes’). These three processes have resulted in China’s various spheres of development experiencing complex and rapid changes. These changes have become an elemental driving force for social development, generating substantial spatiotemporal changes (Liang et al., 2021; Chen, 2010a; Qu et al., 2019; Long et al., 2010; Yu et al., 2007). Meanwhile, owing to the high uniformity of urban and rural development within Chinese prefecture-level cities, a prefecture-level city is often considered a regional complex that integrates urban and rural areas (Liu et al., 2021; Liu et al., 2020; Li et al., 2020; Tu and Long, 2017; Zhao, 2013). Spatial correlations in domestic prefecture-level cities drives similarities and differences in the urban–rural integral landscape among prefecture-level cities. Consequently, understanding the characteristics, spatial state, and mechanisms of various developmental fields through the lens of prefecture-level cities has important practical implications.

Prior to the COVID-19 pandemic (since the early 2010s), China’s economy had been in a state of rapid growth. Specifically, the development of leading industries, primary industries, and the bubble economy played a significant role in facilitating the rapid development (Liu et al., 2021; Duan et al., 2017; Wu et al., 2014). In the early 2010s, the development of Chinese prefecture-level cities was generally evolutionary; it was both ordered and chaotic. This complex pattern arose due to the systematic patterns inherent in the three processes (Gong et al., 2021; Jiao et al., 2020). Therefore, an in-depth analysis of years across this period will significantly elucidate the developmental trajectory of China before the epidemic (Chen, 2008; Allen and Sanglier, 1981; Arthur, 1999; Batty, 2000). According to previous research on the developmental complexity of all major regions in China, the overall development of Chinese prefecture-level cities was found to be complicated, possessing certain spatial regularities (Chen, 2014, 2016). That is, because of their influence, all prefecture-level cities easily maintain a dynamic balance and achieve external diffusion (Chen, 2010b). Many developmental fields possess multifractal characteristics within a pre-defined range of prefecture-level cities (Song et al., 2021a, 2021b; Song et al., 2020; Chen and Wang, 2013; Chen, 2010b); therefore, the evolutionary dynamics of prefecture-level cities affects macroregions at high levels. Strengthening or weakening evolutionary patterns of different levels/degrees occur in multiple developmental fields in China at the macro–meso–micro scales. The trend of coupling is holistically geared toward maximising the performance of the entire nation/major regions under government regulations (Chen, 2008), and the three processes in prefecture-level cities are complex processes that combine self-organising evolution with government regulations across multiple developmental fields. Consequently, the development of prefecture-level cities should manifest as a nationwide filling process for the optimum urban development mode/process. Simultaneously, rural properties undergo a cutting process, which often

leads to spatial autocorrelation, chaos, and bifurcation. Spatial autocorrelation typically generates multifractal results. In the early 2010s, the rapid development at the micro- (such as the income of urban and rural residents), meso- (built-up area), and macrolevels (per capita GDP) equated to increased complexity. Otherwise, the complexity remains low or even causes the disappearance of the spatial correlation.

This study uses the cross-sectional data that is representative of the year 2012 and employs a traditional mathematical method (power-law analysis) and multifractal technology (one-*dimensional* multifractal detrended fluctuation analysis, MDFA). By thoroughly reviewing the data from 288 prefecture-level cities in mainland China, this study aims to unveil the overall characteristics/states and mechanisms underlying spatial evolution in the context of China's rapid development in the early 2010s. Overall, there is a research gap in the analysis of non-ideal spatial characteristics. The study of the hierarchy and strength–weakness differences in the development of various localities is also conducive to spatial planning or the adjustment of governmental efforts at all levels. Our study is rooted in multifractal analysis derived from nonlinearity; therefore, it should be able to accurately condense complexities within an economic geospatial space, yielding more realistic approaches.

As such, the highlights of this paper can be summarised as follows:

- 1) **Describing** the spatial state of various developmental fields in the early 2010s in China and a variety of spatial dissymmetries.
- 2) **Addressing** the complexity and different intensities of volatility of socioeconomic activities at the prefecture level and the spatial agglomeration and diffusion of different areas and levels at the prefecture level.
- 3) **Analysing** multiple fractals within the context of one-dimensional data fluctuations. In this study, we demonstrate the superiority of our approach in economic geography.
- 4) **Overcoming** limitations in conventional techniques, which solely depend on the traditional mathematical methods.
- 5) **Reveal** the specific features and local details in the process of China's macrodevelopment using the multifractal dimension spectrum  $h_q$ , singularity exponent  $H_q$ , and generalised fractal dimension value  $D_q$ .

The theoretical framework of this study can be summarised as follows: the power-law analysis is utilised to obtain data segments representative of regular changes in significant socioeconomic activities. This is done to create a preliminary understanding of the developmental status of China's prefecture-level cities at the time (Sect. 3). Specifically, multifractal analysis is used to develop a more detailed mathematical description of the development status based on specified data segments (Sect. 4) and to reveal patterns across multiple developmental fields in prefecture-level cities (Sect. 5).

The remainder of this paper is organised as follows: Sect. 2 introduces the data sources and methods used to perform the study. Section 3 presents a fundamental

mathematical analysis of developmental multiple fields. Section 4 presents the multifractal analysis. Section 6 presents the results and discusses them. Finally, Sect. 7 concludes the study and discusses future work.

## Data and Methods

### Data Sources and Study Regions

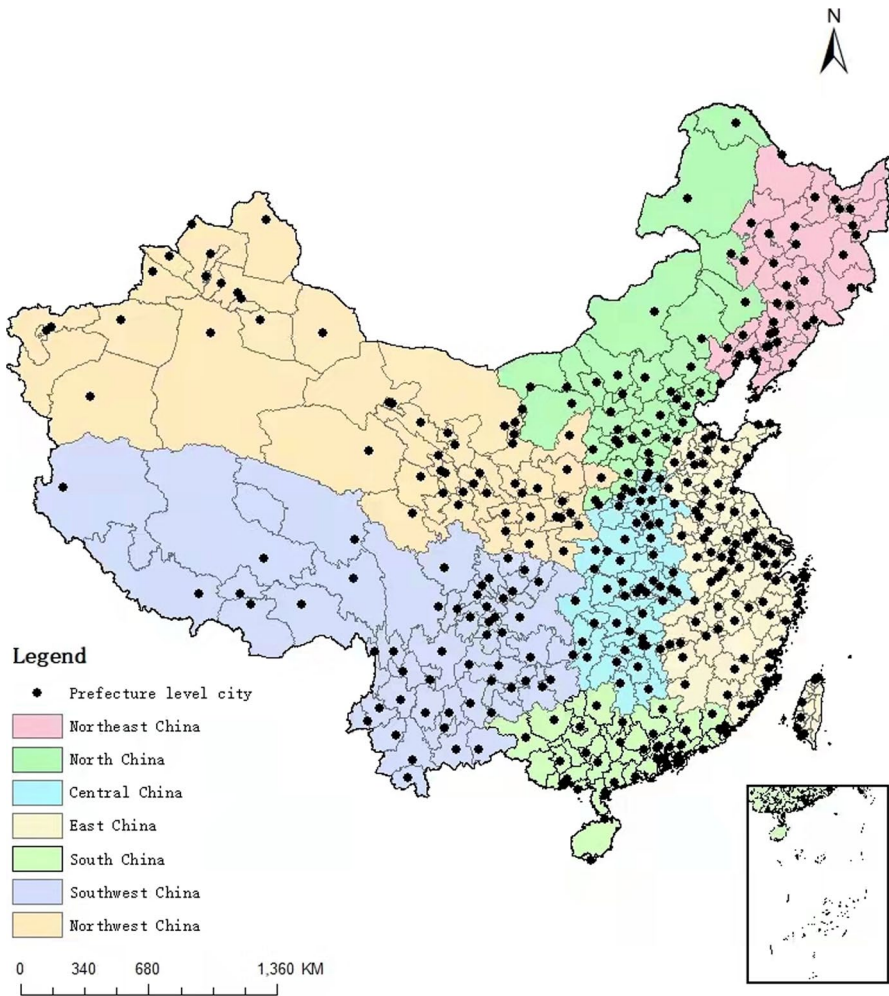
The data for this study were obtained from the China City Statistical Yearbook 2012 (Chinese National Bureau of Statistics, 2012). This data was chosen because China's economy was in a state of rapid and regular evolution in 2012, and the cross-sectional data for this year were more representative than that in other years. This quality of the data will facilitate multifractal detrended fluctuation analysis to describe the research objects in detail and further deepen the understanding of the above-mentioned salient features.

Strictly speaking, cities in the dataset are urban–rural regional complexes, that is, prefecture-level cities outside built-up areas. This database provides relatively detailed and accurate statistics on the socioeconomic development of 288 prefecture-level cities in 31 provincial administrative regions in mainland China. For this study, we chose 22 representative indices from the database as the objects of analysis. These indices include employment status, such as the number of people in the manufacturing, primary business, and service industries; social security status, such as the average wage of workers and number of unemployed urban residents; information on macrodevelopment, such as the total output value of large-scale industrial enterprises, gross domestic product (GDP), and per capita GDP; and social operating conditions, such as total social retail sales and fixed asset investments. Data from these fields could encapsulate China's economic development in the early 2010s across macro–meso–micro scales.

This paper aims to reflect the spatial evolution and overall characteristics of China's socioeconomic trends. Figure 1 shows the spatial layout of more than 300 prefecture-level cities in mainland China. However, western China is home to many deserts and plateaus and is characterised by a small population density. Thus, prefecture-level cities in western China are relatively less developed than those in other regions. Thus, many of these regions were not included in the related power-law and multifractal analyses. Most of the 288 prefecture-level cities in the 31 provincial administrative regions of mainland China in this analysis are located in China's eastern and middle regions.

