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ORIGINAL PAPER

Design model for assembly lines including fractional tasks and parallel workstations

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Abstract

In recent years, the use of robots and cobots allow to increase productivity and quality of products. Due to the higher investment, the robustness and efficiency of flow lines are crucial to reduce the throughput loss. The solution of installing buffers between stations increases costs and factory space. To improve the efficiency and robustness of assembly lines, the literature proposed some variants to the simple assembly line balancing problem. The introduction of fractional tasks and parallel workstations are two promising models proposed in recent works to reduce throughput loss caused by short failures. The potential of the two approaches has been studied individually, but no work has evaluated the integration of fractional and parallel tasks can further improve the efficiency of the production lines. This paper proposes a matheurstic method to design assembly lines integrating fractional tasks and parallel workstations. The approach proposed aims to reduce the computational complexity of the design of the assembly lines and provides a series of design alternatives. The simulation model tests the robustness of the design alternatives against short failures. The numerical results highlight how the proposed model improves the performance and the robustness of the assembly line when unforeseen events such as failures occur. The integration of fractional tasks and parallel tasks can improve the robustness against short failures. This benefit is relevant for robotic assembly lines, and the increasing use of cobots that are mainly used in the automotive, electronics sector, and metal machinery industries.

Keywords Assembly line · Fractional tasks · Parallel workstations · Robustness · Simulation

1 Introduction

Assembly lines are widely used when the volume to satisfy is higher, and the products can be manufactured or assembled by several tasks. In an assembly line, the flow of the items is unidirectional through several consecutive stations that perform the tasks to obtain the finished product.

The most widespread design approach of assembly lines is the assignment of the task to the stations minimizing the total idle time under the precedence constraints, and the sum of the tasks time assigned to each station is lower than the cycle time [1].

The introduction of automation and the collaboration human–robot [2] is significant in recent years to improve the performance of assembly lines [3].

In this context, the effect of short failures [4] cause blocking or starvation state of the stations reducing the throughput of the assembly line.

In the literature are proposed some approaches to improve the robustness of assembly lines such as the introduction of buffers with preventive maintenance [5, 6], redundancy of the tasks [7, 8], the fractional tasks allocation models [9], and the introduction of some parallel stations [10].

The introduction of buffers increases the costs of items in queues and reduces the benefits of assembly lines such as reduced work in process and space occupied.

The redundancy approach has to improve the workers' capability and duplicate equipment in some stations to perform more tasks. Then, it is necessary to duplicate some equipment among stations.

The fractional tasks models can be used when some tasks can be divided into sub-tasks and shared among stations of the assembly line [11]. The development of these approaches follows restrictions such as two stations can share a task and a limitation on the number of possible shared tasks.

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The introduction of the parallel workstation in some stations of the assembly line allows one to perform the tasks assigned to these stations in *m* parallel workstations keeping the total number of workstations required in the only serial line [10]. The total time allocated to the workstations in the parallel arrangement can be larger than the cycle time, but the workstation working in parallel can achieve the desired assembly line throughput. The introduction of some parallel workstations can improve the efficiency of the assembly line reducing total idle time.

These two approaches improve the efficiency of the assembly line without impact on the main characteristics of assembly lines.

The works proposed in the literature explained the possible improvement of fractional or parallel tasks separately, while any works evaluated the integration of them. This research fills this gap by proposing an assembly design model that includes the introduction of fractional tasks and parallel workstations together. Most line balancing problems are NP-hard [12], and the introduction of parallel workstations [10] and fractional tasks [9] increases further the computational complexity. For this reason, the solution of the model developed is obtained by an original matheurstic approach to reduce the computational complexity and solve larger problems that are more suitable for industrial applications. The main drawback of the proposed model that integrates fractional and parallel tasks is the duplication of the possible configurations of the flow line. The number of configurations that need to be restricted follows a guideline (as described in the manuscript) to detect the best configuration in a reasonable time. The models proposed in the literature with fractional tasks and parallel workstations are used as benchmarks to highlight how the proposed model can further improve the performance of assembly lines.

The comparison is mainly conducted with simulation models to test the robustness of the assembly lines designed against short failures.

The article is structured as follows. Section 2 discusses the main recent works proposed in the literature about fractional tasks and parallel workstation to support assembly line design. Section 3 presents the formulation of the assembly line problems and the proposed model. Section 4 discusses the numerical results and the main findings. Finally, the conclusions and future developments are presented in Sect. 5.

2 Literature review

A complete survey on parallel assembly lines is provided by Aguilar et al. [13], while this research concerns the possibility that some stations consist of parallel workstations. Battaïa and Dolgui [14] reviewed the hybridization of line balancing with other optimisation problem proposed during the last decade. Among the solutions studied, parallel workstations and fractional tasks emerged as alternatives in recent years. This section discusses recent works on parallel workstations and fractional tasks assembly line problems.

Öztürk et al. [15] developed a mixed integer programming model for simultaneous balancing and scheduling of flexible mixed model assembly lines with parallel stations. Due to the complexity of the problem, they proposed a decomposition scheme to solve large-size industrial applications.

Tiacci [16] proposed an innovative approach solving the Mixed Model Assembly Line Balancing Problem (MALBP) with stochastic task times and parallel workstations. Due to the computational complexity, a genetic algorithm is used to solve the problem. The objective of the model proposed is the minimization of the design cost, that is the total annual cost for labor and equipment costs of the line configuration. This work was extended to include the introduction of buffers [17].

Lopes et al. [11] presented a mixed-integer linear programming formulation to solve simultaneous balancing and cyclical scheduling problems. The numerical tests highlighted how the introduction of parallel workstations can improve the throughput of the lines.

Álvarez-Miranda et al. [10] studied the SALBP problem with the introduction of parallel workstations. The objective is to minimise the number of parallel workstations to achieve the maximum theoretical efficiency of the assembly line. Due to the computational complexity of the parallel SALBP formulation, they proposed a heuristic approach based on a variable neighborhood search (VNS) metaheuristic framework.

Grzechca and Foulds [18] presented an industrial case study where it is possible to split certain tasks among more than one station. The numerical test highlighted how splitting some tasks improve the throughput. The splitting of tasks seems more effective nearer the beginning of the assembly line.

