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Correction to: Some functional inequalities on non-reversible Finsler manifolds

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Abstract We correct the proof of the Sobolev-type inequality in [2] for $1 < p < 2$ (called the Beckner inequality).

Keywords. Finsler geometry; Ricci curvature; Sobolev inequality.

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In [2, Theorem 5.6], we state the following Sobolev-type inequality on a Finsler manifold (M, F) equipped with a measure $\mathfrak{m}$.

Theorem 1. *Assume that* $\mathrm{Ric}_N \geq K > 0$ *for some* $N \in [n, \infty)$ *and* $\mathfrak{m}(M) = 1$. *Then we have*

$$\frac{\|f\|_{L^p}^2 - \|f\|_{L^2}^2}{p-2} \leq \frac{N-1}{KN} \int_M F^2(\nabla f)\, d\mathfrak{m} \tag{1}$$

for all $1 \leq p \leq 2(N+1)/N$ *and* $f \in H^1(M)$.

The proof in [2] is, however, incorrect for $1 < p < 2$ (precisely, the final approximation procedure requires $p > 2$). Instead, we can apply the argument in [1] to show (1) for $1 < p < 2$ (such an inequality is called the *Beckner inequality*). Furthermore, the argument in [1] gives the following generalization of Theorem 1.

Theorem 2. *Assume that* $(M, F, \mathfrak{m})$ *is compact and satisfies* $\mathrm{Ric}_N \geq K > 0$ *for some* $N \in (-\infty, -2)$ *and* $\mathfrak{m}(M) = 1$. *Then we have*

$$\frac{\|f\|_{L^p}^2 - \|f\|_{L^2}^2}{p-2} \leq \frac{N-1}{KN} \int_M F^2(\nabla f)\, d\mathfrak{m}$$

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for all $1 \leq p \leq (2N^2 + 1)/(N - 1)^2$ *and* $f \in H^1(M)$.

We refer to a forthcoming book [3] for details and further discussions.

References

[1] Gentil I and Zugmeyer S, A family of Beckner inequalities under various curvature-dimension conditions, *Bernoulli* **27** (2021) 751–771

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[3] Ohta S, Comparison Finsler geometry, in preparation