### Research Methods

The development of each field in cities generally tends to increase or decrease, which drives regional spatial patterns and structural changes, in turn leading to regional hierarchical evolution. In terms of analytical methods, traditional mathematical analysis (especially power law) guarantees the discrimination of regularity



**Fig. 1** Sketch map of the spatial layout of prefecture-level cities in China

in regional development, but it cannot elucidate the complexity of evolutionary processes. However, complexity theory, represented by fractals, provides a powerful technical tool for deepening research into regional evolution and is widely applied in related research (Chen, 2008; Kantelhardt et al., 2002; Ihlen, 2012). The multifractal technique was developed at the end of the last century; it can use a spectral function to describe the evolutionary characteristics of different levels of a fractal system. Such an approach can also use spectrum width and spectrum truncation to comprehensively characterise a system. Specifically, the variation in the system can be described by a series of genealogical parameters. In this study, the development of all regions in prefecture-level cities appeared to exhibit both strengthening and weakening trends. Fractal dimensions are likely to be generated at different levels.

The multifractal characteristics lead to structural changes in multiple fields and drive spatial evolution. Therefore, multifractal characteristics could reveal spatial complexity in multiple fields as a result of the three processes in China and explain the evolutionary processes, as well as their corresponding resultant landscape differences in prefecture-level cities. In addition, various fields of socioeconomic development are nonstationary time-series systems with time-varying mean or variance, making the MDFA technique an ideal approach within the context of this study (Shin and Lee, 2003).

In practice, the space fractal formed by the complex regional space, certain land types, and certain socioeconomic attributes often exist in many forms, such as time series systems (such as the Mandelbrot set, Julia set, and Burning Ship fractal), spatial sequence systems (such as the Cantor set, Koch snowflake, Selpinsky triangle, and Peano curve), and attractors under iterative action (such as the Lorenz attractor). Current research on fractal dimension analysis, radial fractal analysis, and multifractal deterministic fluctuation analysis is based on spatial sequences and time series. Its purpose is to reveal phenomena such as a region's horizontal spatial expansion/shrinking and the hierarchical differentiation in vertical development attributes. (Song et al., 2021a; Ariza-Villaverde et al., 2013; Ihlen, 2012; Chen, 2008; Batty and Longley, 1994; Kantelhardt et al., 2002; Appleby, 1996). Correspondingly, it is different from the distribution of the 'physical density field' of land patches in two-dimensional space because the analysis of cross-sectional data mainly uses multifractal deterministic fluctuation analysis based on one-dimensional time series, becoming an important means to analysing the hierarchical evolution of a region. The multifractal characteristics of multi-domain hierarchies in prefecture-level cities, based on this method, can clearly reveal the spatial complexity of China's three processes and help explain the state processes and the emergent landscape differences in prefecture-level cities.

## Mathematical Analysis

### Mathematical Description of Development and Evolution in Multiple Fields

The basic function that describes correlation changes among multiple fields is the power function, which also serves as a basis for multifractal analysis (Ihlen, 2012; Chen, 2008; Kantelhardt et al., 2002; Lee, 1989). Hence, this study judges the spatial complexity of prefecture-level cities in China with an important foundation: whether such complexity fits the conditions of the power law.

In reality, the development of a country involves a variety of development fields, which influence and restrict each other and will generate many new and emergent characteristics through coupling; therefore, the China City Statistical Yearbook 2012 possesses hundreds of indicators. This study mainly selects indicators that are relevant within the context of industrial classification and adopts 22 commonly used dominant, basic, and typical indicators while ignoring most of the production and life detail indicators (such as the number of urban telephones and per capita green space area) and the number of students in primary and secondary schools).

The selection of the 22 indicators was mainly based on the commonly used indicators in the study of China's development in economics and economic geography. These indicators represent the main areas of China's development and reflect their basic development status. Based on the multifractal detrended fluctuation analysis (Ihlen, 2012, this study identifies and arranges **22 indices** of 288 prefecture-level cities in 31 provincial administrative regions in mainland China in successive descending order, as shown in Fig. 2 and Table 1, in which, the 22 indexes are those that have been changing the most and are, therefore, representative of the main areas of China's development. This study used cross-sectional data and represented a transient period of regional development. Because China's development has been extremely complex and rapid (not only is there a difference in the type of development, but there is also a dynamic change in each type), the amount of data in the middle- to long-term is too large, so this study focuses only on one year (i.e. 2012).

Then, all or part of the data were tested with various functions, and the findings are as follows:

- (1) The power function is used to fit the rank distribution of the 22 indicators, and the goodness of fit ( $R^2$ , column 6 of Table 1) of all indicators exceeded or was close to 0.95. Hence, the spatial regularity of socioeconomic activities in Chinese prefecture-level cities is universal and evident, leading to the spatial integration of multiple fields through prefecture-level cities, which are urban–rural regional complexes.

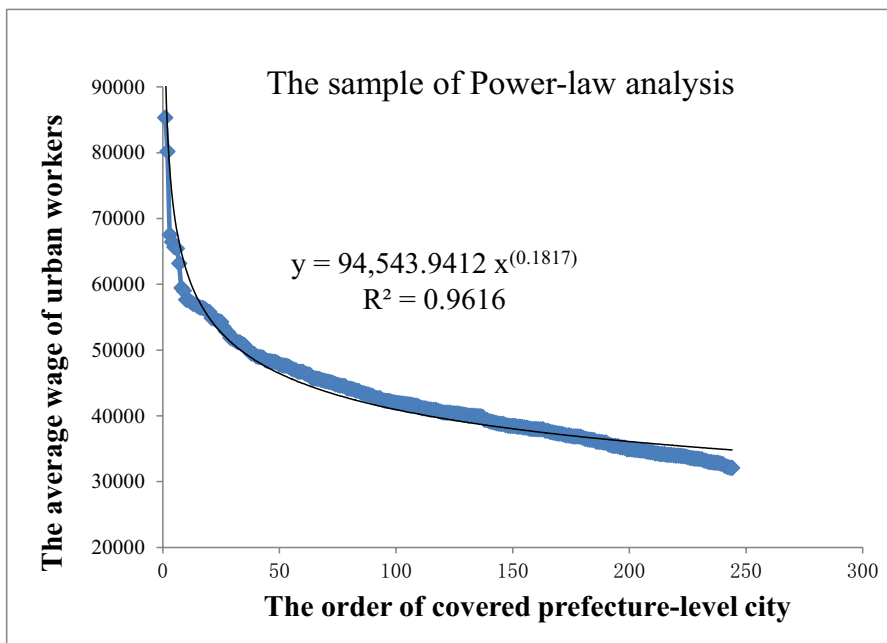


Fig. 2 Power-law analysis of the 31 provincial administrative regions

**Table 1** Mathematical description of 22 indexes from multiple fields in the context of Chinese prefecture-level cities

Number	Index	Total amount	The biggest scale range			The second scale range			Other scale ranges
			The order of covered prefecture-level city	Power exponent	Goodness-of-fit	The order of covered prefecture-level city	Power exponent	Goodness-of-fit	
1	The number of people in lodging and catering industry	3.3659 million	1–216	-1.1838	0.9900	●		▲	
2	The number of people in basic business and service industry	2.9889 million	1–198	-1.0953	0.9900	●		▲	
3	Real estate investment	7002.8 billion yuan	1–259	-1.0077	0.9503	●		Nil	
4	The number of employees in real estate	2.8201 million	1–196	-1.0023	0.9901	●		▲	
5	The number of scientific research personnel	3.2078 million	4–243	-0.9723	0.9699	●		▲	
6	The number of people in wholesale and retail industry	8.218 million	4–172	-0.9530	0.9930	●		▲	
7	The number of people in transportation industry	6.0091 million	1–205	-0.9331	0.9918	206–278	-0.3156	0.9794 Nil	



Table 1 (continued)

Number	Index	Total amount	The biggest scale range			The second scale range			Other scale ranges
			The order of covered prefecture-level city	Power exponent	Goodness-of-fit	The order of covered prefecture-level city	Power exponent	Goodness-of-fit	
8	The number of insured people in urban basic endowment insurance	248.47 million	1-259	-0.8825	0.9532	●		Nil	
9	The total output value of large-scale industrial enterprises	90,289.1 billion yuan	1-231	-0.8338	0.9502	●		▲	
10	Building area	35,634 Km <sup>2</sup>	1-286	-0.8146	0.9510	●		Nil	
11	The total amount of social consumer goods	20,479.1 billion yuan	1-230	-0.8060	0.9635	231-284	-5.2425	0.9563 Nil	
12	The profits of large-scale industrial enterprises	6122.5 billion yuan	1-181	-0.8000	0.9500	●		▲	
13	The number of people in manufacturing industry	43,241 million	22-163	-0.7403	0.9533	1-21	-0.5477	0.9617 ▲	
14	GDP	57,404.9 billion yuan	1-208	-0.7284	0.9786	209-287	-0.2516	0.9400 Nil	

Table 1 (continued)

Number	Index	Total amount	The biggest scale range			The second scale range			Other scale ranges
			The order of covered prefecture-level city	Power exponent	Goodness-of-fit	The order of covered prefecture-level city	Power exponent	Goodness-of-fit	
15	The employment situation of urban labors	151.61 million	2–254	-0.7180	0.9770	●			Nil
16	The number of employees in financial industry	5.1074 million	1–221	-0.7108	0.9903	222–281	-0.2789	0.9577	Nil
17	Fixed-asset investment	34,955.8 billion yuan	1–261	-0.7102	0.9498	●			Nil
18	The number of people in information industry like computer	2.339 million	5–218	-0.6992	0.9948	219–271	-0.3565	0.9524	Nil
19	The size of urban population	403.2 million	2–250	-0.6759	0.9678	251–281	-4.5916	0.9754	Nil
20	The number of employees in construction industry	20.3387 million	25–185	-0.5490	0.9206	1–24	-0.4035	0.9695	◆
21	Per capita GDP	Vacancy	1–180	-0.4030	0.9501	●			▲
22	The average wage of urban workers	Vacancy	1–257	-0.1873	0.9507	●			Nil

●: Do not conform to the requirements of the power law. ▲: The fitting effect is not ideal. ◆: Have the third scale range (192–282)

- (2) The fitting range of the data from the different fields exhibited significant distinctions. All the data on some indices (such as total wages of urban workers) exhibit evident regularity, whereas the different data segments of some indexes (such as the number of people in the manufacturing industry) exhibit different evolutionary rules. Some data segments did not exhibit any regularity. Overall, most of the first 200 data items from the 22 indices show evident regularity, which suggests that prefecture-level cities from the upper and middle levels possess strong regularity. By contrast, data from the lowest level (such as prefecture-level cities in western China) do not show marked regularity due to their secondary position.
- (3) Different power exponents could reveal evolvement intensity in different fields, which interweave and influence one another in certain regions, together forming a regional development system.