Jeong and Jeon [19] studied the design of assembly lines with work-sharing among stations. The work-sharing is supported by floating workers where a portion of tasks or workers is shifted to the succeeding station respectively. They proposed a mixed mathematical model to design assembly lines and highlight those floating workers is more efficient than only floating works.

Lopes et al. [9] addressed the problem of SALBP problem with fractional tasks. They proposed a mixed integer linear program to describe the problem studied. The model presented has been developed to achieve two main objectives: one model to minimize the cycle time and the other two models to minimize internal storage required to realize a given cycle time value. The numerical result shows how fractional tasks can be an attractive solution to improve performance and reduce costs.

Renna [20] proposed a mixed-integer linear programming model to design assembly lines with fractional and redundancy possibilities for the tasks. The design model is integrated with a control policy that allocates dynamically the fractional tasks between two consecutive stations. The simulation results highlight how the proposed model allows to improve performance with a limited number of shared tasks.

Table 1 classifies the main issues of the literature review considering: Meta-heuristic, genetic algorithm, or Mixed-Integer Linear Programming (MILP) to solve the mathematical problem proposed; the objective to balance, schedule, or minimize the costs of the line, the introduction of buffer, the split only of tasks, the sharing of workers related to the fractional tasks, dynamic task assignation between two consecutive stations; finally, the case study is discussed only in the case of fractional tasks in the recent literature.

As highlighted in Table 1, the works proposed in the literature are completely focused on parallel workstations or fractional tasks, no works proposed in the literature studied the integration of fractional tasks and parallel workstation models in assembly lines. Furthermore, the effect of the models on energy consumption was not studied in any works; for this reason, this issue is not reported in Table 1. The evaluation of the most mathematical models proposed highlights how the computational complexity is high for applications in larger industrial cases.

The research proposed in this paper overcomes the limits of the literature with a MILP model to design assembly lines with fractional tasks and parallel workstations together. Then, a framework to integrate the SALBP model with the proposed model has developed to reduce the computational complexity and solve the larger problem. The simulation model tests the proposed model compared to classical SALBP, fractional tasks, and parallel workstations evaluating the throughput, energy consumption, and robustness to short failures.

The first research question of this paper is the following:

RQ1*: Can the design model that integrates fractional tasks and parallel workstations improve significantly the performance of assembly lines*?

The studies proposed in the literature proposed meta-heuristic or genetic algorithms to solve the mathematical problem due to the computational complexity.

Matheuristics are the hybridization of mathematical models with heuristic/metaheuristic algorithm and simulation techniques to improve the solutions of known mathematical programming models. This allows to obtain good solution in a reasonable computational time [21].

Then, the second research question is the following:

RQ2: *Is the matheuristic framework proposed adapt to solve the proposed design mathematical problem with suitable computational time?*

3 Problem formulations

This section presents the formulations of the studied problem and the proposed mathematical model to integrate fractional tasks and parallel workstations.

The main simplifying assumptions of the models are the following:

- The precedence relations among operations are known and invariable;
- All equipment used by the stations are available, and costs are not considered;
- The tasks can be shared by two consecutive stations for the fractional case;
- the introduction of parallel stations leads to keeping the same total number of stations;
- The line is balanced for a single product.

The notation used is reported in Table 2.

3.1 The simple assembly line problem

The first problem presented is the classical simple assembly line problem (SALBP) denominated Model 1.

$$\text{Mimize CT} \tag{1}$$

Subject to

$$\sum_{s=1}^{S} x_{t,s} = 1 \forall t \in T \tag{2}$$

$$\sum_{s=1}^{S} s * x_{t,s} \le \sum_{s=1}^{S} s * x_{p,s} \forall P_{t,p} = 1 \tag{3}$$

$$\sum_{t=1}^{T} x_{t,s} * PT_t \le CT \forall s \in S \tag{4}$$

$$Total\ Idle\ Time = \sum_{s=1}^{S} CT - \left(\sum_{t=1}^{T} x_{t,s} * PT_t \right) \tag{5}$$

$$x_{t,s} \in \{0,\ 1\} \forall t \in T,\ s \in S \tag{6}$$

The objective function is the minimisation of the cycle time (expression 1). Constraint 2 assures that each task is assigned only to one station. Constraint 3 assures the precedence constraints of the tasks. The processing time due to the tasks assigned to each station is constrained by the cycle time (constraint 4). Expression 5 computes the total idle time of the assembly line. The variables x must be integer values (expression 6).

Table 1 Classification of the literature review

Parallel workstations

	Meta-heuristic	Genetic algorithm	MILP	Balancing	Scheduling	Costs	Buffer
[15]	X			X	X		
[16]		X		X		X	
[17]		X		X		X	X
[11]			X	X	X		
[10]	X			X			

Fractional tasks

	Split tasks	Workers sharing	MILP	Balancing	Buffer	Dynamic task sharing	Case study
[18]	X			X			X
[19]		X	X	X			
[9]			X	X	X		
[20]			X	X		X	

3.2 The parallel simple assembly line problem

The second problem presented is a modified Parallel Simple Assembly Line Balancing (PSALB) proposed in [10] denominated Model 2.

$$\text{Mimize CT} \tag{7}$$

$$\sum_{s=1}^{Ns} x_{t,s} = 1 \forall t \in T \tag{8}$$

$$\sum_{s=1}^{Ns} s * x_{t,s} \le \sum_{s=1}^{Ns} s * x_{p,s} \forall P_{t,p} = 1 \tag{9}$$

$$\sum_{t=1}^{T} x_{t,s} * PT_t \le CT * (1 + PP_s) \forall s \in Ns \tag{10}$$

$$Total\,Idle\,Time = \sum_{s=1}^{S} CT - \left(\sum_{t=1}^{T} \frac{x_{t,s} * PT_t}{(1 + PP_s} \right) \tag{11}$$

$$CT \le CTmax \tag{12}$$

$$\sum_{s=1}^{Ns} PP_s + Ns \le S \tag{13}$$

$$x_{t,s} \in \{0,\ 1\} \forall t \in T,\ s \in Ns \tag{14}$$

$$PP_s \in Z^{\ge 0} \tag{15}$$

The objective function (expression 7) and constraints 8,9 are the same as in model 1. The processing time due to the tasks assigned to each station is constrained by the cycle time that takes into account the parallel workstation that increases the production capacity of the station (constraint 10). Expression 11 computes the total idle time of the assembly line considering the parallel workstations. Expression 12 constrains the cycle time to the value minimized by model 1. Then, this model is used after the solution of model 1, and this allows to reduce the computational time. The total number of stations with parallel stations does not exceed the number of stations of model 1 (expression 13) to keep the same equipment. This model is solved at different numbers of Ns to explore different configurations (see sub-section framework). The expressions 12 and 13 allow solving the model with lower computational time. The variables x must be integer values (expression 14) and the variables PP must be positive integers.

