Think It Over

This section of *Resonance* presents thought-provoking questions, and discusses answers a few months later. Readers are invited to send new questions, solutions to old ones and comments, to ‘Think It Over’, *Resonance*, Indian Academy of Sciences, Bangalore 560 080. Items illustrating ideas and concepts will generally be chosen.

Two Simple Partial Fractions*

Problem:

Find the partial fraction expansion of proper rational function of types $\frac{1}{x^n(x^m+1)}$ and $\frac{1}{x^n(x^m-1)}$.

Solution:

We may assume without loss of generality that m divides n by multiplying with a suitable power of x ,

$$\begin{aligned} \frac{1}{x^n(x^m+1)} &= \frac{1}{x^n} - \frac{1}{x^{n-m}} + \frac{1}{x^{n-2m}} - \frac{1}{x^{n-3m}} \\ &+ \dots + (-1)^{\left(\frac{n}{m}-1\right)} \cdot \frac{1}{x^m} + (-1)^{\frac{n}{m}} \cdot \frac{1}{(x^m+1)}. \end{aligned}$$

$$\begin{aligned} \frac{1}{x^n(x^m-1)} &= -\frac{1}{x^n} - \frac{1}{x^{n-m}} - \frac{1}{x^{n-2m}} - \frac{1}{x^{n-3m}} \\ &- \dots - \frac{1}{x^m} + \frac{1}{x^m-1}. \end{aligned}$$

Note that the number of terms is just $\left(\frac{n}{m} + 1\right)$. These formulas can be easily checked by mathematical induction. x could even be like y^π and this would give partial fraction in terms of y^π .

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