Shaped by the three processes, the data show that the spatial evolution of regularity in various regions in China is diverse. Concretely speaking, as a result of the adaptability of the system and government regulations, the spatial layout of Chinese prefecture-level cities does not appear to follow the ideal single power law but is likely characterised by a regional autocorrelation evolution dominated by various power laws (Liang et al., 2021; Yang et al., 2019; Duan et al., 2017; He and Wu, 2007).

### Basic Law of Evolution in Multiple Fields

After arranging all the indices in descending order, their power functions become decreasing functions in the first quadrant and their related curves include relatively fitting data (shown in the fourth row in Table 1). The analysis is as follows:

- (1) Microactivities differed significantly. Large to medium and small cities are represented by the number of people in the lodging and catering industries (the power exponent is  $-1.1838$ ), and those in the basic business and service industry ( $-1.0953$ ) are mainly driven by the market. The difference in the development of the traditional business and service industry is relatively large, followed by indices in real estate (real estate investment,  $-1.0077$ ; number of employees in real estate,  $-1.0023$ ), whereas real estate was one of the hot economic activities in the early 2010s. These outcomes represent the primary developmental trend in Chinese prefecture-level cities.
- (2) Some indices show remarkable evidence of government regulation. Among the 22 indices, those with low-power exponents exhibit huge practical differences, whereas those with high-power exponents reveal evident macro characteristics. The indices reflecting macro development, such as the average wage of urban workers ( $-0.1873$ ) and per capita GDP ( $-0.4030$ ), are usually related to government regulations, such as industrial policies. In contrast, the scaling indices, such as the GDP of a prefecture-level city ( $-0.7284$ ) and the total amount of

- social consumer goods ( $-0.8060$ ), are ranked at the average level among the 22 indices.
- (3) The data distribution of many indexes features wide scale ranges (i.e. different data segments indicate other power laws) but is primarily based on two-scale ranges. Hence, related social and economic activities among prefecture-level cities are multifractal. In addition, except for the data segments with the widest scale ranges, the other data segments in some indices are difficult to fit, hence the row ‘the fitting effect is not ideal’ in Table 1. Although almost all the data in some indexes show regularity, they do not have other scale ranges (as shown in the last row ‘nil’ of Table 1).
  - (4) The agglomeration of large- and medium-sized cities is greater than that of other cities, whereas the hierarchy of medium- and small-sized cities is remarkably declining. Regarding the agglomeration, although some indexes (such as the number of scientific research personnel and number of people in the information industry) cover too few cities and cannot be considered a scale range, they can be regarded as a set of representative cities within the scale range (mainly consisting of large cities, such as Beijing, Shanghai, Guangzhou, and Xi’an). The hierarchy (the eighth line in Table 1) carries a high overall power exponent (except for indices such as the total amount of social consumer goods and the size of the urban population). This result suggests that indices related to medium- and small-sized cities do not exhibit considerable differences. Similarly, the existence of several scale ranges has become an important indicator of the multifractal characteristics of multiple fields in prefecture-level cities.

## Multifractal Analysis

### Process of Multifractal Analysis

To unveil the state characteristics and mechanisms underlying the evolution of multiple developmental fields in the 2012 Chinese context, the basic unit of which is a prefecture-level city, a multifractal analysis was conducted via MATLAB using the data of the 22 indices arranged in descending order (Ihlen, 2012; Kantelhardt et al., 2002). The analysis of the multifractal dimension spectrum  $h_q$ , singularity exponent  $H_q$ , generalised fractal dimension value  $D_q$ , and mass exponent  $\tau_q$  is presented in Fig. 2. In the summary of Fig. 2, 8 indices at the macro–meso–micro scale present multifractal characteristics in the scale range shown in Table 1 and within certain data segments (from the second to the third line in Table 2) after conducting related tests. The other 14 indices do not present multifractal characteristics in the entire dataset and within any scale range. Therefore, their analysis is not shown in Fig. 2. As presented in the analysis in the section on ‘mathematical analysis’, although regularity changes in these 14 fields with the indices above, they are generally in chaos or coupling processes.

Specifically, in Eq. (1), the generalised fractal dimension  $D_q$  can be stated in Eq. (1), in which  $q$  denotes the order of moment ( $q = \dots, -2, -1, 0, 1, 2, \dots$ ), and we

**Table 2** The main characteristics of indexes with multifractal characteristics

Index	Data range	Whether fits with the Table 1	Spectrum width	Spectrum truncation	D <sub>0</sub>	D <sub>1</sub>	D <sub>2</sub>
The average wage of urban workers	1–257	Yes	1.5785	2.1600	1.000003	0.6771	-0.1659
The total output value of large-scale industrial enterprises	1–231	Yes	1.8744	2.2141	1	0.4745	0.0223
The number of people in manufacturing industry	1–163*	Yes	3.8361	2.4280	1.000007	0.4592	0.0906
GDP	1–208	Yes	2.1259	2.2484	1.000001	0.4467	0.0109
The employed people of social labors	2–254	Yes	3.5144	2.2778	1.000002	0.3193	-0.0437
The size of urban population	2–250	Yes	1.8939	2.1858	1.000001	0.3098	-0.0604
The building area that over 50Km <sup>2</sup>	1–205	No	5.1151	2.4689	1.000013	0.4800	0.0547
The number of people in basic business and service industry	1–109	No	1.9316	2.1104	1	0.7004	0.1635

Notes: The prefecture-level cities covered by the indexes' multifractal of the number of people in manufacturing industry include the biggest and second scale range, while others are the biggest scale ranges

divide the phase space into  $N(s)$  hypercubes of edge  $s$ . The probability of finding a point in the  $i$ th hypercube is denoted by  $P_i$ .

$$D_q = \frac{1}{q-1} \lim_{s \rightarrow 0} \frac{\log \sum_{i=1}^{N(s)} P_i(s)^q}{\log s} = -\lim_{s \rightarrow 0} \frac{I_q(s)}{\log s} \quad (1)$$

In Eq. (2),  $\tau_q$  refers to the mass exponent of order  $q$ , and  $D_q$  and  $\tau_q$  comprise the set of global parameters of the multifractal sets.

$$\tau_q = (q-1)D_q \quad (2)$$

We can transform the global parameters  $\tau_q$  and  $D_q$  into local parameters  $H_q$  and  $h_q$ , as shown in Eqs. (3) and (4).

$$H_q = D_q + (q-1) \frac{dD_q}{dq} \quad (3)$$

$$h_q = qH_q - (q-1)D_q \quad (4)$$

The specific analysis shows that the comprehensive index (the scale of GDP) and micro–macro-scale indices present significant multifractal characteristics. The meso-scale index (built-up area) presents multifractal characteristics using normalised data (thus, the analysis process is not shown in Fig. 4-2). Table 2 lists the capacity dimension ( $D_0$ ), information dimension ( $D_1$ ), and correlation dimension ( $D_2$ ) of the four indices. The capacity dimensions ( $D_0$ ) are all approximately equal to one, which shows that this study corresponds to the analysis of one-dimensional data fluctuations.

### Parameter Analysis of Multifractal Characteristics

Based on meeting the power-law characteristics, this study conducts multifractal analysis on the broadest scale range of a prefecture-level city (the fourth and seventh lines in Table 1), as stated in Sect. 3.1. The study chose other data segments to test according to the power law if the entire scale range did not exhibit multifractal characteristics. The study found that prefecture-level cities with over 50 km<sup>2</sup> building area (1–205) and cities with a certain number of people in the primary business and service industry (1–109) exhibit multifractal characteristics. Using MATLAB software, we obtained the spectrum width, spectrum truncation, and representative dimension ( $D_q$ ) of the multifractal spectrum of the eight indices (Table 2).  $D_1$  is equal to 1, which suggests that changes in all the indices of the Chinese prefecture-level cities are one-dimensional, which corresponds to the level differentiation of each indicator. The hierarchical evolution of multifractal systems is described by the corresponding mathematical changes in the  $q$ -values of the above parameters, and this level of specificity is the hierarchical evolution of multidomains in China. By summarising the above-mentioned analysis process and various parameters, we deduce the following:

- (1) The multifractal spectrum of many indices ( $h_q$ , the lower right corner of every drawing in Fig. 4) is not a regular parabola (such as the number of people in the manufacturing industry and building area, as can be seen in Fig. 3-4 and 2). This indicates that the characteristics of practical development are highly complicated and do not appear regularly in the multifractal spectrum.
- (2) Spectrum width ( $h_{qmax} - h_{qmin}$ ) and spectrum truncation of each index multifractal