3.3 The simple assembly line problem with fractional tasks

The third problem presented concerns the SALB with fractional tasks [9] denominated model 3:

$$\text{Mimize CT} \tag{16}$$

$$\sum_{s=1}^{S} (x_{t,s} + y_{t,s}) = 1 \forall t \in T \tag{17}$$

$$\sum_{s=1}^{S} z_{t,s} = 1 \forall t \in T \tag{18}$$

Table 2 Notation

Notation	Definition
Indices	
T	It is the total number of tasks to assemble the product
t	It is the index of the tasks t = 1,..T
S	It it the number of stations of the assembly line
s	It is the index of the station s = 1,..,S
Ns	Number of stations in series for parallel workstations cases
Parameters	
PT_t	It is the processing time of the task t
$P_{t,p}$	It is a binary value that is equal to 1, if the task t must precede the task p and 0 otherwise
CTmax	Upper bound of the cycle time
Decision Variables	
$X_{t,s}$	It is a binary value that is equal to 1, if the task t is assigned to station s and 0 otherwise
$Y_{t,s}$	It is a binary value that is equal to 1, if the task t is shared between station s and s + 1, and 0 otherwise. This is because the task can only be shared between two adjacent stations
$Z_{t,s}$	It is the percentage of the task t performed by the station s; this value is equal to 0, if the task is not shared between two stations (Yt,s = 0)
PP_s	It is an integer value that denotes the number of additional parallel workstations for the station s
CT	Cycle time

$$\sum_{s=1}^{S} s * (x_{t,s} + y_{t,s}) \le \sum_{s=1}^{S} s * (x_{p,s} + +y_{p,s}) \forall P_{t,p} = 1 \tag{19}$$

$$\sum_{s=1}^{S} s * z_{t,s} \le \sum_{s=1}^{S} s * z_{p,s} \forall P_{t,p} = 1 \tag{20}$$

$$\sum_{t=1}^{T} z_{t,s} * PT_t \le CT \forall s \in S \tag{21}$$

$$Total\ Idle\ Time = \sum_{s=1}^{S} CT - \left(\sum_{t=1}^{T} z_{t,s} * PT_t \right) \tag{22}$$

$$CT \le CTmax \tag{23}$$

$$\sum_{t=1}^{T} \left(y_{t,s-1} + y_{t,s} \right) \le 1 \forall s \in S\ with\ s > 1 \tag{24}$$

$$z_{t,s} \ge x_{t,s} \forall t \in T, \forall s \in S \tag{25}$$

$$z_{t,s} \le x_{t,s} + y_{t,s} + y_{t,s-1} \forall t \in T, \forall s \in S\ with\ s > 1 \tag{26}$$

$$z_{t,1} \le x_{t,1} + y_{t,1} \forall t \in T \tag{27}$$

$$y_{t,Ns} = 0 \forall t \in T \tag{28}$$

$$K * z_{t,s} \in Z^+ \tag{29}$$

$$x_{t,s} \in \{0,\ 1\} \forall t \in T,\ s \in Ns \tag{30}$$

$$y_{t,s} \in \{0,\ 1\} \forall t \in T,\ s \in Ns \tag{31}$$

$$x_{t,s} \ge 0 \forall t \in T,\ s \in Ns \tag{32}$$

The objective function (expression 16) is the same as the above models. Constraint 17 assures that each task is assigned to a station or shared between two stations. The sum of fractional allocations for each task must be 100% (constraint 18). Expressions 19 and 20 assure that the precedence constraints of the tasks are both assigned or shared. The processing time assigned to each station must be under the cycle time (constraint 21). Expression 22 computes the total idle time of the assembly line designed. Expression 23 constraints the cycle time to the value obtained by model 1. Expression 24 states that a station can share one task with both the previous and following neighbor. The relations among the variables x,y, and z are defined in the expressions (25)–(27).

Expression (29) allows the variable z to be a fraction of a given integer denominator K. For instance, if K = 4, the fractional allocations variables *zt, s* can only assume the values 0, 0.25, 0.5, 0.75, and 1.

3.4 Proposed mathematical model and matheuristic framework

The fourth problem presented is completely original and integrates the fractional tasks and parallel stations denominated model 4:

$$\text{Mimize CT} \tag{33}$$

$$\sum_{s=1}^{Ns} (x_{t,s} + y_{t,s}) = 1 \forall t \in T \tag{34}$$

$$\sum_{s=1}^{Ns} z_{t,s} = 1 \forall t \in T \tag{35}$$

$$\sum_{s=1}^{Ns} s * (x_{t,s} + y_{t,s}) \le \sum_{s=1}^{Ns} s * (x_{p,s} + +y_{p,s}) \forall P_{t,p} = 1 \tag{36}$$

$$\sum_{s=1}^{Ns} s * z_{t,s} \le \sum_{s=1}^{Ns} s * z_{p,s} \forall P_{t,p} = 1 \tag{37}$$

$$\sum_{t=1}^{T} z_{t,s} * PT_t \le CT * (1 + PP_s) \forall s \in Ns \tag{38}$$

$$Total\, Idle\, Time = \sum_{s=1}^{S} CT - \left(\sum_{t=1}^{T} \frac{z_{t,s} * PT_t}{(1 + PP_s} \right) \tag{39}$$

$$CT \le CTmax \tag{40}$$

$$\sum_{s=1}^{Ns} PP_s + Ns \le S \tag{41}$$

$$\sum_{t=1}^{T} \left(y_{t,s-1} + y_{t,s} \right) \le 1 \forall s \in Ns\, with\; s > 1 \tag{42}$$