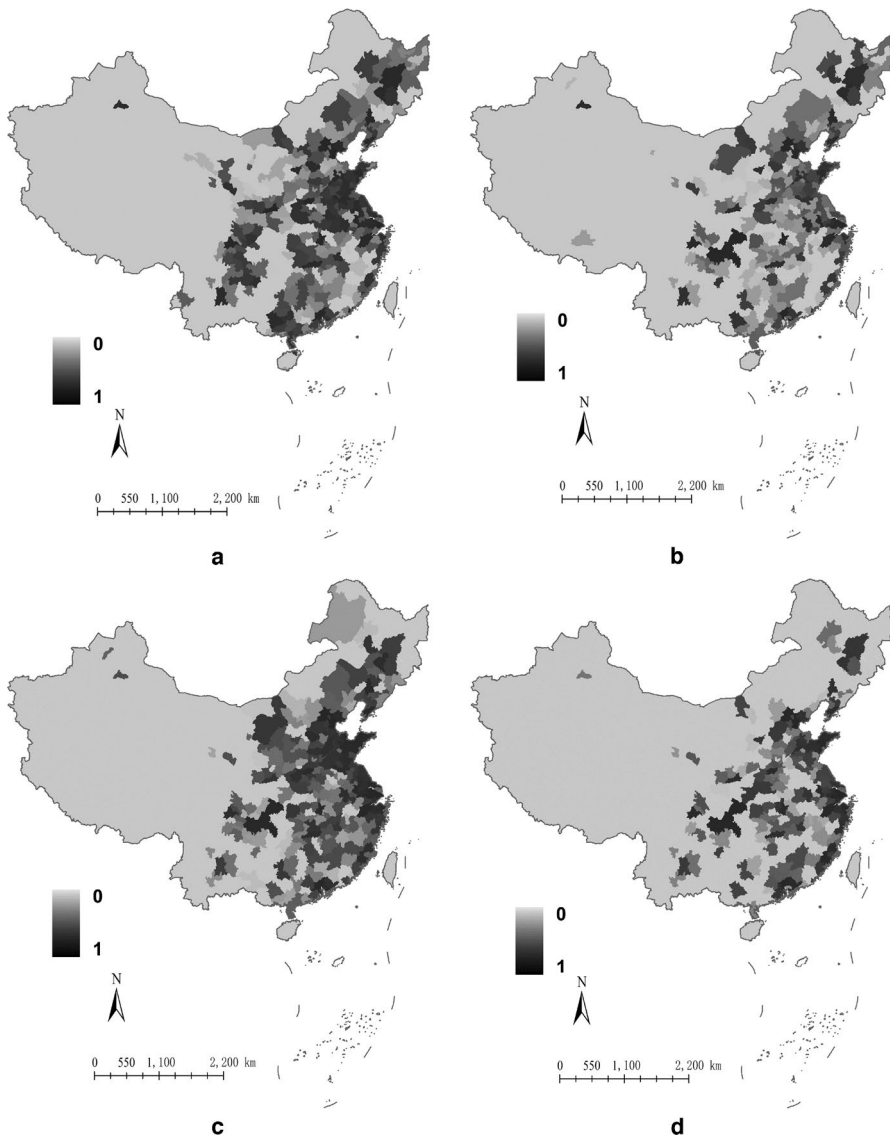
Notes: The prefecture-level cities covered by the indice multifractal of the number of people in the manufacturing industry include the largest and second-largest scale ranges, while others are the largest scale ranges.

spectrum shows the complexity of the index evolution and data volatility (this study standardises all indices to achieve a convenient comparison of the multifractal spectra of the eight indexes).

- (3) Data with fractal characteristics in multiple fields include two forms: downward radiation from top cities and agglomeration from entirety to medium–small cities. The former can be observed in the radiation from the basic business and service industries in big cities to those in small cities. The latter can be observed in the state's inclination to set worker wages in medium and small cities to narrow the income gap with big cities. Thus, the evolution of socioeconomic activities is a complicated system that combines self-organisation (market-driven) and hetero-organisation (government regulations).
- (4) Three conditions are identified in the existential state of each index. **First**, the eight indices show multifractal characteristics in related ranges of prefecture-level cities and could be used as the foundation to explain the development and evolution of associated fields. **Second**, if we leave out the limit of every scale range in Table 1, some data segments in the other 14 indices could show multifractal characteristics. However, their power function characteristics are no longer evident or may change (e.g., a change in the exponential function). A relevant analysis was not conducted in this work owing to the heavy workload it entails and the immature analysis techniques available. **Third**, most fields meet the requirements of the power law; however, they do not exhibit multifractal characteristics and are still in the chaos or coupling state of evolution. Thus, the three conditions become intertwined and represent a detailed state of China's development at the macro–meso–micro scale.

## Developmental characteristics of multiple fields in prefecture-level cities

To analyse the development state of 288 prefecture-level cities in the state of **prefecture-level cities in China**, this study divides the 22 indices into five categories. This categorisation aims to reflect the characteristics of, and changes in, the related social and economic fields from different perspectives.



**Fig. 3** Spatial layout map of 8 indices of Chinese prefecture-level cities based on coarse-graining processing (In the map, the deeper the colour, the higher the level of the prefecture-level city). **a.** Urban spatial layout of urban population size. **b.** Urban spatial layout of building area. **c.** Urban spatial layout of large-scale. **d.** Urban spatial layout of employed people in manufacturing industry enterprises' total output value. **e.** Urban spatial layout of people in basic business. **f.** Urban spatial layout of urban employed and service industry people. **g.** Urban spatial layout of GDP. **h.** Urban spatial layout of urban workers' average wage



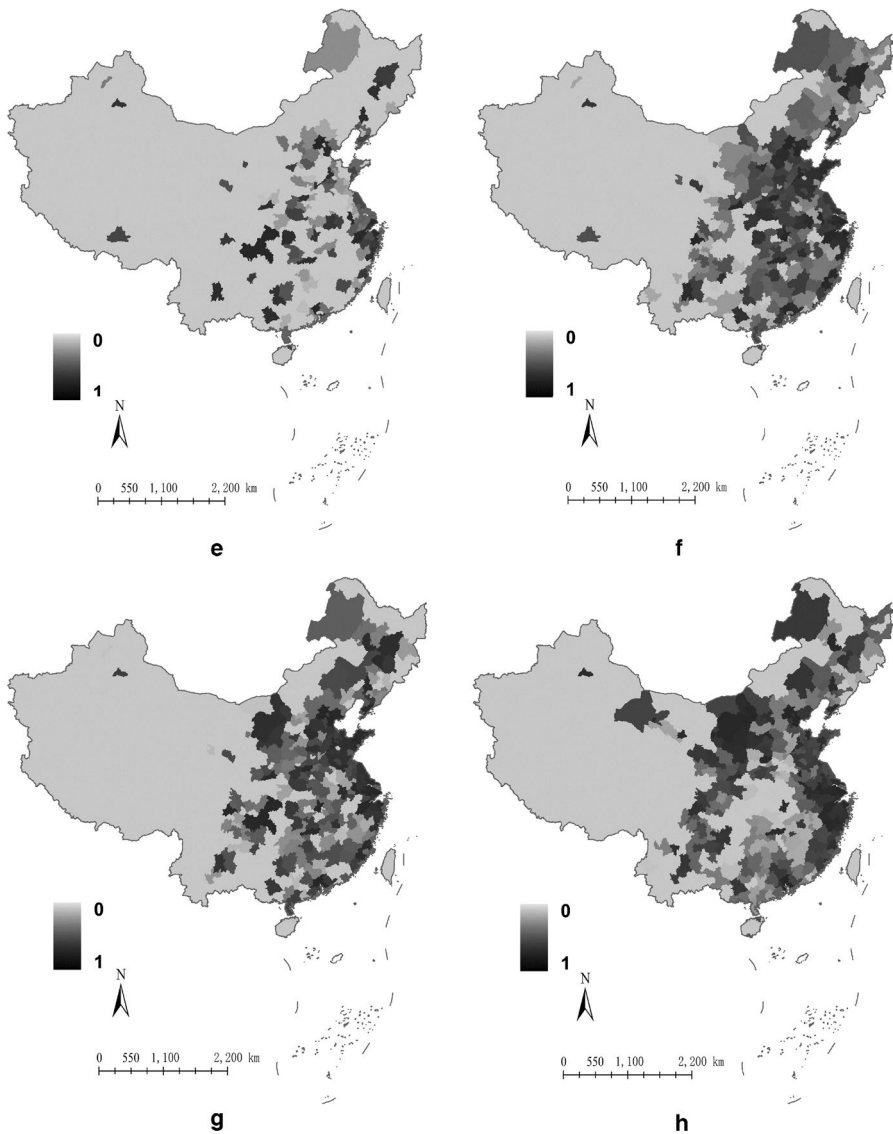


Fig. 3 (continued)

### Basic Indices

This index type includes aggregate indices (size of urban population, building area, and fixed asset investment) and background indices (number of insured people under urban basic endowment insurance and number of employed urban labourers). Analysing basic indices of Chinese prefecture-level cities deepens our understanding of the development and evolution of other fields.

## (1) Aggregate index

The layout of urban populations is a typical research object for analysing the regional hierarchy system (Ihlen, 2012; Chen, 2008; Kantelhardt et al., 2002). According to the above analyses, Chinese prefecture-level cities meet the power-law and multifractal conditions. From the 2nd to 250th prefecture-level cities (the first city, Chongqing, comprises considerably large areas and is thus excluded), the spectrum width of the indexes' multifractal characteristics is 1.8939 (as seen in the right box of the small graph in Fig. 4-1, the same as that provided below). This value **indicates** that the indices rank 6th among the eight indices with multifractal characteristics and medium-level complexity. Meanwhile, the left spectrum truncation (2.1858) ranks sixth, which indicates the weak volatility of the data on the size of the urban population. Furthermore, this index reflects the typical distribution of carriers (people) in various activities, including complexity and volatility. Complexity and volatility reflect the typicality of all types of activity carriers (people), which usually become the basis of the evolutionary trajectories of other indices. As indicated by the spatial distribution (Fig. 4-1), prefecture-level cities with high urban populations are concentrated in municipalities (Beijing, Shanghai, and Tianjin) and provinces (especially Jiangsu, Shandong, and Guangdong provinces) in eastern and central China but rarely in western China.

Building area is a standard index used to analyse the urban hierarchy system in economic geography (Ihlen, 2012; Chen, 2008; Kantelhardt et al., 2002). The power function fitting 286 prefecture-level cities in mainland China (missing two data items) is good, which indicates strong overall regularity. This study further chooses the first 205 prefecture-level cities with areas over 50 km<sup>2</sup> to conduct a multifractal analysis (Fig. 4-2), which shows a spectrum width of 5.1151 and a right spectrum truncation of 2.4689 and which belong to the indices with the strongest complexity and volatility among the eight indices. This index proves the entry of the Chinese development to the 'overurbanisation' stage. Further, it clarifies the bubble in real-estate indices (i.e. the amount of real-estate investment and the number of employed people in the real estate and construction industry). Multifractal characteristics can reflect the bubble characteristics of varying degrees among prefecture-level cities. Compared with the rank of the urban population size, the rank of the building area of prefecture-level cities is used to show that this rank status of rising medium- and small-sized prefecture-level cities in the whole nation and its chaos relative to other indexes.

Considering the two typical indices, the value of urban population size declines slowly in accordance with rank (the scale range is wide). Hence, this study refers to it as a typical index with a relatively generalised trend in terms of rank. In contrast, the value of building area declined quickly. The study refers to it as a typical index with a nongeneralised trend in terms of data (as seen in the comparison in Sect. 5.6, rank generalisation is not equal to spatial homogenisation and nongeneralisation does not equate to spatial agglomeration). According to the fifth line in Table 1, on a national scale, many indexes lean toward nongeneralised evolution, whereas few indexes lean toward generalised evolution.

To visualise spatial differences within the context of a macroscale (Fig. 3), we drew a spatial layout map of the eight indices with the goal of analysing this spatial difference from the micro- and mesoperspectives, as discussed above. Therefore, this paper presents a schematic of the spatial pattern of the eight indices with multifractal characteristics, to analyse this spatial difference more intuitively (Fig. 3). For example,

- Fig. 3-a and h show that prefecture-level cities with multifractal characteristics of urban population size and average employee salary occupy a larger space and have more high-level cities, and their mediocre trends can be graphically and intuitively displayed. On the contrary:
- Fig. 3-d and e show that prefecture-level cities with multifractal characteristics of the number of manufacturing workers and the number of basic commercial service workers are smaller in scope and fewer in high-level cities, and the trend of nonbanality is apparent.

This verifies the power-law characteristics of the different developmental fields described in Sect. 3.1 and graphically reflects the difference in the evolution intensity of the different developmental fields described in Sect. 3.2.

The size of a fixed asset investment is an important index that reflects the existing and future development of a prefecture-level city. According to the analysis of this index, the goodness-of-fit of the power function of the first 261 prefecture-level cities meets the requirements but does not indicate multifractal characteristics. In other words, although the evolution of this index is regular and complicated, it has no autocorrelation characteristics (thus, this study does not draw a corresponding spatial layout map, as shown below) in the chaotic or chaotic development state. However, from the ranking of fixed asset investment, its overall development tends to be generalised and is most intensive in the coastal areas of northern and central China (Bohai Rim and Yangtze River Delta).

## (2) Background Indices

The number of insured persons under urban basic endowment insurance represents the basic protection for residents. We passed the power function fitting of the first 259 prefecture-level cities in the index, but the result did not show multifractal characteristics. In practice, these multifractal characteristics should be the result of the comprehensive influence of multiple fields, and the overall development of the field should be nongeneralised. Specifically, except for Chongqing, the power exponent of the urban labour employment data from the 2<sup>nd</sup> to the 254<sup>th</sup> year is  $-0.718$ , which suggests that the change in the number of employed people is smaller than that in the number of insured persons under basic urban endowment insurance (the power exponent is  $-0.8825$ ). This phenomenon could be explained as follows. Given that the basic amount of national endowment insurance is uniform, prefecture-level cities with a small number of employed people tend to not pay endowment insurance, thus increasing the difference in this background index among prefecture-level cities. Because of the influence of the entanglement between the abovementioned indices,

it does not prevent multifractal characteristics for the index of the number of insured people in prefecture-level cities.

In summary, the overall development background and basis of the early years of the 2010s in Chinese prefecture-level cities are diversified. However, the two important indices (size of urban population and building area) are still very typical (including the typicality of real estate bubbles).

## Secondary Industry

Chinese farming and rural areas have long been in inferior positions, and their evolution has been passive. Meanwhile, there are few indices concerning rural areas in the database, and the correlation between rural landscapes and the development of prefecture-level cities is high. Therefore, this study lacks a comprehensive analysis of the development and evolution of the primary industry.

For the secondary industry, the Chinese industrial structure system is integrated and has a large scale, making China the 'world's processing plant'. However, the level of development of the Chinese secondary industry, as well its prefecture-level cities, show considerable differences. The related conditions can be reflected on the total output value of large-scale enterprises, number of people in the manufacturing industry, profit for large-scale enterprises, and other corresponding indices (real-estate investment and number of employed people in the construction industry).

## Development and evolution of the industry

The total output value of large-scale enterprises can reflect the scale difference of all prefecture-level cities, with the scale of the Yangtze River Delta and Pearl River Delta being the largest. According to Tables 1 and 2, the power exponent value of the first 231 prefecture-level cities of the index is at the middle level; that is, it is within the average level in terms of the differences among prefecture-level cities. This index exhibits multifractal characteristics (Fig. 4-3), a spectrum width of 1.8744 (ranked seventh), and left spectrum truncation of 2.2141. This development field exhibits different evolutionary regularities in prefecture-level cities with varying ranks, but their complexity and volatility are not considered high. For eastern China (some cities in eastern China are mining cities), the multifractal characteristics of the index of their prefecture-level cities are similar to those of their urban population size (see Fig. 3-a and c). Nevertheless, unlike the ranking of the urban population, the top ranking of prefecture-level cities in terms of the total output of large-scale enterprises is wide in eastern and central China. In other words, the essence of the Chinese urban economy lies in the eastern coastal areas, but important industrial areas can be found in eastern and central China.

The above analysis was performed from the perspective of the enterprises. From the perspective of practitioners, the difference in the number of employed people in the manufacturing industry in prefecture-level cities is likewise unsatisfactory and can be divided into two scale ranges (Table 1). (1) The first 21 prefecture-level cities

(second scale range) mainly include large-scale industrial cities in eastern and central China (such as Shenzhen, Suzhou, Chengdu, and Zhengzhou) and some metropolises (such as Shanghai and Tianjin). Compared with other indices, the difference in the number of employed people at this scale was not significant. (2) The 22<sup>nd</sup> to 163<sup>rd</sup> prefecture-level cities (most scale range) are mostly medium cities whose positions as industrial cities have been significantly reduced (see details in the difference in colour in Fig. 3-d). The power exponent value for most of the scale range was smaller than that for the second scale range. However, considering the multifractal characteristics for the 1<sup>st</sup> to 163<sup>rd</sup> prefecture-level cities (Fig. 4-4), the spectrum width (3.8361) ranks second, and the spectrum truncation (2.4280) is irregular. This result indicates that the scale layout of the Chinese manufacturing industry is complicated and differs from that of the urban population. It is even different from the distribution of the total output value of large-scale enterprises among prefecture-level cities (comparing Fig. 3-c and d, the latter appears as a large area of low-value zones in the west). At the prefecture-level city level, the three aforementioned indices also show apparent spatial dissymmetry, which could be the result of the difference between the scale layout of a single manufacturing industry and the layout of the overall social economy and the industry and enterprise profit among prefecture-level cities.

However, the profits of large-scale enterprises best illustrate the efficiency of industrial development among prefecture-level cities. According to Table 1, the profit of the first 181 prefecture-level cities (the largest scale range) is characteristic of power law, which is the overall reflection of the economic benefits of large- and state-owned enterprises. In addition, the power exponents of these prefecture-level cities are roughly equal to those of the two aforementioned indices. Their changes are all within the average level and do not appear markedly generalised or even non-generalised; thus, their scale range is acceptable. Nevertheless, this performance index (i.e. the difference in industrial and corporate benefits among prefecture-level cities) does not show multifractal characteristics, suggesting that the development of enterprises is in an adjusting or coupling state.

The three indices are indicative of the industrial layout of Chinese prefecture-level cities. Without considering the spatial complexity and volatility displayed by the multifractal dimension spectrum and considering the prefecture-level city rank of the power function shown in Fig. 3, we obtained the widely accepted conclusion, that is, the three indices generally rank first in the Yangtze River Delta, Pearl River Delta, and prefecture-level cities of Shandong province. This ranking indicates that the coastal areas in eastern China have a concentrated distribution in practice. According to the above analysis based on the power-law and multifractal characteristics, we observed the following. (1) In these three fields, the industrial layout of all prefecture-level cities shows regular changes and generates an autocorrelation action in different fields as well as mutual spatial dissymmetry. (2) The organised, disorganised, and chaotic evolution in multiple fields jointly shaped China's developmental trends in the early 2010s, influencing its industrial development. (3) The Shanghai–Suzhou area may be the largest industrial centre in China, but its industrial output does not indicate absolute superiority. This suggests that the top industrial zone in China is not isolated but coexists with many industrial zones at the

same level. (4) Although many urbanised prefecture-level cities (e.g. Beijing) are important economic centres, they are not industrial centres. Thus, we can deduce that industrial development may not be the driving force of Chinese development but only plays the role of a pillar industry. However, other factors may affect and complicate the spatial state of Chinese industry.

## Development and evolution of secondary industries

This study classifies the real estate investment index into secondary industry because several original industrial and mining warehouse lands are being converted into residential lands in China, thus further involving some enterprises. Moreover, this field is prone to economic bubbles. The power exponent of the first 259 prefecture-level cities of this index reaches  $-1.0077$ , but it does not exhibit multifractal characteristics. Presumably, this type of investment mainly centred on large and medium cities, and gradually spread to medium and small cities. However, they generally exhibit chaotic spatial activity. For example, compared with the urban population size rank, the bottom rank of Guizhou Province in inland China and Hainan Province in southern China indicates strong real estate investment. Thus, the layout of real estate investment is incompatible with the national industrial layout and the overall development of major fields.

In this context, the first scale range (1–24) of the index of the number of employed people in the construction industry mainly consists of developed prefecture-level cities, but also partly includes several prefecture-level cities with an evident bubble economy. Compared with the former, the power exponent of the second scale range (25–185) declined rapidly and tended to be normal. Compared with the other indices, it has poorer regularity ( $R^2=0.9206$ ). The index also has third and fourth scale ranges, but their goodness-of-fit does not meet the requirements. In summary, as influenced by real-estate investment, the development of the construction industry demonstrates polarisation and a multilayered state but is overall in a chaotic state.

Comprehensively viewing the development of the Chinese secondary industry, we find that the traditional secondary industry, represented by the manufacturing industry, forms a generalised spatial structure with its own development characteristics. However, its development still involves coupling and adjustment processes. Nonetheless, related activities centred on the construction industry are generally in a chaotic and nongeneralised state.