$$z_{t,s} \ge x_{t,s} \forall t \in T,\ \forall s \in Ns \tag{43}$$

$$z_{t,s} \le x_{t,s} + y_{t,s} + y_{t,s-1} \forall t \in T,\ \forall s \in Ns\, with\; s > 1 \tag{44}$$

$$z_{t,1} \le x_{t,1} + y_{t,1} \forall t \in T \tag{45}$$

$$y_{t,Ns} = 0 \forall t \in T \tag{46}$$

$$K * z_{t,s} \in \mathrm{Z}^{+} \tag{47}$$

$$x_{t,s} \in \{0,\ 1\} \forall t \in T,\ s \in Ns \tag{48}$$

$$y_{t,s} \in \{0,\ 1\} \forall t \in T,\ s \in Ns \tag{49}$$

$$x_{t,s} \ge 0 \forall t \in T,\ s \in Ns \tag{50}$$

$$PP_s \in \mathrm{Z}^{\ge 0} \tag{51}$$

The model proposed is an integration of the above models, and the expressions are derived from the expressions explained for models 1,2, and 3.

The original contribution of this research concerns also a framework to integrate the use of the formulations presented to provide several alternative configurations for the design of the assembly line.

Figure 1 shows the matheuristic framework proposed and the interactions among the model formulations. The first step is the use of model 1 (classical SALP formulation) to provide a base configuration and the upper bound of the cycle time. The first solution of model 1 has the function objective of cycle time minimization. Then, the model 1 is solved with this value as the upper bound and the objective value is the minimization of the total idle time. This provides the first assembly line configuration and the upper bound level of the cycle time for the other models.

Models 2 and 4 which include the possibility of parallel workstations are solved for different values of the total stations in series Ns. The strategy to solve the models for different values of Ns does not consider Ns a variable that reduces the computational time. Ns changes between Ns = S-1 and Ns > S/2; these values concern a series line with different stations that include parallel stations. The lower bound is Ns > S/2 because the value of Ns = S/2 leads to a complete parallel configuration of the assembly line. Models 2,3 and 4 are solved to minimize the cycle time and, then with the optimized cycle time the total idle time is minimized. This provides several solutions for the assembly configurations that allow to evaluate the potential benefits derived from the fractional tasks, parallel workstations, or integration of them. The different configurations are evaluated by simulation models.

4 Numerical experiments

The proposed formulation and the matheuristic framework are tested using an illustrative example extracted from the assembly line balancing dataset [22, 23] instance_n = 20_525. The focus is to highlight the application of the proposed matheuristic framework and how the integration of fractional tasks and parallel workstations can work together. For this objective is used only one instance. Table 3 reports the processing time and precedence constraints of the tasks of the numerical example.

The mathematical model solutions are provided by the Lingo® software package. The design of the assembly line considers three assembly line dimensions: 4, 8 and 12 stations. Figure 2 shows the configurations of the parallel workstations case changing Ns value for 4 and 8 stations.

The parallel workstations cases for 12 stations are the following:

- S = 12; Ns = 11; one possible parallel workstation;
- S = 12; Ns = 10; two possible parallel workstations;
- S = 12; Ns = 9; three possible parallel workstations;
- S = 12; Ns = 8; four possible parallel workstations;
- S = 12; Ns = 7; five. possible parallel workstations;

The solutions for the fractional tasks' models consider two fractional percentages 25% and 10% (parameter K).

The complete cases derived from the application of the matheuristic framework are shown in the Figs. 3,4 and 5.

Table 4 reports the solutions for the 4 stations case (fract. denotes the possible percentage of the fractional tasks).