## Tertiary industry

### (1) Development and evolution of the traditional business and service industries

The distribution of the number of people in the basic business and service industry among prefecture-level cities shows power-law characteristics; this non-generalised

phenomenon is also highly evident (Table 1). The power exponent of the first 198 prefecture-level cities (first-scale range) was  $-1.0953$ . According to this index's prefecture-level city rank, its data drops rapidly from the rank of a metropolis to that of medium and small cities and shows little difference below the medium city level. Table 1 further suggests that its non-generalized evolution is even stronger than the evolution of the prefecture-level city's building area. According to further analysis, data from all and the first 198 prefecture-level cities do not display multifractal characteristics, indicating that, for most Chinese prefecture-level cities, their development is relatively chaotic. However, the number of people in the basic business and service industries of the first 109 prefecture-level cities displays multifractal characteristics (Fig. 4-5), a spectrum width of 1.9316 (ranks fifth), and medium-level complexity. At the same time, it has a weaker left-spectrum truncation (2.1104), which suggests that it is the least fluctuant index among the eight indices with multifractal characteristics. This is congruent with characteristics that indicate that the micro activity at the primary level is relatively stable. According to Fig. 3-e, the size and development of the primary business and service industry in the early 2010s seem to be stable among high-level prefecture-level cities. In other words, this activity forms relatively normal autocorrelation with multifractal characteristics, but its development is highly evident in the rank diffusion of large and medium cities. The  $D_I$  value (0.7004) of this index is the largest, further suggesting that, in large cities, the employment situation of the basic business and service industry tends to be significant and could attract additional employees within and out of the city.

The first scale range of the number of people in the lodging and catering industry included the first 216 prefecture-level cities. Its power exponent is  $-1.1838$ , which is the strongest index with non-generalized evolution among the prefecture-level cities. However, this index does not exhibit multifractal characteristics. As seen in the spatial layout, the index's average value is high in Hainan, Guangdong, and Zhejiang provinces in eastern and southern China, where business, service, and light industries are relatively developed. However, the functions of these departments reduce rapidly from metropolises to medium and small cities. Particularly in small cities, the influence of the lodging and catering industry on prefecture-level cities is minimal.

Compared with the above two indices, in terms of the number of people in the wholesale and retail industries, the power exponent of the largest scale range's prefecture-level city (excluding the 4th–172nd after Chongqing, Beijing, and Shanghai) increases to  $-0.9530$  and does not display multifractal characteristics. This result is similar to those pertaining to the number of people in lodging and catering industries. Employed people in the financing, transportation, and real-estate industries face similar contexts. Overall, although these economic activities have certain complexities, they are generally in a chaotic or coupled state. In particular, the power exponent of the number of employed people in the real estate industry is  $-1.0023$ , which suggests a huge difference among prefecture-level cities.

## (2) Total number of social consumer goods

The largest scale range of the index for prefecture-level cities (from the 1st to the 230th) represents prominent characteristics related to the power law. Its power exponent ( $-0.8060$ ) ranks at the medium level, reflecting the tertiary industry's comprehensive development in the prefecture-level city. According to the analysis, this index did not exhibit multifractal characteristics. As presented in the analysis in Sect. 5.3 (1), the power exponent of the number of employed people in the lodging, catering, basic business and service, and real estate industries is less than  $-1$ , which suggests that the total amounts of social consumer goods in all prefecture-level cities are in a chaotic development state (except for several indices).

Summarising the development of the tertiary industry, we can deduce that this type of activity, mainly driven by the market, tends to exhibit nongeneralised development. This difference is generally complicated; although some indices form multifractal autocorrelation characteristics, most of the indices are still in the spatial-unordered or coupling state.

Comparing the number of employed people in all departments of the secondary and tertiary industries, we can see the following: On the one hand, the number of employed people in the manufacturing industry (43.24 million) and construction industry (20.34 million) exceeds 20 million; in fact, the construction industry with the characteristics of a bubble economy has become the second largest industry in terms of employment. On the other hand, the typical wholesale and retail industry (8.22 million), transportation industry (6.01 million), and lodging and catering industry (3.37 million) based on the individual and private economy have suddenly emerged as new forces and have thus become the 'big reservoir' of labourers and new growth points since the reform and opening up. They develop rapidly and exhibit a certain regularity but do not generally show ordered and layered multifractal characteristics. This type of phenomenon represented by the tertiary industry is notably the normal state and mainstream of Chinese microactivities. However, the same could be said for other matters; as a basic-level employment channel with low prospective earnings, the basic business and service industry (2.99 million employees) exhibits not only regularity and medium-level complexity, but also multifractal characteristics. In practice, compared with the chaos indices, the multifractal characteristics of the number of employed people in the basic business, service, and manufacturing industries tend to indicate a certain stable state of Chinese prefecture-level cities in the development process. Therefore, this subject deserves further analysis.

The 2nd to the 254th employed urban population (except Chongqing) from various industries all show multifractal characteristics (Fig. 4-5). The spectrum width (3.5144) indicates its strong complexity in general situations and suggests that the employed population is further inclined to enter prefecture-level cities and industries with the goal of profit maximisation. Its irregular spectrum (2.2778) indicates that the number of employed people generates medium-level volatility among the different prefecture-level cities. In fact, the practical employment situation reflects practical and profit-seeking economic situations, but the government's macroregulation (Sect. 5.3 (1)) is a slow process in an inferior position. According to Fig. 3-f, the employed population of all prefecture-level cities is similar to the spatial layout



of the urban population (Fig. 3-a), and their  $D_j$  values (the seventh line in Table (2)) are extremely close. The correlation between them is remarkable, but the complexity of the employed urban population is more evident, considering its influence on additional social factors. Furthermore, the  $D_j$  value of the employed urban population is small (0.3193), which means that the employment situation is weak and the prospect is a cause for concern in medium and small cities.

## Quaternary Industry

The quaternary industry is a new technological industry. This study not only refers to the number of people in the information industry (e.g., computer evaluation index) but also considers the number of scientific personnel, which traditionally belong to the tertiary industry, because it is an important sign of the connotative development of a prefecture-level city.

The number of people in the information industry, such as the computer industry, represents the technological capacity of a prefecture-level city. From the perspective of the power law, differences exist among research-and-development cities, popularisation cities, and backward cities in all prefecture-level cities. Beijing, Chongqing, Xi'an, and Shanghai are research-and-development centres; the power exponent of such prefecture-level cities in the second scale range (from the 5th to the 218th) was  $-0.6992$ . Thus, they do not differ greatly and should be considered popularised cities. Cities in the third-scale range (the rest of the cities) were extremely irregular.

The number of scientific personnel represents the capacity of the scientific research and technology of a prefecture-level city. According to the power law, the first scale range includes Beijing, Shanghai, and Xi'an; the power exponent of such prefecture-level cities in the second scale range (from the 4<sup>th</sup> to the 243<sup>rd</sup>) is  $-0.9723$ . The fitting of prefecture-level cities in the third-scale range was not ideal. This result confirms the analysis of the number of employed people in the information industry and indicates that both (the number of people in the information industry and that of scientific personnel) support each other. However, these data suggest that none of the samples have multifractal characteristics.

From a macrospatial layout, prefecture-level cities ranked high in the quaternary industry (such cities, excluding the aforementioned ones, include Guangzhou and Shenzhen) and were located in the coastal areas of eastern China. Further, they were more concentrated than the top prefecture-level cities in the building-area index. In terms of development, the development of these metropolises further meets the criteria of 'industry incubator'.

## Comprehensive Index

The comprehensive index is a closely integrated reflection of the secondary and tertiary industries and their related social and economic activities; thus, it is neither a primary index nor a simple sum of all industry indices.

## (1) GDP and per capita GDP

The GDP index is suitable for analysing the economic scale of a prefecture-level city from meso- and macroperspectives. The power exponent of the GDP of the first 208 prefecture-level cities is  $-0.7284$ , which is the average range of the economic scale of Chinese prefecture-level cities. Comparing the power exponents of the different indexes, we can observe the following: The change in the index across all departments tends to be solid or nongeneralised in the tertiary industry, real-estate industry, and the total output value of large-scale enterprises driven by the market (Table 1). In contrast, the change in the indexes of the secondary industry (the number of people in the manufacturing and construction industry and the profit of large-scale enterprises) and the average indexes within their scale ranges tend to be weak. In large cities, the aforementioned vital indices generally develop well, but the weak indices show poor development. In contrast, it does not represent the reverse change in medium and small cities but only represents the difference from large cities. Combining the changes in GDP in prefecture-level cities at all levels reveals their multifractal characteristics (4-7) with comprehensive medium-level complexity (spectrum width is 2.1259) and volatility (left spectrum truncation, 2.2484). (On this basis, the levels of GDP in each hierarchy also exhibit multiple fractal features; complexity (score 1.8744) and volatility (left side spectrum, 2.2141) are at a comprehensive medium level.) Fig. 3-g further shows that this index with overall scale characteristics in this field implies that the Chinese economic aggregate displays a spatial layout dominated by the coastal areas of eastern China and is based on eastern and central China. Despite the chaotic or coupling state that appears in specific development fields, the overall economic development of Chinese prefecture-level cities exhibits multifractal characteristics of spatial autocorrelation.

However, this type of characteristic prompts changes in the per capita GDP index. (1) The per capita GDP of productive prefecture-level cities (e.g. resource-based cities in central China) is relatively high, whereas that of comprehensive prefecture-level cities is relatively low. (2) The power exponent of the per capita GDP is  $-0.4030$ , indicating that the difference in the per capita scale among prefecture-level cities narrows or becomes generalised. This difference may be attributed to both the market and government. Possibly because of the influence of various factors, the autocorrelation of the per capita GDP is disrupted and, thus, fails to display spatial multifractal characteristics at the prefecture-level city scale.

## (2) Average wages of urban workers

The power exponent of the average wage of the urban workers of the first 257 prefecture-level cities is  $-0.1873$ , which shows the least differences among the 22 indices. Considering the Chinese government's income adjustment policy, it could employ these measures to narrow the income gap in prefecture-level cities and maintain social equity. These measures have led to an evolution characterised by strong regularity and low complexity (spectrum width is the smallest at 1.5785), and low volatility (the value of its spectrum truncation is 2.1600), which also indicates these

multifractal characteristics (4-8). Figure 3-h indicates that this index covers the largest spatial scale, along with small practical differences among residents' incomes; thus, this index exhibits a marked generalised trend.

Comparing the various indices of prefecture-level cities and the average wage of urban workers, the government's income adjustment measures have not only narrowed the differences in multiple development fields among prefecture-level cities, but also further generated ordered multifractal characteristics at the national spatial scale.

### **Spatial Evolution State of Multiple Fields among Prefecture-Level Cities**

Based on the abovementioned analyses, we can draw the following conclusions: generally, indices in the tertiary and real-estate industries show a typical nongeneralised evolution; in contrast, the indices in the secondary industry tend to exhibit nongeneralised evolution. Under government intervention, some basic and comprehensive indices are inclined to show a generalised development. The above analyses, based on the power-law and multifractal characteristics, mainly exhibit the basic state of the rank-scale hierarchical development in eight fields (Chen, 2008) and were used to conduct complicated analysis through related data to determine the regularity of evolutionary processes. However, this regularity is mainly reflected in whether the data are generalised or nongeneralised and whether such data show multifractal characteristics. To further unveil the Chinese spatiotemporal evolution at the macrolevel, we analysed the spatial agglomeration and diffusion of prefecture-level cities within the multifractal scale range. Theoretically, the seemingly disordered spatial chaos in Fig. 3 contains rich spatial entropy and can largely reflect the spatial polarisation and diffusion in various socioeconomic fields. For example, the above eight indices have typical multifractal characteristics. Different forms of spatial variation in the 14 domains of development do not possess typical multifractal features. Owing to space constraints, this study did not conduct in-depth research on this topic.

Clearly, the generalised evolution of certain fields in China corresponds mainly to the spatial diffusion effect. The nongeneralised evolution of certain fields generally leads to a spatial agglomeration, but it does not show a one-to-one correspondence. If the agglomeration of a certain field develops to a certain degree, then the rank transfer phenomenon occurs in the macrofield. For example, in the initial development stage of the business and service industry, several national-level regional centres have emerged in China, including Shanghai, Beijing, and Guangzhou. This change was the result of the accumulation of socioeconomic activities in eastern, northern, and southern China. However, with social development after the reform and opening up, business and service industry centres of the same level emerged successively, including in Wuhan, Dalian, and Chongqing in central, northeastern, and southwestern China, indicating a typical rank transfer phenomenon. In other words, from a broad spatial perspective, it stems from a high-level nationwide diffusion effect. In the context of this study, these eight fields mainly belong to the strengthening urban property, but they could also belong

to the agglomeration and diffusion of different levels or show the logical process of agglomeration → diffusion → agglomeration.

Summarising the analyses from Sects. 5 and Fig. 3, the above-mentioned rank-scale hierarchical evolution law would be reflected in practice; the spatial evolution 现状 of multiple fields in Chinese prefecture-level cities (in the early 2010s) are as follows:

- (1) The developed zones in China remain concentrated in eastern and central China, but unlike in the past, they have surpassed the famous ‘Hu Huanyong Line’ and are expanding to western China, with development in northwestern China being relatively fast. In the spatial layout of the eight indexes of a prefecture-level city with multifractal characteristics (Fig. 3), including four apparent indices, namely, the urban population, total output value of large-scale enterprises, number of social labourers, and GDP, the average wage of urban workers displays a westward spatial distribution with a propensity for homogenisation. However, the three indices of the number of people in the basic business and service industry, number of people in the manufacturing industry, and building area have not generally broken the ‘Hu Huanyong Line’, suggesting that the fast-developing areas of China are still concentrated in the east of this line. As indicated in the analysis and spatial layout of the early 2010s, the Yangtze River Delta and northern China had the strongest autocorrelation ability, followed by the Pearl River Delta.
- (2) As seen in the spatial distribution covered by eight indices with multifractal characteristics, the keys to ‘western expansion’ are Shanxi, Gansu, and Ningxia in northwestern China and the Yunnan–Guizhou Plateau in southwestern China. Considering the regions covered by the eight fields, the two regions are not completely covered. The evolution of related development fields is suggested to be uncoordinated. In particular, the development of the tertiary industry in Shanxi, Gansu, and Ningxia has yet to keep up with the development of multiple social and economic activities represented by the secondary industry. The Yunnan–Guizhou Plateau has mainly become the development ‘basin’ of eastern and central China.  
However, eastern and central China do not have absolute superiority. Furthermore, Figure 3 indicates that the previous heavy-industry base in northeastern China is losing its old status. The number and scale of prefecture-level cities with autocorrelation ability in this region are not as good as before.
- (3) As a result of its position as a hub, central China has naturally obtained the rank transfer + radiation of multiple fields and has formed a strong autocorrelation ability. Hence, it plays an extremely important role in national macrospatial development.
- (4) Numerous medium and small cities serve as the foundation and support for the evolution of prefecture-level cities with autocorrelation characteristics. According to the results, the overall difference in the development of these cities was not extremely significant.
- (5) Fig. 3 shows that the number of people in the basic business and service industry, as well as the number of people in the manufacturing industry and building

area, generally seem to show a spatial distribution that diverges from a fluid state at the national scale but results from spatial agglomeration within all regions. Although the average wage of urban workers tends to balance regional differences, it is shaped by government regulations. Its spatial distribution differs from the spatial distribution that diverges from the fluid state of the aforementioned fields. Meanwhile, the spatial layout of all three indices for the number of urban labourers, total output value of large-scale enterprises, and size of the urban population largely belong to the spatial diffusion layout that forms group development within all regions. The spatial layout of GDP tends to be an integration of these three scenarios.

In summary, the generalised and nongeneralised evolution of data from all prefecture-level cities' fields show high uniformity with regards to spatial agglomeration and diffusion. Both supplemented each other and showed a significant correlation.

## Discussion

### Spatial Development State in China

Since the reform and opening up, the three processes have widened the gap among Chinese prefecture-level cities and provided a developmental space for its heterogeneous evolution. The overall development basis (the size of the urban population) and economic scale (GDP) show regularity and multifractal characteristics, particularly among prefecture-level cities of different scales. However, in more specific developmental fields, the chaotic or coupling state is the norm in most fields in China.

Based on the above characteristics, the early 2010s China had formed an agglomeration of social and economic activities represented by coastal developed areas, such as the Yangtze River Delta, northern China, and the Pearl River Delta, whereas the development of inland western China was relatively poor. This resulted in complexity, volatility, and spatial irregularities in the development of multiple fields in prefecture-level cities. The multiple-field evolution among prefecture-level cities objectively realises the radiation/hierarchical diffusion development from eastern to central, then to western China, as well as from large to medium cities and then to small towns. Tables 1 and 2 and Figs. 4 and 3 further show the autocorrelation characteristics of spatial evolution in multiple fields during this process. Furthermore, spatial dissymmetry among multiple fields results from this process. The power exponent value and all multifractal parameters obtained through the analysis reveal these spatial relations with complicated system characteristics. Specifically, they formed the spatial evolution characteristics of multiple fields, as discussed in Sect. 5.6.

Compared with existing studies, the present study determines the evolutionary regularity of multiple fields in Chinese prefecture-level cities through power-law and multifractal analysis. We analyse the basic characteristics and

spatial layout of the overall operation of Chinese society and the economy with a prefecture-level city as the basic unit. Comprehensively viewing the spatial quantitative analysis based on multifractal characteristics, we note that the theoretical contributions of this work lie in the analysis and evaluation of the early 2010s's development of Chinese prefecture-level cities and the interaction among multiple fields from the nonEuclidean geometry perspective. The practical contributions of this study lie in the revelation of local details and concrete characteristics in the development of prefecture-level cities, thereby practically elucidating the Chinese macrolevel development. The practical significance of this study is that it provides a quantitative basis for related research and decision making (such as overall regional planning, industry planning, and urban planning).

### Spatial Development in Chinese Prefecture-level Cities

- (1) The evolution of multiple fields in large and medium cities driven by urban properties underlies Chinese prefecture-level cities, whereas the evolution of vast rural areas and medium and small cities only plays a minor role in the overall development of prefecture-level cities. Consequently, the urban and rural landscapes of Chinese prefecture-level cities are determined by the development of the secondary and tertiary industries. Figure 3 reflects the differences in the development landscape of Chinese prefecture-level cities (urban–rural regional complexes) from multiple perspectives.
- (2) In general, the development pillar of all prefecture-level cities is the secondary industry, but the development orientation is the tertiary industry. According to the power-law analysis, the evolution of the tertiary industry shows strong sensitivity and tends to generate overly rapid or overly slow development. In the long run, the important orientation of the connotative development of prefecture-level cities is the development of a high-technology industry.
- (3) The tertiary industry's overall development has promoted spatial agglomeration among prefecture-level cities at all scales, thereby complicating the development of the tertiary industry in all prefecture-level cities. Although some fields in this industry have formed multifractal autocorrelation characteristics, the evolution of most fields is still in a chaotic state. Coupling these industries would lead to the spatial agglomeration and diffusion effect of different fields and scales in prefecture-level cities, thereby prompting the nationwide autocorrelation evolution of social and economic activities with regularity and even multifractal characteristics.

Socioeconomic activities based on secondary and tertiary industries further influence the development of Chinese macroprimary and comprehensive indices and cause them to possess multifractal characteristics of varying degrees of complexity and volatility.

- (4) In the early 2010s, the traditional secondary industry, represented by the manufacturing industry, formed a spatial structure with autocorrelation characteristics and balanced the spatial agglomeration resulting from the development of the tertiary industry in central and eastern China (to some extent). However, its development remains limited to the coupling process. The construction industry exhibits severe bubble characteristics but is in a relatively chaotic state.
- (5) The Chinese government has taken various measures to reduce income differences among residents of prefecture-level cities and narrow the per capita GDP gap among prefecture-level cities. Doing so encourages homogenous development of different prefecture-level cities (or relatively fair development; practically, strict fairness is impossible).