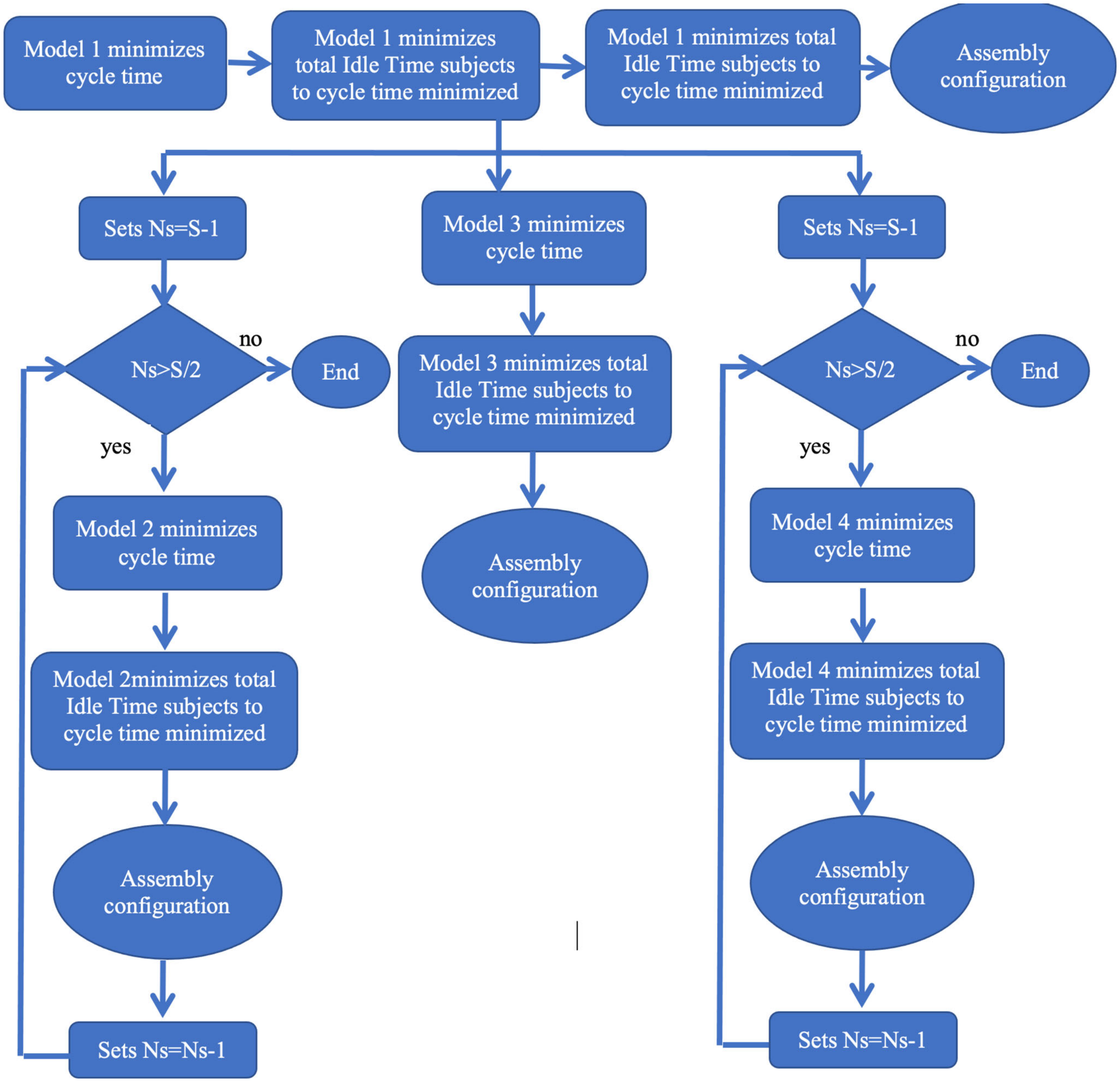

Fig. 1 Design models of the assembly line

The reduction of the cycle time is limited and the model 2,3 and 4 have closer value. The main benefits of the models compared to the classical SALBP problem is the reduction of the total idle time. The introduction of the fractional tasks allows to zero the total idle time. In this case, when the dimensions of the assembly line are lower the benefits of the proposed model is limited.

Table 5 reports the solutions for the 8 stations case (s-p) denotes the stations in series s and p in parallel of the models).

For the 8 stations case, the reduction of the cycle time is more relevant than the case with 4 stations.

The better values of cycle time and total idle time is the case with fractional tasks and one workstation in parallel. This result highlights how the integration of fractional tasks and parallel workstations improves the performance of the assembly line. One station with parallel workstations is enough to obtain better performance. Table 6 reports the solutions for the 12 stations case.

In this case, the reduction of the cycle time is lower than the case with 8 stations. The integration between fractional and parallel allows for reducing drastically the total idle time of the assembly line.

Table 3 Process time and precedence of the tasks

Task	Time [sec]	Precedence	Task	Time [sec]	Precedence
1	71	–	11	124	1-2-3-6
2	40	–	12	82	10
3	173	–	13	214	11
4	53	–	14	37	11
5	176	–	15	182	11
6	226	5	16	224	11
7	67	4	17	92	12
8	177	5	18	131	15
9	81	5	19	88	15
10	267	5	20	200	17
Total processing time 2705					

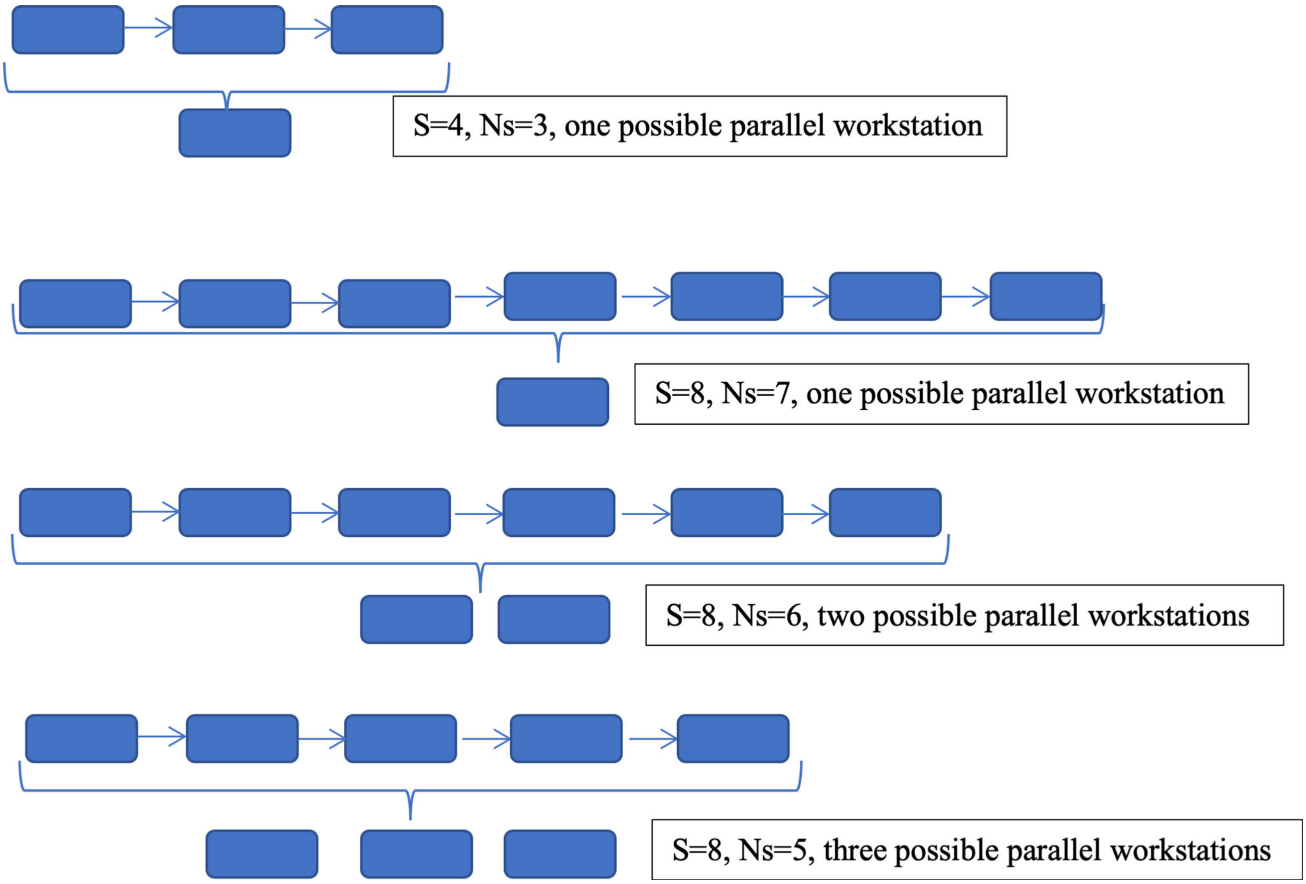

Fig. 2 Parallel workstations configurations

In this case, the increase of the potential parallel workstation leads to better results (4 and 5 stations with parallel workstations and fractional tasks.