## Conclusions and Future Work

A significant theoretical contribution of this study is the analysis of the development status of a multifield complex in China based on one-dimensional cross-sectional data and using multifractal technology. The analysis reveals that the spatial fractal system is often affected by the development of the evolution of the domain presenting a hierarchy, and the shape, peak, and bias of each fitting curve in the fractal analysis can be used to describe the differentiated changes in its multidomain, thus showing its nonlinear and spatial characteristics in the early 2010s. The results show that multifractal analysis, derived from nonlinearity, is more explanatory than traditional methods and can reflect the complexity of economic geospatial space, yielding much more practical results. Practically, governments at all levels should plan, adjust, or guide at different spatial levels according to the level of dislocation in different developmental fields and the advantages and disadvantages of local-level cities.

Due to the spatial complexity of the Multifractal Characteristics Analysis for the proposed system and model, it can be observed that the existing approaches proposed have their limitations. Such as many parameters, variables, computing resource requirements, etc. In our future work, we are planning to go deeper into the following areas:

- (1) high-performance design for the multifractal analysis using computational intelligence-aided design (CIAD) (Chen and Li, 2018; Song et al., 2021b);
- (2) one further step research on AI-driven platform based on CIAD framework for spatial analysis and design;
- (3) the virtual and physical verification and validation for the multifractal analysis, especially in understanding the multifractal characteristics, spatial state, and mechanisms of various developmental fields through the lens of prefecture-level cities, has important practical implications using digital twins technology (Song et al., 2021b);

Appendix

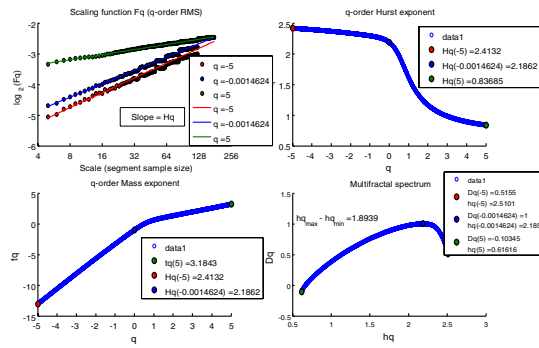


Figure 4-1 Multifractal analysis of urban population from the second to the 250<sup>th</sup>

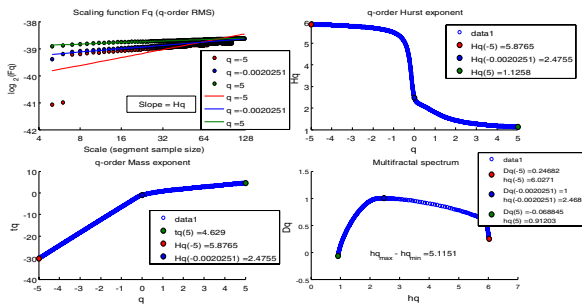


Figure 4-2 Multifractal analysis of the first 205 building area

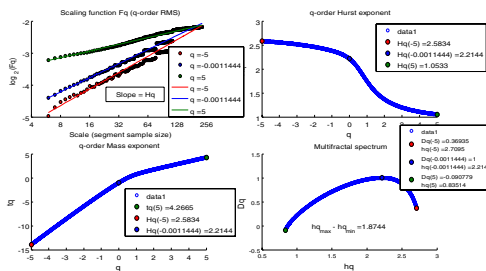


Figure 4-3 Multifractal analysis of the first 231 total output value of large-scale industrial enterprises

Fig. 4a Multifractal analysis process diagram of 8 indexes of Chinese prefecture-level cities (1)



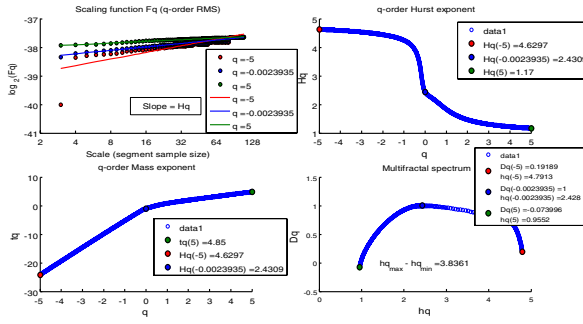


Figure 4-4 Multifractal analysis of the first 163 people in manufacturing industry

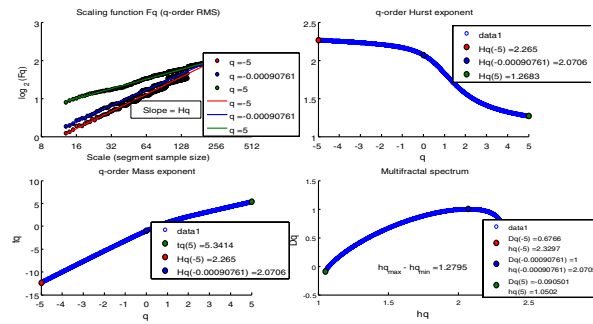


Figure 4-5 Multifractal analysis of the first 109 people in basic business and service industry

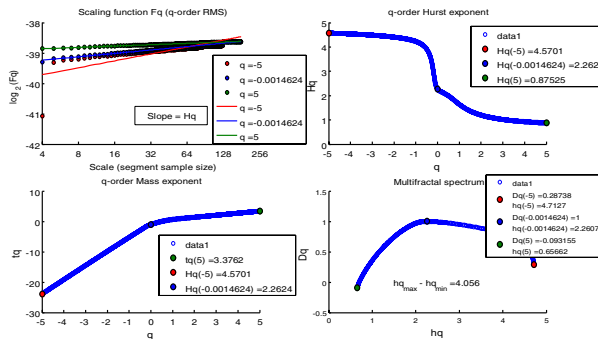


Figure 4-6 Multifractal analysis of the employed population from the second to the 254<sup>th</sup>

Fig. 4b Multifractal analysis process diagram of 8 indexes of Chinese prefecture-level cities (2)

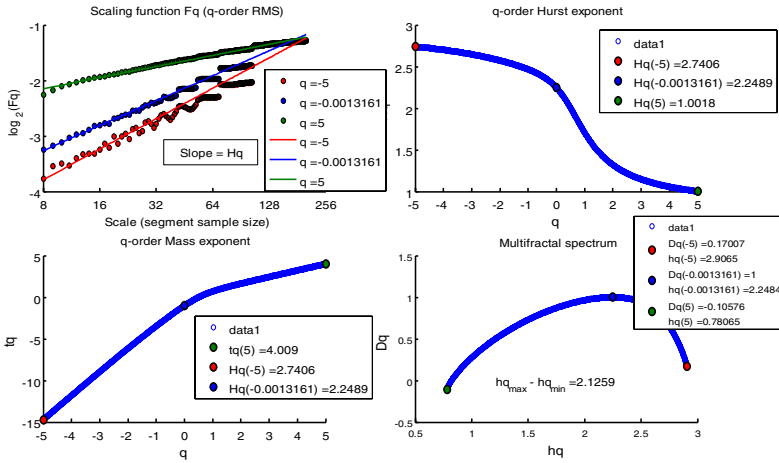


Figure 4-7 Multifractal analysis of the first 208 GDP

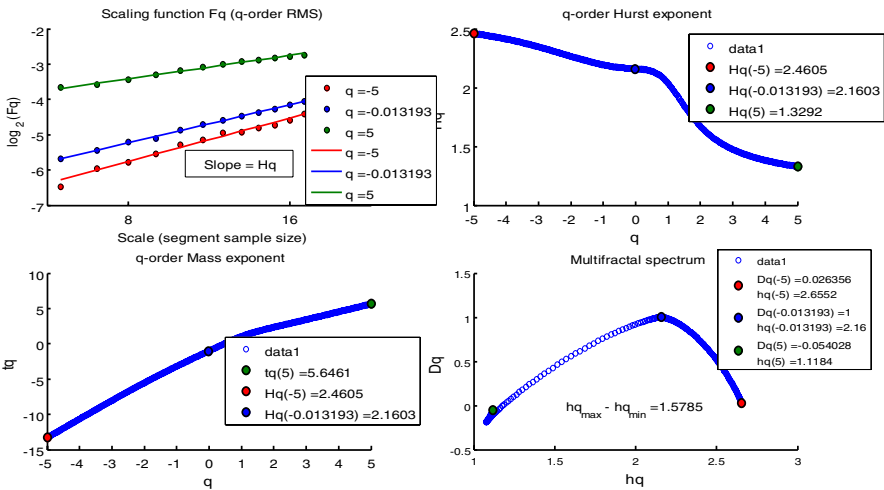


Figure 4-8 Multifractal analysis of the first 257 average wages of urban workers

Fig. 4c Diagram of multifractal analysis process of 8 indexes of Chinese prefecture-level cities (3)

**Acknowledgements** The research was supported by the National Natural Science Foundation of China (No.42071156) and the XJTLU Research Development Funding RDF-21-02-019 .

**Author Contributions** Z.-J. S. and Y. C. conceived the idea, Z.-J. S., M.-Y. B. and Y.C. provided the theoretical analysis, Z.-J. S. and Y. C. wrote a major portion of the manuscript, Z.-J. S., M.-Y. B. and Y.C. performed the numerical simulations and prepared all the figures, and M.-Y. B. wrote a minor portion of the manuscript. All authors reviewed the manuscript.

**Data Availability** The datasets analysed during the current study were derived from the following public domain resources: 1. China City Statistical Yearbook 2013 "<http://cyfd.cnki.com/N2014050073.htm> <http://tongji.cnki.net/kns55/Nav/YearBook.aspx?id=N2014050073&floor=1>"><http://tongji.cnki.net/kns55/Nav/YearBook.aspx?id=N2014050073&floor=1> 2. China Statistical Yearbook for Regional Economy 2013 [http://www.stats.gov.cn/tjsj/tjcbw/201505/t20150526\\_1076522.html](http://www.stats.gov.cn/tjsj/tjcbw/201505/t20150526_1076522.html)">[http://www.stats.gov.cn/tjsj/tjcbw/201505/t20150526\\_1076522.html](http://www.stats.gov.cn/tjsj/tjcbw/201505/t20150526_1076522.html) <http://www.yearbookchina.com/navibooklist-N2014030146-1.html>"><http://www.yearbookchina.com/navibooklist-N2014030146-1.html>

## Declarations

**Competing Interest** The authors declare that they have no competing financial and non-financial interests.

**Conflicts of Interest** The authors declare that they have no financial or non-financial conflicts of interest.

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