The above results show how the higher benefits of the proposed model can be obtained for the assembly line composed of 8 stations.

The solutions gained from only the matheuristic model do not take into account the item flows and dynamic events, then the use of the simulation can test the solution in a dynamic context providing an evaluation more realistic of the performance measures.

To evaluate the performance and the impact on energy consumption, the assembly line configurations for 4 and 8 stations are developed using simulation models in Simul8® with the higher throughput possible when raw items are always available for the first station. The simulation length is related

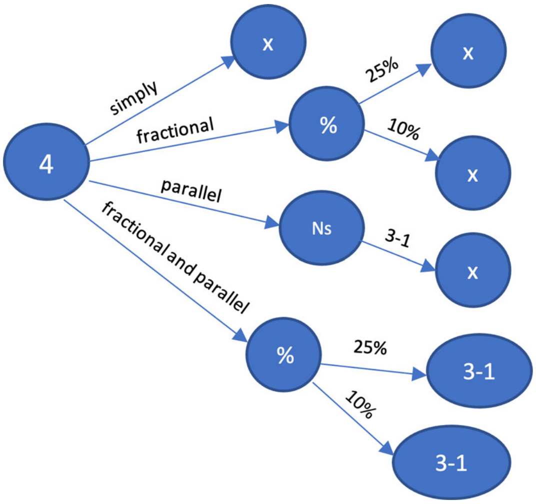

Fig. 3 Four stations cases

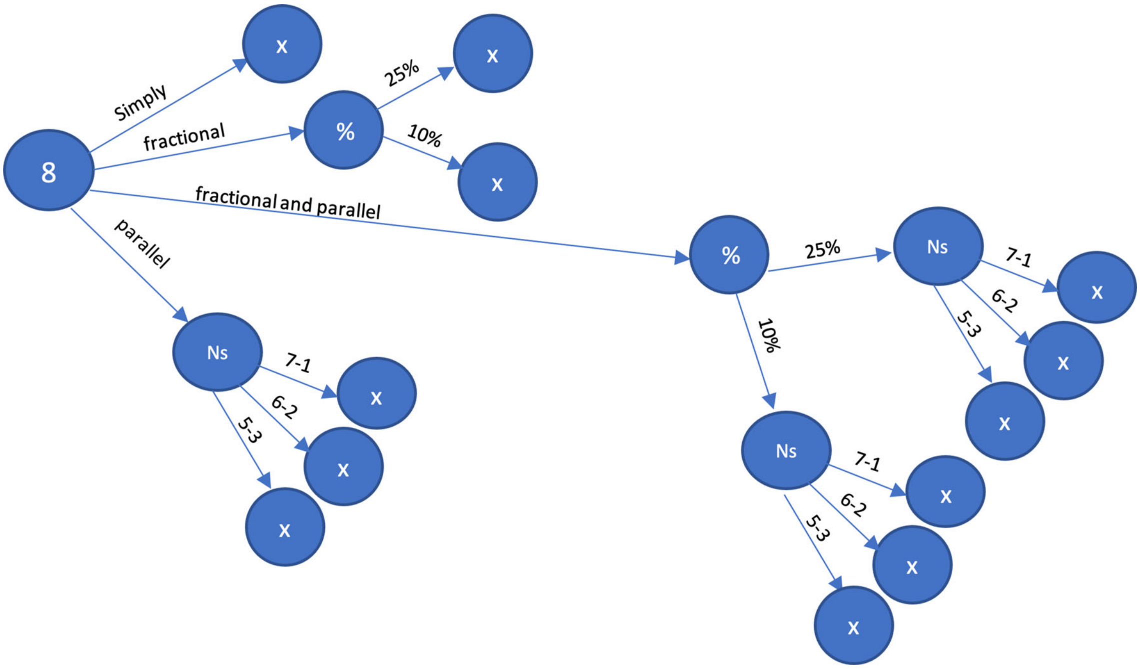

Fig. 4 Eight stations cases

to a month of working time is 28,800 min obtained from 24 h per day, five days a week, and four weeks. This length of the simulation assures enough amount of production items for the stability of each simulation run for the analysis of the terminating simulations conducted.

The simulations are conducted following the terminating simulation approach [24]. For each experimental class, a number of replications able to assure a 5% confidence interval and 95% of confidence level for each performance measure have been conducted.

The simulations consider one scenario without failures and a second scenario with short failures of the stations. The Mean Time Between Failures (MTBF) and Mean Time To Repair (MTTR) follow an exponential distribution to consider a relevant coefficient of variation as 1 for the exponential distribution. The parameters of the exponential distributions are 500 min for the MTBF and 10 min (closer to the cycle time of 4 stations case) for the MTTR.

The performance measures investigated with the simulations are the following:

- throughput rate of the assembly line;
- the average time of the items in the assembly line;

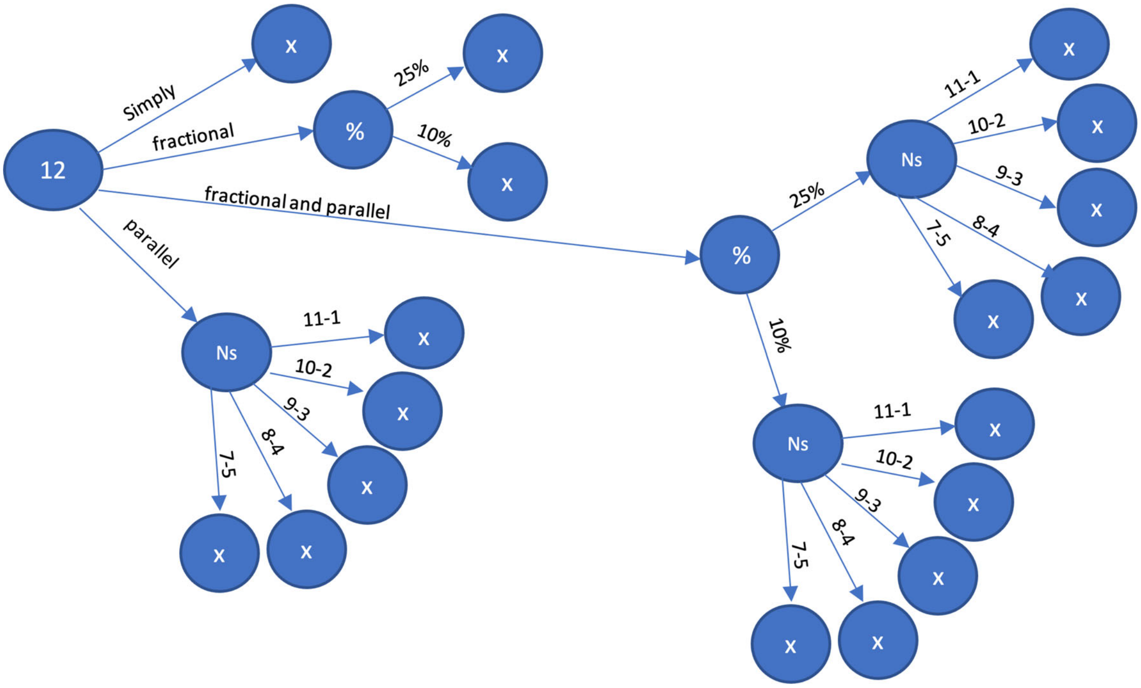

Fig. 5 Twelve stations cases

Table 4 Four stations solutions

[seconds]	Model 1	Model 2	Model 3 fract. 25%	Model 3 fract. 10%	Model 4 fract. 25%	Model 4 Fract. 10%
Cycle Time	679	676.5	676.25	676.3	676.25	676.3
Total idle time	11	1	0	0.2	0	0

Table 5 Eight stations solutions

[seconds]	Model 1	Model 2 (7–1)	Model 2 (6–2)	Model 2 (5–3)	Model 3 Fract 25%	Model 3 Fract 10%
Cycle time	469	348	348	343	340	338.9
Total idle time	1047	74	44.333	22	15.5	7
	Model 4 (7–1) Fract. 25%	Model 4 (7–1) Fract. 10%	Model 4 (6–2) Fract. 25%	Model 4 (6–2) Fract. 10%	Model 4(5–3) Fract. 25%	Model 4(5–3) Fract. 10%
Cycle time	338.5	338.2	339.75	338.5	339	338.4
Total idle time	2.125	0.6	9.625	3	4.125	5.1333

Table 6 Twelve stations solutions

	Model 1	Model 2 (11–1)	Model 2 (10–2)	Model 2 (9–3)	Model 2 (8–4)	Model 2 (7–5)
Cycle time	270	244	246.5	236	231	232.167
Total idle time	535	203.5	253	126.3	56.6	81.02

	Model 3 Fract. 25%	Model 3 Fract. 10%	Model 4 (11–1) Fract. 25%	Model 4 (11–1) Fract. 10%	Model 4 (10–2) Fract 25%	Model 4 (10–2) Fract 10%
Cycle time	229.5	226.9	227	226.5	226.75	226
Total idle time	49	17.8	19.125	8.65	15.167	5.75

	Model 4 (9–3) Fract. 25%	Model 4 (9–3) Fract. 10%	Model 4 (8–4) Fract. 25%	Model 4 (8–4) Fract. 10%	Model 4 (7–5) Fract 25%	Model 4 (7–5) Fract 10%
Cycle time	233	229.65	228.65	225.72	226.25	225.7
Total idle time	69.08	50.08	73.6	3.64	8.9583	3.4

Table 7 Simulation results 4 stations and no failures

	Model 2	Model 3 fract. 25%	Model 3 Fract. 10%	Model 4 fract. 25%	Model 4 Fract. 10%
throughput	0.39%	0.43%	0.43%	0.43%	0.43%
time in system	− 0.39%	− 0.40%	− 0.40%	− 0.40%	− 0.40%
time in system DV	7.90%	− 1.58%	− 1.55%	− 1.58%	− 1.57%
energy consumption	0.30%	0.32%	0.32%	0.32%	0.32%

- the standard deviation of the average time of the items in the assembly line;
- the energy consumed for unit of final product considering 5 Kw for the working state and 1 Kw for the idle state of the stations.

Table 7 reports the performance measures for 4 stations and no failures of the stations considering the percentage difference compared to model 1.

As the reader can notice, the percentage differences are very low; then, for the case studied and the restricted number of stations that composes the assembly line, the introduction of fractional tasks or parallel workstations are not relevant approaches.

The introduction of short failures (see Table 8) improves slightly the throughput with the introduction of fractional tasks and parallel workstations. The integration of fractional and parallel has a relevant impact on the reduction of the standard deviation of the average time in the system with the improvement of about 1.5% of the throughput. Then, the proposed model improves slightly the throughput with the higher stability of the average time in the system.

Table 9 reports the performance measures for 8 stations and no failures of the stations considering the percentage difference compared to model 1.

The improvement of the throughput is relevant with a significant reduction of the average and standard deviation of the time in the system. Also, the energy consumption for the unit of a final product has a relevant reduction. The main benefits are obtained with the introduction of fractional tasks than the parallel workstations. The integration between fractional tasks and parallel workstations does not improve compared to the only fractional tasks case.

The introduction of short failures (see Table 10) shows how parallel workstations lead to better results than fractional tasks. In this case, the integration between fractional tasks and parallel workstations allows to improve all the performance of the assembly line.

Table 11 reports the best cases for the simulation test. The integration of fractional tasks and parallel workstation is a promising approach when short failures occur increasing

Table 8 Simulation results 4 stations with failures

	Model 2	Model 3 fract. 25%	Model 3 Fract. 10%	Model 4 fract. 25%	Model 4 Fract. 10%
throughput	1.46%	0.51%	0.50%	1.55%	1.49%
time in system	− 0.86%	− 0.51%	− 0.49%	− 1.07%	− 1.45%
time in system DV	− 9.83%	− 2.72%	− 2.60%	− 14.33%	− 20.17%
energy consumption	− 0.29%	− 0.10%	− 0.10%	− 0.31%	− 0.30%

Table 9 Simulation results 8 stations and no failures

	Model 2 (7–1)	Model 2 (6–2)	Model 2 (5–3)	Model 3 fract 25%	Model 3 fract 10%
throughput	26.51%	27.98%	31.42%	37.95%	38.38%
time in system	− 26.26%	− 26.32%	− 27.07%	− 27.52%	− 27.72%
time in system DV	− 78.31%	− 73.48%	− 77.95%	− 81.60%	− 81.90%
Energy consumption	− 6.67%	− 6.67%	− 7.01%	− 7.13%	− 7.19%

	Model 4 (7–1) fract. 25%	Model 4 (7–1) fract. 10%	Model 4 (6–2) fract. 25%	Model 4 (6–2) fract. 10%	Model 4 (5–3) fract. 25%	Model 4 (5–3) fract. 10%
throughput	38.54%	38.68%	38.19%	38.54%	38.35%	38.60%
time in system	− 82.08%	− 82.13%	− 80.14%	− 82.11%	− 77.08%	− 78.84%
time in system DV	− − 7.19%	− 7.21%	− 7.14%	− 7.19%	− 7.14%	− 7.20%

Table10 Simulation results 8 stations with failures

	Model 2 (7–1)	Model 2 (6–2)	Model 2 (5–3)	Model 3 fract 25%	Model 3 fract 10%
throughput	26.51%	27.98%	31.42%	27.25%	27.57%
time in system	− 21.79%	− 22.13%	− 23.93%	− 22.65%	− 22.83%
time in system DV	9.35%	5.65%	− 15.74%	12.91%	12.45%
Energy consumption	− 5.68%	− 5.92%	− 6.52%	− 5.81%	− 5.87%

	Model 4 (7–1) fract. 25%	Model 4 (7–1) fract. 10%	Model 4 (6–2) fract. 25%	Model 4 (6–2) fract. 10%	Model 4 (5–3) fract. 25%	Model 4 (5–3) fract. 10%
throughput	29.26%	29.39%	30.70%	30.95%	32.64%	32.51%
time in system	− 23.47%	− 23.43%	− 23.80%	− 24.12%	− 24.12%	− 24.07%
time in system DV	3.63%	4.54%	− 1.76%	− 5.62%	− 10.75%	− 5.14%
Energy consumption	− 6.13%	− 6.15%	− 6.35%	− 6.39%	− 6.63%	− 6.64%

the robustness of the assembly line and reducing the energy consumption for a unit of the final product.

The matheuristic framework proposed provides the design alternatives for the assembly line to choose the suitable configuration. Three main issues are significant from the numerical results obtained by the matherustic framework. The reduction of idle time is relevant for the energy consumption of the equipment/machines when they are in the idle state because the modern equipment/machines during idle periods have a high energy request to maintain the ready-for-process conditions. The second issue concerns the robustness of the assembly line configurations; the integration of fractional tasks and parallel workstations improves mainly the robustness of the assembly line to the short failures.

The third issue regards the number of stations that leads to better improvements. The 8 stations configuration is the

Table 11 Best cases

	4 stations	8 stations
No failures	All models 3 and 4	Model 4
Short failures	Model 4 (7–1) Fract. 10%	Model 4 (5–3) Fract. 25–10%

better configuration because allows for proper distribution of fractional tasks and parallel stations compared to the 4 and 12 stations configurations considering keeping the same total number of stations. A low dimension (4 stations) doesn't allow introduction of more fractional tasks and parallel stations, while with a higher dimension (12 stations) the introduction of fractional tasks and parallel stations can introduce limited benefits.

5 Conclusions and future development path

This research starts from models proposed in the literature to design assembly lines with fractional tasks or the introduction of parallel workstations to propose an approach to integrate the fractional and parallel issues. A mathematical model is proposed to include parallel workstations and fractional tasks; the proposed model and the models proposed in the literature are linked together to reduce the computational complexity and provide a series of design alternatives for the assembly line. Then, the simulation is used to test the design alternatives with the introduction of short failures.

Then, it responds to the first research question asked: "*Can the design model that integrates fractional tasks and parallel workstations improve significantly the performance of assembly lines ?*".

The numerical results of the simulation show how the integration between fractional tasks and parallel workstations is relevant when failures occur improving the robustness of the assembly line and reducing the energy consumption. Furthermore, the number of stations that composes the assembly line has a relevant influence on the potential improvements of the proposed model. For the cases studied, the improvements are lower when the dimension is lower (4 stations) or higher (12 stations) than the medium dimension (8 stations). When the failures are not relevant, the fractional tasks approach is an approach that improves the performance more than the parallel workstation. This is due to the reduction of the cycle time and throughput time assured by the fractional tasks, while the parallel stations provide an alternative routing of the items when the failures occur.

The second answer to the research question asks: "*Is the matheuristic framework proposed adapt to solve the proposed design mathematical problem with suitable computational time*".

The solutions of the model using the framework proposed to show how the mathematical model can be solved for a different number of variables without increasing the computational time.

At more strategic level, the research shows how the proposed framework can support the manager of the manufacturing systems to evaluate the better configuration and the relative investment (fractional tasks or parallel workstations or together) to improve the performance of assembly lines and reduce energy consumption. This reduces the risks of an investment in the configuration selected.

The required investment costs in equipment or workforce learning to share tasks and introduce parallel workstations is the main limitation of the proposed research. Then, future development will introduce the economic issues in the design model to extend the evaluation to the profitable economics of the assembly configurations. Another future development concerns the introduction of reconfigurable equipment/machines to reconfigure the assembly line moving the parallel workstations or the shared tasks can improve the robustness of the assembly line face to demand variability. These issues will allow to plan an extensive numerical analysis to conduct an ANOVA test to verify the statistical differences among the factors investigated.

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